Natural SUSY Dark Matter: A Window on the GUT Scale

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Is this a natural explanation for dark matter?

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This sounds like fine-tuning.

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... and remember that the MSSM is an effective theory.

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If $\Delta_a^{\Omega} = 100$, a 1% change in a gives a 100% change in $\Omega_{CDM} h^2$.

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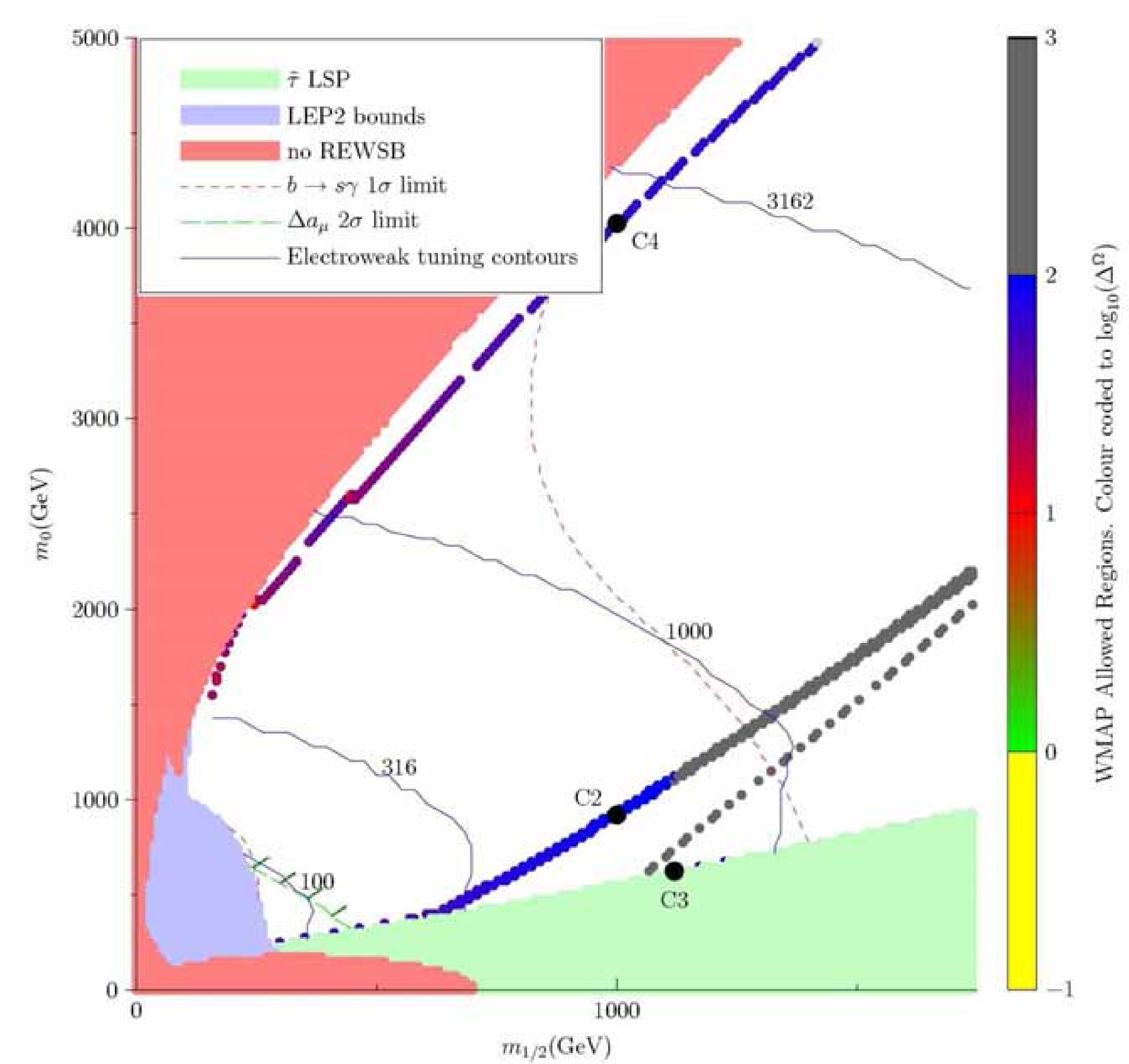
The masses are set at m_{GUT} and run (using `SoftSusy`) to m_{EW}.

