

# $D^0-\bar{D}^0$ Mixing – Theory

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# **Experimental Results**

## **Evidence for D mixing at last!**

Presented at Moriond EW on Mar 13. . .

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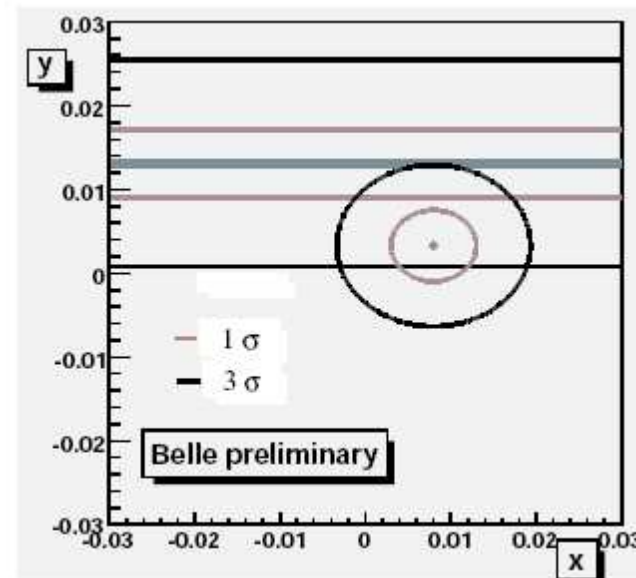
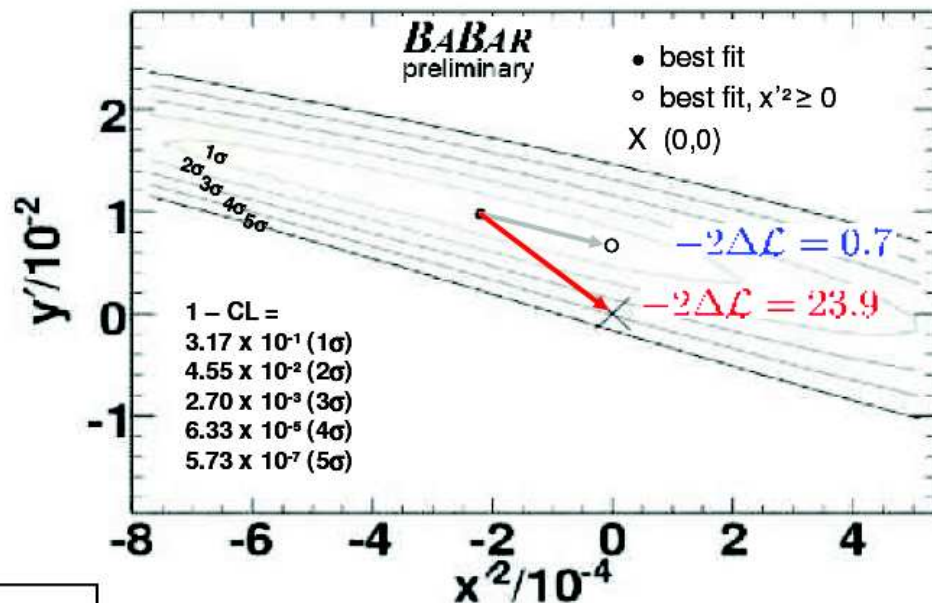
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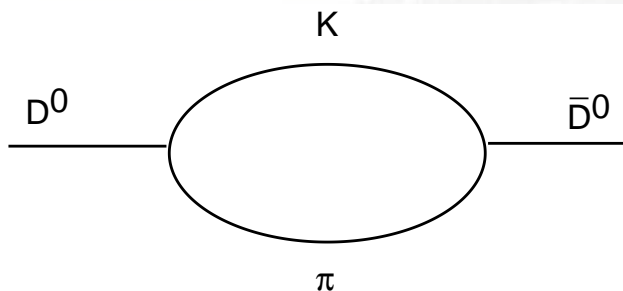
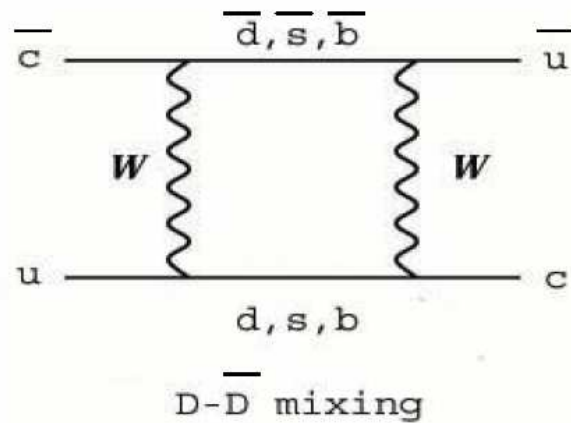
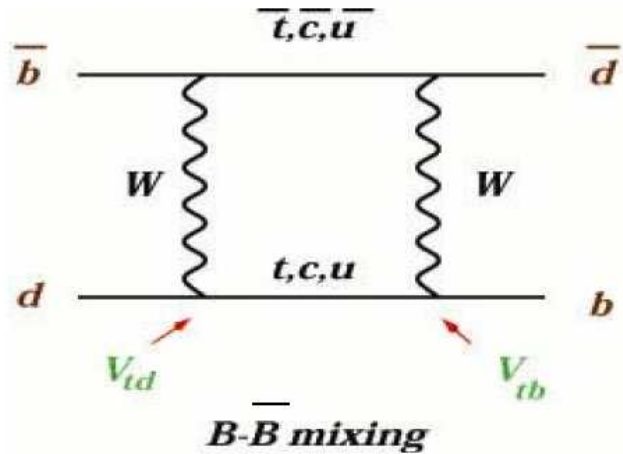
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Sorry... What is it you are talking about?

