Precision Muon Physics Updates: G_F, g_P, and $g-2$

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I report on a trio of precision muon experiments aimed at improvements in fundamental standard model parameters or in the search for new physics. New results from PSI yield an improved Fermi constant and the first unambiguous determination of the nucleon weak pseudoscalar coupling. These two experiments both involve precise measurements of the muon lifetime: τ_{μ^+} obtains G_F, while τ_{μ^-} in ultra-pure protium gas leads to g_P. The techniques and results are described. A planned next-generation muon anomalous magnetic moment experiment at BNL aims to significantly reduce the uncertainty in this test of new physics. The current status and a brief review of the physics case is given.

1 Introduction

The predictive power of the standard model depends on well-measured input parameters. In turn, tests of the standard model depend on comparisons of measurements to theoretical expectations. The difference is partially semantic; it can depend on what is known and when it is known. In the three vignettes presented below, we span the extremes. In the first chapter, a new precision muon lifetime measurement is described that updates the Fermi constant. Next, we report on a novel measurement of the μp singlet capture rate, which is used to deduce the weak-nucleon pseudoscalar coupling; QCD-inspired theory predicts this quantity quite precisely. Finally, the muon anomaly is compared to the field-theory predictions that incorporate all known standard model processes. The difference between theory and experiment can be ascribed to new, missing, physics. In all three situations, the physics output implicitly depends on many "constants" and standard model parameters that are assumed to be known well enough, and the theory confidence is based on prior authenticity tests. We won't dwell further on that subtlety here. The three sections which follow report a new G_F, a new g_P, and the physics reach to be expected in the LHC era from an improved $(g-2)$ experiment.

2 The Fermi Constant

At this Electroweak Conference, we have repeatedly seen the Fermi constant, G_F, embedded in the predictions of the theory. That's because G_F is related to the strength of the weak interaction, much like the fine-structure constant α is to electromagnetism or "big G" is to gravity. With the assumption of universality in the weak interaction, any sufficiently precise weak measurement could be re-cast as a determination of G_F, as long as other parameters are well-enough established. By far, the most precise measure of G_F is through ordinary muon decay, $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$. The G_μ extracted from the lifetime (see below) is usually taken as the universal G_F for any weak interaction rate. It is, in this sense, truly an "input" parameter so that the theory can predict less well measured quantities.

The last update of G_F was made in 1984, despite vast improvements—in other EW measurements including the discovery of the weak bosons (M_Z to 23 ppm!) and the top quark—since that time. The Fermi constant is related[1] to the muon lifetime, τ_{μ^+} by

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1+\Delta q\right), \tag{1}$$

where Δq is the sum of phase space and both QED and hadronic radiative corrections. The relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}\left(1+\Delta r\right) \tag{2}$$

connects G_F to the electroweak gauge coupling, g. Here, Δr represents the weak-boson-mediated tree-level and radiative corrections, which have been computed to second order[2].

The MuLan experiment[3] uses an optimized "pulsed-muon-beam" technique to acquire the more than 10^{12} decays required for a 1 ppm measurement—the goal of the experiment. At PSI, the πE3 beamline is tuned to deliver $\sim 8\times10^6$ μ^+/s at ~ 28 MeV/c in dc operation. A customized time structure is applied to the beam using a fast kicker built for this experimental program. It works by applying a transverse electric field along a 1.5-m length of the beamline where the beam divergence is minimized. A 25 kV potential, switched on in 60 ns, is applied across two series-connected sets of aluminum plates to produce the field. When the plates are energized, the beam flux is reduced by a factor of more than 800. A 5 μs "accumulation period" T_A is followed by a 22 μs "measuring period" T_M (see Fig. 1). Muons are stopped in a thin ferromagnetic target, which has a high internal magnetic field to rapidly precess and decohere the initial polarization. At these rates and cycle, the experiment can accumulate muon decays at approximately 600 kHz. In 2006, 10^{12} decays were recorded, and in 2007 a similar data set will be collected using a different stopping target strategy.

