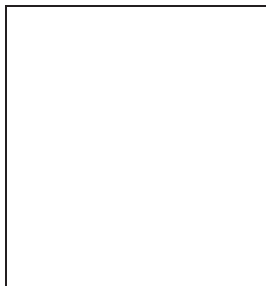


Natural SUSY Dark Matter: A Window on the GUT Scale

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One of the key motivations for supersymmetry is that it provides a natural candidate for dark matter. For a long time the density of this candidate particle fell within cosmological bounds across much of the SUSY parameter space. However with the precision results of WMAP, it has become apparent that the majority of the SUSY parameter space no longer fits the observed relic density. This has given rise to claims that supersymmetry no longer provides a natural explanation of dark matter. We address this claim by quantifying the degree of fine-tuning required for the different dark matter regions. We find that the dark matter regions vary widely in the degree of tuning required. This degree of tuning can then be used to provide valuable insights into the structure of SUSY breaking at the GUT scale.

1 Introduction

Supersymmetry at the TeV scale is one of the most compelling candidates for physics beyond the Standard Model (SM). A primary motivation for supersymmetry is that it removes the need to fine-tune the bare Higgs mass. It also naturally provides a candidate for cold dark matter. If we are to avoid fast proton decay, we must introduce a symmetry that constrains the interactions of particles with their supersymmetric partners. The most common form of this symmetry is R-parity. This forbids the decay of a single superpartner into purely SM matter. One result of this is that the lightest superparticle (LSP) is absolutely stable. If SM matter and superparticles were in thermal equilibrium in the early universe, the cooling universe would leave behind a relic density of superparticles.

This has given rise to many claims that SUSY naturally accounts for dark matter. However, having a candidate particle is one thing whereas naturally accounting for the observed relic density is quite another. In fact as WMAP has improved the constraints on the relic density the regions of the SUSY parameter space that fit the observed relic density have begun to look

very slender. This has led to recent claims that low energy supersymmetry requires significant fine-tuning to fit the data that others claim it accounts for ‘naturally’.

One could ignore such a war of words over what is or is not natural. However SUSY derives a significant portion of its motivation from questions of tuning and naturalness. Therefore this question deserves to be taken seriously. Here we present a quantitative study of the fine-tuning required to access the different dark matter regions of the Minimal Supersymmetric Standard Model (MSSM). We discuss the implications of these different degrees of tuning for the MSSM. Finally, we highlight how considerations of tuning allow us to compare GUT scale models of SUSY breaking from LHC data.

2 Fine-tuning and Dark Matter

To quantitatively study fine-tuning we need a measure. The fine-tuning required for electroweak symmetry breaking has a long history of quantitative study. We follow Ellis and Olive¹ in using an analogous measure to study the fine-tuning of dark matter^a:

$$\Delta_a^\Omega = \left| \frac{\partial \ln(\Omega_{CDM} h^2)}{\partial \ln(a)} \right| \quad (1)$$

where $\{a\}$ are the free parameters of the theory. This provides a measure of the sensitivity of the dark matter relic density to the inputs. For example, if $\Delta_a^\Omega = 10$, a 1% variation in a would result in a 10% variation in $\Omega_{CDM} h^2$.^b

3 The Constrained Minimal Supersymmetric Standard Model (CMSSM)

The MSSM is notorious for having over 100 free parameters. However, many of these parameters are already constrained by experiment to be zero. Furthermore, the parameters are only free if we leave the mechanism of SUSY breaking entirely unspecified. In a realistic theory, we would expect all the MSSM parameters to be set in terms of a smaller set of more fundamental parameters. In the absence of a specific theory of SUSY breaking we can still make progress. The most frequently studied SUSY model is the CMSSM with the parameters:

$$a_{CMSSM} \in \{m_0, m_{1/2}, A_0, \tan\beta \text{ and } \text{sign}(\mu)\} \quad (2)$$

Here m_0 is the common soft mass of all the scalar particles, $m_{1/2}$ is the common mass for all the gauginos, A_0 sets the third family trilinear couplings, $\tan\beta$ is the ratio of the two Higgs vevs and μ is a bilinear Higgs mass term.