# D Mixing: why do we care?



D mixing	B mixing
intermediate <b>down-type</b> quarks	intermediate <b>up-type</b> quarks
SM: $b$ quark contribution is <b>negligible</b> due to $V_{ub}$	SM: $t$ quark contribution is <b>dominant</b>
$\Delta M \sim [\text{SU}(3) \text{ breaking}]^2$ (Falk hep-ph/0402204)	$\Delta M \propto m_t^2$ and sizeable
sensitive to <b>long distance QCD</b>	described by <b>local</b> Lagrangian

D prefers to **decay rather than mix**: off-shell intermediate states very relevant, outdo short-distance box diagram.

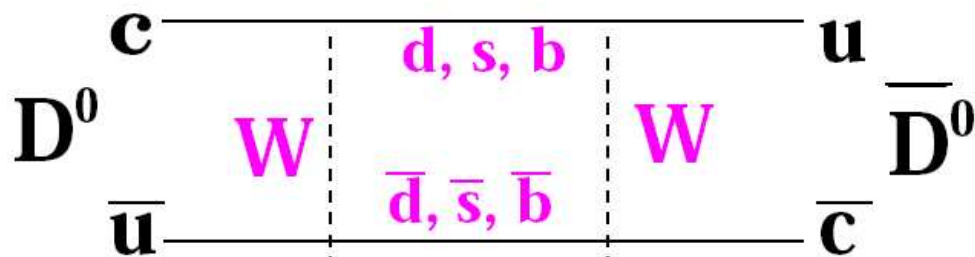
# D Mixing: a story of $x$ and $y$

$$\frac{\partial}{\partial t} \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix} = -i \left( M - i\frac{\Gamma}{2} \right) \begin{pmatrix} D^0 \\ \bar{D}^0 \end{pmatrix}$$

Mass eigenstates:  $|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle$ ,  $|D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle$ ,  
 $|p|^2 + |q|^2 = 1$ .  $|p/q| \neq 1$  sign for CP violation.

Mass and lifetime differences:

$$x \equiv \frac{\Delta M}{\Gamma} = \frac{M_2 - M_1}{\Gamma}, \quad y \equiv \frac{\Delta\Gamma}{2\Gamma} = \frac{\Gamma_2 - \Gamma_1}{\Gamma}.$$