The CMSSM with $A_0 = 0$, $\tan\beta = 50$ from hep-ph/0609147, S.F.King, J.R.



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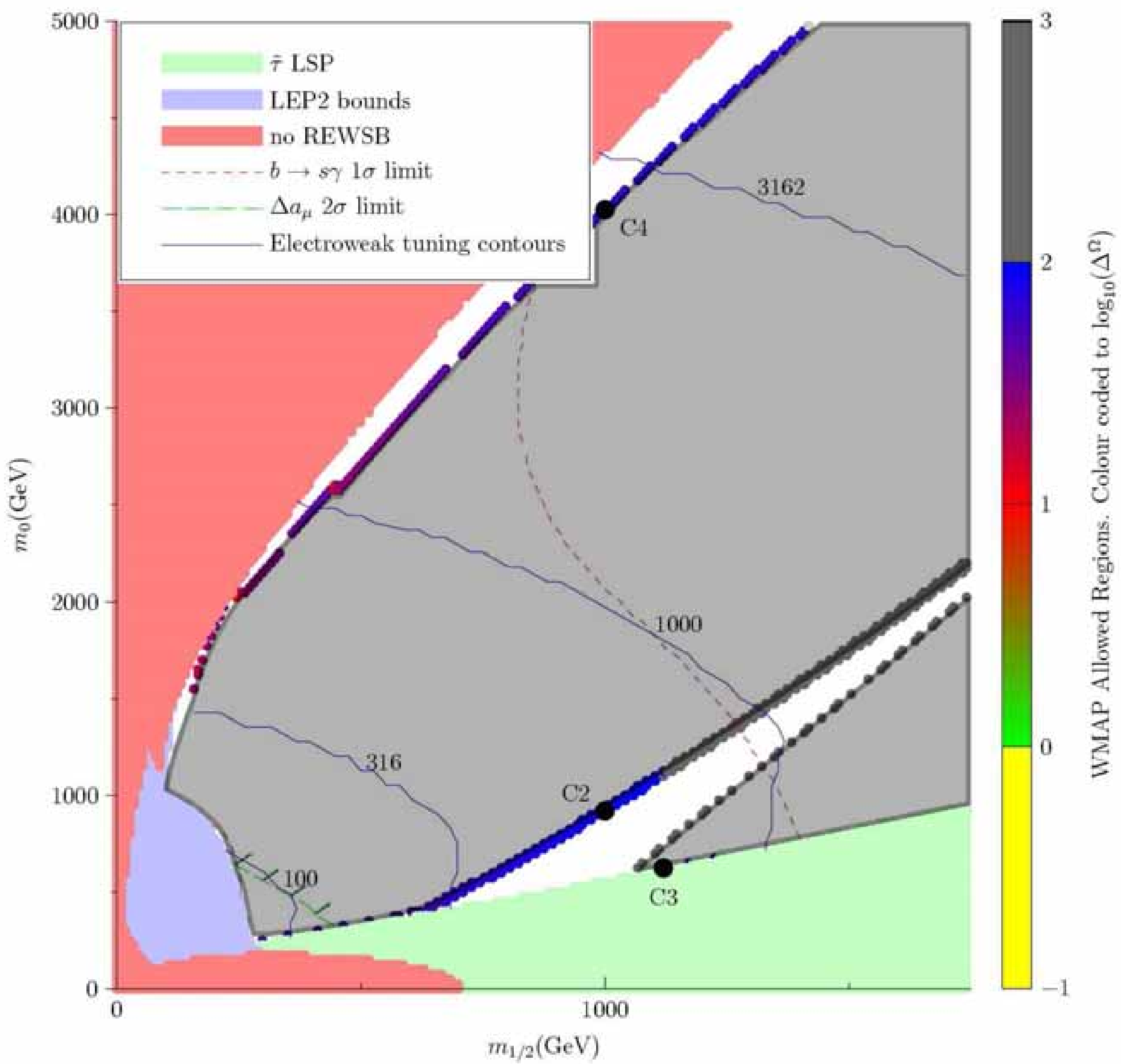
