The $B \to X_s\gamma$ BF at NNLO

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XLII Rencontre de Moriond

Why a NNLO calculation?

- Exp:

$$\mathrm{BR}(B \to X_s\gamma) = \left(3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03\right) 10^{-4}$$

latest HFAG E_γ>1.6 GeV

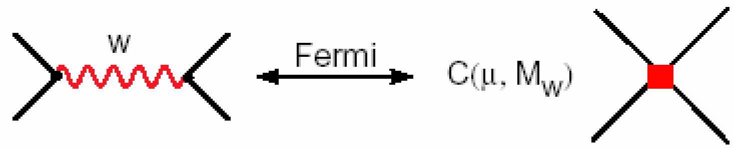
- Until recently: SM prediction at NLO in QCD + leading EW and non-perturbative effects with ~ 10% theory error

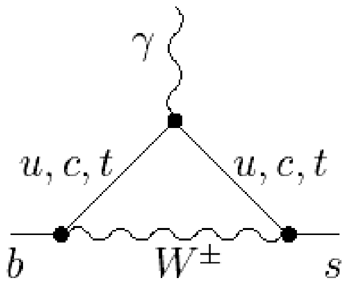
- Need to match ~ 5% exp error at end of B factories: first results of a collective effort for the NNLO calculation in **hep-ph/0609232**

- **authors**: M. Misiak, H.M. Asatrian, K. Bieri, M. Czakon, A. Czarnecki, T. Ewerth, A. Ferroglia, P. Gambino, M. Gorbahn, C. Greub, U. Haisch, A. Hovhannisyan, T. Hurth, A. Mitov, V. Poghosyan, M. Slusarczyk and M. Steinhauser.

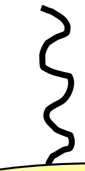
$b \rightarrow s$ transitions

$\Lambda_{QCD} \ll m_b \ll M_W$





at $O(1/M_W^2)$



$$O_7 = m_b \bar{b}_R \sigma_{\mu\nu} F^{\mu\nu} s_L$$

But many more operators appear adding gluons

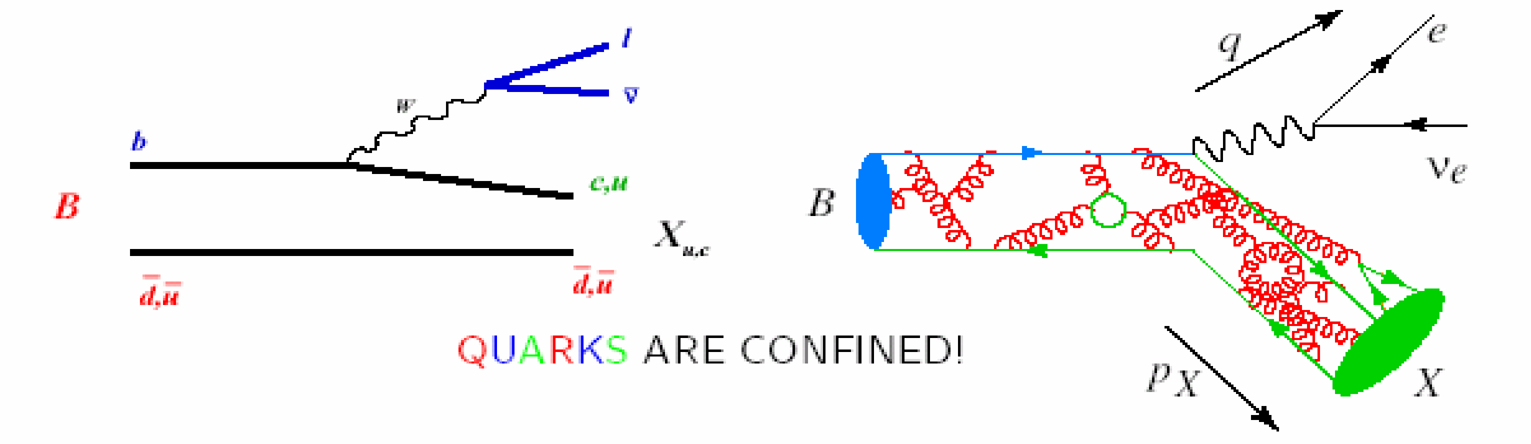
The tensor current is not conserved and runs between M_W and m_b

We have (at least) 3 scales: M_W, m_b, Λ_{QCD}

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i + \dots$$

L=ln m_b/M_W must be resummed. LO: $\alpha_s^n L^n$, NLO: $\alpha_s^n L^{n-1}$, NNLO: $\alpha_s^n L^{n-2}$

The advantage of being inclusive



The decay of a B meson is not the decay of a free b quark!