In Fig. 1 we show the $(m_{1/2}, m_0)$ plane of the CMSSM parameter space for $\tan\beta = 10$, $A_0 = 0$ and μ positive. Low m_0 is ruled out (light green) as it results in a $\tilde{\tau}$ LSP. This would result in a charged relic which would have been observed in searches for anomalously heavy nuclei. Low $m_{1/2}$ is ruled out (light blue) as this results in a light Higgs $m_h < 111$ GeV. In the remaining parameter space we plot the SUSY contribution to $(g-2)$ of the muon, δa_μ . We take the current observation of a deviation from the Standard Model seriously and plot the region in which we agree with the measurement at 2σ (long dashed green lines) and 1σ (short dashed green lines). It is clear that to explain the observed value of δa_μ we require light soft SUSY masses and thus light superpartners. Finally we plot the band that fits the observed relic density

^aIn contrast to Ellis and Olive, we take the total fine-tuning of a point to be equal to the largest individual tuning $\Delta^\Omega = \max(\Delta_a^\Omega)$.

^bThere many alternative measures of fine-tuning that have been proposed in the literature. We use this simple sensitivity measure as it is easy to understand, and allows for a straightforward comparison to the electroweak fine-tuning price of SUSY models.

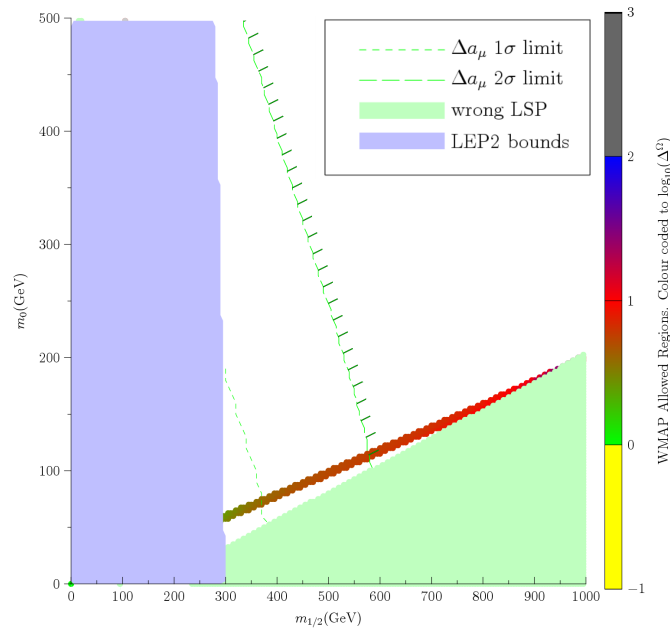


Figure 1: The $(m_{1/2}, m_0)$ plane of the CMSSM parameter space with $\tan\beta = 10$, $A_0 = 0$ and μ positive.

of dark matter within 2σ . For every point that lies in this band we calculate the fine-tuning and plot the point in a colour that corresponds to the log-scale on the right of the plot.

The only dark matter region in Fig. 1 lies alongside the region in which the LSP is the $\tilde{\tau}$. Above this region the LSP is the bino (the partner to the $U(1)$ gauge boson of the Standard Model). Bino LSPs annihilate very weakly and normally give $\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$. Thus, above the dark matter strip we have too much dark matter and WMAP rules out the CMSSM. The remaining parameter space is very slender. In the remaining parameter space, the $\tilde{\tau}$ and the lightest neutralino, $\tilde{\chi}_1^0$, are very close in mass. This results in a large number density of both particles in the early universe. The resulting coannihilation of staus and neutralinos greatly enhances the annihilation rate of SUSY matter, greatly lowering the resulting relic density. This is very sensitive to the mass difference between the stau and the lightest neutralino. Thus we would expect a coannihilation region to be fine-tuned. However note that the band is green at low $m_{1/2}$ and red at large $m_{1/2}$. This corresponds to a tuning of 3 – 10. This is a surprisingly small degree of fine-tuning. The reason is that, for low m_0 and low $\tan\beta$, the mass of the stau and the lightest neutralino are both primarily dependent on $m_{1/2}$. Therefore the masses of the coannihilating particles are coupled and the majority of the fine-tuning is removed.