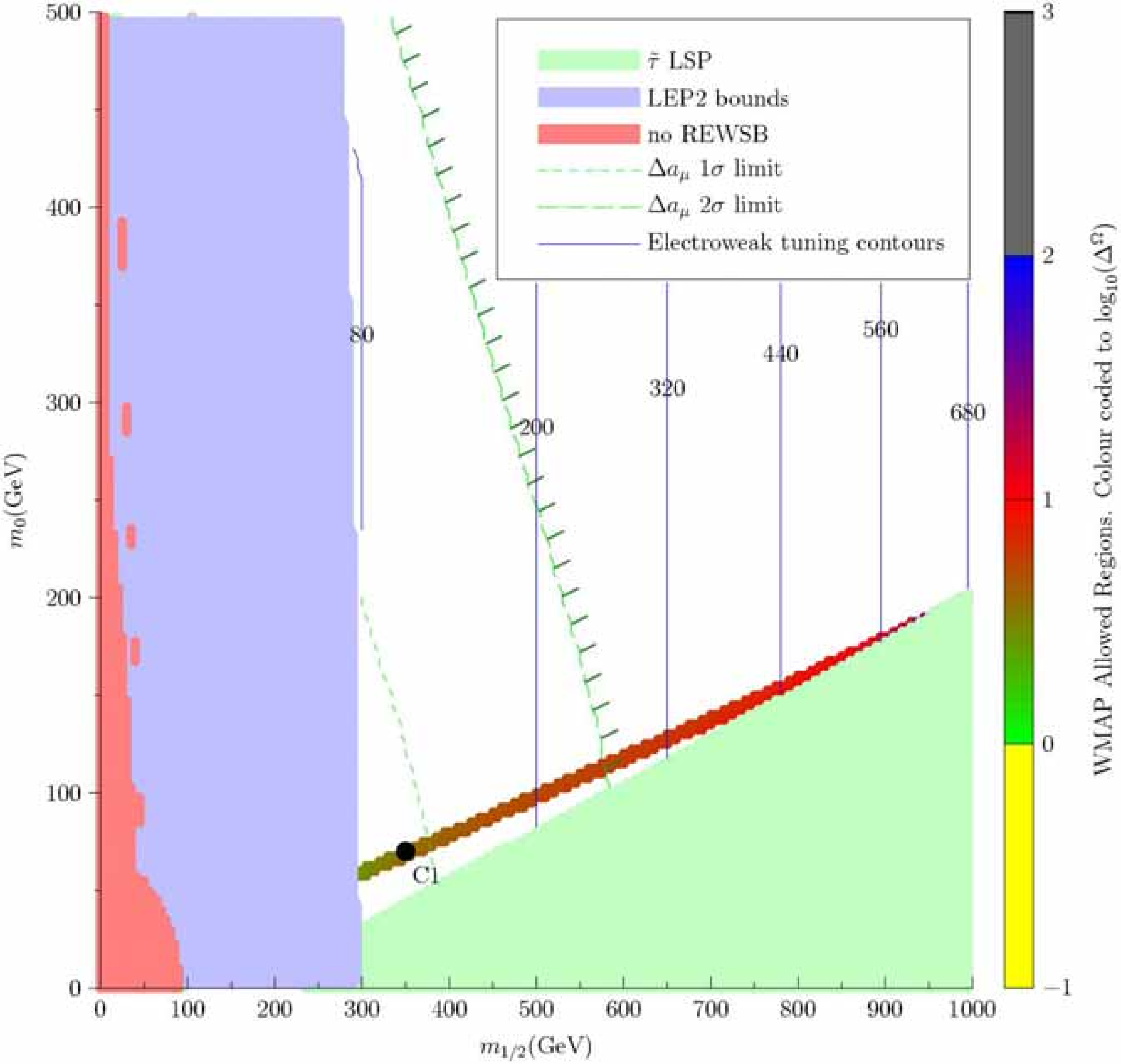
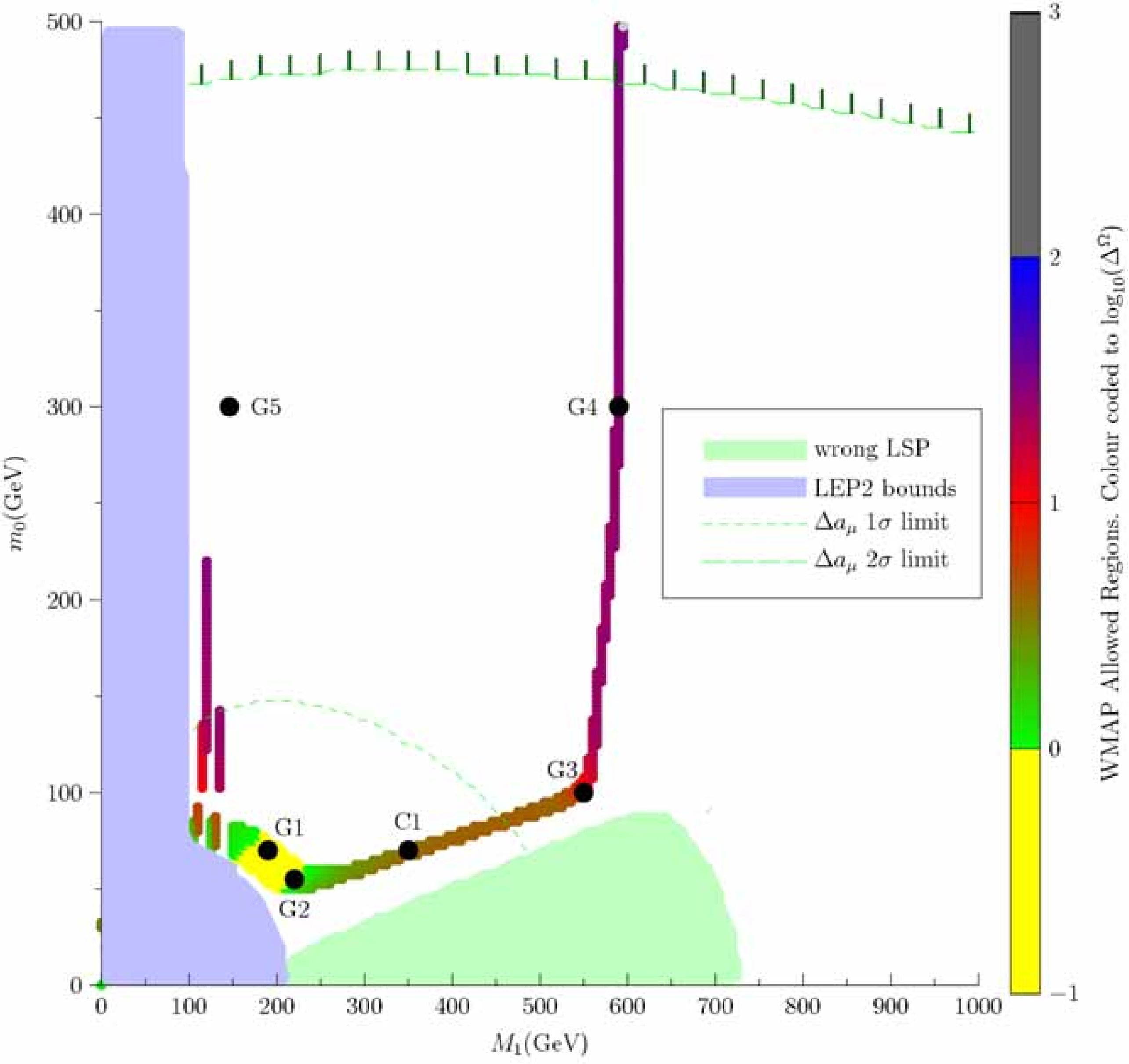