However $\Lambda_{QCD} \ll m_b$: inclusive decays via OPE admit systematic expansion in α_s and Λ_{QCD}/m_b starting with Λ^2_{QCD}/m_b^2.

➔ Leading term is parton model.

➔ Non-pert corrections are generally small and can be controlled

Dominant (magnetic operator) contributions to inclusive radiative B decays are described by OPE

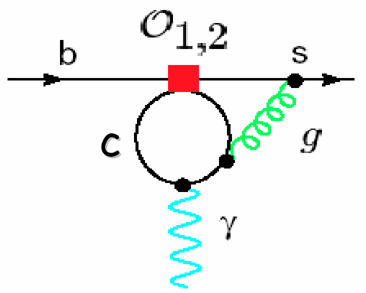
The main perturbative ingredients

Process independent:

- **The Wilson coefficients C_i** (encode the short distance information, initial conditions at scale M_W)
- **The Anomalous Dimension Matrix** (mixing among operators, determines the evolution of the coefficients, allowing to resum large logs of m_b/M_W)

Process dependent:

- **Matrix elements** of O_i, for ex:



Until recently: QCD at NLO, plus EW & power corrections

Status of the NNLO enterprise

- **3-loop Wilson coefficients for O_7 and O_8**

  Misiak, Steinhauser, NPB 683 (2004) 277

- **3-loop mixing in $(O_1,...,O_6)$ & (O_7,O_8) sectors**

  Gorbahn, Haisch, NPB 713 (2005) 291
  Gorbahn, Haisch, Misiak, PRL 95 (2005) 102004

- **4-loop mixing $(O_1,...,O_6)$ into O_7 and O_8**

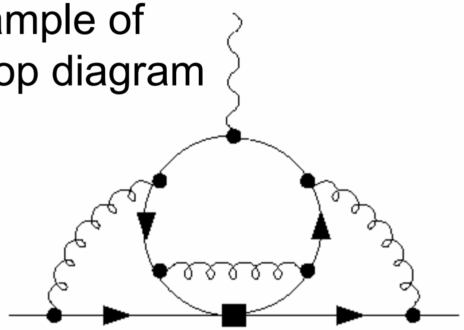
  Czakon,Haisch,Misiak, hep-ph/0612329

- **2-loop matrix elements of O_7 (real & virt), BLM for others**

  Bieri,Greub,Steinhauser, PRD 67 (2003) 114019
  Blokland, Czarnecki, Misiak, Slusarczyk,Tkachov, PRD 72 (2005) 033014
  Asatrian,Ewerth,Greub,Hurth,Hovhannisyan,Poghosyan, NPB 749 (2006)325
  Melnikov, Mitov, PLB 620 (2005) 69
  Asatrian, Ewerth, Ferroglia, PG, Greub, hep-ph/0607316
  Asatrian,Ewerth, Gabrielyan, Greub, hep-ph/0611123

- **3-loop matrix elements of O_1,O_2**

  Bieri, Greub,Steinhauser, PRD 67 (2003) 114019
  Misiak,Steinhauser,hep-ph/0609241 (**interpolation**)



Results of 4-Loop Mixing

- numerical effect of $\mathcal{O}(\alpha_s^3)$ mixing of Q_{1-6} into Q_7 on $\mathcal{B}_{\rm SM}$ is -2.4% to -4.4% depending on scale
- impact of remaining $\mathcal{O}(\alpha_s^3)$ mixing of Q_{1-6} into Q_8 is found to be $\approx$10 times smaller

$$\gamma^{(2)}_{1-6,7} = \begin{pmatrix} \frac{150994745}{1062882} + \frac{1272596}{6561}\zeta_3 \\ \frac{138336202}{177147} - \frac{2713672}{2187}\zeta_3 \\ -\frac{58397866}{177147} + \frac{3236560}{2187}\zeta_3 \\ -\frac{5108749081}{2125764} + \frac{2007886}{6561}\zeta_3 \\ \frac{5824017302}{177147} + \frac{112180720}{2187}\zeta_3 \\ \frac{3603565835}{531441} + \frac{15361912}{6561}\zeta_3 \end{pmatrix}$$