This is a special case. In Fig. 2 we show the $(m_{1/2}, m_0)$ plane of the CMSSM with $\tan\beta = 50$, $A_0 = 0$ and μ positive. We have also extended the range of m_0 and $m_{1/2}$. Many of the bulk features remain the same. Low m_0 is still ruled out by a $\tilde{\tau}$ LSP. Low m_0 and $m_{1/2}$ is ruled out by a light Higgs. A new bound rules out large m_0 and low $m_{1/2}$ (light red). Here the parameter space gives $\mu^2 < 0$ which is unphysical and corresponds to a failure of radiative electroweak symmetry breaking (REWSB).

The dark matter phenomenology is noticeably more complex. As before, the LSP is bino across the majority of the parameter space and thus mostly gives $\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$. The exceptions to this are marked by the thin WMAP strips. Once again we have a coannihilation strip lying along the side of the $\tilde{\tau}$ LSP region. In contrast to Fig. 1, this band is plotted in purple, corresponding to $\Delta^{\Omega} \approx 50$.

This band is broken by two bands that go up in both m_0 and $m_{1/2}$. These lie on either side of the line along which $2m_{\tilde{\chi}_1^0} = m_A$ and the neutralinos annihilate via an s-channel pseudoscalar

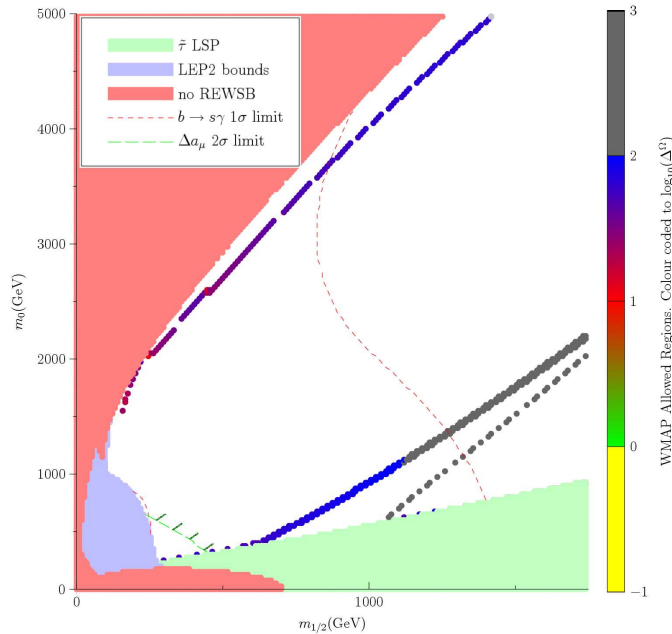


Figure 2: The $(m_{1/2}, m_0)$ plane of the CMSSM parameter space with $\tan\beta = 50$, $A_0 = 0$ and μ positive.

Higgs boson. As could be expected, such a process enormously enhances the annihilation of dark matter. We only fit the dark matter relic density with *just enough* resonant annihilation. This sounds like fine-tuning and indeed the lines are mostly plotted in grey indicating $\Delta^\Omega > 100$.

