Status of V_{us}

M. Jamin, ICREA & IFAE, UA Barcelona

Status of V_{us}

1) V_{us} from K_{l3} decays
2) V_{us} from F_K/F_π ratio
3) V_{us} from hadronic τ decays
4) V_{us} from hyperon decays

V_{us} from K_{l3} decays

Theoretically, the K_{l3}-decay rate is given by:

$$\Gamma[K \to \pi l \nu_e(\gamma)] = \frac{G_F^2 M_K^5}{192\pi^3} C_K^2 S_{\rm EW} I_K(\{\lambda\}) |V_{us}|^2 f_+^2(0)$$

where with $t = (p-p')^2$:

$$\langle \pi^-(p') | \bar{s}\gamma_\mu u | K^0(p) \rangle = (p+p')_\mu f_+(t) + (p-p')_\mu f_-(t)$$

The largest theoretical uncertainty resides in the hadronic form factor $f_+(0)$. $\Rightarrow$ First determine: $|V_{us}|\, f_+(0)$

Recent new measurements by E865, KTeV, NA48 and KLOE:

$$|V_{us}|\, f_+(0) = 0.2169(9) \qquad \text{(PDG 06)}$$

$$|V_{us}|\, f_+(0) = 0.21673(46) \qquad \text{(FLAVIAnet } K \text{ WG 06)}$$

$K\pi$ form factors

Current data on K_{l3} form-factor slopes

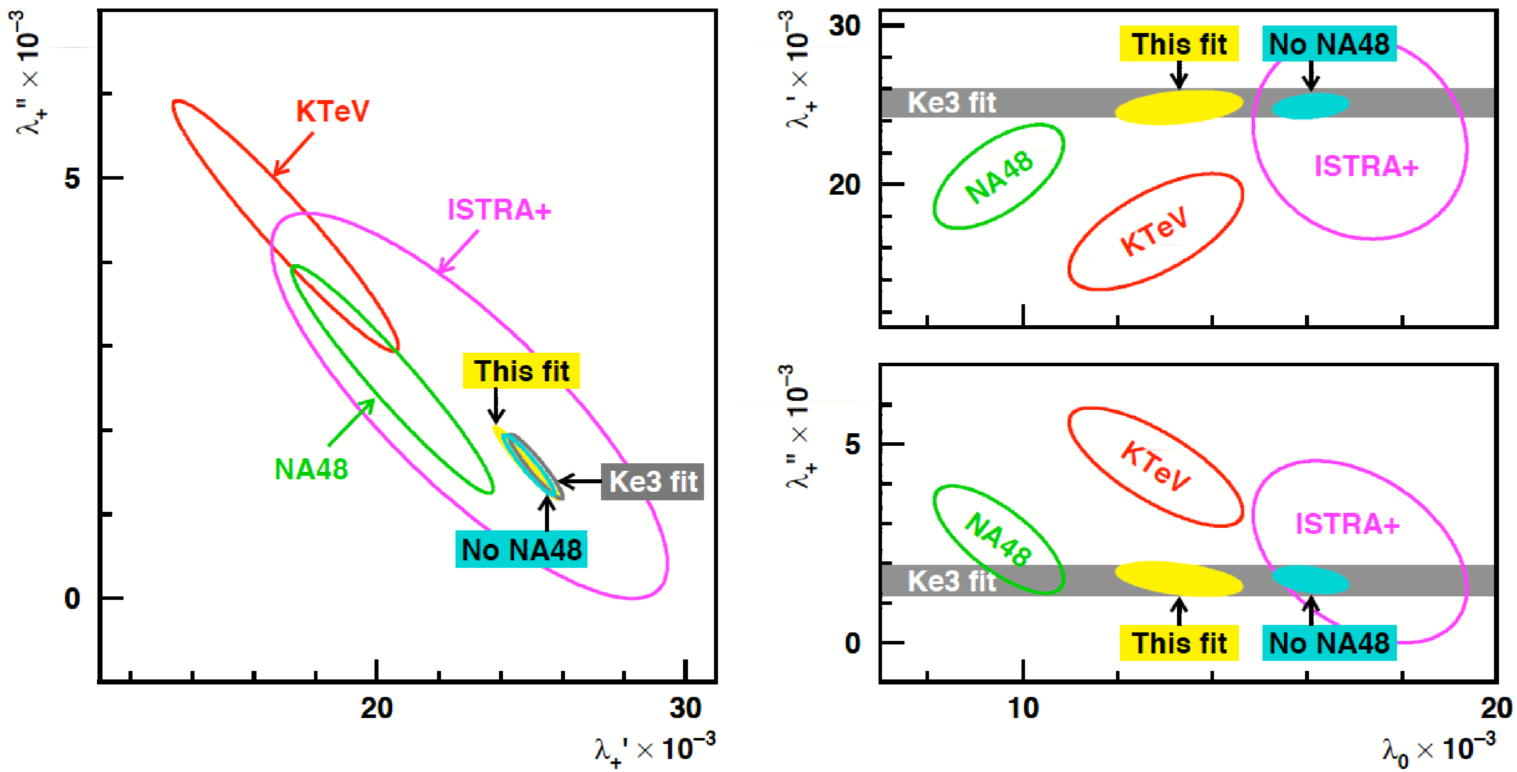
	Note	$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$	$\lambda_0 \times 10^3$
KTeV PRD 70 (2004)	$K_L e3$	21.7 ± 2.0	2.9 ± 0.8	
	$K_L \mu3$	17.0 ± 3.7	4.4 ± 1.5	12.8 ± 1.8
KLOE PLB 636 (2006)	$K_L e3$	25.5 ± 1.8	1.4 ± 0.8	
NA48 PLB 604 (2004)	$K_L e3$	28.0 ± 2.4	0.4 ± 0.9	
NA48 hep-ex/0703002	$K_L \mu3$	20.5 ± 3.3	2.6 ± 1.4	9.5 ± 1.4
ISTRA+ PLB 581 (2004)	$K^- e3$	24.9 ± 1.7	1.9 ± 0.9	
ISTRA+ PLB 589 (2004)	$K^- \mu3$	23.0 ± 6.4	2.3 ± 2.3	17.1 ± 2.2