Czakon, Haisch, Misiak

Ulrich Haisch, University of Zurich, "Flavour in the era of the LHC" workshop, 3rd meeting, 15-17 May, 2006, CERN

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- **3-loop Wilson coefficients for O_7 and O_8**
  Misiak, Steinhauser, NPB 683 (2004) 277
- **3-loop mixing in $(O_1,\ldots,O_6)$ & (O_7,O_8) sectors**
  Gorbahn, Haisch, NPB 713 (2005) 291
  Gorbahn, Haisch, Misiak, PRL 95 (2005) 102004
- **4-loop mixing $(O_1,\ldots,O_6)$ into O_7**
  Czakon,Haisch,Misiak in writing
- **2-loop matrix elements of O_7 (real & virt), partially of O_8**
  Bieri,Greub,Steinhauser, PRD 67 (2003) 114019
  Blokland, Czarnecki, Misiak, Slusarczyk,Tkachov, PRD 72 (2005) 033014
  Asatrian,Ewerth,Greub,Hurth,Hovhannisyan,Poghosyan, NPB 749 (2006)325
  Melnikov, Mitov, PLB 620 (2005) 69
  Asatrian, Ewerth, Ferroglia, PG, Greub, hep-ph/0607316
  Asatrian,Ewerth, Gabrielyan, Greub, hep-ph/0611123 (not yet included)
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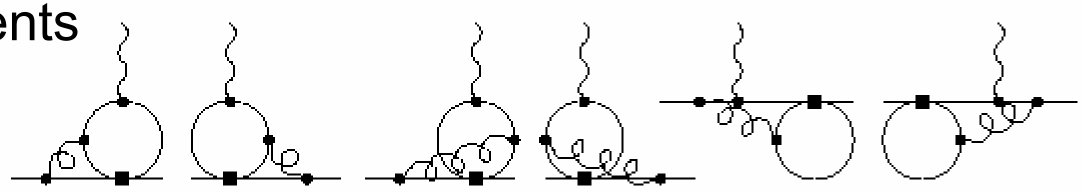
The charm mass problem

m_c enters the phase factor due to normalization

$$C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e\bar{\nu}]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]} \approx 0.58$$

and the NLO matrix elements



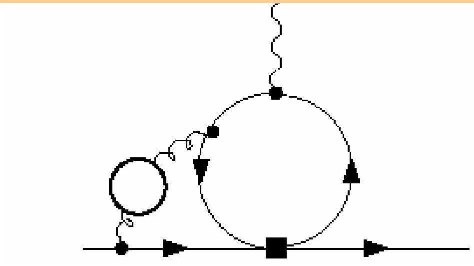
LO diagrams vanish: **definition of m_c is a NNLO issue**.
Numerically very important because these are large at NLO:

$m_c(m_c)$=1.25±0.10 GeV $m_c(m_b)$=0.85±0.11 GeV m_c(pole)~1.5GeV

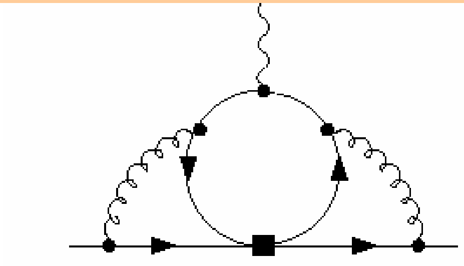
But pole mass has nothing to do with these loops

Changing m_c/m_b from 0.29 (pole) to 0.22 (MSbar) increases BRγ by 11%
0.22 ±0.04 **DOMINANT 6% theory error at NLO**

Interpolation in m_c (Misiak & Steinhauser)