The CMSSM with $A_0 = 0$, $\tan\beta = 10$



Relaxing the CMSSM: non-universal gauginos



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Region	Typical Δ^{Ω}
Mixed bino/wino	~ 30
Mixed bino/higgsino	$30 - 60$
Mixed bino/wino/higgsino	$4 - 60$
Bulk region (t-channel $\tilde{f}$ exchange)	< 1
slepton coannihilation (low M_1, m_0)	$3 - 15$
slepton coannihilation (large M_1, m_0, $\tan\beta$)	~ 50
Z-resonant annihilation	~ 10
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Therefore the MSSM allows for **natural dark matter**.

Going from the EW scale to the GUT scale

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The dark matter tuning with respect to a_{string}, $\Delta^{\Omega}_{a_{string}}$ is directly related to $\Delta^{\Omega}_{a_{MSSM}}$ via the relation:

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If we minimise the coefficients, we minimise the dark matter tuning.

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3. The same measures of sensitivity can be used to relate LHC data to $\Omega_{CDM}h^2$.
   - By studying sensitivity to electroweak scale SUSY parameters we can find the sensitivity required of LHC measurements to disprove SUSY.

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- Dark matter fine-tuning allows us to weigh up the different GUT models.
- We can identify models that provide the most natural explanation of the observed phenomena.
- We can then make novel predictions for both the LHC, ILC and dark matter detection experiments to test the theory.

Backup Slides

Resonant Annihilation

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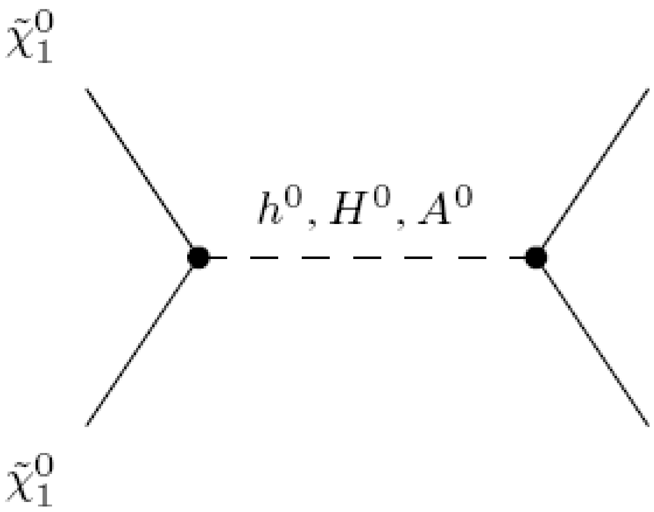

