

BASICS OF $D^0-\bar{D}^0$ MIXING

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Complementing the presentations, at this conference, of the first experimental evidence for D mixing found at BaBar and Belle, I discuss the theoretical status of D mixing.

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The highlight of this year’s Moriond conference on electroweak interactions and unified theories arguably was the announcement by BaBar and Belle of experimental evidence for D^0 - \bar{D}^0 mixing^{1,2}, accompanied by an experimental paper³ and, less than one week after the event, a theoretical analysis⁴ – very likely to be followed by many others. As the experimental result came as a surprise to everyone, including the conference organisers, no theoretical talk on the topic had been organised. I was asked to fill the gap and give a quasi-impromptu talk on the theory basics of D mixing, whose written form is presented in these pages. Excellent reviews on the topic can be found in Refs.^{5,6}, and an enlightening reminder of the importance of charm physics in Ref.⁷.

In complete analogy to B mixing, D mixing in the SM is due to box diagrams with internal quarks and W bosons. In contrast to B , though, the internal quarks are down-type. Also in contrast to B mixing, the GIM mechanism is much more effective, as the heaviest down-type quark, the b , comes with a relative enhancement factor $(m_b^2 - m_{s,d}^2)/(m_s^2 - m_d^2)$, but also a large CKM-suppression factor $|V_{ub}V_{cb}^*|^2/|V_{us}V_{cs}^*|^2 \sim \lambda^8$, which renders its contribution to D mixing $\sim 1\%$ and hence negligible. As a consequence, D mixing is small in the SM, which makes it very sensitive to the potential intervention of new physics (NP), but on the other hand, it is also more difficult to accurately calculate the SM “background”, as the loop-diagrams are dominated by s and d quarks and hence sensitive to the intervention of resonances and non-perturbative QCD, see Fig. 1. The quasi-decoupling of the 3rd quark generation also implies that CP violation in D mixing is extremely small in the SM, and hence any observation of CP violation will be a clear-cut signal of new physics, independently of hadronic uncertainties.

The theoretical parameters describing D mixing can be defined in complete analogy to those for B mixing: the time evolution of the D^0 system is described by the Schrödinger equation

$$\frac{\partial}{\partial t} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = -i \left(M - i\frac{\Gamma}{2} \right) \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} \quad (1)$$

with Hermitian matrices M and Γ . The off-diagonal elements of these matrices, M_{12} and Γ_{12} , describe, respectively, the dispersive and absorptive parts of D mixing. The flavour-eigenstates $D^0 = (c\bar{u})$, $\bar{D}^0 = (u\bar{c})$ differ from the mass-eigenstates $D_{1,2}$; they are related by

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle \quad (2)$$

with

$$\left| \frac{q}{p} \right|^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}. \quad (3)$$

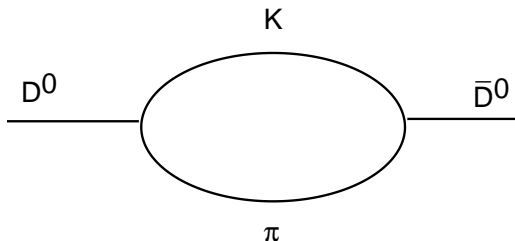


Figure 1: Resonance contribution to D mixing. Figure taken from Ref.⁵.

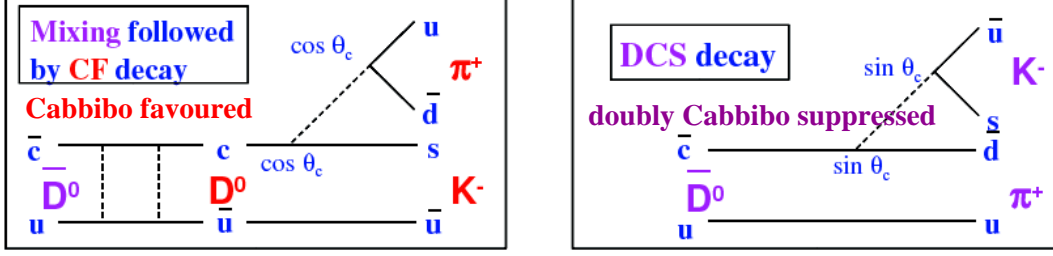


Figure 2: The two amplitudes contributing to the wrong-sign decay $\bar{D}^0 \rightarrow K^- \pi^+$. Figure adapted from Ref. ².

