

# Low-energy non unitary leptonic mixing

Belen Gavela  
Universidad Autónoma de Madrid and IFT

The complete theory of  $\nu$  masses is unitary.

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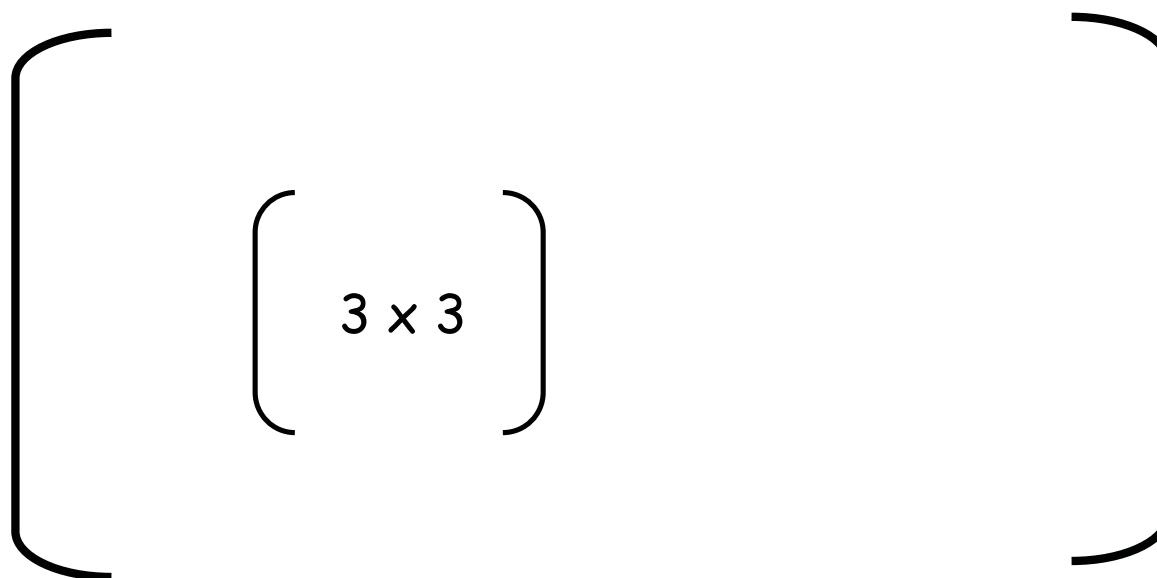
*Low-energy non-unitarity may result  
from new physics contributing to  
neutrino propagation.*

The complete theory of  $\nu$  masses is unitary.

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*Low-energy non-unitarity may result from new physics contributing to neutrino propagation.*

i.e., a neutrino mass matrix larger than  $3 \times 3$



# $\nu$ masses beyond the SM

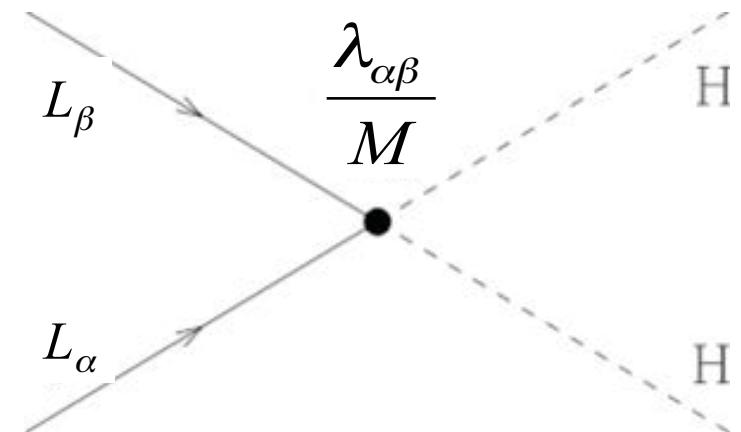
Favorite options: BSM theory at higher scale  $M$

Heavy fields manifest in the low energy effective theory (SM) via higher dimensional operators

Dimension 5 operator:

$$\frac{\lambda}{M} (L L H H) \rightarrow \frac{\lambda v^2}{M} (\nu \nu)$$

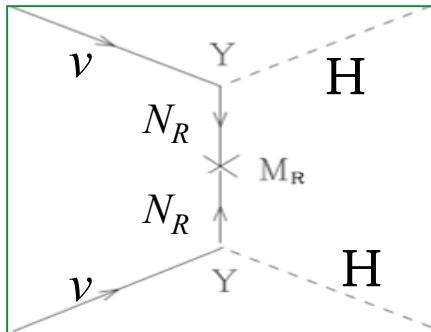
It's unique  $\rightarrow$  very special role of  $\nu$  masses:  
lowest-order effect of higher energy physics



This mass term violates lepton number  
 $\rightarrow$  Majorana neutrinos

# $\nu$ masses beyond the SM

## ★ Tree-level realizations

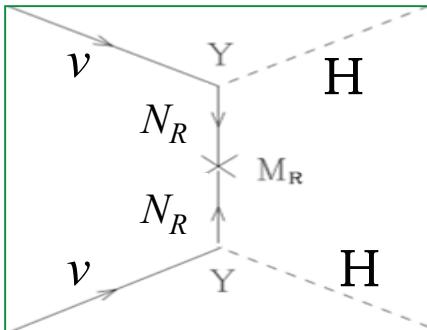


Heavy fermion singlet  $N_R$

(Type I See-Saw) Minkowski, Gell-Mann, Ramond, Slansky, Yanagida, Glashow, Mohapatra, Senjanovic

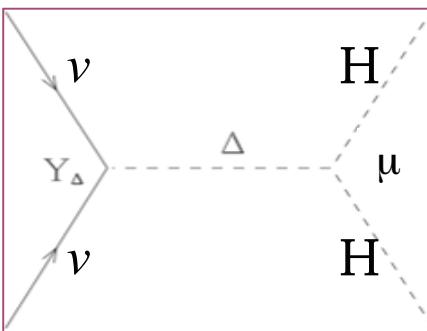
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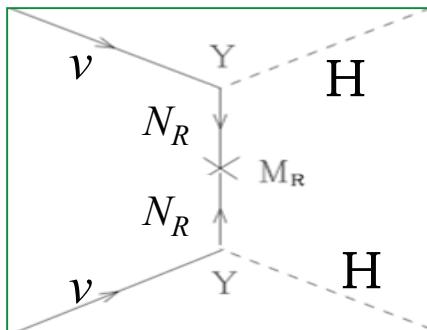


Heavy scalar triplet  $\Delta$

Magg, Wetterich, Lazarides, Shafi, Mohapatra, Senjanovic, Schechter, Valle

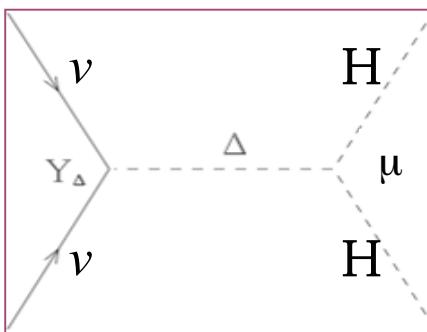
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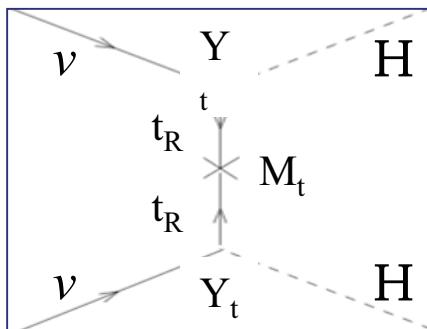
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Heavy fermion triplet  $t_R$

Ma, Roy, Senjanovic, Hambye et al., ...

## A general statement...

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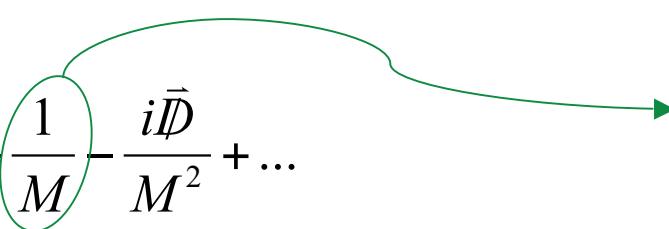
We have unitarity violation whenever we integrate out heavy fermions:

$$\frac{1}{i\bar{D} - M} = -\frac{1}{M} - \frac{i\bar{D}}{M^2} + \dots$$

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There's a  $\gamma^\mu$ : it connects fermions with the same chirality → correction to the kinetic terms

The propagator of a scalar field does not contain  $\gamma^\mu$  → if it generates neutrino mass, it cannot correct the kinetic term

# Low-energy theory

After EWSB, in the flavour basis:

$$L = \frac{1}{2} \left( i\bar{\nu}_\alpha \not{\partial} K_{\alpha\beta} \nu_\beta - \bar{\nu}_\alpha^c M_{\alpha\beta} \nu_\beta + h.c. \right) +$$



$M_{\alpha\beta} \rightarrow$  diagonalized  $\rightarrow$  unitary transformation

$K_{\alpha\beta} \rightarrow$  diagonalized and normalized  $\rightarrow$  unitary transf. + rescaling

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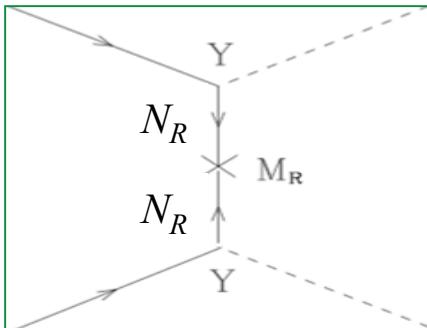
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*N* non-unitary

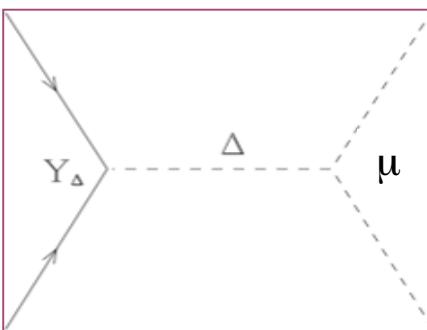
# $\nu$ masses beyond the SM

## ★ Tree-level realizations



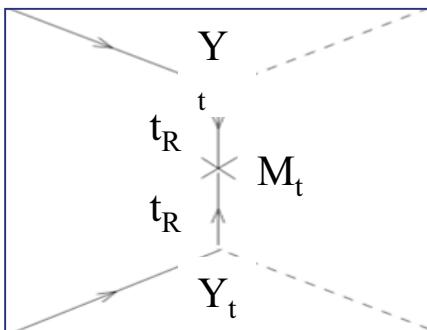
Heavy fermion singlet  $N_R$  (Seesaw I)

→ deviations from unitarity



Heavy scalar triplet  $\Delta$

→ no deviations from unitarity



Heavy fermion triplet  $t_R$

→ deviations from unitarity

# Non-unitarity from see-saw

$$L = L_{SM} + i \overline{N}_R \cancel{d} N_R - Y_\nu \overline{L} H N_R - M N_R N_R$$