The new result, alluded to in the introduction, is based on a much smaller sample of data obtained in 2004, when the experiment was being commissioned. A sample of 1.8×10^{10} events, fit with a $\chi^2/\mathrm{dof} = 452.5/484$ gives

$$\tau_\mu(\mathrm{MuLan}) = 2.197\,013(21)(11)\ \mu\mathrm{s} \quad (11.0\ \mathrm{ppm}). \tag{3}$$

The first error is statistical and the second is systematic. The largest set of systematic uncertainties are absent for 2006 because of experimental changes in electronics and better performance of the kicker. However, challenging new systematic considerations enter at the precision of a 2 ps lifetime measurement. The 2006 data are presently being analyzed, and, just as in the 2004 case, the analysis is blinded by using a clock oscillator whose accurate and precise frequency is kept secret from the Collaboration until the analysis is complete. The new world average lifetime leads to an improved Fermi constant

$$G_F = 1.166\,371(6)\times10^{-5}\ \mathrm{GeV}^{-2} \quad (5\ \mathrm{ppm}). \tag{4}$$

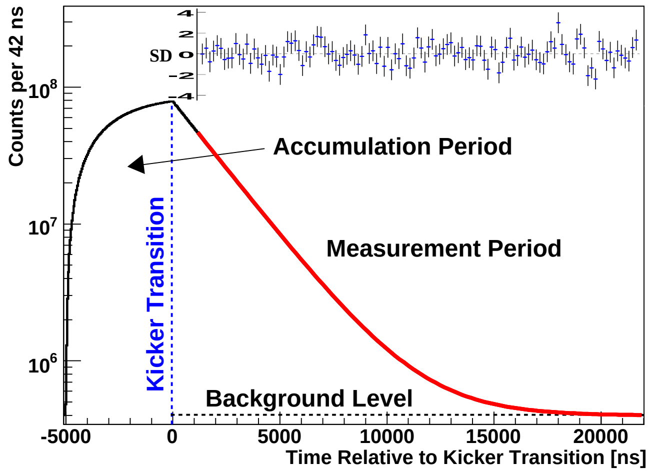

Figure 1: The accumulation and measurement periods created by the kicker cycle. The fit region is indicated in the inset, which displays the normalized residuals. (Fig. courtesy Ref. 3.)

3 The Weak Pseudoscalar Coupling

Muon capture on the proton—a weak interaction within a hadronic system—is a fundamental process whose rate is predicted[4] following the symmetries of QCD. The process retains the familiar current-current $V - A$ form, but requires modified vector and axial vector pieces associated with form factors and having momentum transfer dependence[5]. The rate for

$$\mu^- + p \rightarrow n + \nu_\mu \tag{5}$$

with the $\mu^- p$ atom in the singlet state is termed Λ_S. It is characterized by a matrix element where the vector and axial terms, assuming Lorentz and T-invariance (and no 2nd-class currents), can be written[5] as

$$V^\alpha = \bar{u}_n \left(g_V(q^2)\gamma^\alpha + \frac{i g_M(q^2)}{2m_N}\sigma^{\alpha\beta} q_\beta \right) u_p \tag{6}$$

$$A^\alpha = \bar{u}_n \left(g_A(q^2)\gamma^\alpha \gamma_5 + \frac{g_P(q^2)}{m_\mu} q^\alpha \gamma_5 \right) u_p. \tag{7}$$

Here g_V, g_M, g_A are the vector, magnetic and axial-vector form factors, and g_P is the induced pseudoscalar coupling, the least well known and the subject of a new measurement[6] by the MuCap Collaboration.

The sensitivity of Λ_S to these form factors is:

$$\frac{\delta\Lambda_S}{\Lambda_S} = 0.47\frac{\delta g_V}{g_V} = 0.024\% \qquad \frac{\delta\Lambda_S}{\Lambda_S} = 0.15\frac{\delta g_M}{g_M} = 0.01\% \tag{8}$$

$$\frac{\delta\Lambda_S}{\Lambda_S} = 1.57\frac{\delta g_A}{g_A} = 0.38\% \qquad \frac{\delta\Lambda_S}{\Lambda_S} = 0.18\frac{\delta g_P}{g_P} \approx 5\% \tag{9}$$

where it is clear that only g_P is poorly known; a precision measurement of Λ_S effectively deduces g_P.