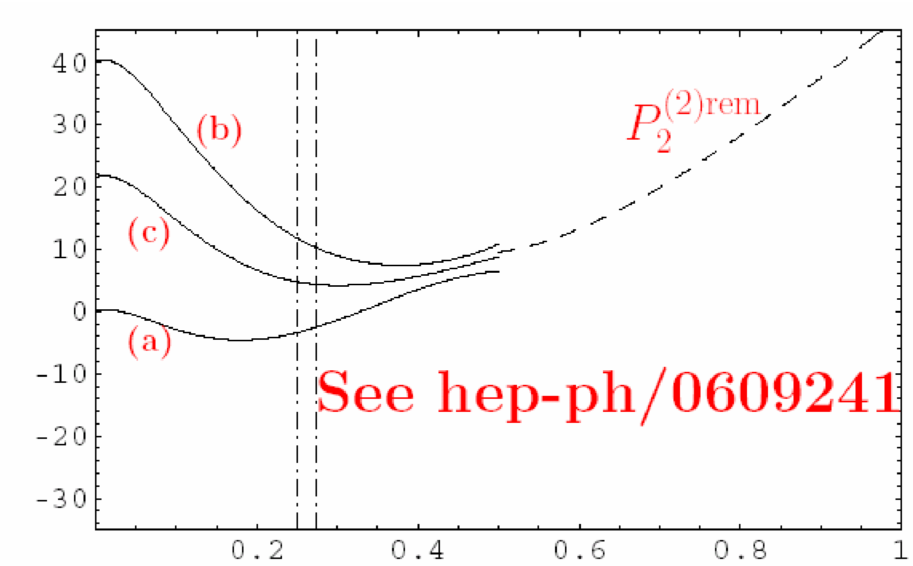
$O(\alpha_s^2 n_f)$ diagrams → BLM contribution, known at small and large m_c Bieri et al



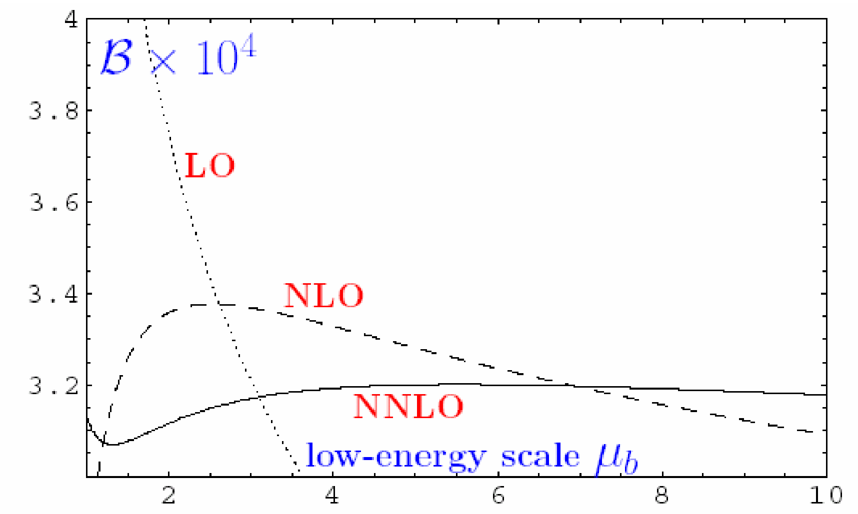
non-BLM contribution only for m_c» $m_b/2$
Misiak & Steinhauser

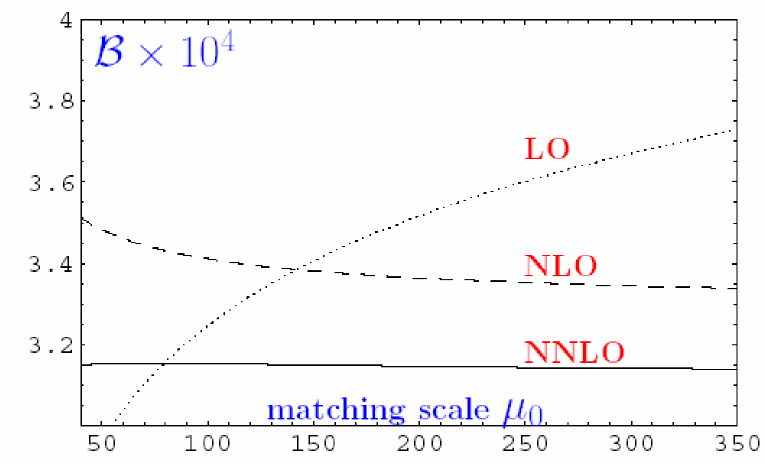
BLM contributions generally dominant. Since small-heavy m_c interpolation works very well at NLO and for NNLO-BLM use it at NNLO as a first step