Integrate out  $N_R$        $L^{eff} = L_{SM} + \frac{1}{M} L^{d=5} + \frac{1}{M^2} L^{d=6} + \dots$

$$YY^T/M (L L H H)$$

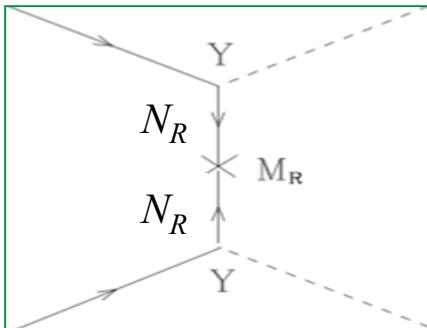
$$YY^+/M^2 (\overline{L} H) \cancel{d}(H L)$$

d=5 operator  
it gives mass to  $\nu$

d=6 operator  
it renormalises kinetic energy

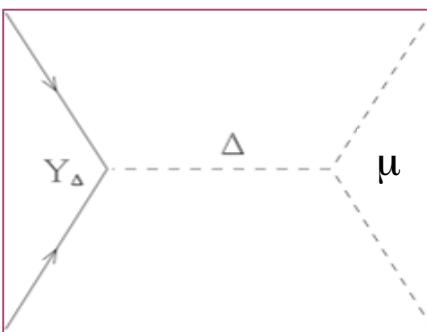
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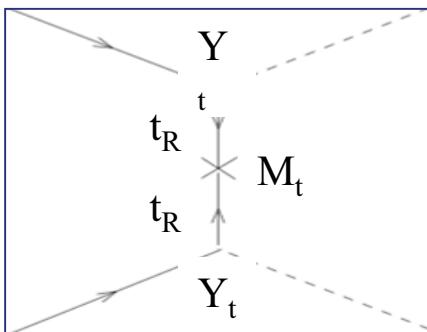
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→ deviations from unitarity  
Broncano, Gavela, Jenkins 02



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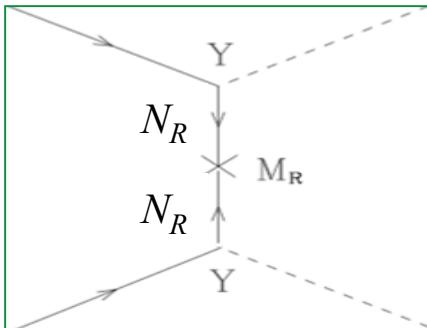


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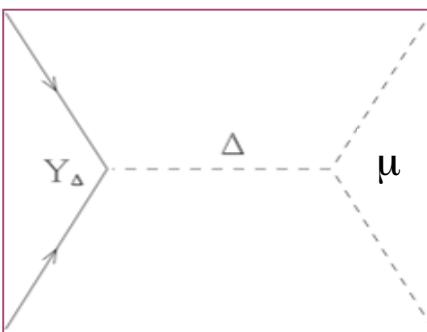
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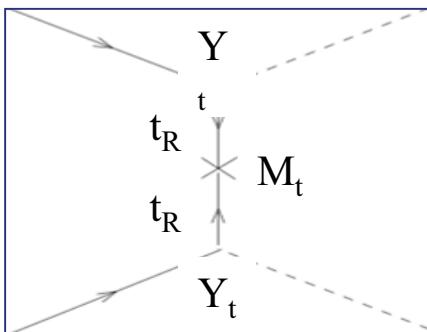
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Broncano, Gavela, Jenkins 02



Abada,  
Antusch,  
Biggio,  
Bonnet,  
Hambye,  
M.B.G.



## *In short*

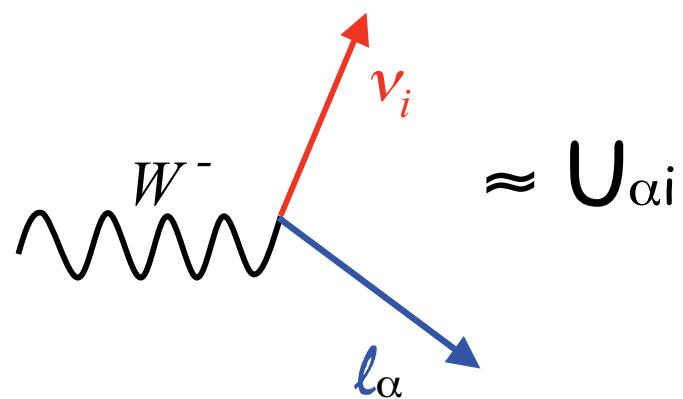
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- Unitarity violations arise in models for  $\nu$  masses with heavy fermions
- *Worthwhile to analyze neutrino data relaxing the hypothesis of unitarity of the mixing matrix*

Antusch, Biggio, Fernández-Martínez, López-Pavón, M.B.G. 06

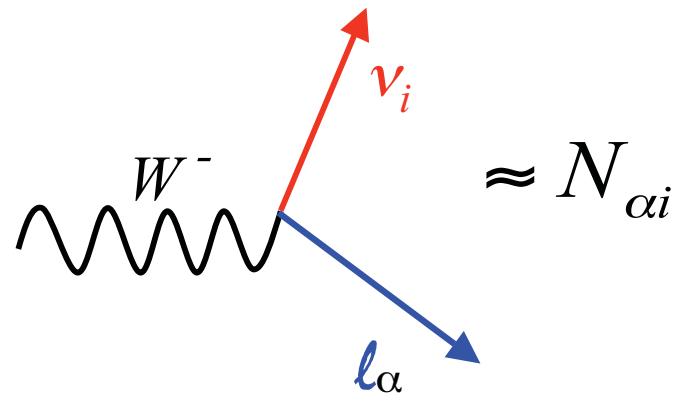
## The general idea.....

$$U = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12}e^{-i\delta} & c_{12}c_{23} - s_{23}s_{13}s_{12}e^{-i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{-i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha} \\ e^{i\beta} \\ 1 \end{pmatrix}$$



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$$N = \begin{pmatrix} N_{e1} & N_{e2} & N_{e3} \\ N_{\mu 1} & N_{\mu 2} & N_{\mu 3} \\ N_{\tau 1} & N_{\tau 2} & N_{\tau 3} \end{pmatrix}$$

**M(inimal) U(nitarity) V(olation) :**

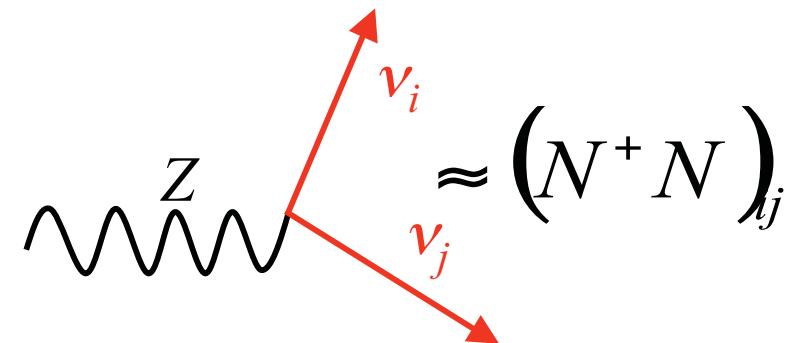
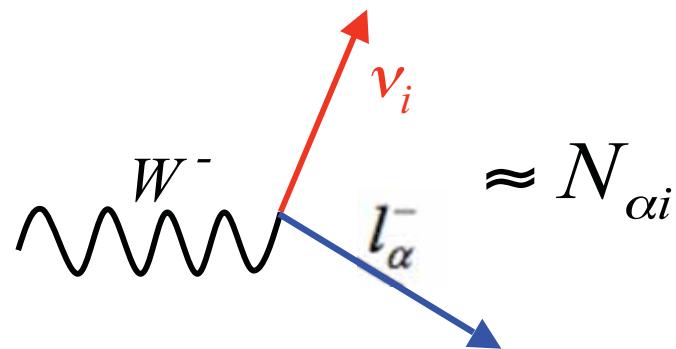
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$$L = i\bar{\nu}_i \not{d} \nu_i + \bar{\nu}_i m_{ii} \nu_i - \frac{g}{\sqrt{2}} \left( W_\mu^+ \bar{l}_\alpha \gamma^\mu P_L N_{\alpha i} \nu_i + h.c. \right) - \frac{g}{\cos \theta_W} \left( Z_\mu \bar{\nu}_i \gamma^\mu P_L (N^+ N)_{ij} \nu_j + h.c. \right) + \dots$$

with only 3 light  $\nu$

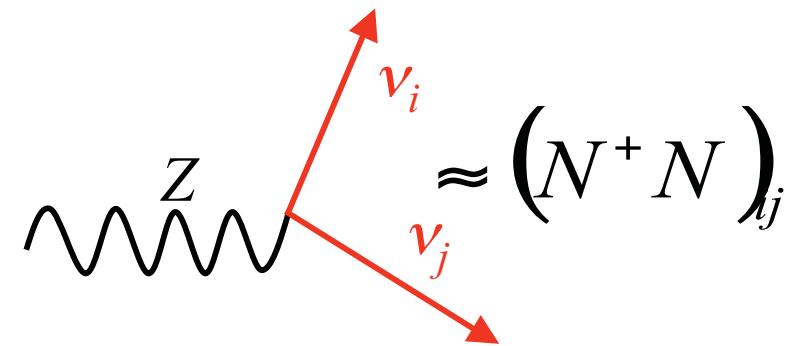
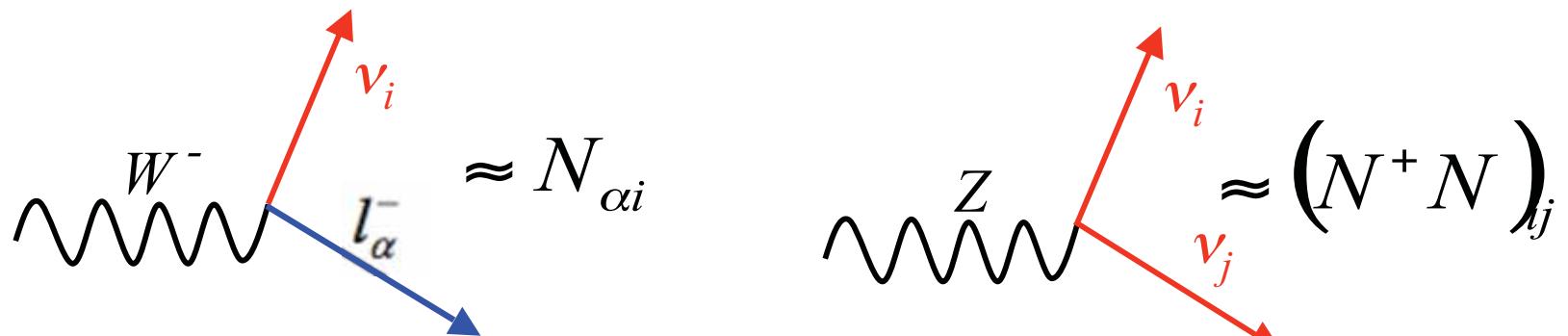
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... appear in the interactions