Kindly provided by Matthew Moulson; Update from CKM06.

$$f_+(t) = f_+(0)\left[1+\lambda'_+ \frac{t}{M_\pi^2}+\lambda''_+ \frac{t^2}{2M_\pi^4}+\ldots\right]$$

Fit to K_{l3} form-factor slopes

Slopes from KTeV $K\mu3$ ISTRA+ $K\mu3$ NA48 $K\mu3$ Fit to $Ke3$ data



$Kl3$ fit, no NA48 $K\mu3$: χ^2=11.9/9 (21.7%) $Kl3$ fit, all data, χ^2=58/12 (10^{-6})

Kindly provided by Matthew Moulson; Update from CKM06.

K_{l3} form-factor slopes: Fit results

Although compatibility poor, no *a priori* reason to exclude NA48 $K\mu3$ data
Inconsistency parameterized by scale factors for fit results

Slope parameters × 10³:

$\lambda'_+ = 24.84 \pm 1.10 \quad S = 1.4$

$\lambda''_+ = 1.61 \pm 0.45 \quad S = 1.3$

$\lambda_0 = 13.30 \pm 1.35 \quad S = 2.1$

$\chi^2/\text{ndf} = 52/12 \ (10^{-6})$

Integrals	
$I(K^0e3)$	0.15452(29)
$I(K^+e3)$	0.15887(30)
$I(K^0\mu3)$	0.10207(34)
$I(K^+\mu3)$	0.10501(35)

Correlation coefficients:

	λ'_+	λ_0
λ'_+	−0.94	+0.31
λ''_+		−0.42

These results used to evaluate $|V_{us}|\, f_+(0)$ for all modes

Kindly provided by Matthew Moulson; Update from CKM06.