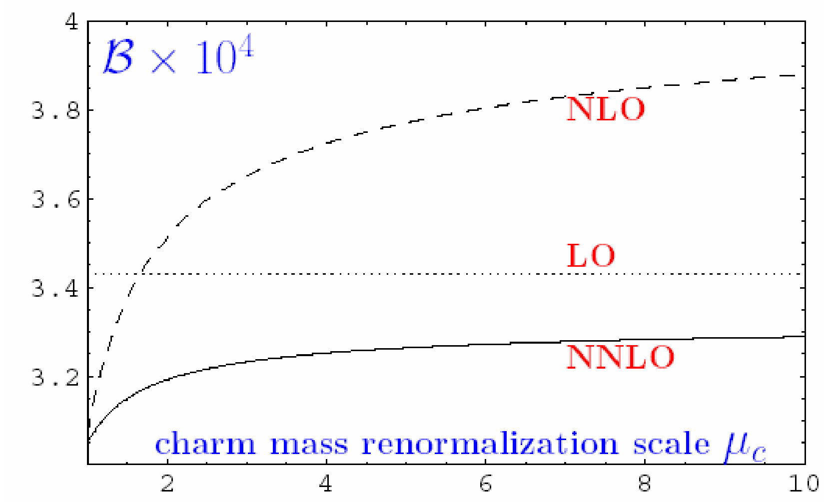
Three ansatzt for matching non-BLM terms at m_c=0 are employed and averaged:
3% uncertainty on the BR



Residual scale dependence







Clear reduction of scale dependence
Charm mass dependence remains visible
Higher orders uncertainty ~3%

central values: μ_b=2.5GeV μ_c=1.5GeV and μ_0=m_t

NLLO “estimate” hep-ph/0609232

Including known power corrections, pure OPE result is at the moment

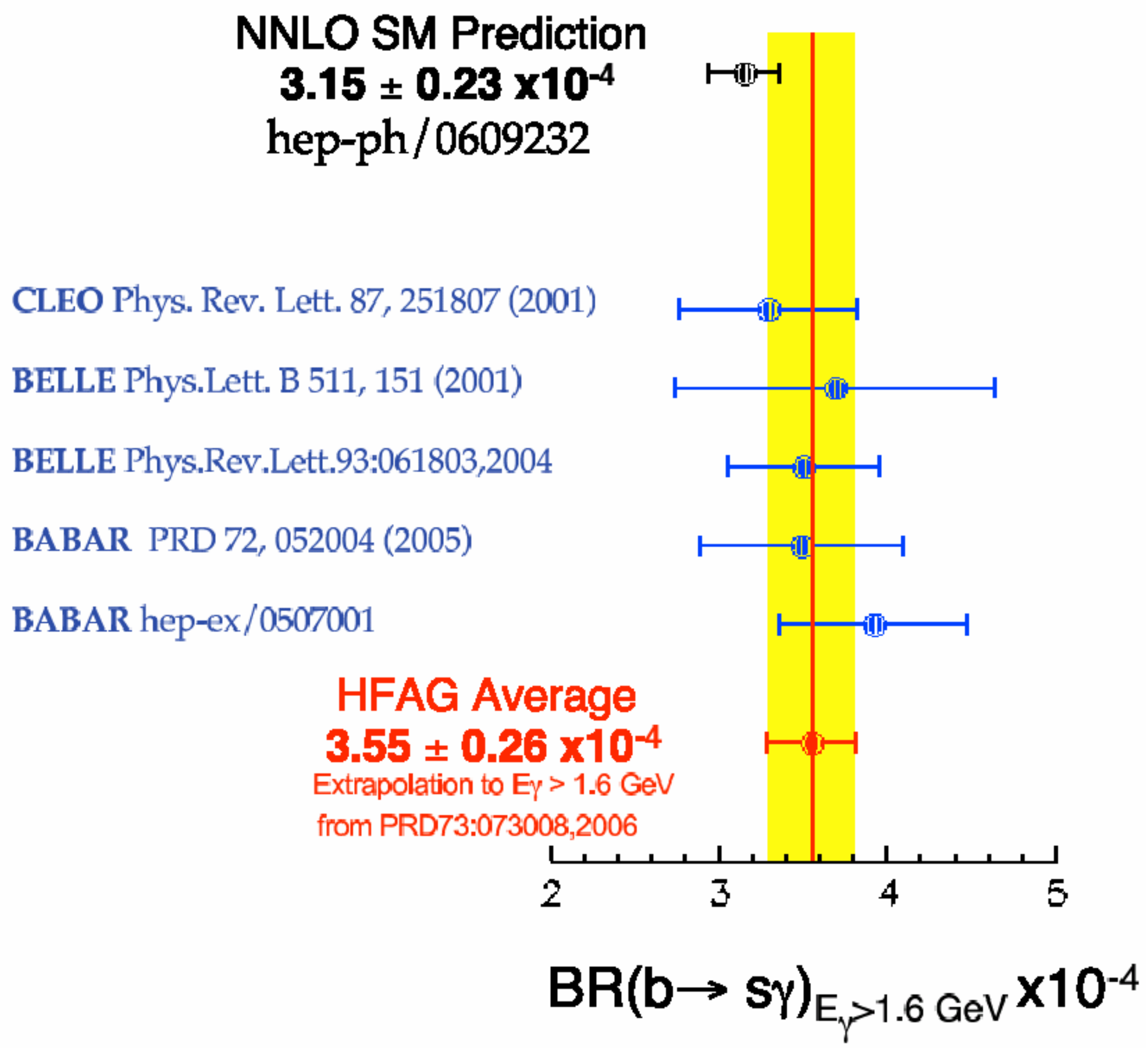
$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm NNLO}_{E_\gamma>1.6\ {\rm GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$