Finally there is a dark matter band that runs alongside the region in which $\mu^2 < 0$. Along the edge of this region we have low μ and the higgsino fraction of the lightest neutralino increases. As bino dark matter gives $\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$, and higgsino dark matter generally gives $\Omega_{CDM}h^2 \ll \Omega_{CDM}^{WMAP}h^2$, it is not surprising that somewhere in between we manage to fit the observed dark matter relic density. However, the requirement that the composition of the LSP include *just enough* higgsino is an indication of fine-tuning and indeed the line is plotted in purple and red indicating a tuning $\Delta^\Omega \approx 30 - 60$.

4 Breaking the Constraints

We have shown the typical tunings of dark matter in the CMSSM, and highlighted the problem that the bino LSP results in $\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$, ruling out the majority of the CMSSM parameter space. However, there are few compelling theoretical reasons to remain within the confines of the CMSSM. Indeed there are many good reasons to relax a number of the constraints. In previous work² we study the implications for fine-tuning of relaxing the constraint of universal scalar masses and universal gaugino masses. In Fig. 3 we consider a model in which we allow the gaugino masses to vary independently of one another. Such a model has the parameters:

$$a_{CMSSM+M_i} \in \{m_0, M_1, M_2, M_3, A_0, \tan\beta \text{ and } \text{sign}(\mu)\} \quad (3)$$

where $M_{1,2,3}$ set the GUT scale soft SUSY breaking mass of the superpartners to the $U(1)$, $SU(2)$ and $SU(3)$ gauge bosons respectively. Such a break from gaugino mass universality can arise naturally in string models³ and GUT models⁴.

In Fig. 3, we show the (M_1, m_0) plane of a model with non-universal gaugino masses where we have fixed $M_{2,3} = 350$ GeV, $A_0 = 0$ and $\tan\beta = 10$ with μ positive. As before there is a region at low m_0 that is ruled out by a $\tilde{\tau}$ LSP. There is a region ruled out at low $m_{1/2}$ due to light neutralinos and a region ruled out at light m_0 and $m_{1/2}$ due to light sleptons.

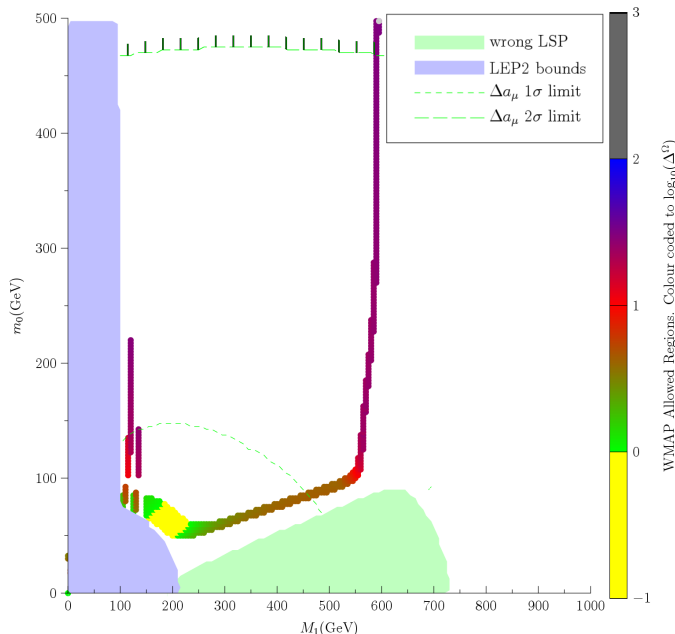


Figure 3: The (M_1, m_0) plane of the $CMSSM + M_i$ parameter space with $M_{2,3} = 350$ GeV, $\tan \beta = 10$, $A_0 = 0$ and μ positive.