- CP violation in mixing induced by  $\phi = \arg(M_{12}/\Gamma_{12})$

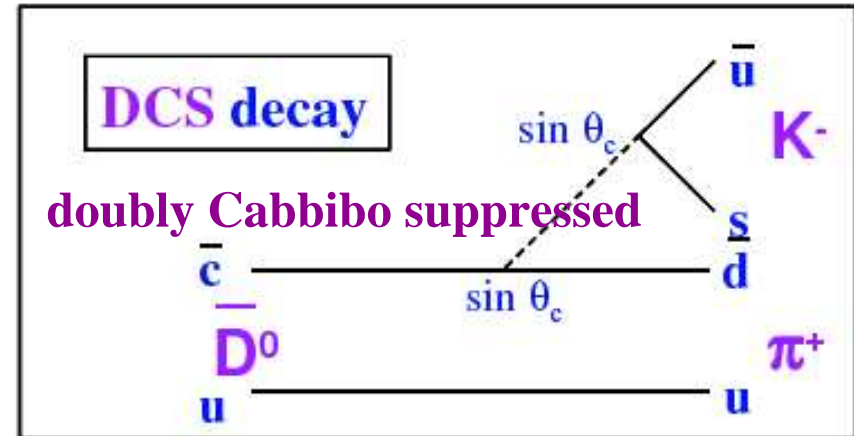
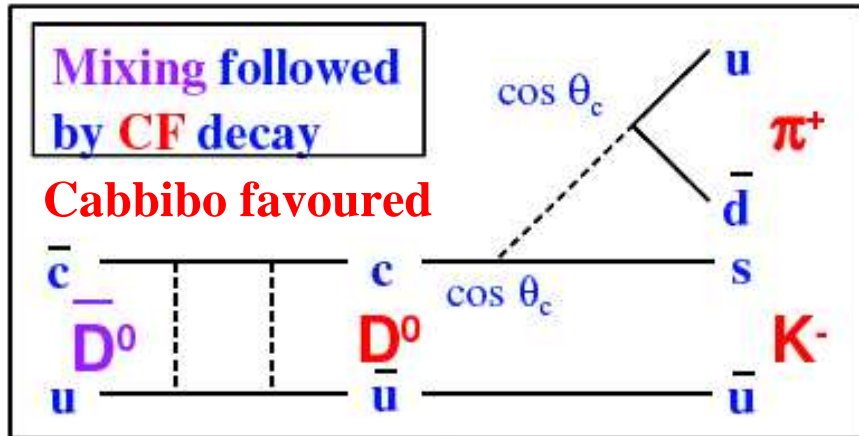
- $\phi \sim \lambda^4$  in SM: extremely small, excellent place for BSM to manifest itself



# How BaBar and Belle measure $D$ mixing

Time-dependent decay rates

$$\Gamma(D^0(t) \rightarrow K^+\pi^-) \text{ and } \Gamma(\bar{D}^0(t) \rightarrow K^-\pi^+)$$



Neglect CP violation, expand to 2nd order in  $(\Delta M, \Delta\Gamma) \ll \Gamma$ :

$$\Gamma(D^0(t) \rightarrow K^+\pi^-) \propto e^{-\Gamma t} \left[ R_D + \sqrt{R_D} y' (\Gamma t) + \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right]$$

$R_D \sim \tan^4 \theta_C \sim 0.3\%$ : ratio of DCS/CF rate;  
 $x', y'$ : differ from  $x, y$  by strong phase  $\delta$

## $x', y'$ vs. $x, y$

$\delta$  : relative **strong** phase between DCS  $A(D^0 \rightarrow K^+ \pi^-)$  and CF  $A(\bar{D}^0 \rightarrow K^+ \pi^-)$ ; enters decay rate as

$$x' = x \cos \delta + y \sin \delta, \quad y' = -x \sin \delta + y \cos \delta$$

$$\Gamma(D^0(t) \rightarrow K^+ \pi^-) \propto e^{-\Gamma t} \left[ R_D + \sqrt{R_D} y' (\Gamma t) + \frac{x'^2 + y'^2}{4} (\Gamma t)^2 \right]$$

Extract  $y'$  and  $x'^2$  from time-dependent rates.

$\delta$  can be (but hasn't been) measured from CP-tagged  $D_{\text{CP}}^0 \rightarrow K^\pm \pi^\mp$  decays at Cleo-c.

Another interesting idea (M. Neubert):  $\delta_{\pi K}$  could be accessible in the  $B^0$  system from the time-dependent rates in  $B^0 \rightarrow K \pi$  (measured).

# Theory Predictions for $x$ and $y$

Main difference to  $B$  system:

$x$ , i.e.  $\Delta M$ , dominated by light-quark loops  $\Rightarrow$  long-distance (non-local) contributions