$K\pi$ form factors

A description of the $K\pi$ vector form factor can be obtained within chiral perturbation theory with resonances (RχPT):

(MJ, Pich, Portolés 2006)

$$F_+^{K\pi}(s) = \frac{M_{K^*}^2 \, e^{\frac{3}{2}\mathrm{Re}\left[\widetilde{H}_{K\pi}(s)+\widetilde{H}_{K\eta}(s)\right]}}{M_{K^*}^2 - s - iM_{K^*}\Gamma_{K^*}(s)} \,.$$

where

$$\Gamma_{K^*}(s) = \frac{G_V^2 M_{K^*} s}{64\pi F_\pi^4}\left[\sigma_{K\pi}^3(s) + \sigma_{K\eta}^3(s)\right] .$$

The parameters of this model, namely M_{K^*} and G_V, can be fitted from experimental data for p-wave $K\pi$ scattering.

Also a second resonance contribution can easily be included.

$K\pi$ form factors

As a prediction of the model, we obtain the slope and the curvature of the vector form factor $F_+^{K\pi}(s)$:

$$\lambda_+' = 25.6 \cdot 10^{-3}\,, \qquad \lambda_+'' = 1.31 \cdot 10^{-3}\,,$$

to be compared with the experimental result:

$$\lambda_+' = (24.8 \pm 1.1) \cdot 10^{-3}\,, \quad \lambda_+'' = (1.61 \pm 0.45) \cdot 10^{-3}\,.$$

Likewise, the scalar form factor $F_0^{K\pi}(s)$ can be obtained from a dispersion relation analysis of S-wave $K\pi$ scattering data.

(MJ, Oller, Pich 2002/04/06)

Here, the respective result is:

$$\lambda_0 = (14.7 \pm 0.4) \cdot 10^{-3} \Leftrightarrow \lambda_0^{\exp} = (13.3 \pm 1.4) \cdot 10^{-3}\,.$$

V_{us} from K_{l3} decays

	Reference	$f_+^{K^0\pi^-}(0)$
Quark model	[Leutwyler & Roos, 1984]	0.961 (8)
Lattice	[Becirevic et al, 2005]	0.960 (9)
	[MILC Collab., 2005]	0.962 (11)
	[Dawson et al, 2006]	0.968 (11)
	[UKQCD/RBC Collab., 2006]	0.961 (5) !
	[Bijnens & Talavera, 2003]	0.976 (10)
Kπ scalar f.f.	[Jamin, Oller & Pich, 2004]	0.974 (11)
Large-N_C	[Cirigliano et al, 2005]	0.984 (12)

Kindly provided by Jorge Portolés.

IFIC - Instituto de Física Corpuscular

$$\Rightarrow \quad f_+(0) = 0.966(5) \quad \Rightarrow \quad |V_{us}| = 0.2244(13)$$

V_{us} from F_K/F_π

From the leptonic decays $\Gamma[K\to l\nu_l(\gamma)]/\Gamma[\pi\to l\nu_l(\gamma)]$, one can predict: (Marciano 2004)

$$\frac{|V_{us}|}{|V_{ud}|}\,\frac{F_K}{F_\pi} = 0.27618 \pm 0.00048\,.$$

Together with the recent lattice result (MILC 2006)

$$\frac{F_K}{F_\pi} = 1.208(2)(^{+7}_{-14}) \quad \text{and} \quad V_{ud} = 0.97377(27)$$

(nuclear β-decay)

this leads to:

$$|V_{us}| = 0.2226\,(^{+26}_{-14})$$

Unitarity and V_{ud} imply: $F_K/F_\pi = 1.182(6)$! LR84: $1.22(1)$

V_{us} from τ decays

Consider the physical quantity R_τ: (Braaten, Narison, Pich (1992))

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \mathrm{hadrons}\,\nu_\tau(\gamma))}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau(\gamma))} = 3.640 \pm 0.010\,.$$

R_τ is related to the QCD correlators $\Pi^{T,L}(z)$: $(z \equiv s/M_\tau^2)$

$$R_\tau = 12\pi \int_0^1 dz\,(1-z)^2 \Big[(1+2z)\,\mathrm{Im}\Pi^T(z) + \mathrm{Im}\Pi^L(z)\Big]\,,$$

with the appropriate combinations

$$\Pi^J(z) = |V_{ud}|^2 \Big[\Pi_{ud}^{V,J} + \Pi_{ud}^{A,J}\Big] + |V_{us}|^2 \Big[\Pi_{us}^{V,J} + \Pi_{us}^{A,J}\Big]\,.$$