The most notable feature is the explosion in the complexity of the dark matter regions. Now, rather than the three dark matter regions of the CMSSM, we have five distinct dark matter regions. Firstly, there is the familiar band along the edge of the $\tilde{\tau}$ LSP region due to $\tilde{\tau}-\tilde{\chi}_1^0$ coannihilation. As before, this exhibits low fine-tuning. This band is interrupted at $M_1 \approx 570$ GeV. At larger M_1 the neutralino is wino rather than bino. A wino LSP generally gives $\Omega_{CDM} h^2 \ll \Omega_{CDM}^{WMAP} h^2$. For $M_1 < 580$ GeV the neutralino is bino so $\Omega_{CDM} h^2 \gg \Omega_{CDM}^{WMAP} h^2$. Around $M_1 \approx 570$ GeV the neutralino has *just enough* bino and wino to fit the observed dark matter density. As in the mixed bino/higgsino case, this requires a delicate balance and thus the region exhibits a tuning $\Delta^\Omega \approx 30$.

At low M_1 there are two distinct peaks at $M_1 = 110$ GeV and $M_1 = 130$ GeV. These correspond to neutralino annihilation via an s-channel Z or light Higgs boson respectively.

Finally, there is a wide band that fits the observed dark matter relic density at low m_0 and low $m_{1/2}$. It lies alongside the region that is ruled out by LEP searches for light sleptons. In this band, the sleptons are light enough to enhance the decay of neutralinos via t-channel slepton exchange to the point where we suppress the dark matter relic density enough to fit the observed data. This decay process is remarkably insensitive to the precise value of the soft masses. This translates to $\Delta^\Omega < 1$, corresponding to no fine-tuning.

Fig. 3 presents one example of the dark matter phenomenology of the wider MSSM. By relaxing the constraints of the CMSSM we can find the typical tunings of the different dark matter regions that exist within the MSSM. We list these in Table 1. Thus we conclude that each region has a typical tuning, and that there remain regions of the MSSM that require no tuning to accommodate the observed relic density.

5 Conclusions: interpreting fine-tuning

We must be careful in our interpretation of these results. Just because an MSSM dark matter region exhibits significant fine-tuning, does not mean that such a region will not be found at a future collider. These tunings have been calculated with respect to the MSSM, which is an

Table 1: The typical tunings for dark matter regions within the MSSM.

Region	Typical Δ^Ω
Mixed bino/wino	~ 30
Mixed bino/higgsino	30 – 60
Mixed bino/wino/higgsino	4 – 60
Bulk region (t-channel \tilde{f} exchange)	< 1
slepton coannihilation (low M_1, m_0)	3 – 15
slepton coannihilation (large $M_1, m_0, \tan\beta$)	~ 50
Z-resonant annihilation	~ 10
h^0 -resonant annihilation	10 – 1000
A^0 -resonant annihilation	80 – 300

effective theory. The MSSM does not specify the mechanism of SUSY breaking, instead we parameterise our ignorance with soft SUSY breaking masses. We expect that these masses should be set by a deeper theory. Thus a region that is tuned in the MSSM may have a very different tuning within a specific model of SUSY breaking.

This variation of tuning between models provides us with a useful tool. If the LHC finds signals for new physics in the form of new particles and large quantities of missing energy, many will interpret this as a SUSY mass spectrum. There will be many different high energy models that fit the data, and probably many models that will also fit the observed dark matter relic density. However this raw mass spectrum will do little to tell us what relations must obtain between high energy parameters.

If we analyse the sensitivity of the dark matter relic density in such a scheme we test precisely this dependence between high energy parameters. For example, it is only because the $\tilde{\tau}$ mass and the $\tilde{\chi}_1^0$ mass in the CMSSM coannihilation region are both dominantly set by $m_{1/2}$ that such a region has low tuning. Therefore we would have to favour such an explanation of an observed coannihilation region than a model that set both masses independently.

After we identify the relations that mitigate the fine-tuning, we can go on to make further predictions. Thus questions of fine-tuning can help to narrow down the candidate explanations for a given experimental signal. Having done this, novel predictions can be made on the basis of hypothesised GUT scale relations between the soft masses, and these can be tested in future experiments. Thus fine-tuning and naturalness should allow us to analyse and compare GUT scale physics using LHC energy data.

Acknowledgments

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