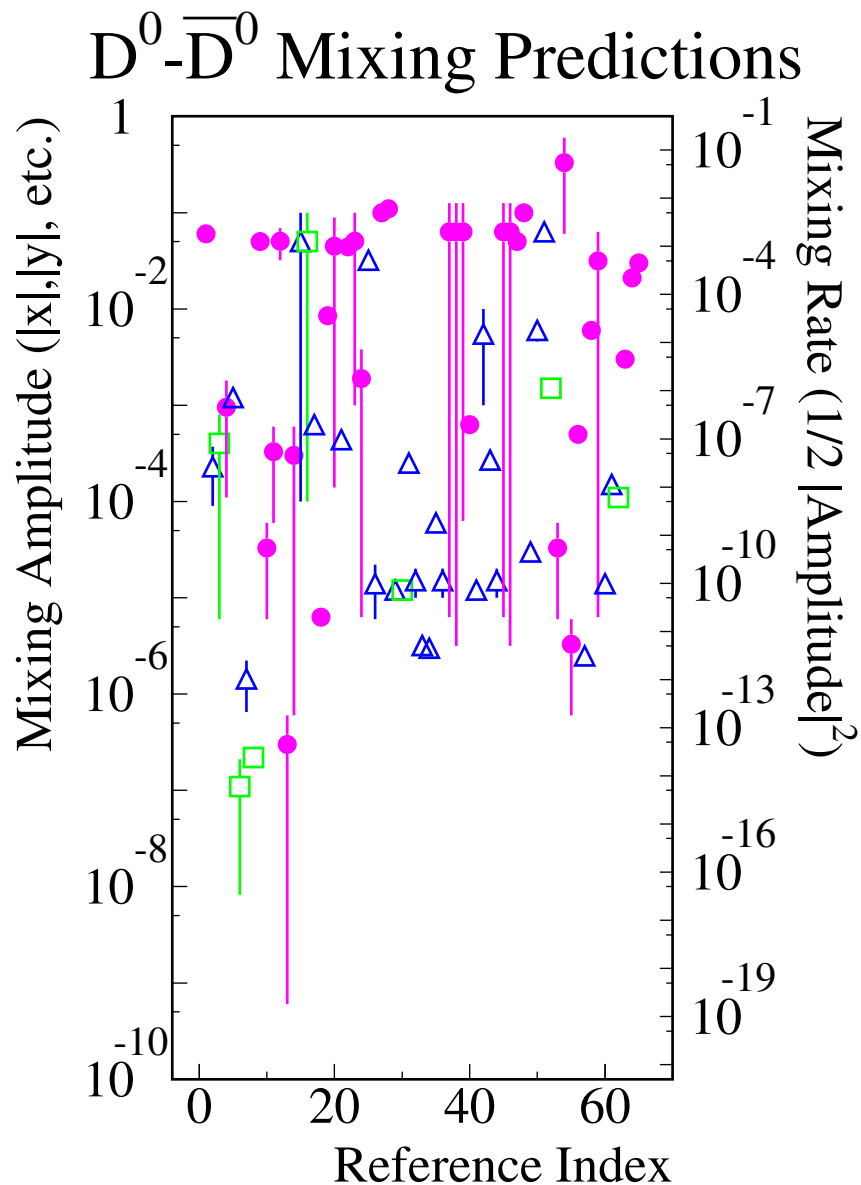
$x, y$  given in terms of  $M_{12}, \Gamma_{12}$  with





$$M_{12} = \underbrace{\langle \bar{D}^0 | \mathcal{H}_{\text{eff}}^{\Delta C=2} | D^0 \rangle}_{\text{possible new physics contributions!}} + P \underbrace{\sum_n \frac{\langle \bar{D}^0 | \mathcal{H}_{\text{eff}}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{\text{eff}}^{\Delta C=1} | D^0 \rangle}{m_D^2 - E_n^2}}_{\text{SM dominated}}$$

$$\Gamma_{12} = \underbrace{\sum_n \rho_n^{\text{ph.sp.}} \langle \bar{D}^0 | \mathcal{H}_{\text{eff}}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{\text{eff}}^{\Delta C=1} | D^0 \rangle}_{\text{SM dominated}}$$

# Assorted Theory Predictions for $x$ and $y$

H. Nelson, hep-ex/9909021



-  open triangles:  $x$  in SM
-  open squares:  $y$  in SM
-  solid circles:  $x$  in various BSM
-   $y$  assumed to be not affected by new physics (see, however, Golowich hep-ph/0610039)

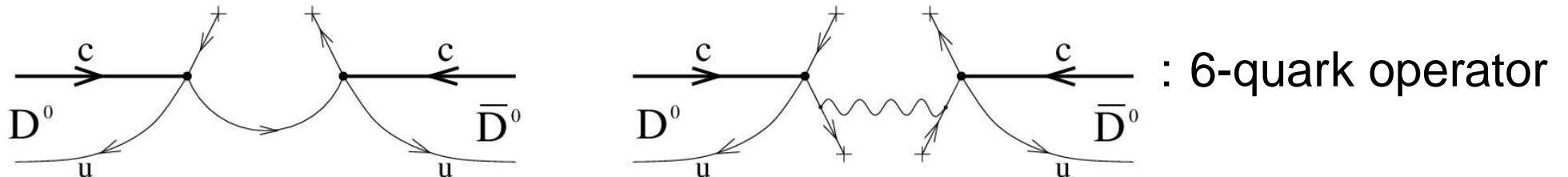
 numbers slightly outdated!

# Theory Calculations I: “inclusive”

- assume  $D$  is **heavy enough** for local operator product expansion (OPE) to hold:  $m_c \gg \Lambda_{\text{hadronic}}$  (Georgi 92, Ohl 93, Bigi 00)
- power-expand e.g.  $\Gamma_{12}$  in  $\Lambda_{\text{hadronic}}/m_c$ :

$$\Gamma_{12} = \frac{1}{2M_D} \text{Im} \langle \bar{D}^0 | i \int d^4x T \{ \mathcal{H}_{\text{eff}}^{\Delta C=1}(x) \mathcal{H}_{\text{eff}}^{\Delta C=1}(0) \} | D^0 \rangle$$

- leading contribution from box highly **GIM-suppressed**,  $\Delta M \sim m_s^4$ ,  $x_{\text{box}} \sim 10^{-5}$ ,  $\Delta\Gamma \sim m_s^6$ ,  $y_{\text{box}} \sim 10^{-7}$
- suppression relieved by **higher-dimensional operators**, e.g.