On the theoretical side, g_P is known to a few percent[7], giving $g_P(thy) = 8.26 \pm 0.23$. The experimental picture is murky and has been so for nearly 30 years. The reasons are associated with experimental complications from "muon chemistry." The μp system can form a $p\mu p$ molecule in the ortho ($J = 1$; pp spins aligned) state, which can transition to the para ($J = 0$;

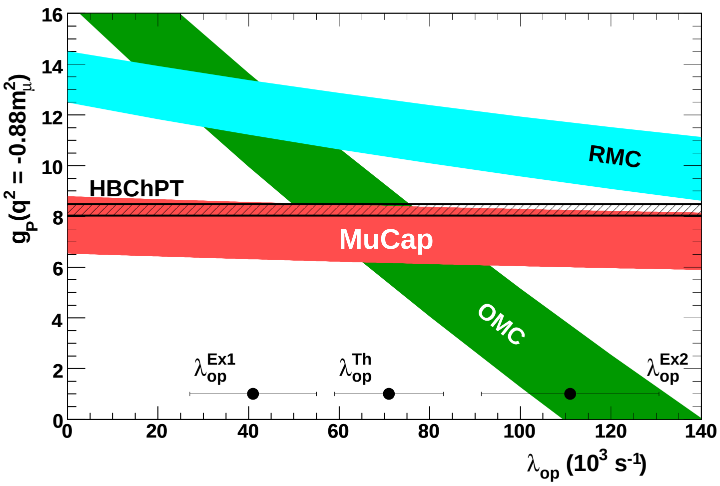

Figure 2: g_P versus the ortho-para transition rate required to interpret the experimental result. See text for full explanation. (Fig. courtesy Ref. 6)

pp spins anti-aligned) state at the rate λ_{op}. That means three muon-proton systems exist and their relative populations change with time. Further, the capture rate is different in all of them. In Fig. 2, g_P at the fixed $q^2 = -0.88\ m_\mu^2$ of muon capture is given on the vertical axis. The heavy baryon chiral perturbation theory prediction is represented by the black band. The horizontal axis is λ_{op}, with two measurements and one theoretical calculation of λ_{op} spanning the range $20 - 130 \times 10^3\ \mathrm{s}^{-1}$. Because of this large range, the ordinary muon capture (OMC) experiment[8] performed in liquid hydrogen cannot be used to determine g_P—it is too sensitive to the unknown λ_{op}. The TRIUMF radiative muon capture (RMC) experiment[9] is better, but its "high" value compared to theory has been controversial for many years. Is it an experimental issue or a challenge to theory? Could λ_{op} be very high? The new MuCap result confirms the theory and is relatively immune to the uncertainty in λ_{op}.

The MuCap method uses a 10-bar, ultra-pure protium gas TPC target to image a muon stop in the gas vessel, well away from walls. The $p\mu p$ molecular formation is limited owing to the 1% density compared to liquid hydrogen. The TPC is surrounded by cylindrical wire chambers and a scintillating barrel hodoscope, which are used for tracking and timing of the decay electrons. A high-precision (~ 33 ppm) negative muon lifetime experiment is performed, where the difference between τ_{μ^-} and the free muon lifetime measured in MuLan is attributed to the capture rate, a difference of $\approx 0.15\%$. The rich technical challenges and solutions of MuCap are far too expansive to report here. The reader is urged to consult the PRL[6] for details.

The capture rate is $\Lambda_S = 725.0 \pm 17.4\ \mathrm{s}^{-1}$. When systematics are accounted for and minor (known) corrections are applied, we find $g_P(\mathrm{MuCap}) = 7.3 \pm 1.1$. Significantly more data have been obtained and the uncertainty will be halved in the future.