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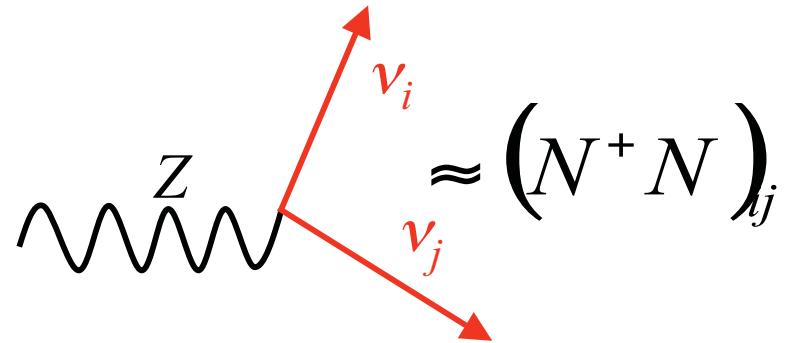
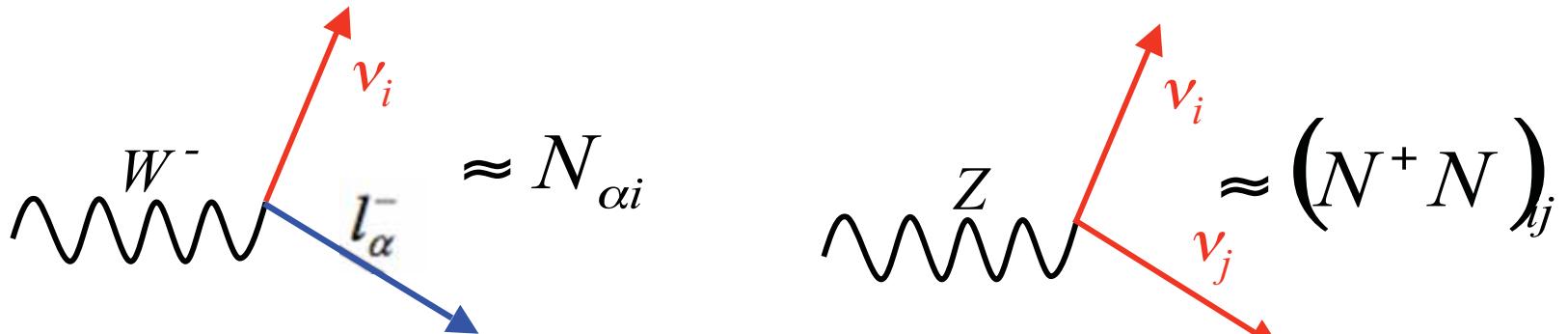
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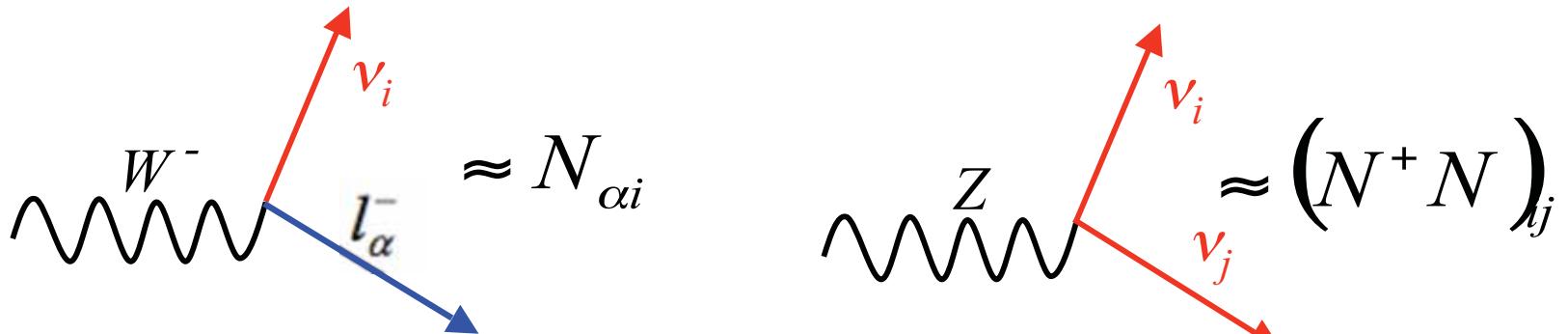
This affects weak decays...

$$\Gamma = \Gamma_{SM} \sum_i |N_{\alpha i}|^2 = \Gamma_{SM} (NN^+)_\alpha^\alpha$$

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... and oscillation probabilities...

$$P_{\alpha\beta}(E, L) = \frac{\left| \sum_i N_{\alpha i}^* e^{i P_i L} N_{\beta i} \right|^2}{\left( \langle N N^\dagger \rangle_{\alpha\alpha} \langle N N^\dagger \rangle_{\beta\beta} \right)}$$

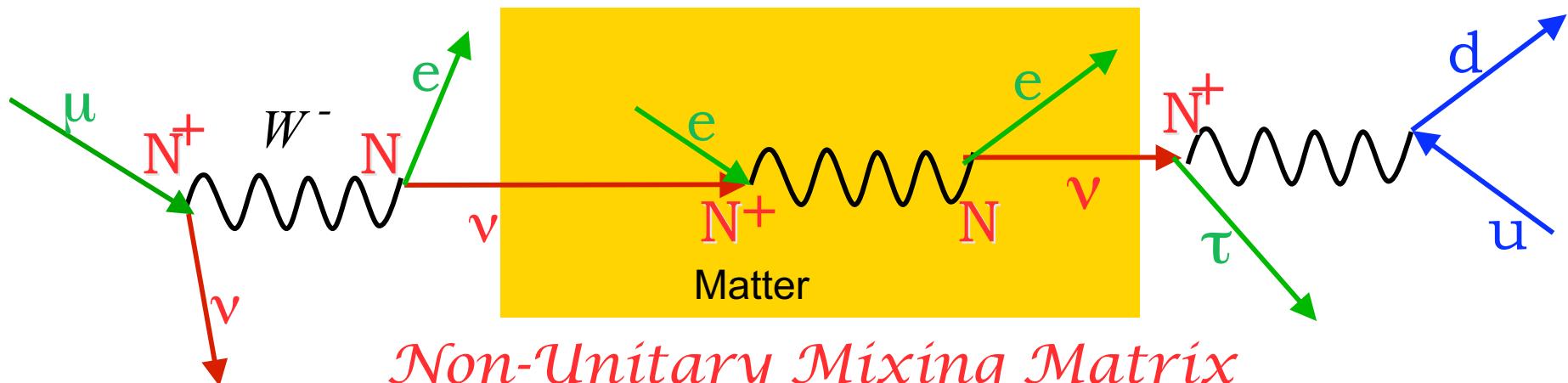
Zero-distance effect at near detectors:

$\rightarrow P(\nu_\alpha \rightarrow \nu_\beta; 0) \propto \left| \sum_i N_{\alpha i}^* N_{\beta i} \right|^2 \neq \delta_{\alpha\beta}$

$$P_{\alpha\beta}(E, L) = \frac{\left| \sum_i N_{\alpha i}^* e^{i P_i L} N_{\beta i} \right|^2}{\left( \bar{N} \bar{N}^\dagger \right)_{\alpha\alpha} \left( \bar{N} \bar{N}^\dagger \right)_{\beta\beta}}$$

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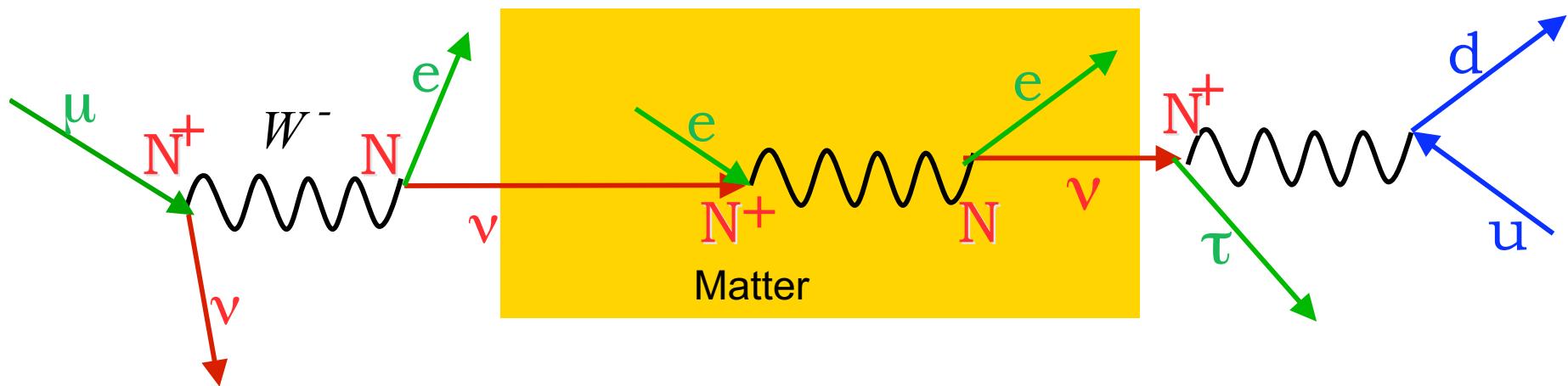
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In matter

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = N^* \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} (N^*)^{-1} + \begin{pmatrix} (V_{CC} - V_{NC}) \sum_i |N_{ei}|^2 & -V_{NC} \sqrt{\frac{\sum_i |N_{ei}|^2}{\sum_i |N_{ui}|^2}} \sum_i N_{ei}^* N_{ui} \\ (V_{CC} - V_{NC}) \sqrt{\frac{\sum_i |N_{ei}|^2}{\sum_i |N_{ui}|^2}} \sum_i N_{ei}^* N_{ui} & -V_{NC} \sum_i |N_{ui}|^2 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$N = \begin{pmatrix} N_{e1} & N_{e2} & N_{e3} \\ N_{\mu 1} & N_{\mu 2} & N_{\mu 3} \\ N_{\tau 1} & N_{\tau 2} & N_{\tau 3} \end{pmatrix}$$

# $N$ elements from oscillations: $e$ -row

CHOOZ       $P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx \left( |N_{e1}|^2 + |N_{e2}|^2 \right) + |N_{e3}|^4 + 2\left( |N_{e1}|^2 + |N_{e2}|^2 \right)|N_{e3}|^2 \cos(\Delta_{23})$

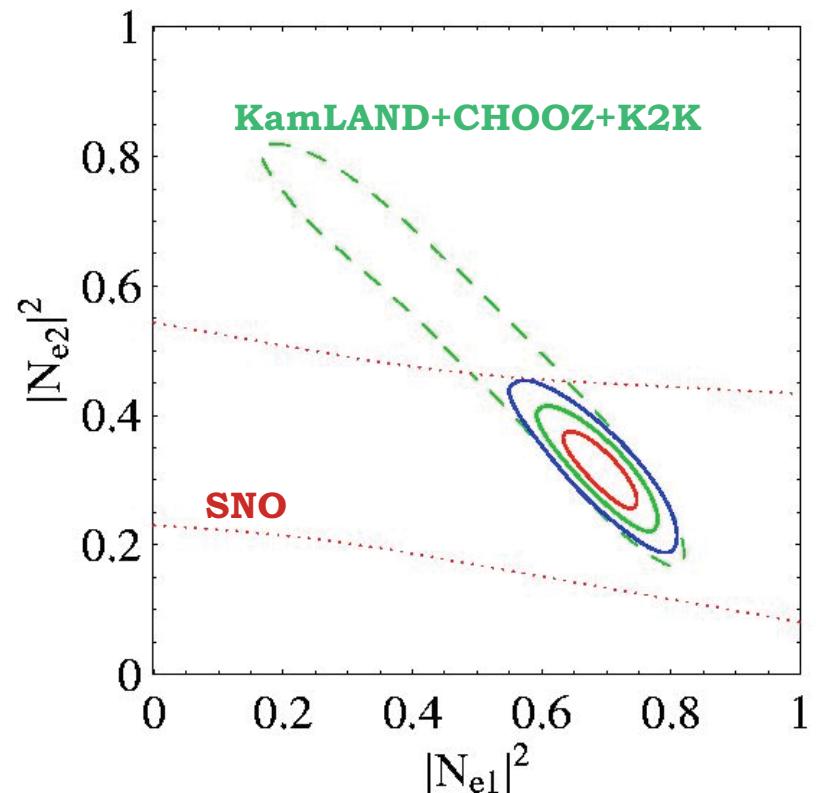
KamLAND:     $\hat{P}(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx |N_{e1}|^4 + |N_{e2}|^4 + |N_{e3}|^4 + 2|N_{e1}|^2|N_{e2}|^2 \cos(\Delta_{12})$