- final estimate:  $x_{\text{OPE}} \lesssim y_{\text{OPE}} \lesssim 10^{-3}$  : with large uncertainties from **bad convergence of OPE**

# Theory Calculations II: “exclusive”

(Falk/Grossman/Ligeti/Petrov 02, 04)

- D mixing is **2nd order in SU(3) breaking**
- assume D is **light enough** for sum over exclusive states to be dominated by only few terms:

$$y = \sum_n \eta_{\text{CKM}}(n) \eta_{\text{CP}}(n) \cos \delta_n \sqrt{\mathcal{B}(D^0 \rightarrow n) \mathcal{B}(\bar{D}^0 \rightarrow n)}$$

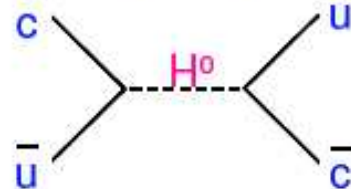
- include effects of SU(3) breaking from **phase-space**, but not from matrix elements and strong phases
- obtain  $x$  from  $y$  via a dispersion relation
- final estimate:

$$y_{\text{excl}} \sim +1\%, \quad |x_{\text{excl}}| \sim 10^{-3} - 10^{-2}, \quad x_{\text{excl}} \text{ and } y_{\text{excl}} \text{ with opposite sign}$$

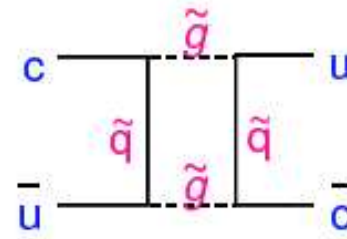
- $D \rightarrow 4P$  dominant contribution, uncertainties from measured branching fractions etc.

# Possible New Physics Contributions

- Possible enhancements to mixing due to new particles and interactions in new physics models
- Most new physics predictions for  $\mathbf{x}$ 
  - Extended Higgs, tree-level FCNC
  - Fourth generation down-type quarks
  - Supersymmetry: gluinos, squarks
  - Lepto-quarks



FCNC



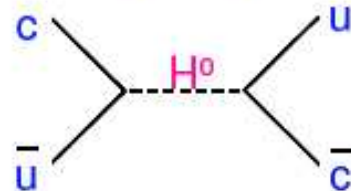
supersymmetry

- Large possible SM contributions to mixing require observation of either a CP-violating signal or  $|\mathbf{x}| \gg |\mathbf{y}|$  to establish presence of NP
  - [Ann.Rev.Nucl.Part.Sci 53 431-499 \(2003\)](#)

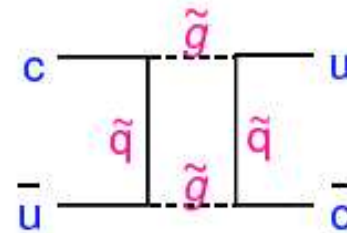


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supersymmetry

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# CP Violation

- CP violation in D mixing described by weak phase

$$\phi_D = \arg(M_{12}/\Gamma_{12}); \quad \phi_D \sim \lambda^4 \sim 10^{-3} \text{ in SM}$$

- time-dependent rates:

$$\frac{\Gamma(D^0 \rightarrow K^+ \pi^-)}{\Gamma(\bar{D}^0 \rightarrow K^+ \pi^-)} = R_D + \sqrt{R_D} \left| \frac{q}{p} \right| (y' \cos \phi_D - x' \sin \phi_D) (\Gamma t) + \left| \frac{q}{p} \right|^2 \frac{x'^2 + y'^2}{4} (\Gamma t)^2$$