4 The Muon Anomaly

The muon anomaly is one of the most sensitive tests of the standard model because it can be measured and calculated precisely. Experiment and theory boast similar impressive uncertainties of ≈ 0.5 ppm. When the numbers are compared, using the theoretical update given in the review of Miller, Roberts and deRafael[11], they do not agree, suggesting missing physics in the standard model evaluation or (let us hope not) a mistake in either of the numbers. The comparison gives

$$\Delta a_\mu^{(\mathrm{today})} = a_\mu^{(\mathrm{Exp})} - a_\mu^{(\mathrm{SM})} = (29.5 \pm 8.8) \times 10^{-10}. \qquad (10)$$

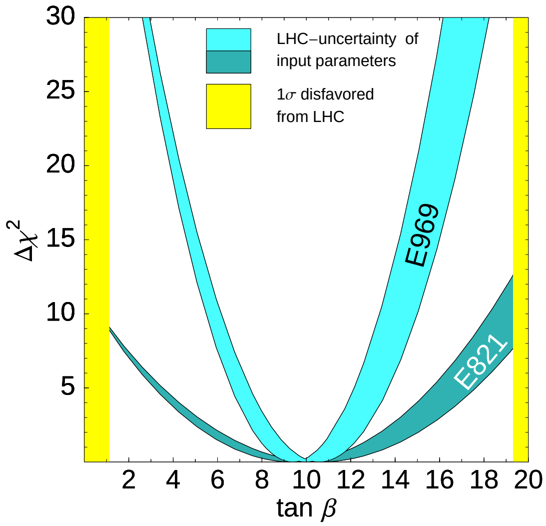

Figure 3: Future "blueband" plot of $\Delta\chi^2$ versus $\tan\beta$ when the present (dark blue) or future (light blue) precision of a_μ is considered in the global fit. In contrast, the yellow bands bracket the limits with no a_μ considered. (Courtesy D. Stöckinger).

At this conference, Z. Zhang reviewed the theoretical contributions that make up the standard model value. These include QED, weak, and hadronic loops, with the latter carrying the largest uncertainty. Two main categories contribute nearly equally to the total theory uncertainty. The 1st-order hadronic vacuum polarization $a_\mu(\mathrm{HVP})$ is $690.1 \pm 4.7 \times 10^{-10}$. It comes from data—the absolute cross section for $e^+e^- \rightarrow hadrons$, together with a well-known dispersion relation. Hadronic light-by-light (HLbL) is a second-order 4-point function that must be evaluated using a QCD-like model. The present best summary[12] of many efforts gives $a_\mu(\mathrm{HLbL}) = (11.0 \pm 4.0) \times 10^{-10}$. Improvements—and even independent verification—of both numbers is important.

What could an anomalous moment having a $\sim 30 \times 10^{-10}$ departure from the standard model imply? Certainly, it points to new physics. Beyond that, it is only a part of the necessary clues required to "fingerprint" the source. Additional information will hail from direct measurements of masses and branching ratios at the LHC, from limits (or signals) from new charged lepton flavor violation experiments, from different EDM searches, and possibly other precision measurements, such as Möller scattering. Consider the landscape in the LHC era, by which time many of these projects will mature. For $(g-2)$, a new experiment[13] E969 is approved at BNL but awaits funding. Its goal is a 2.5 or higher reduction in uncertainty. Improvements in theory are already on track with additional HVP data expected from radiative-return experiments at BaBar, KLOE and Belle. Some reduction in HLbL can be anticipated as well but here the path is less clear at the moment. For the sake of discussion, a 3.9×10^{-10} uncertainty on the *comparison* of experiment to theory is used as a future benchmark for new physics sensitivity.

In a recent White Paper[14], the physics case for such a scenario of improved precision is outlined. Let's look at just one example[15] appropriate to this meeting—SUSY. Imagine a future in which the SPS1a reference point[16] is realized and the LHC has measured masses and a global fit has been performed to establish this model. Still, $\tan\beta$ will be largely unconstrained. In Fig. 3 a "blueband" plot is made with the reduction in χ^2 versus $\tan\beta$ for the a_μ present (dark blue) and future (light blue) precisions considered when a_μ is included in the global fit. Compared to the LHC-alone limits (inside yellow bands), adding a_μ helps impressively. Other examples are given in the White Paper study.

5 Summary

In this brief report, we have announced two new measurements involving the muon lifetime. They update the Fermi constant and, for the first time, demonstrate unambiguous agreement between g_P and fundamental QCD-inspired predictions. The new physics reach of the muon anomaly is already impressive with a 3.4 σ significance on a deviation from the standard model. Plans for improved experiment and theory match nicely to the expected discoveries in the LHC era.

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