$$\Delta_{ij} = \Delta m_{ij}^2 L / 2E$$

SNO:

$$\hat{P}(\nu_e \rightarrow \nu_e) \approx 0.1|N_{e1}|^2 + 0.9|N_{e2}|^2$$

→ all  $|N_{ei}|^2$  determined



# $N$ elements from oscillations: $\mu$ -row

Atmospheric + K2K:  $\Delta_{12} \approx 0$

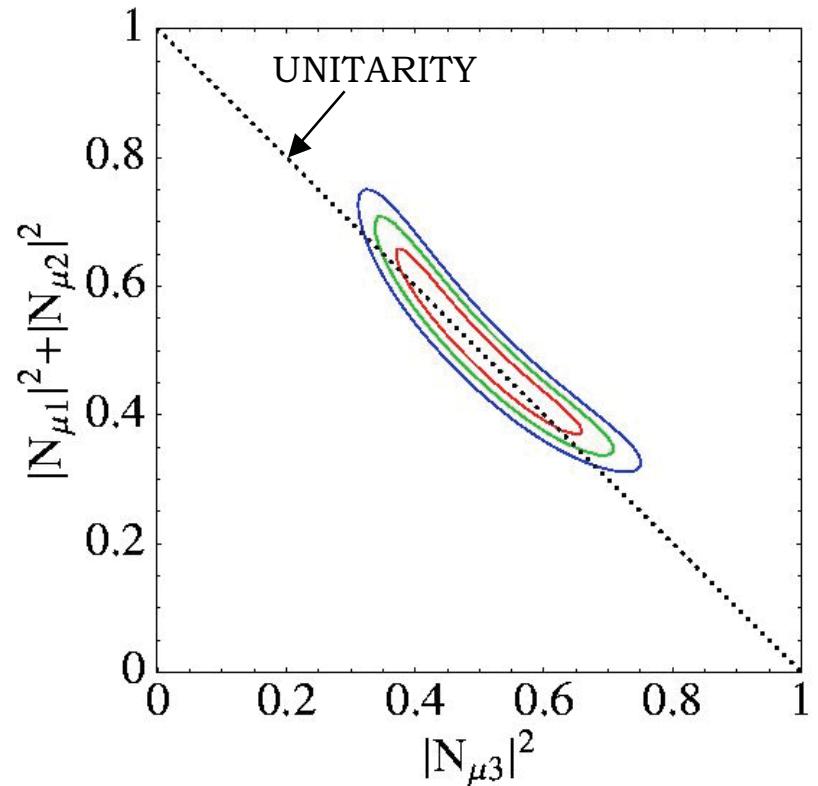
$$\hat{P}(\nu_\mu \rightarrow \nu_\mu) \approx \left( |N_{\mu 1}|^2 + |N_{\mu 2}|^2 \right) + |N_{\mu 3}|^4 + 2 \left( |N_{\mu 1}|^2 + |N_{\mu 2}|^2 \right) |N_{\mu 3}|^2 \cos(\Delta_{23})$$

1. Degeneracy

$$|N_{\mu 1}|^2 + |N_{\mu 2}|^2 \leftrightarrow |N_{\mu 3}|^2$$

2.  $|N_{\mu 1}|^2, |N_{\mu 2}|^2$

cannot be disentangled



# $N$ elements from oscillations only

without unitarity  
OSCILLATIONS  
**MUV**

$$|N| = \begin{cases} .76 - .89 & .45 - .66 & < .36 \\ [(|N_{\mu 1}|^2 + |N_{\mu 2}|^2)^{1/2} = 0.57-0.85] & .57 - .85 \\ ? & ? & ? \end{cases}$$

$3\sigma$

with unitarity  
OSCILLATIONS

$$|U| = \begin{cases} .79 - .89 & .47 - .61 & < .20 \\ .19 - .52 & .42 - .73 & .58 - .82 \\ .20 - .53 & .44 - .74 & .56 - .81 \end{cases}$$

M. C. Gonzalez Garcia hep-ph/0410030

# Unitarity constraints on $(NN^+)$ from:

## \* Near detectors...

- MINOS:  $(NN^\dagger)_{\mu\mu} = 1 \pm 0.05$
- BUGEY:  $(NN^\dagger)_{ee} = 1 \pm 0.04$
- NOMAD:  $(NN^\dagger)_{\mu\tau} < 0.09$      $(NN^\dagger)_{e\tau} < 0.013$
- KARMEN:  $(NN^\dagger)_{\mu e} < 0.05$

## \* Weak decays...

- W decays  $\rightarrow \frac{(NN^+)_{\alpha\alpha}}{\sqrt{(NN^+)_{ee}} \sqrt{(NN^+)_{\mu\mu}}}$
- Universality tests  $\rightarrow \frac{(NN^+)_{\alpha\alpha}}{(NN^+)_{\beta\beta}}$
- Invisible Z  $\rightarrow \frac{\sum_{ij} (N^+ N)_{ij}}{\sqrt{(NN^+)_{ee}} \sqrt{(NN^+)_{\mu\mu}}}$
- Rare leptons decays  $\rightarrow \frac{|(NN^+)_{\beta\alpha}|^2}{(NN^+)_{\alpha\alpha} (NN^+)_{\beta\beta}}$

## → Limits on $NN^+$

Global fit

$$90\% \text{ cl} \quad |NN^+| \approx \begin{pmatrix} 1.002 \pm 0.005 & < 7.2 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.2 \cdot 10^{-5} & 1.003 \pm 0.005 & < 1.3 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.3 \cdot 10^{-2} & 1.003 \pm 0.005 \end{pmatrix}$$

→ N is unitary at the % level

# $N$ elements from oscillations & decays

**MUV**  
without unitarity  
**OSCILLATIONS**  
+**DECAYS**

$$|N| = \begin{pmatrix} .75 - .89 & .46 - .66 & <.20 \\ .19 - .55 & .41 - .73 & .57 - .82 \\ .10 - .57 & .32 - .76 & .54 - .84 \end{pmatrix}$$

$3\sigma$

with unitarity  
**OSCILLATIONS**

$$|U| = \begin{pmatrix} .79 - .88 & .47 - .61 & <.20 \\ .19 - .52 & .42 - .73 & .58 - .82 \\ .20 - .53 & .44 - .74 & .56 - .81 \end{pmatrix}$$

# In the future...

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## TESTS OF UNITARITY (90%CL)



### Rare leptons decays (present)

- $\mu \rightarrow e\gamma$   $|\sum_i N_{ei} N_{\mu i}^*|^2 < 7.2 \cdot 10^{-5}$
- $\tau \rightarrow e\gamma$   $|\sum_i N_{ei} N_{\tau i}^*|^2 < 0.016$
- $\tau \rightarrow \mu\gamma$   $|\sum_i N_{\mu i} N_{\tau i}^*|^2 < 0.013$

### ZERO-DISTANCE EFFECT Near detector at a $\nu$ factory

- $\nu_e \rightarrow \nu_\mu$   $|\sum_i N_{ei} N_{\mu i}^*|^2 < 2.3 \cdot 10^{-4}$
- $\nu_e \rightarrow \nu_\tau$   $|\sum_i N_{ei} N_{\tau i}^*|^2 < 2.9 \cdot 10^{-3}$
- $\nu_\mu \rightarrow \nu_\tau$   $|\sum_i N_{\mu i} N_{\tau i}^*|^2 < 2.6 \cdot 10^{-3}$

# In the future...

## TESTS OF UNITARITY (90%CL)



### Rare leptons decays (present)

- $\mu \rightarrow e\gamma$   $|\sum_i N_{ei} N_{\mu i}^*|^2 < 7.2 \cdot 10^{-5}$
- $\tau \rightarrow e\gamma$   $|\sum_i N_{ei} N_{\tau i}^*|^2 < 0.016$
- $\tau \rightarrow \mu\gamma$   $|\sum_i N_{\mu i} N_{\tau i}^*|^2 < 0.013$

### ZERO-DISTANCE EFFECT Near detector at a $\nu$ factory

- $\nu_e \rightarrow \nu_\mu$   $|\sum_i N_{ei} N_{\mu i}^*|^2 < 2.3 \cdot 10^{-4}$
- $\nu_e \rightarrow \nu_\tau$   $|\sum_i N_{ei} N_{\tau i}^*|^2 < 2.9 \cdot 10^{-3}$
- $\nu_\mu \rightarrow \nu_\tau$   $|\sum_i N_{\mu i} N_{\tau i}^*|^2 < 2.6 \cdot 10^{-3}$

O  
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like

# Can we measure the phases of N ?

E. Fdez-Martinez, J.Lopez, O. Yasuda, M.B.G.

If we parametrize  $N = (1 + \varepsilon) \cdot U$  with  $U \approx U_{PMNS}$

and

$$\varepsilon = \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix}$$

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If  $L/E$  small

$$P_{\alpha\beta} = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right) - 2 \operatorname{Im}(\varepsilon_{\alpha\beta}) \sin(2\theta) \sin\left(\frac{\Delta m^2 L}{2E}\right) + 4|\varepsilon_{\alpha\beta}|^2$$

SM      CP violating interference      Zero dist. effect

→ New CP-violation signals  
even in the two-family approximation

i.e.  $P(\nu_\mu \rightarrow \nu_\tau) \neq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$

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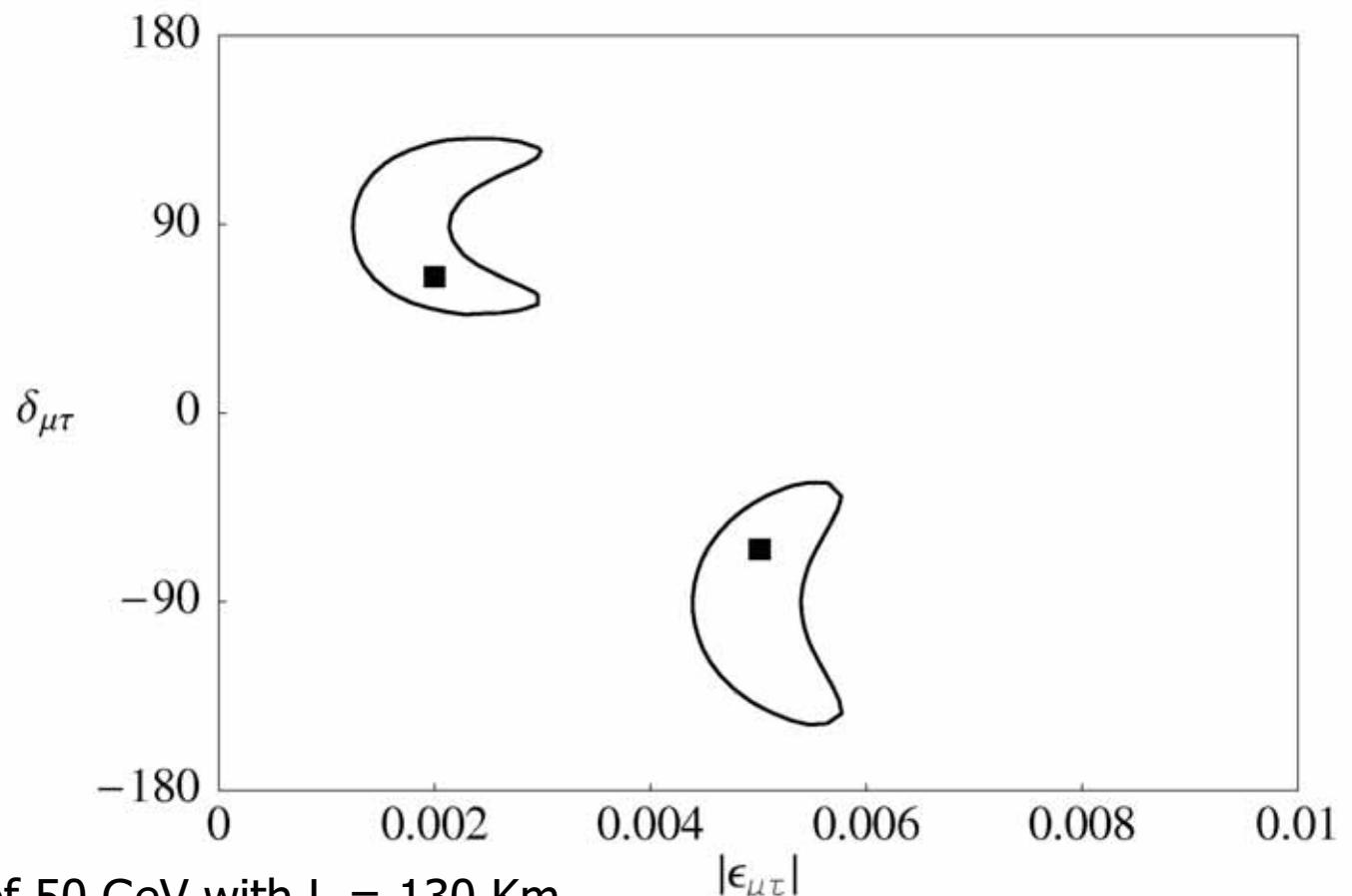
i.e.  $P(\nu_\mu \rightarrow \nu_\tau) \neq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$