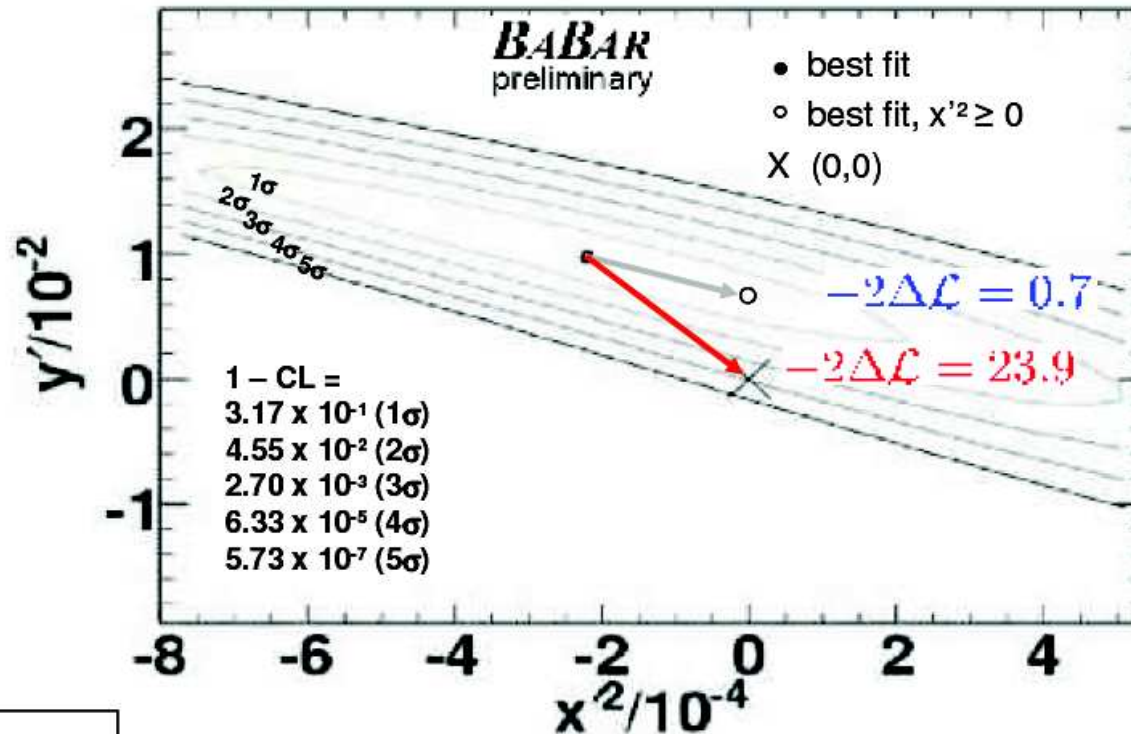
$$\frac{\Gamma(\bar{D}^0 \rightarrow K^- \pi^+)}{\Gamma(D^0 \rightarrow K^- \pi^+)} = R_D + \sqrt{R_D} \left| \frac{p}{q} \right| (y' \cos \phi_D + x' \sin \phi_D) (\Gamma t) + \left| \frac{p}{q} \right|^2 \frac{x'^2 + y'^2}{4} (\Gamma t)^2$$

- BaBar/Belle found no evidence for  $|p/q| \neq 1$ , but did not include **mixing induced CP violation**  $\phi_D \neq 0$

# Experimental Results – 2nd try

## Mixing Contours from $D^0 \rightarrow \pi^- K^+$

- $y'$ ,  $x'^2$  contours computed by change in log likelihood
  - Best-fit point is in non-physical region  $x'^2 < 0$ , but 1-sigma contour extends into physical region
  - correlation: -0.94
- Contours include systematic errors



- **Accounting for systematic errors, the no-mixing point is at ~4-sigma contour**

$$R_D: (3.03 \pm 0.16 \pm 0.06) \times 10^{-3}$$

$$x'^2: (-0.22 \pm 0.30 \pm 0.20) \times 10^{-3}$$

$$y': (9.7 \pm 4.4 \pm 2.9) \times 10^{-3}$$

# Experimental Results – 2nd try



$D^0 \rightarrow K\pi$  (Belle,  $400 \text{ fb}^{-1}$ )