The basic observables in D mixing are the mass and lifetime differences of $D_{1,2}$, which are usually normalised to the average lifetime $\Gamma = (\Gamma_1 + \Gamma_2)/2$:

$$x \equiv \frac{\Delta M}{\Gamma} = \frac{M_2 - M_1}{\Gamma}, \quad y \equiv \frac{\Delta \Gamma}{2\Gamma} = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}. \quad (4)$$

While previously only bounds on x and y were known, both BaBar and Belle have now obtained evidence for a non-vanishing mixing in the D system. BaBar has obtained this evidence from the measurement of the doubly Cabibbo-suppressed decay $D^0 \rightarrow K^+ \pi^-$ (and its CP conjugate), yielding

$$y' = (0.97 \pm 0.44(\text{stat}) \pm 0.31(\text{syst})) \times 10^{-2}, \quad x'^2 = (-0.022 \pm 0.030(\text{stat}) \pm 0.021(\text{syst})) \times 10^{-2}, \quad (5)$$

while Belle obtains

$$y_{\text{CP}} = (1.31 \pm 0.32(\text{stat}) \pm 0.25(\text{syst})) \times 10^{-2} \quad (6)$$

from $D^0 \rightarrow K^+ K^-, \pi^+ \pi^-$ and

$$x = (0.80 \pm 0.29(\text{stat}) \pm 0.17(\text{syst})) \times 10^{-2}, \quad y = (0.33 \pm 0.24(\text{stat}) \pm 0.15(\text{syst})) \times 10^{-2} \quad (7)$$

from a Dalitz-plot analysis of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$. Here $y_{\text{CP}} \rightarrow y$ in the limit of no CP violation in D mixing, while the primed quantities x', y' are related to x, y by a rotation by a strong phase δ , see below.

CP violation in $D^0 \rightarrow f$ decays, which is predicted to be extremely small in the SM, can be characterised by non-vanishing values of

$$A_M = \left| \frac{q}{p} \right| - 1, \quad \phi = \arg(M_{12}/\Gamma_{12}), \quad (8)$$

where A_M measures CP violation in the mixing amplitude, while ϕ plays a rôle in the interference between the decays $D^0 \rightarrow f$ and $\bar{D}^0 \rightarrow f$.

Various D^0 decay channels are sensitive to D mixing. The evidence found by BaBar relies on $D^0 \rightarrow K^+ \pi^-, \bar{D}^0 \rightarrow K^- \pi^+$, which are “wrong sign” decays (the dominant transition is $c \rightarrow s$ and produces a K^- in D^0 , and a K^+ in \bar{D}^0 decays) and receive contributions from two amplitudes: a doubly Cabibbo-suppressed amplitude $\bar{D}^0 \rightarrow K^- \pi^+$, i.e. $\bar{c} \rightarrow \bar{d} u s$, and a two-step process via the oscillation $\bar{D}^0 \rightarrow D^0$, followed by the Cabibbo-favoured process $D^0 \rightarrow K^- \pi^+$, see Fig. 2. The relevant point here is that the amplitude with no oscillation is heavily suppressed which makes it competitive with the oscillated amplitude. The wrong-sign time-dependent

decay rate $D^0(t) \rightarrow K^+\pi^-$ is usually normalised to the Cabbibo-favoured rate $D^0 \rightarrow K^-\pi^+$. Expanding the ratio of suppressed vs. favoured amplitudes to second order in x, y , one finds

$$\begin{aligned} \frac{\Gamma(D^0(t) \rightarrow K^+\pi^-)}{\Gamma(D^0 \rightarrow K^-\pi^+)} &= \Gamma e^{-\Gamma t} \left[R_D + \left| \frac{q}{p} \right| \sqrt{R_D} (y' \cos \phi - x' \sin \phi) (\Gamma t) + \left| \frac{q}{p} \right|^2 \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right], \\ \frac{\Gamma(\bar{D}^0(t) \rightarrow K^-\pi^+)}{\Gamma(D^0 \rightarrow K^+\pi^-)} &= \Gamma e^{-\Gamma t} \left[R_D + \left| \frac{p}{q} \right| \sqrt{R_D} (y' \cos \phi + x' \sin \phi) (\Gamma t) + \left| \frac{p}{q} \right|^2 \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right], \end{aligned} \quad (9)$$

where the overall factor Γ ensures the correct normalisation upon integration over t . Here $R_D^{1/2}$ is the modulus of the ratio of the doubly Cabbibo-suppressed amplitude vs. the favoured one and x', y' contain the effect of the relative strong phase δ between the two amplitudes:

$$\frac{A(D^0 \rightarrow K^+\pi^-)}{A(\bar{D}^0 \rightarrow K^+\pi^-)} = -R_D^{1/2} e^{-i\delta}, \quad (10)$$

$$x' = x \cos \delta + y \sin \delta, \quad y' = y \cos \delta - x \sin \delta; \quad (11)$$

δ vanishes in the SU(3) limit. The minus sign in (10) originates from the sign of V_{us} relative to V_{cd} . Note that the 2nd term in brackets in (9) comes from the interference of the two decay amplitudes with and without mixing. BaBar has obtained R_D , y' and x'^2 from the fit of their experimental results to the above formulas in the case of (a) CP conservation, i.e. $|q/p| \rightarrow 1$, $\phi \rightarrow 0$, and (b) CP violation, i.e. different coefficients R_D^\pm , y'_\pm etc. for D^0 and \bar{D}^0 decays. The difference of the latter proved to be compatible with 0, so there is no evidence for CP violation.