→ Increased sensitivity to the moduli  $|N|$

In  $P_{\mu\tau}$  there is no  $\sin \theta_{13}$  or  $\Delta_{12}$  suppression:

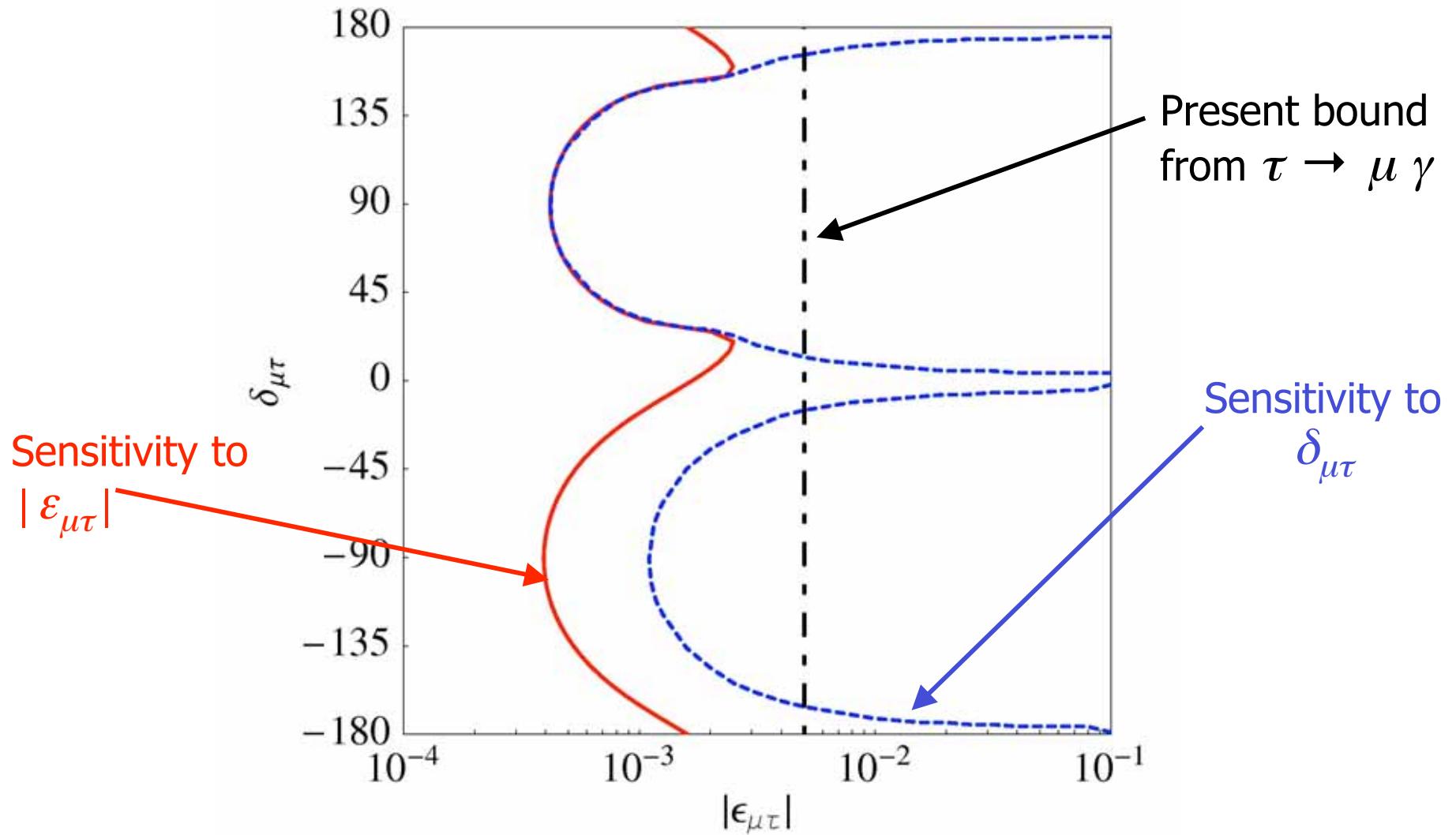
$$P_{\mu\tau} - P_{\bar{\mu}\bar{\tau}} = -4 \operatorname{Im}(\epsilon_{\mu\tau}) \sin(2\theta_{23}) \sin\left(\frac{\Delta m_{23}^2 L}{2E}\right)$$

The CP phase  $\delta_{\mu\tau}$   
can be measured



At a Neutrino Factory of 50 GeV with  $L = 130$  Km

# Measuring non-unitary phases

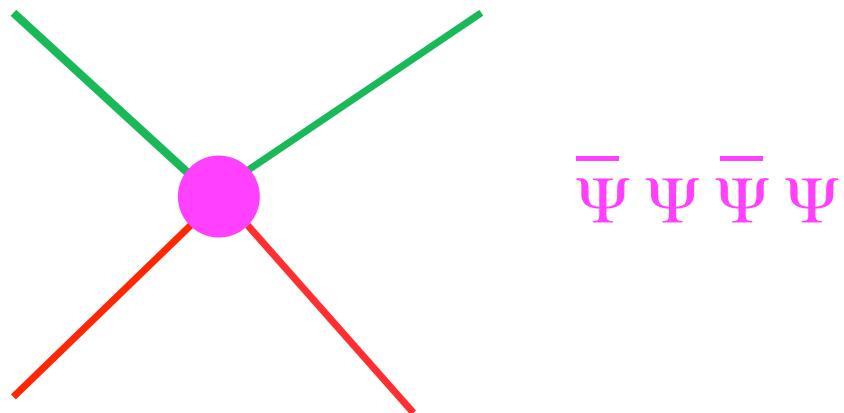


For non-trivial  $\delta_{\mu\tau}$ , one order of magnitude improvement for  $|N|$

Our analysis will also apply to ``non-standard'' or ``exotic'' neutrino interactions.

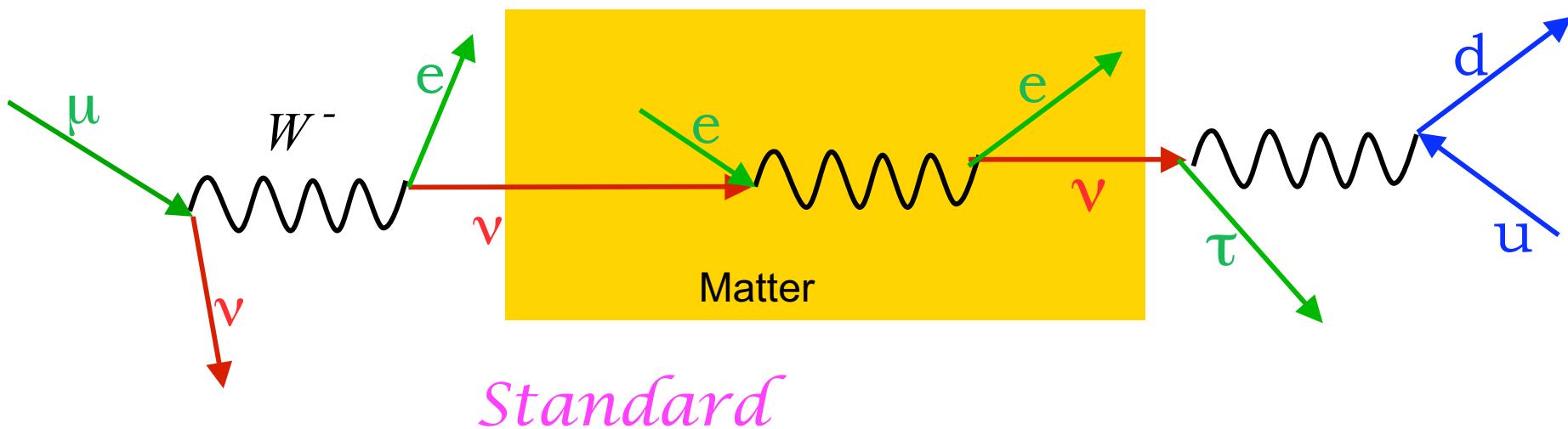
Grossman, Gonzalez-Garcia et al., Huber et al., Kitazawa et al., Davidson et al. Blennow et al...)