Unbinned fit to time distribution

## Results

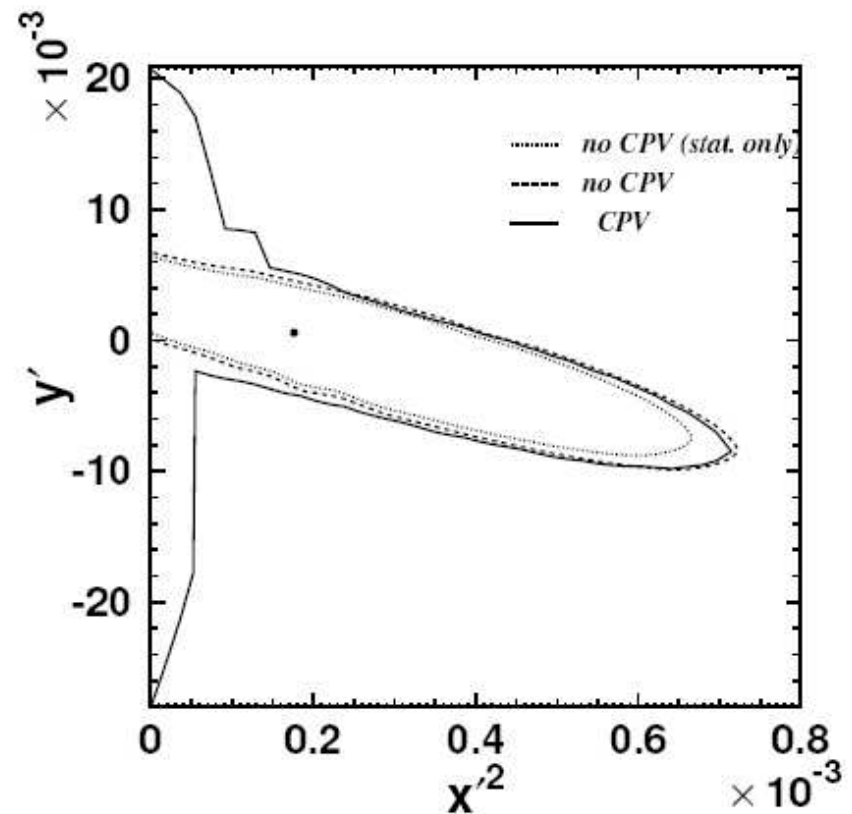
- ◆ Assuming CP conservation

$$R_D = (0.364 \pm 0.017)\%$$

$$x'^2 = (0.18_{-0.23}^{+0.21}) \times 10^{-3}$$

$$y' = (0.6_{-3.9}^{+4.0}) \times 10^{-3}$$

- ◆ CP asymmetries consistent with 0  
→ no evidence for CPV



$$R_M < 0.40 \times 10^{-3} \quad @ 95\% \text{ C.L.}$$

$$R_M = (x'^2 + y'^2)/2$$

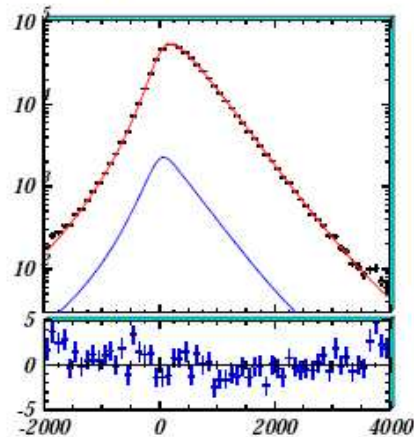
# Experimental Results – 2nd try



$D^0 \rightarrow K_s^0 \pi^+ \pi^-$  Dalitz (Belle,  $540 \text{ fb}^{-1}$ )



Time fit (in projection)



Systematics

Largest contributions ( $\times 10^{-4}$ )

x	y	
+14.6	+7.8	Model dependence
-13.6	-8.8	
+8.5	+6.6	Time fit
-6.8	-11.6	

Total ( $\times 10^{-4}$ )

x	y
+16.9	+10.2
-15.2	-14.6

Results (preliminary)

$$x = 0.80 \pm 0.29 \pm 0.17 \%$$

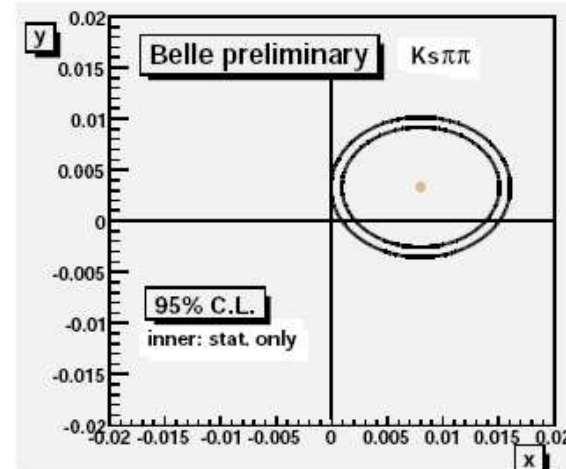
$$y = 0.33 \pm 0.24 \pm 0.15 \%$$

most stringent limits on x up to now

Cleo, PRD 72, 012001 (2005):

$$x = 1.8 \pm 3.4 \pm 0.6\%$$

$$y = -1.4 \pm 2.5 \pm 0.9\%$$





# Experimental Results – 2nd try



$D^0 \rightarrow K^+ K^-, \pi^+ \pi^-$  (Belle,  $540 \text{ fb}^{-1}$ )

- ◆ Measurement of lifetime difference between  $D^0 \rightarrow K^- \pi^+$  and  $K^+ K^-, \pi^+ \pi^-$

▷ mixing parameter:  $y_{CP} = \frac{\tau(K^- \pi^+)}{\tau(K^+ K^-)} - 1$

▷ in CP conservation limit:  $y_{CP} = y = \Delta\Gamma/\Gamma$

- ◆ If CP not conserved, difference in lifetimes of  $D^0/\bar{D}^0 \rightarrow K^+ K^-, \pi^+ \pi^-$