Total error ~7% = ± 3% (interpolation) ± 3% (parametric)
± 3% (higher orders) ± 5% (non-pert)

At NLO: BR(E_g>1.6 GeV)=3.58 10^{-4} (Misiak, PG 2001) and the NNLO error was 6% (m_c scale)+4% (other NNLO)

The present NNLO result (given the same inputs) is not far from the edge of that range (-12%)

Various effects (notably but not only BLM, 4loop ADM) lower the BR; charm scale is not set. Two scales (μ_c and μ_b) interplay.


NNLO SM Prediction
3.15 ± 0.23 x10-4
hep-ph/0609232
CLEO Phys. Rev. Lett. 87, 251807 (2001)
BELLE Phys.Lett. B 511, 151 (2001)
BELLE Phys.Rev.Lett.93:061803,2004
BABAR PRD 72, 052004 (2005)
BABAR hep-ex/0507001
HFAG Average
3.55 ± 0.26 x10-4
Extrapolation to Eγ > 1.6 GeV
from PRD73:073008,2006
2
3
4
5
BR(b→ sγ)Eγ>1.6 GeV x10-4

The disadvantage of cuts

The OPE breaks down close to boundaries: unreliable unless cut is low enough.

In the endpoint region $E_\gamma \sim 2$ GeV new dynamics: distribution function (Shape Function=SF) and Sudakov resummation.

Nobody questions validity of OPE for $E_{cut} \sim 1\text{-}1.2$ GeV. Experiments cut at 1.8-1.9 GeV. **What happens in between?**

In 2001 we proposed E_{cut} =1.6 GeV as a reference point because SF studies showed O(<1%) SF effects for E_{cut}<1.6 GeV