They add 4-fermion exotic operators to production  
or detection  
or propagation in matter



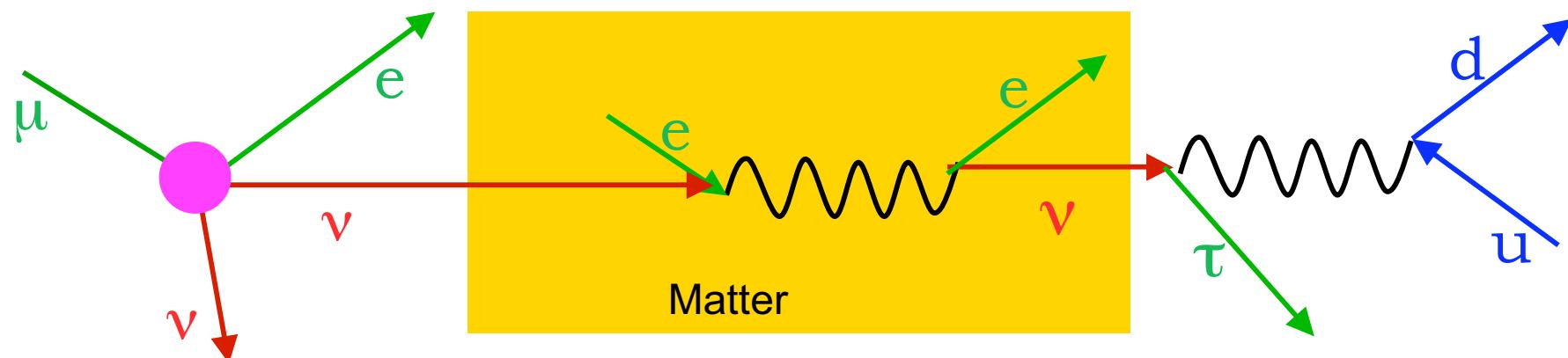
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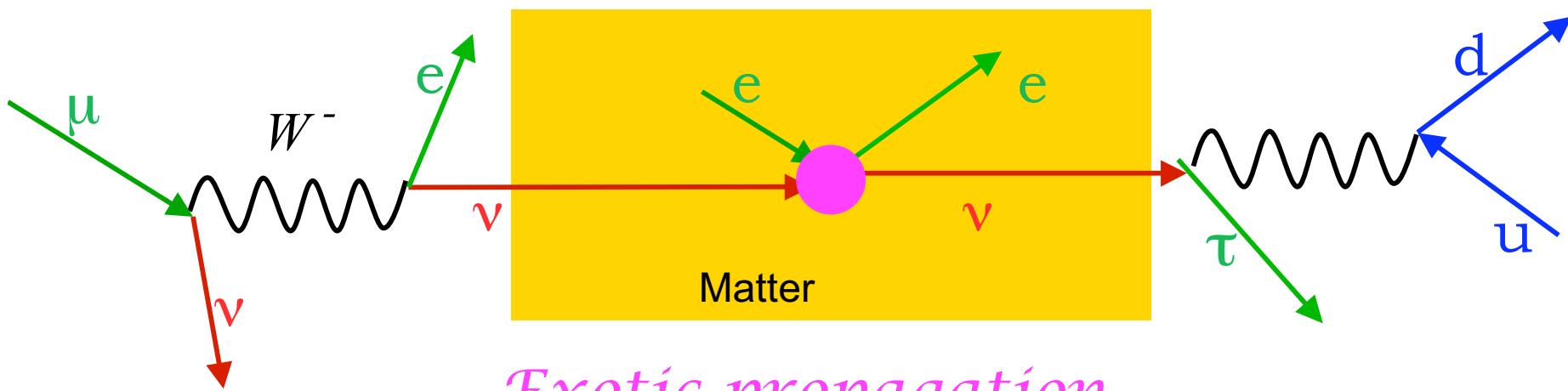
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*Exotic production*

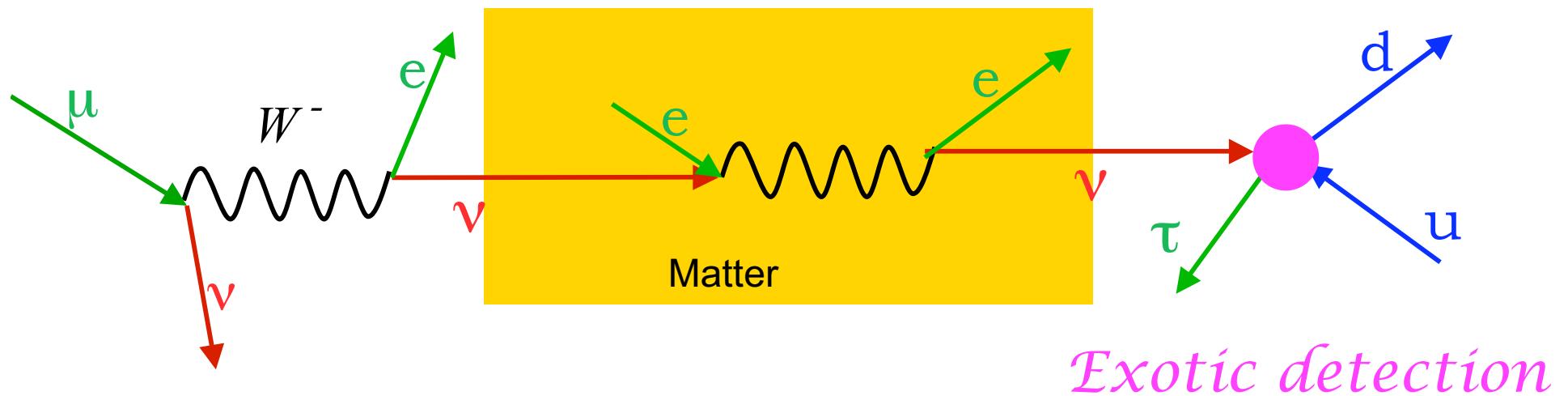
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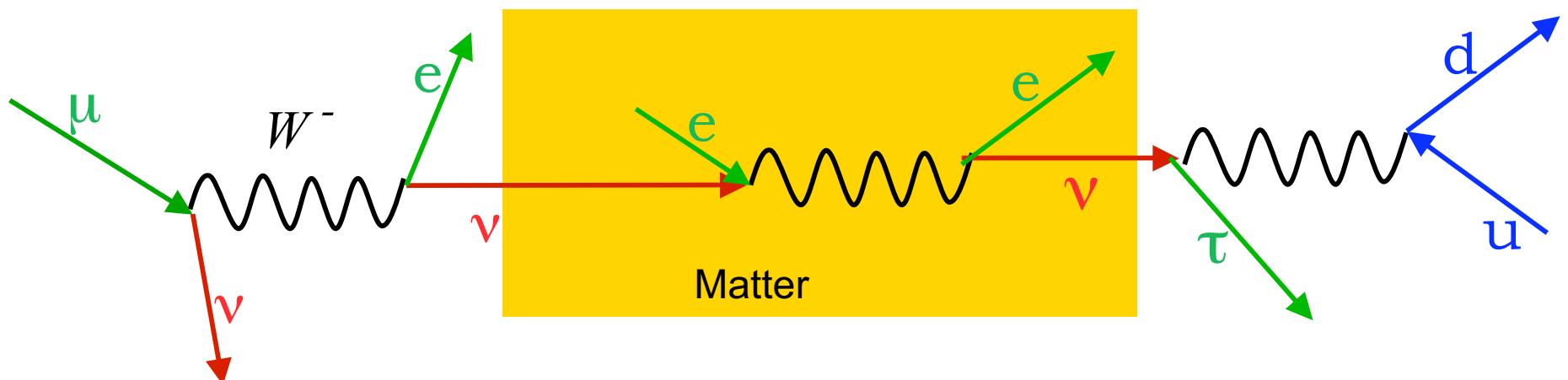
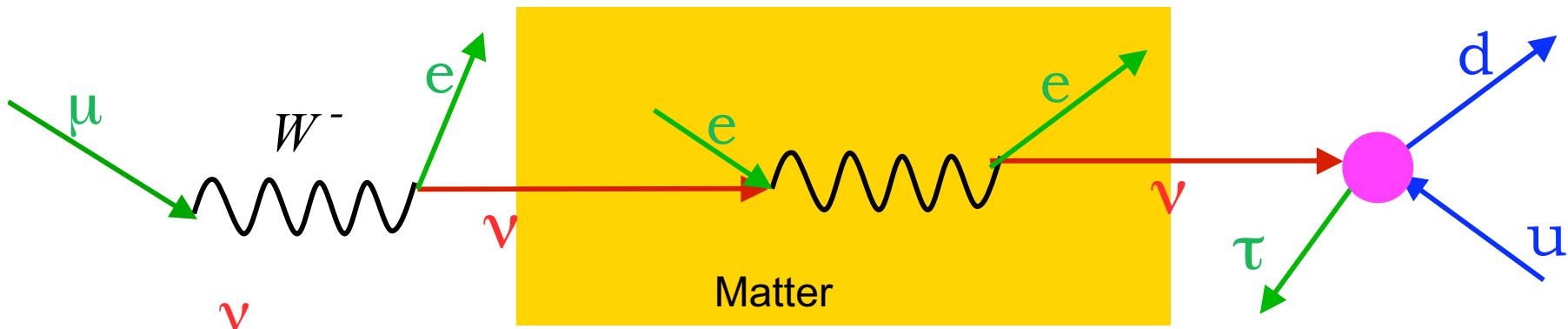
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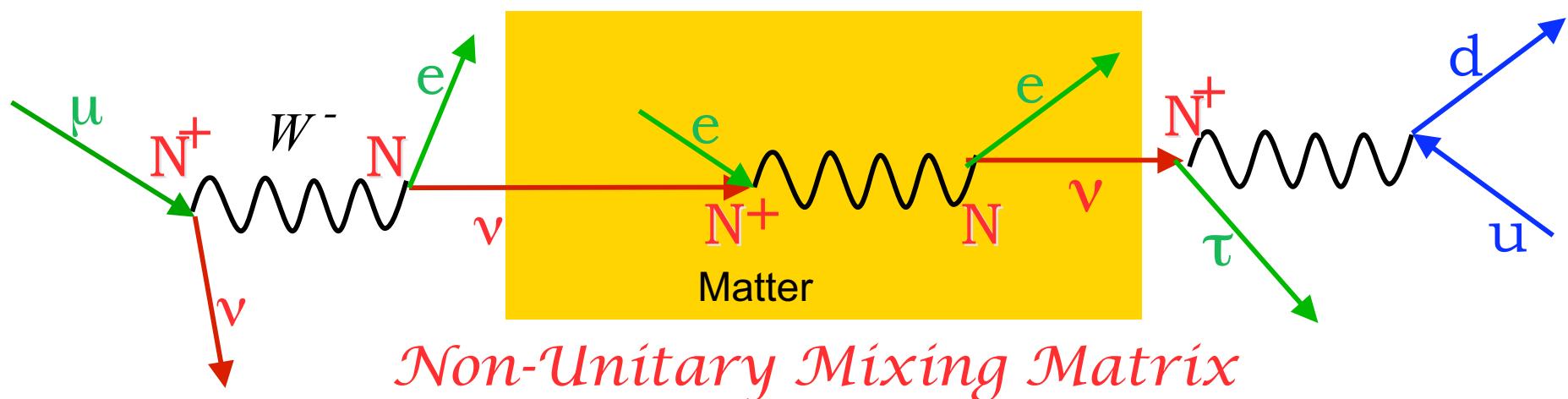
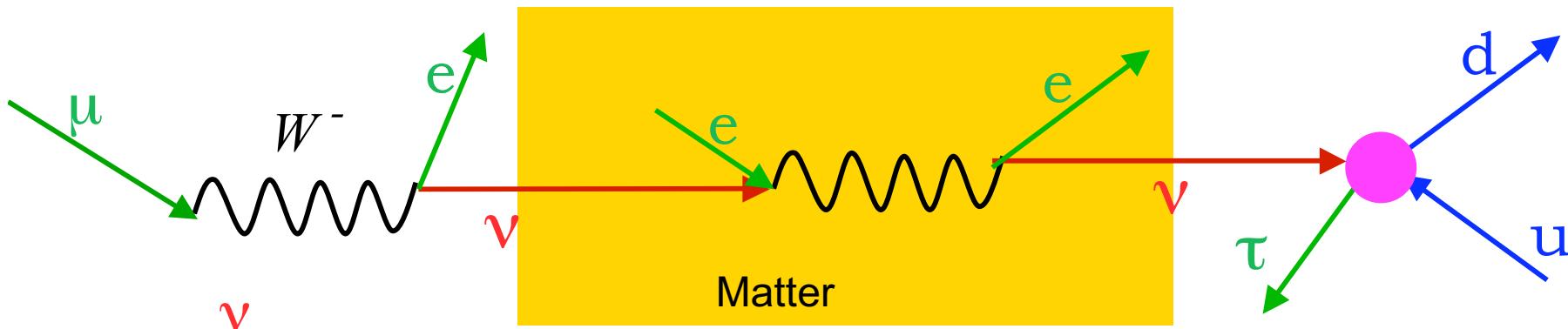
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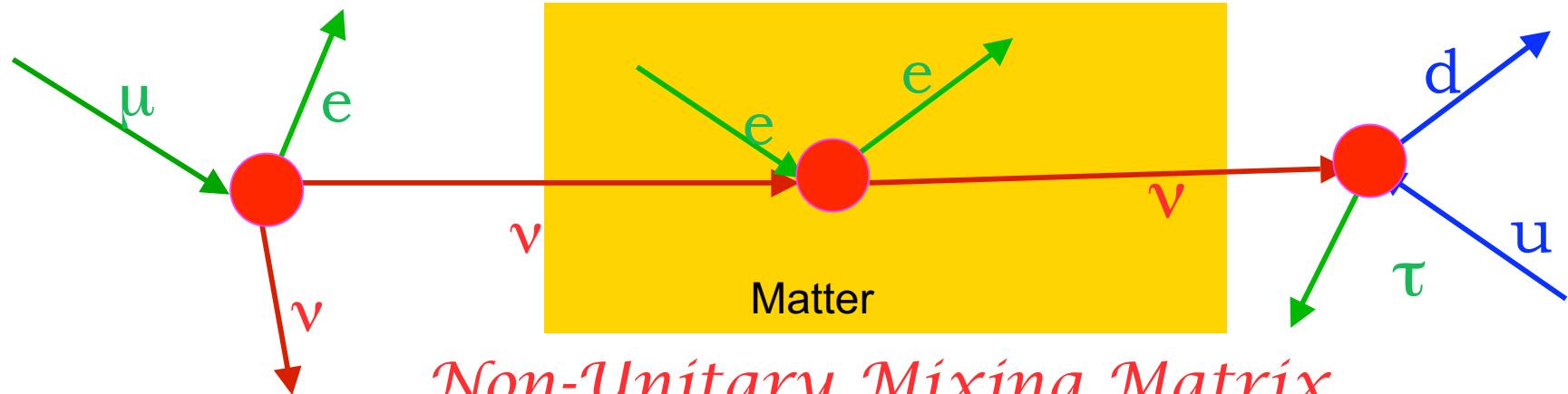
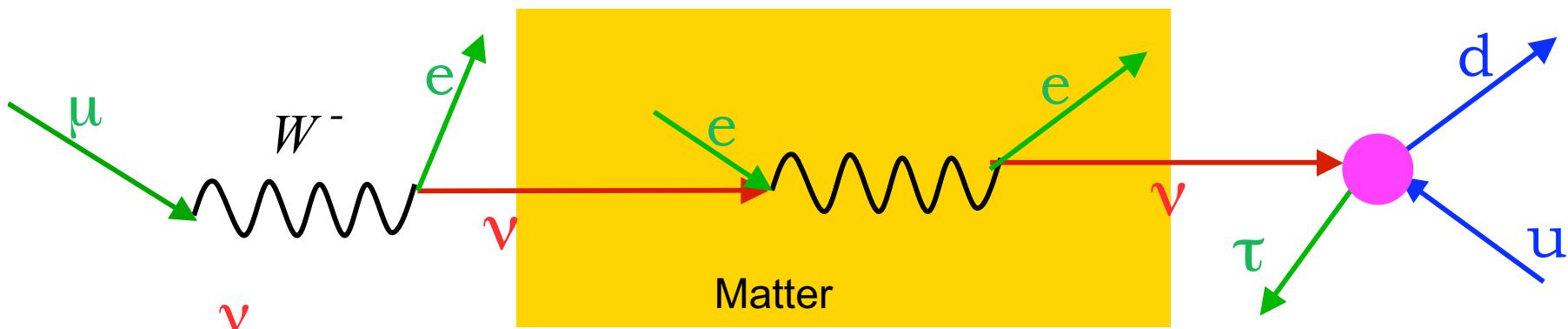
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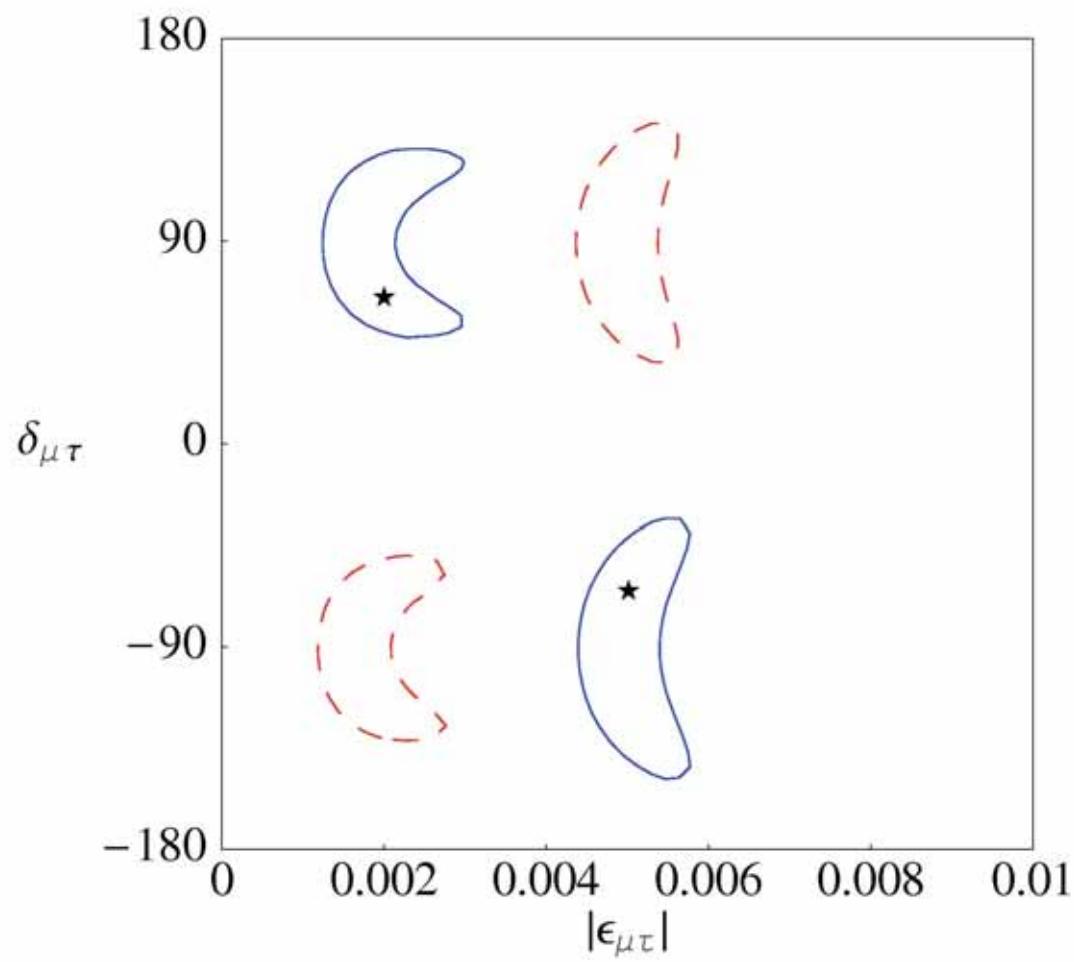