▷ CP violating parameter:  $A_\Gamma = \frac{\hat{\Gamma}(D^0 \rightarrow KK) - \hat{\Gamma}(\bar{D}^0 \rightarrow KK)}{\hat{\Gamma}(D^0 \rightarrow KK) + \hat{\Gamma}(\bar{D}^0 \rightarrow KK)}$

Belle preliminary ( $540 \text{ fb}^{-1}$ )

$y_{CP} = 1.31 \pm 0.32 \pm 0.25 \%$

>  $3\sigma$  above zero

( $4.1\sigma$  stat. only)

first evidence for  $D^0 - \bar{D}^0$  mixing

$A_\Gamma = 0.01 \pm 0.30 \pm 0.15 \%$

no evidence for CP violation

$$y_{CP} = \frac{\Gamma(\text{CP even}) - \Gamma(\text{CP odd})}{\Gamma(\text{CP even}) + \Gamma(\text{CP odd})}$$

$$\approx \frac{\Gamma(D^0 \rightarrow K^+ K^-)}{\Gamma(D^0 \rightarrow K^- \pi^+)} - 1$$

$$= y \cos \phi - x \sin \phi (A_M + A_{\text{prod}})$$

$A_M = |q/p|^2 - 1$ ,  $A_{\text{prod}}$  : production ass. of  $D^0, \bar{D}^0$



# Conclusions

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- new at Moriond: **experimental evidence for  $D$  mixing**
- $\Delta\Gamma \sim y$  **saturates SM predictions** (BaBar/Belle) (ditto for  $x'^2 + y'^2 \equiv x^2 + y^2$ )
- ditto  $\Delta M \sim x$  (Belle)
- interpretation in terms of **large long-distance hadronic contributions**
- long-distance contributions also show up in  $\Delta M \sim x$  and **obscure possible short-distance new-physics contributions**
- tell-tale sign for NP will be **CP violation** (near zero in SM): no indication yet
- prospects for more accurate theory predictions not very good as  $D$  is **too light** to be treated as heavy and **too heavy** to be treated as light. . .

# Backup Slides

# Determination of $\delta_{K\pi}$

At Cleo-c,  $D$  and  $\bar{D}$  produced in **coherent state**.

Use CP-specific decays of one  $D$  to “CP-tag” the other, e.g.  $D \rightarrow \pi^0 K_S$  with  $CP=-1$  implies the other state having  $CP=+1$ .

Neglecting CP violation in mixing (tiny in SM), one has

$$\sqrt{2}A(D_{CP=\pm} \rightarrow K^- \pi^+) = A(D^0 \rightarrow K^- \pi^+) \pm A(\bar{D}^0 \rightarrow K^- \pi^+)$$

This determines  $\cos \delta_{K\pi}$  as

$$1 + R_D \pm 2\sqrt{R_D} \cos \delta_{K\pi} = 2 \frac{\mathcal{B}(D_{CP=\pm} \rightarrow K^- \pi^+)}{\mathcal{B}(D^0 \rightarrow K^- \pi^+)}$$

In the SU(3) limit,  $\delta_{K\pi} = 0$ , but this can be badly broken.



# $y$ from SU(3)-breaking phase-space effects

(Falk/Grossman/Ligeti/Petrov 02)

$y = \frac{1}{\Gamma} \sum_{(F,R)} y_{F,R} \left[ \sum_n \Gamma(D^0 \rightarrow n) \right]$  running over intermediate states belonging to the class **F** (PP,PV,VV etc.) and the SU(3) representation **R**.  $y_{F,R}$  is given by

$$y_{F,R} = \frac{\sum_n \langle \bar{D}^0 | \mathcal{H}_{\text{eff}}^{\Delta C=1} | n \rangle \rho_n^{\text{ph.sp.}} \langle n | \mathcal{H}_{\text{eff}}^{\Delta C=1} | D^0 \rangle}{\sum_n \Gamma(D^0 \rightarrow n)}$$

In the exact SU(3) limit, all  $y_{F,R} \rightarrow 0$  (as  $y \rightarrow 0$ ). In the real world, SU(3) is

broken by different values of the matrix elements and **phase-space effects**.

The biggest such effect occurs **if the sum cannot be completed because some members of R are too heavy for the D to decay into**. While the

estimated values for PP etc., when all PP decays are physical, are

$y_{F,R} \sim 10^{-3}$ , for **F=4P** the cancellation is incomplete ( $D \not\rightarrow 4K$ ) and

$y_{F,R} \sim 10^{-2}$  can be reached, although with large uncertainties.