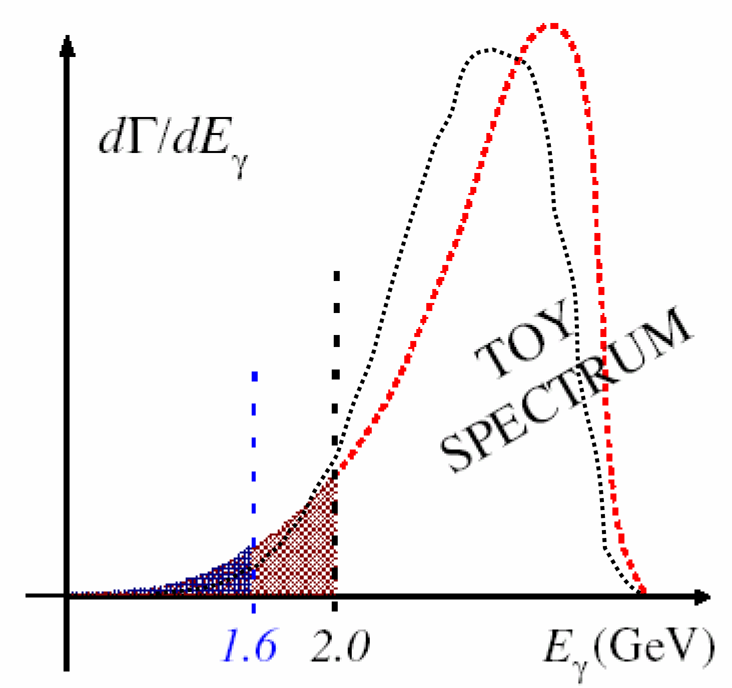
Kagan&Neubert

Confirmed later: we stick to that view

Benson, Bigi, Uraltsev, Andersen& Gardi

But possible effects of scale $\Delta = m_b - 2E_{cut}$ would lower BR(E_{cut}<1.6 GeV) by ~3%. Total tail effect from 1.0 to 1.6 GeV would be ~7±4% at NNLO!

Becher & Neubert



Extrapolation to 1.6 GeV is now performed by HFAG using b→clv input ➔ small 2-3% error counted in the *experimental* budget

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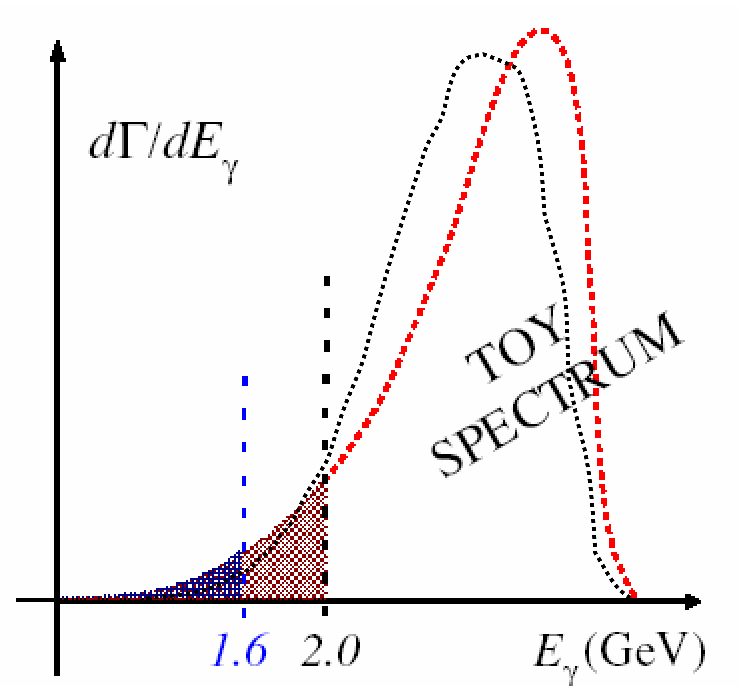


In the future better to compare predictions for E_{cut}=1.8GeV!

More non-perturbative problems

There is **another problem with OPE**: it cannot be applied to the contributions of operators other than O_7 if the photon couples to a light quark.

Some of the additional non-perturbative effects are related to $O_{1,2}$ matrix elements and scale like $1/m_c^2$ (they are included)

Voloshin, Ligeti et al, Buchalla et al

Similar $O(\alpha_s\Lambda/m_b)$ effects in O_7-O_8 have been recently estimated by VIA: 0--3%

Lee, Neubert, Paz

In the presence of a hard gluon, however, the suppression of non-OPE effects in $O_{1,2}$ m.e. is only $\alpha_s\Lambda/m_b$ No estimate at present.
they could be important! → 5% uncertainty in BR