# Conclusions

- A non-unitary mixing matrix is characteristic of models of  $\nu$  mass involving heavy fermions.
- Analyze neutrino data without assuming unitarity.  
We developed a formalism for it and started the first analyses.
- $\nu_\mu - \nu_\tau$  CP-asymmetry is a clean probe of the new phases.
- Our results also apply to *non-standard* or *exotic*  $\nu$  interactions.
- Non-unitary effects in simplest models are too small for nowadays detection, but not in extensions/others: i.e., models with  $M \sim \text{TeV}$ .
  - > keep tracking them in the future.  
They are excellent signals of new physics.

# Back-up slides

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# Measuring unitarity deviations

---

The bounds on

$$|NN^\dagger| = |(1 + \varepsilon)^2| \approx |1 + 2\varepsilon|$$

Also apply to  $\varepsilon$

$$|\varepsilon| \approx \begin{pmatrix} < 2.5 \cdot 10^{-3} & < 3.6 \cdot 10^{-5} & < 8.0 \cdot 10^{-3} \\ < 3.6 \cdot 10^{-5} & < 2.5 \cdot 10^{-3} & < 5.0 \cdot 10^{-3} \\ < 8.0 \cdot 10^{-3} & < 5.0 \cdot 10^{-3} & < 2.5 \cdot 10^{-3} \end{pmatrix}$$

The constraints on  $\varepsilon_{e\mu}$  from  $\mu \rightarrow e \gamma$  are very strong

We will study the sensitivity to the CP violating terms

$\varepsilon_{e\tau}$  and  $\varepsilon_{\mu\tau}$  in  $P_{e\tau}$  and  $P_{\mu\tau}$

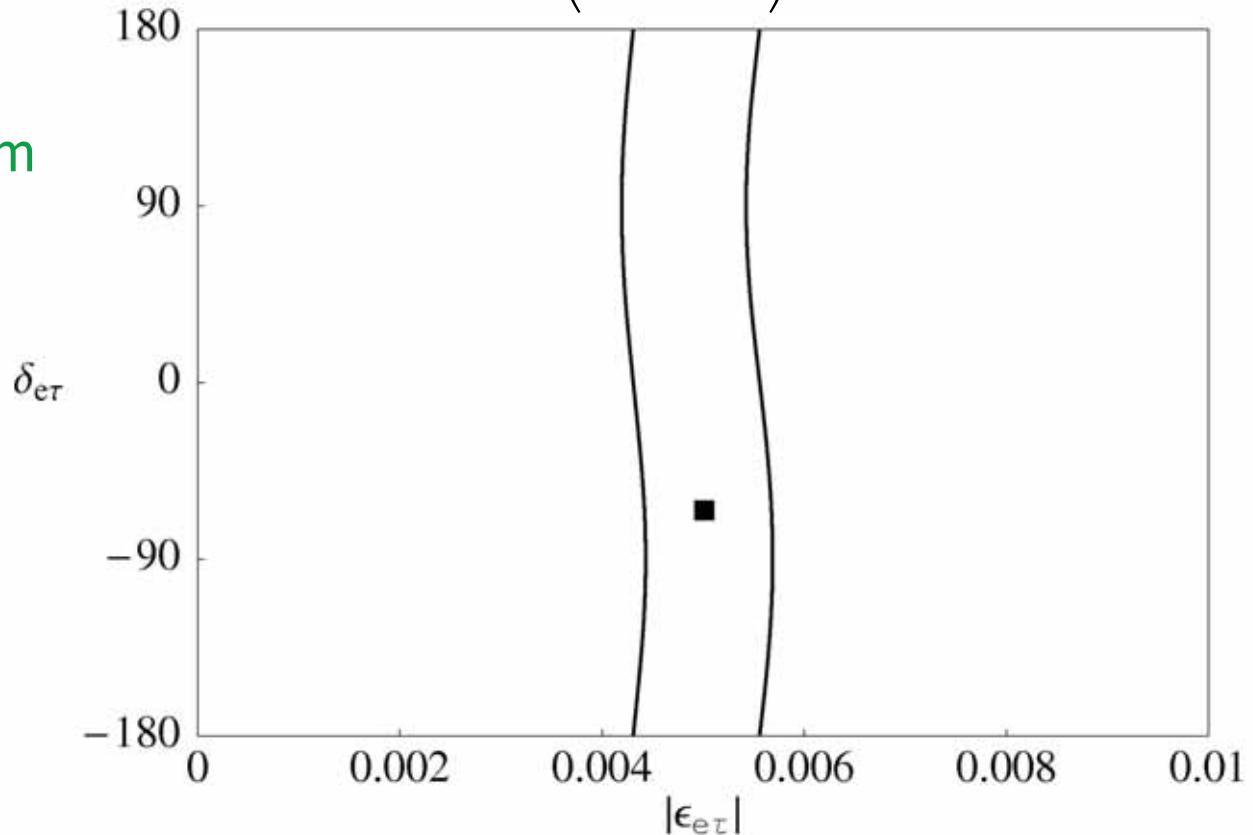
# Measuring unitarity deviations

In  $P_{e\tau}$  the CP violating term is suppressed by

$$\sin \theta_{13} \text{ or } \Delta_{12} \text{ apart from } |\varepsilon_{e\tau}| \sin\left(\frac{\Delta m_{23}^2 L}{2E}\right)$$