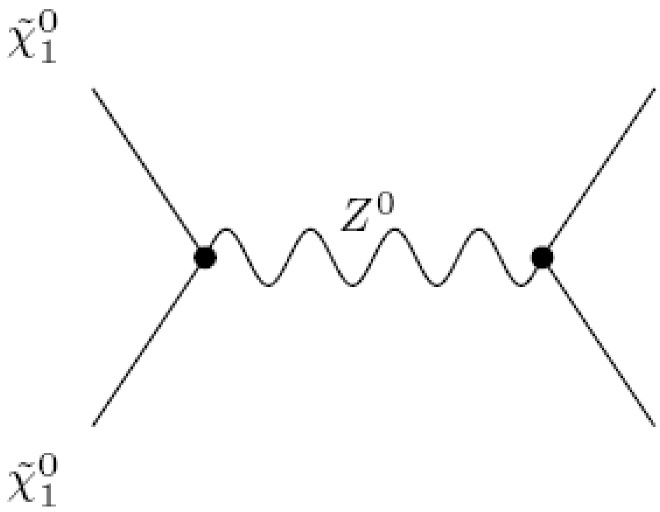

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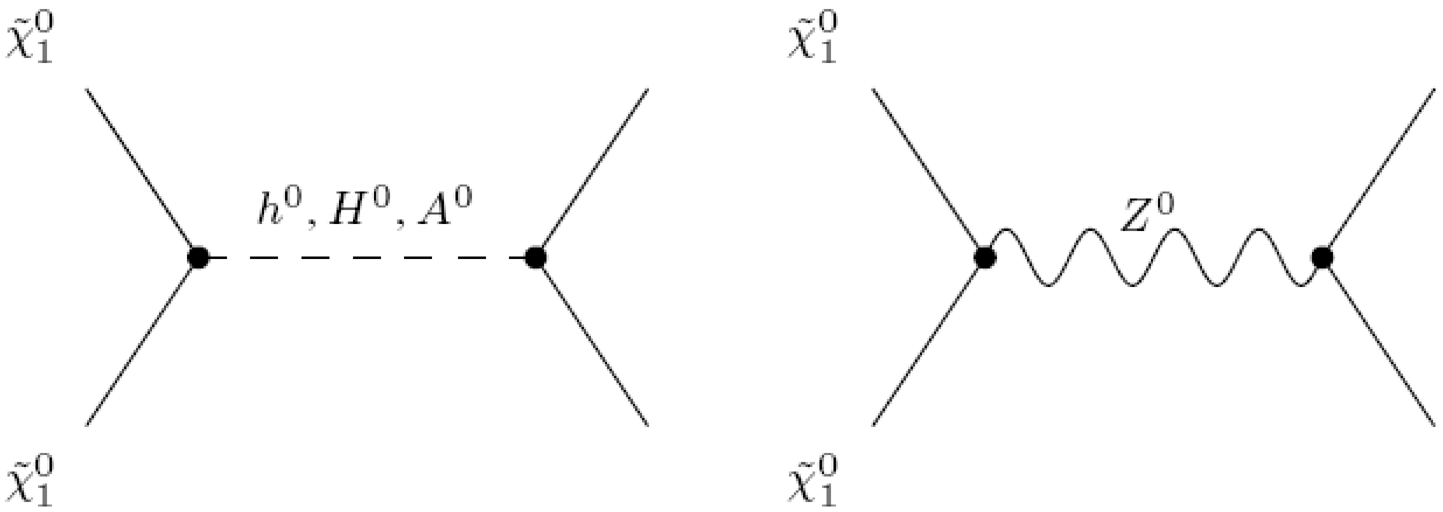

In this case, annihilation is usually **too efficient**: $\Omega_{CDM} h^2 \ll \Omega_{CDM}^{WMAP} h^2$ except on the edges of the resonance.

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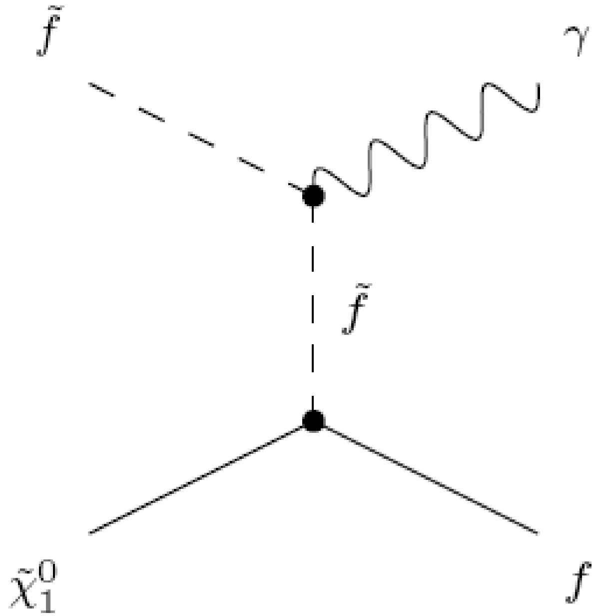

Stau running in the coannihilation region