What's next

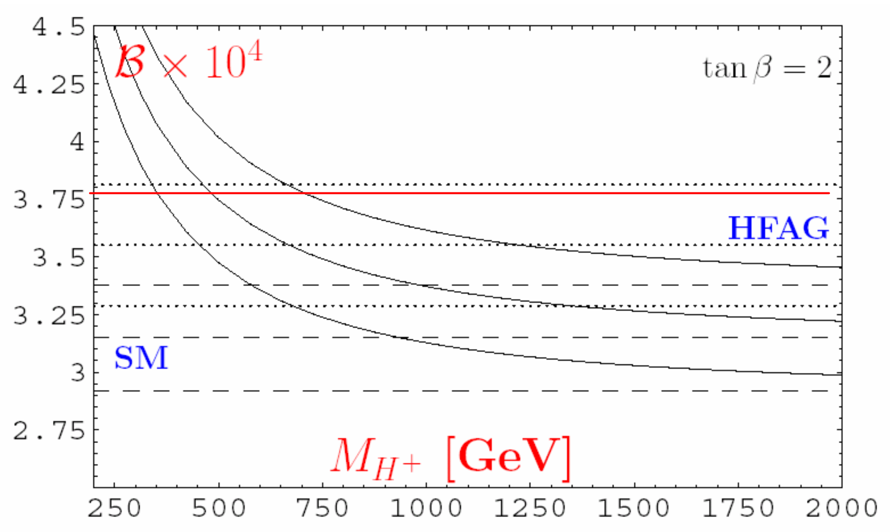
- **Full m_c dependence** of the NNLO contribution to BR beyond BLM (most difficult step: $O_{1,2}$ matrix elements)
- **Study of all $O(\alpha_s\ \Lambda/m_b)$ contributions**: any idea beyond Vacuum Insertion Approximation by Lee, Neubert, Paz?
- Understanding whether the tail of the spectrum can really be computed precisely and how.
- Calculation of **other missing NNLO** matrix elements (78, 88 contributions). Some known BLM corrections (Ligeti et al) to the spectrum are not yet included in our result

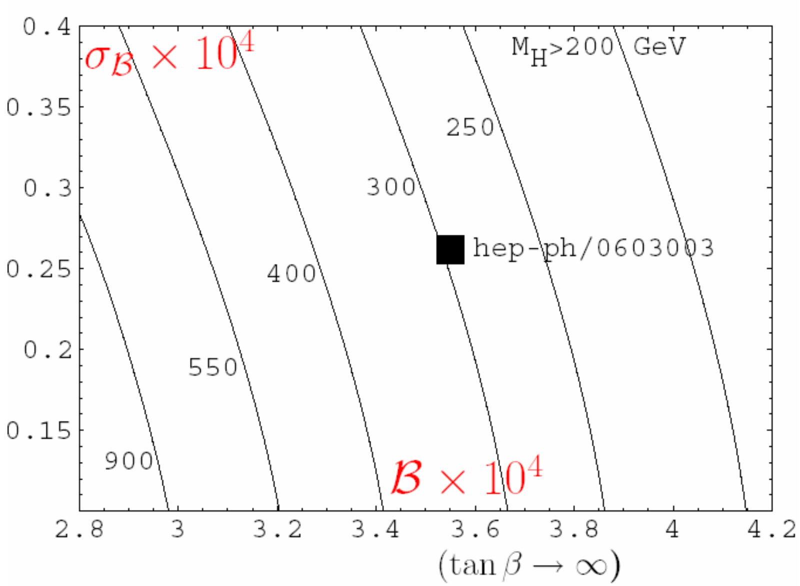
Impact on New Physics?

- Fortunately, no need for NNLO calculations! but higher order effects sometimes important
- However, status of theory predictions in models of new physics not always satisfactory. NLO calculations completed only in a few cases: 2HDM, LR,...
- Recently also MSSM with Minimal Flavor Violation (Degrassi, PG, Slavich, hep-ph/0601135): important dependence on scale at which quark/squark alignment is imposed, relevant effects for light superpartners

Charged Higgs mass in 2HDM-II

Strongest indirect bound from radiative decays, much stronger than direct ones
Mild dependence on tanβ, tighter bounds for small tanβ





Central values favor M_H~650-700 GeV

M_H>295GeV @ 95%CL

remains stringent

Summary

- NNLO calculation of inclusive radiative B decays is quite advanced. New results have decreased the theory uncertainty to level of the experimental one.
- There are a few NNLO contributions still to be computed: most important 3loop charm diagrams.
- Non-perturbative effects are at the moment the dominant source of uncertainty
- Bounds on new physics remain stringent. New NLO calculation in MSSM with MFV, more numerical results soon.