Additional information can be inferred from the moments

$$R_\tau^{kl} \equiv \int_0^1 dz\,(1-z)^k z^l \,\frac{dR_\tau}{dz} = R_{\tau,NS}^{kl} + R_{\tau,S}^{kl} \,.$$

Theoretically, R_τ^{kl} can be expressed as:

$$R_\tau^{kl} = N_c\, S_{\rm EW} \Big\{ (|V_{ud}|^2 + |V_{us}|^2)\big[1 + \delta^{kl(0)}\big] + \sum_{D\geq 2} \Big[|V_{ud}|^2 \delta_{ud}^{kl(D)} + |V_{us}|^2 \delta_{us}^{kl(D)} \Big] \Big\} \,.$$

$\delta_{ud}^{kl(D)}$ and $\delta_{us}^{kl(D)}$ are corrections in the Operator Product Expansion, the most important ones being $\sim m_s^2$ and $m_s\langle \bar{q}q\rangle$.

V_{us} from τ decays

The sensitivity to the strange quark mass can be enhanced by considering the flavour SU(3)-breaking difference:

(Pich, Prades; ALEPH (1998))

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,NS}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = 3\, S_{\rm EW} \sum_{D\geq 2} \left(\delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)} \right).$$

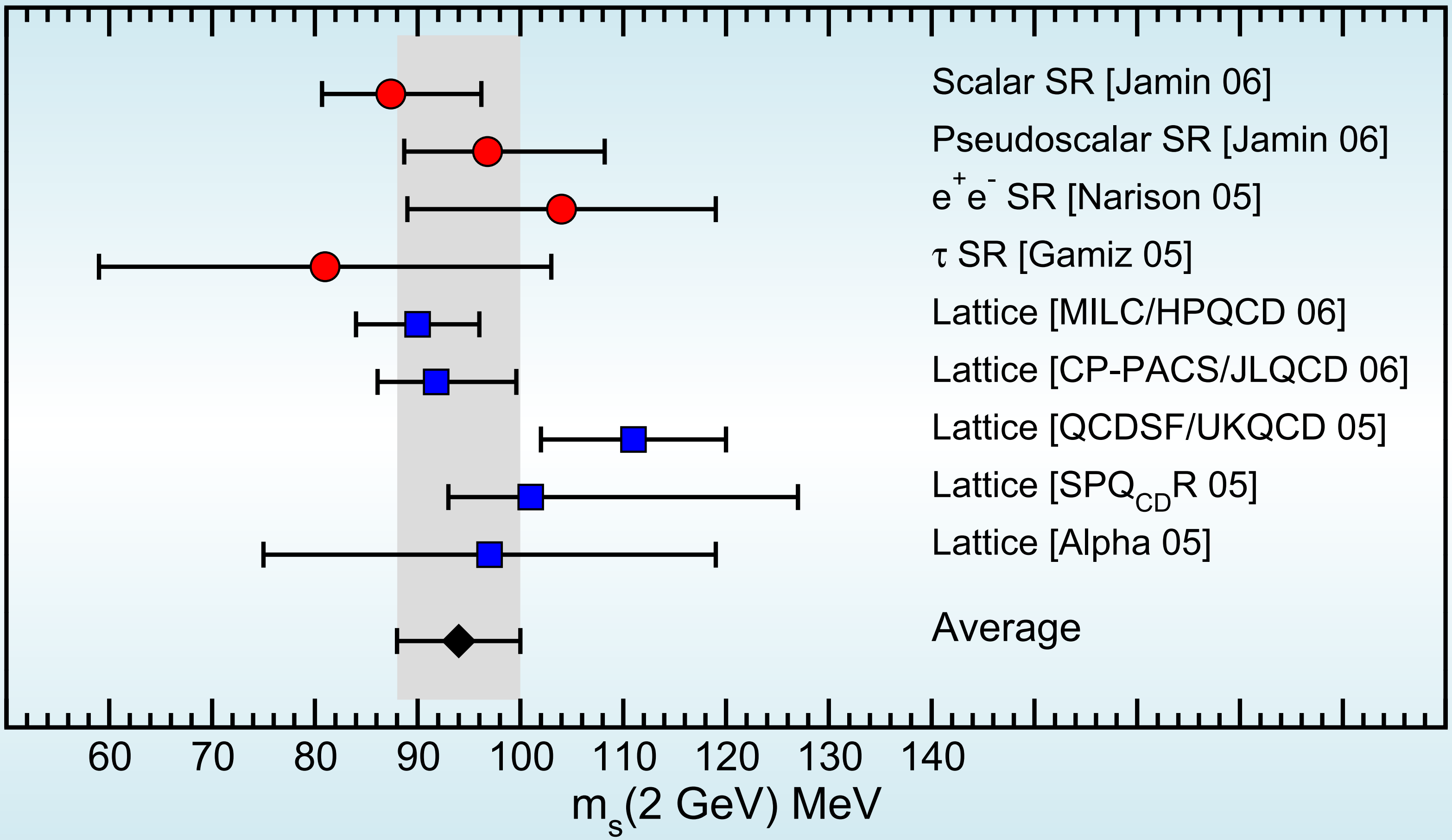
Flavour independent uncertainties drop out in the difference.