The zero distance term  
in  $|\varepsilon_{e\tau}|^2$  dominates

No sensitivity to the  
CP phase  $\delta_{e\tau}$



# Number of events

---

$$n_{ev} \sim \int dE \frac{d\Phi_\alpha(E)}{dE} P_{\alpha\beta}(E, L) \sigma_\beta(E) \varepsilon(E)$$

$\nu$  produced and detected in CC

$$\left\{ \begin{array}{l} \frac{d\Phi_\alpha}{dE} \sim \frac{d\Phi_\alpha^{SM}}{dE} (NN^+)_{\alpha\alpha} \\ \sigma_\beta \sim \sigma_\beta^{SM} (NN^+)_{\beta\beta} \end{array} \right.$$

$$n_{ev} \sim \int dE \underbrace{\frac{d\Phi_\alpha^{SM}(E)}{dE} (NN^+)_{\alpha\alpha}}_{P_{\alpha\beta}(E, L)} P_{\alpha\beta}(E, L) (NN^+)_{\beta\beta} \sigma_\beta^{SM}(E) \varepsilon(E)$$

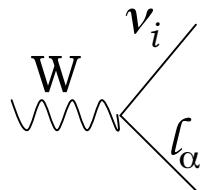
Exceptions:

- measured flux
- leptonic production mechanism
- detection via NC

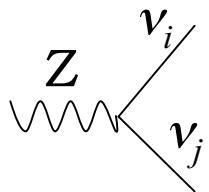
$$\hat{P}_{\alpha\beta}(E, L) = \left| \sum_i N_{\alpha i}^* e^{i P_i L} N_{\beta i} \right|^2$$

# $(NN^\dagger)$ from decays: $G_F$

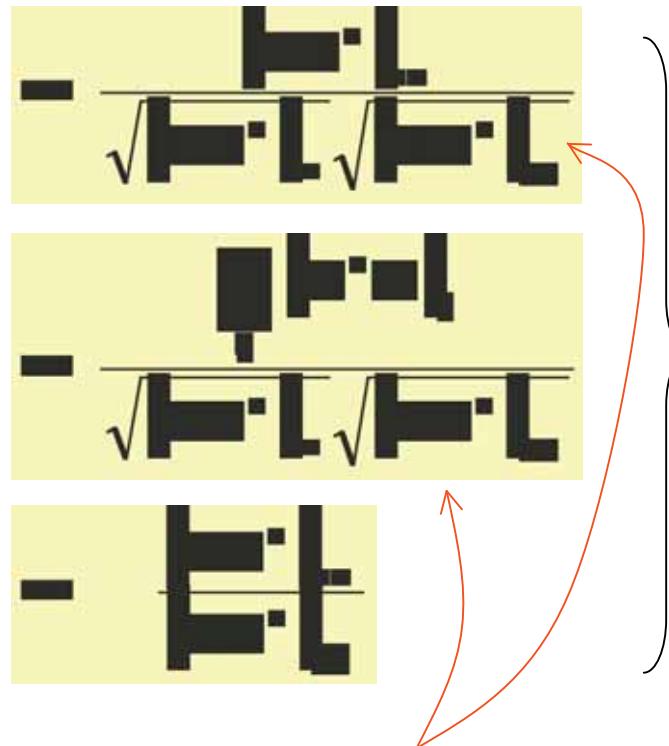
- W decays



- Invisible Z

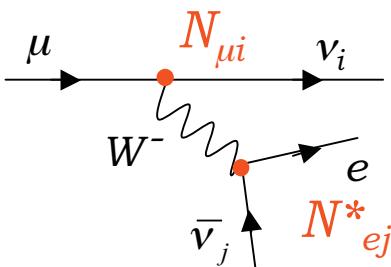


- Universality tests



Info on  
 $(NN^\dagger)_{aa}$

$G_F$  is measured in  $\mu$ -decay

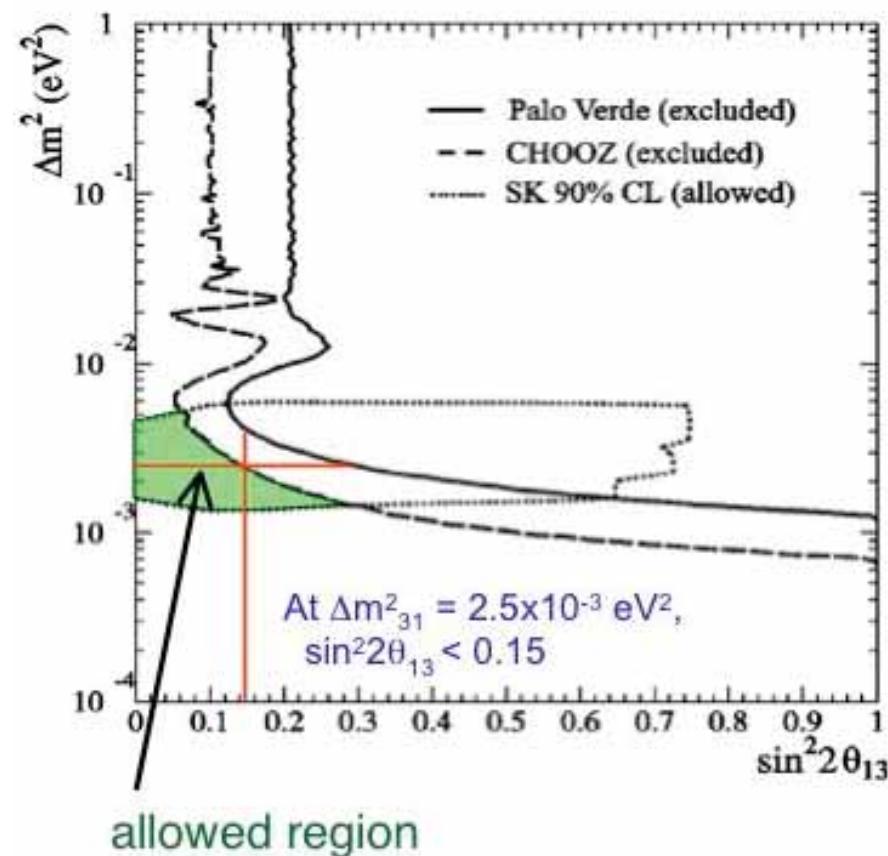


$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} \sum_i |N_{\mu i}|^2 \sum_j |N_{ej}|^2$$



# CHOOZ

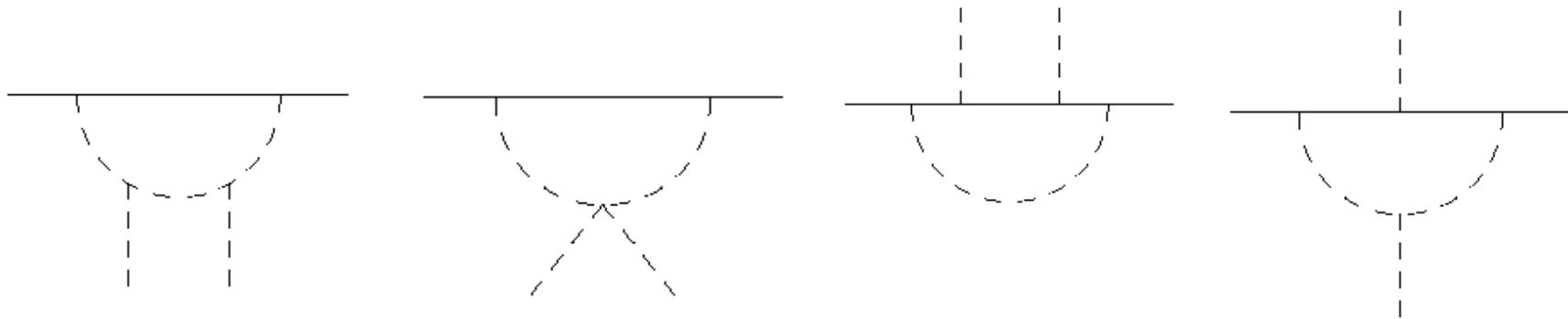
## Direct search



# $\nu$ masses beyond the SM

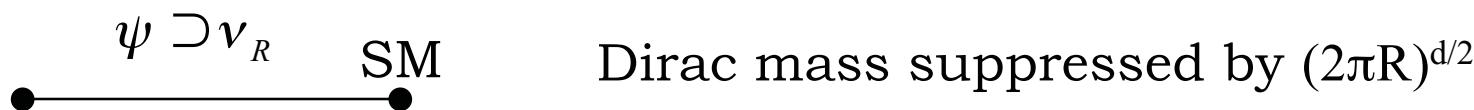
## ★ Other realizations

- radiative mechanisms: ex.) 1 loop:



Frigerio

- SUSY models with R-parity violation
- Models with large extra dimensions: i.e.,  $\nu_R$  are Kaluza-Klein replicas



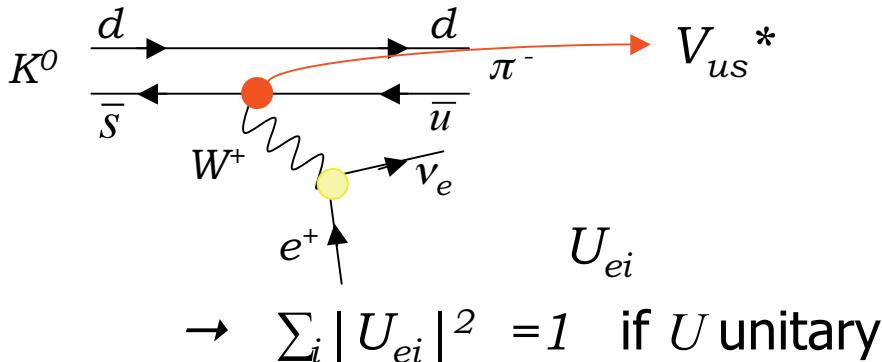
- ...

# Unitarity in the quark sector

Quarks are detected in the final state

→ we can directly measure  $|V_{ab}|$

ex:  $|V_{us}|$  from  $K^0 \rightarrow \pi^- e^+ \nu_e$



$$\rightarrow \sum_i |U_{ei}|^2 = 1 \text{ if } U \text{ unitary}$$

With  $V_{ab}$  we check unitarity conditions:

$$\text{ex: } |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0008 \pm 0.0011$$

→ Measurements of  $V_{CKM}$  elements relies on  $U_{PMNS}$  unitarity

- decays → only  $(NN^\dagger)$  and  $(N^\dagger N)$

With leptons:

- $N$  elements → we need oscillations

- to study the unitarity of  $N$ : no assumptions on  $V_{CKM}$