Let us now turn to the theoretical predictions for x and y in the SM. In terms of hadronic matrix elements, $M_{12} = M_{21}^*$ and $\Gamma_{12} = \Gamma_{21}^*$ can be expressed as

$$\begin{aligned} M_{21} &= \langle \bar{D}^0 | \mathcal{H}_{\text{eff}}^{\Delta C=2} | D^0 \rangle + P \sum_n \frac{\langle \bar{D}^0 | \mathcal{H}_{\text{eff}}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{\text{eff}}^{\Delta C=1} | D^0 \rangle}{m_D^2 - E_n^2}, \\ \Gamma_{21} &= P \sum_n \rho_n^{\text{ph.sp}} \langle \bar{D}^0 | \mathcal{H}_{\text{eff}}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{\text{eff}}^{\Delta C=1} | D^0 \rangle. \end{aligned} \quad (12)$$

While the expression for Γ_{12} is very similar to that in the B system, that for M_{12} differs by the contribution of the second term which is heavily suppressed in B mixing. The sum runs over all decay channels of D^0 ; the contribution to M_{12} includes that of off-shell intermediate states, while only on-shell states contribute to Γ_{12} ; $\rho_n^{\text{ph.sp}}$ is the corresponding phase-space factor. $\mathcal{H}_{\text{eff}}^{\Delta C=2}$ is the local Hamiltonian obtained from the box diagrams, and includes potential contributions from NP, while all terms in $\mathcal{H}_{\text{eff}}^{\Delta C=1}$, the Hamiltonian describing non-leptonic decays of the c quark, are dominated by SM contributions (see, however, Ref. ⁸ for a discussion of NP effects in decay amplitudes). Neglecting long-distance non-perturbative QCD effects, and only including the box diagrams, one finds ⁹ $x_{\text{box}} = O(10^{-5})$, $y_{\text{box}} = O(10^{-7})$, which is far below the experimental results – which indicates that these long-distance effects are extremely important.

There is an extensive literature on estimating x and y within and beyond the SM, see Ref. ¹⁰ for a collection of results. The central problem of all these calculations is that the D is too heavy to be treated as light and too light to be treated as heavy. As a consequence, the two approaches that have been so successful in treating heavy (B) and light (K) meson mixing both are not really applicable to D mixing: the “inclusive” approach is based on operator product expansion and relies on quark-hadron duality. If Λ/m_c , where Λ is a hadronic scale, is considered a small parameter, x and y can be expanded in terms of matrix elements of local operators ¹¹, and the

series can be truncated after a few terms. Such calculations typically yield $x, y \lesssim 10^{-3}$, and the result of both BaBar and Belle, $y \sim 10^{-2}$, is certainly not a generic prediction of such an analysis. In the “exclusive” approach^{9,12}, on the other hand, one sums over intermediate hadronic states, which may be modeled or fit to experimental data. One crucial observation⁹ is that x and y are only generated at second order in SU(3) breaking, which suggests an analysis based on the summation over exclusive states arranged in SU(3) multiplets. As argued in Ref.⁹, the main source of SU(3)-breaking within these multiplets is due to phase-space, or rather, the lack thereof: if the heaviest members of a multiplet are too heavy to be kinematically accessible in the decay, they have to be excluded from the sum over all members of the multiplet (e.g. $D \rightarrow 4\pi$ is kinematically allowed, but $D \rightarrow 4K$ is not) and as a consequence, the cancellation of the sum over all terms, which yields 0 in the SU(3)-limit, is badly broken. The conclusion is that in this way values of $y \sim 10^{-2}$ can be reached – which agrees very well with the experimental result and suggests that these threshold effects may indeed explain the experimental result. The inclusive approach, on the other hand, relies on the duality of hadronic and partonic effects, smeared over sufficiently large energy intervals, and is manifestly insensitive to threshold phenomena – and hence likely to be inapplicable to D decays. In the exclusive approach, x can be related to y via a dispersion relation; the authors of Ref.⁹ find that for $y \sim 1\%$ one expects $|x|$ between 0.1% and 1%, and x and y to be of opposite sign; one should be aware, however, that this calculation is more model-dependent than that of y .

In conclusion, we find that the experimental results on D mixing reported by BaBar and Belle at the 2007 Rencontres de Moriond on electroweak interactions and unified theories present a major step forward in experimental achievement and analysis. The measured value of $y \gtrsim x$ is at the high end of theoretical predictions and indicates large long-distance contributions, which also impact on x , i.e. the short-distance/NP sensitive mass difference. As long as there is no major breakthrough in theoretical predictions for D mixing, which are held back by the fact that the D meson is at the same time too heavy and too light for our current theoretical tools to get a proper grip on the problem, the long-distance SM contributions to x will completely obscure any NP contributions and their detection. The observation of CP violation still presents a theoretically clean way for NP to manifest itself and it is to be hoped that in the near future, i.e. at the B factories or the LHC, at least one of the plentiful opportunities for NP to show up in CP violation¹³ will be realised.

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