In previous analyses a sizeable part of the theoretical error was due to large α_s corrections in the longitudinal contribution.

This uncertainty could be greatly reduced by replacing badly behaved scalar/pseudoscalar correlators with phenomenology.

(Gámiz, M.J., Pich, Prades, Schwab (2003/04))

Strange quark mass



$\Rightarrow$ Average: $m_s(2\,\text{GeV}) = 94 \pm 6$ MeV

V_{us} from τ decays

Given m_s, we are in a position to predict δR_τ^{kl} from theory. Theoretically, the uncertainty is smallest for the $(0,0)$ moment:

$$\delta R_{\tau,th} = \underset{\text{Pheno}}{0.155} + \underset{m_s^2}{0.078} + 0.003 = 0.236 \pm 0.028\,.$$

Let us now reconsider the equation for δR_τ:

$$|V_{us}| = \sqrt{\frac{R_{\tau,S}}{\underbrace{R_{\tau,NS}/|V_{ud}|^2}_{\approx\, 3.658} - \delta R_{\tau,th}}}$$

Thus the theoretically derived quantity $\delta R_{\tau,th}$ only gives a small correction to experimentally measured quantities.

V_{us} from τ decays

Together with the experimental results $R_{\tau,NS} = 3.469 \pm 0.014$ as well as $R_{\tau,S} = 0.1677 \pm 0.0050$, V_{us} can be determined:

$$|V_{us}| = 0.2214 \pm 0.0033_{\rm exp} \pm 0.0010_{\rm th} = \boxed{0.2214 \pm 0.0034}$$

The uncertainty on V_{us} is dominated by the experimental error on $R_{\tau,S}$. The theoretical error by the perturbative expansion.

In the near future, it should be possible to reduce the uncertainty with the τ-data sets from BABAR and BELLE.

If the experimental value $B(\tau \to K\nu_\tau) = (0.686 \pm 0.023)\%$ is replaced by the theoretical prediction $(0.715 \pm 0.004)\%$ based on $K_{\mu 2}$ decays, one finds $|V_{us}| = \boxed{0.2225 \pm 0.0034}$.

Conclusions

$|V_{us}|(K_{l3}) = 0.2244(13)$ $|V_{us}|(\frac{F_K}{F_\pi}) = 0.2226\,(^{+26}_{-14})$

$|V_{us}|(\tau) = 0.2225(34)$ $|V_{us}|(\text{Hyp}) = 0.226(5)$

$\Rightarrow$ **Average:** $|V_{us}| = 0.2240(11)$

Unitarity relation: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta$.

$\Rightarrow$ $\delta = (1.58 \pm 0.72)\cdot 10^{-3}$ $\approx 2.2\,\sigma$ difference.

Or vice versa: $|V_{us}|_{\text{Unit}} = 0.2275(12)$