An introduction

Basis of SUPERSYMMETRY

and of the

SUPERSYMMETRIC STANDARD MODEL

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Is there a

"SUPERWORLD" ?

of new particles ?

Could half of the particles (at least)

have escaped our direct observations ?

STANDARD MODEL

describes

strong, electromagnetic and weak interactions

of quarks and leptons

$SU(3) \times SU(2) \times U(1)$ gauge group

spin-1 gauge bosons:	gluons, $\underbrace{W^+,\ W^-,\ Z,}$ photon
spin-$\frac{1}{2}$ fermions:	**quarks** and **leptons**

$+\,1$ (still unobserved)

spin-0 **Englert-Brout-Higgs boson**

associated with **spontaneous breaking**

of $SU(2) \times U(1)$ **electroweak symmetry**

– remarkably successful

– but leaves many questions unanswered

(a long list ...)

Among which

- why fundamental Higgs fields ?

(do they actually exist ?)

[no fundamental spin-0 Higgs fields ?

$\longrightarrow$ technicolor $\longrightarrow$ extended technicolor, ...]

- why a Higgs potential

$$V(\varphi) = \lambda\,|\varphi|^4 - \mu^2\,|\varphi|^2 \quad ?$$

what is the mass of the Higgs boson ?

($m_{\text{Higgs}} = \mu\sqrt{2} = v\sqrt{2\lambda}$ ...)

what fixes μ ?

what fixes the coupling constant λ ?

- is the Higgs sector (if exists) the one of the SM or a more complicated one ... ?

- do new particles exist ?

(maybe also new forces ?)

[after LEP, we think we know all (sequential) quarks and leptons]

this has become essential,

in view of growing evidence for

non-baryonic dark matter

Other interrogations:

- role of **gravity**

  (related to spacetime through general relativity)

  can it be more closely connected with particle physics?

  can one get a consistent theory of quantum gravity ?

  question of cosmological constant Λ ...

- can interactions be **unified** ?

  approach of **grand-unification**:

  $SU(3) \times SU(2) \times U(1) \quad \subset$ simple gauge group, e.g. $SU(5)$

  $$\begin{cases} \text{gluons} \longleftrightarrow W^{\pm},\ Z,\ \gamma \quad (+\text{ other gauge bosons}) \\ \text{quarks} \longleftrightarrow \text{leptons} \end{cases}$$

  with its own questions ...

  (Higgs potential and symmetry breaking, origin of hierarchy of mass scales, many coupling constants, relations between q and l masses ...

- can one relate particles of different spins?

etc. ...

We now have a "new" tool,

SUPERSYMMETRY

BOSONS $\longleftrightarrow$ FERMIONS

(integer spins) (half-integer spins)

SUPERSYMMETRY ALGEBRA :

$$\begin{cases} \{ Q , \bar{Q} \} = -2 \gamma_\mu P^\mu \\ [Q, P^\mu] = 0 \end{cases}$$

Gol'fand-Likhtman, Volkov-Akulov, Wess-Zumino, 1970-73

P^μ = generator of space-time translations

relation with spacetime

$\longrightarrow$ *connection with general relativity*

$\longrightarrow$ *supergravity* (1976)

a new structure

what to do with it ?

Why was SUSY algebra introduced ?

- Gol'fand and Likhtman, 1970 : SUSY algebra, written as:

$$\{ Q_L , \overline{Q_L} \} = -2 \gamma_\mu \frac{1+i\gamma_5}{2} P^\mu$$

at the origin of parity non-conservation (in weak interact.) ?

- Volkov and Akulov, 1973 :

is the neutrino a Goldstone particle ?

$m_\nu = 0$ from SUSY algebra ... ? (no ...)

V.A. model includes only fermions, no boson ..

($\delta_{\rm SUSY}$ fermion = composite bosonic field made of two fermions ...)

SUSY without bosons !!! ⟶

(SUSY without superpartners ... *not the SUSY we know and like ...*)

- Wess and Zumino, 1973 :

Extend to 4 dim. "supergauge" transformations acting on $2d$ string worldsheet

⟶

Construction of

renormalizable SUSY gauge theories in 4 dim.

Can supersymmetry be a symmetry of the <u>fundamental laws</u> of the world ?

It does not look like it !!!

(Situation in 1974)

Nature is "obviously"
not supersymmetric !

1

bosons and fermions would have equal masses:

$$\underline{m_{\text{boson}} \equiv m_{\text{fermion}}} \;!!$$

Possible answer: break (spontaneously) supersymmetry

But

spontaneous supersymmetry breaking
did not seem possible !

(in contrast with ordinary symmetries)

\+ spontaneous breaking of supersymmetry

would generate massless spin-$\frac{1}{2}$ Goldstone fermion

where is the spin-$\frac{1}{2}$ Goldstone fermion of SUSY ?

– soft explicit breaking (much easier!) considered very early

(e.g. $\underline{m_0^2}$ $(\tilde{q}, \tilde{l})$ in 1976 ...)

– "rules of the game":

generate susy-breaking terms spontaneously,

so that susy may be realized locally (within supergravity, after 1976)

2 Which **bosons** and **fermions** relate ??

$$\textit{photon} \overset{?}{\longleftrightarrow} \textit{neutrino}$$

$$\textit{gluons} \overset{?}{\longleftrightarrow} \textit{quarks}$$

... this does not work ...

3 SUSY theories systematically involve

(self-conjugate) Majorana fermions

while Nature only knows Dirac fermions !

3' Conserved quantum numbers B and L

carried by fermions (quarks and leptons), not bosons !

this cannot be, in a supersymmetric theory ...

seemed to make supersymmetry irrelevant

to the description of the real world !!

- How to deal with Majorana fermions of susy theories ?
- How to construct the Dirac fermions known in Nature ?
- How to attribute them conserved quantum numbers

(B and L) ?

+ • How to obtain a **correct set of interactions**

i.e. weak, electromagnetic and strong interactions

due to the sole exchanges of

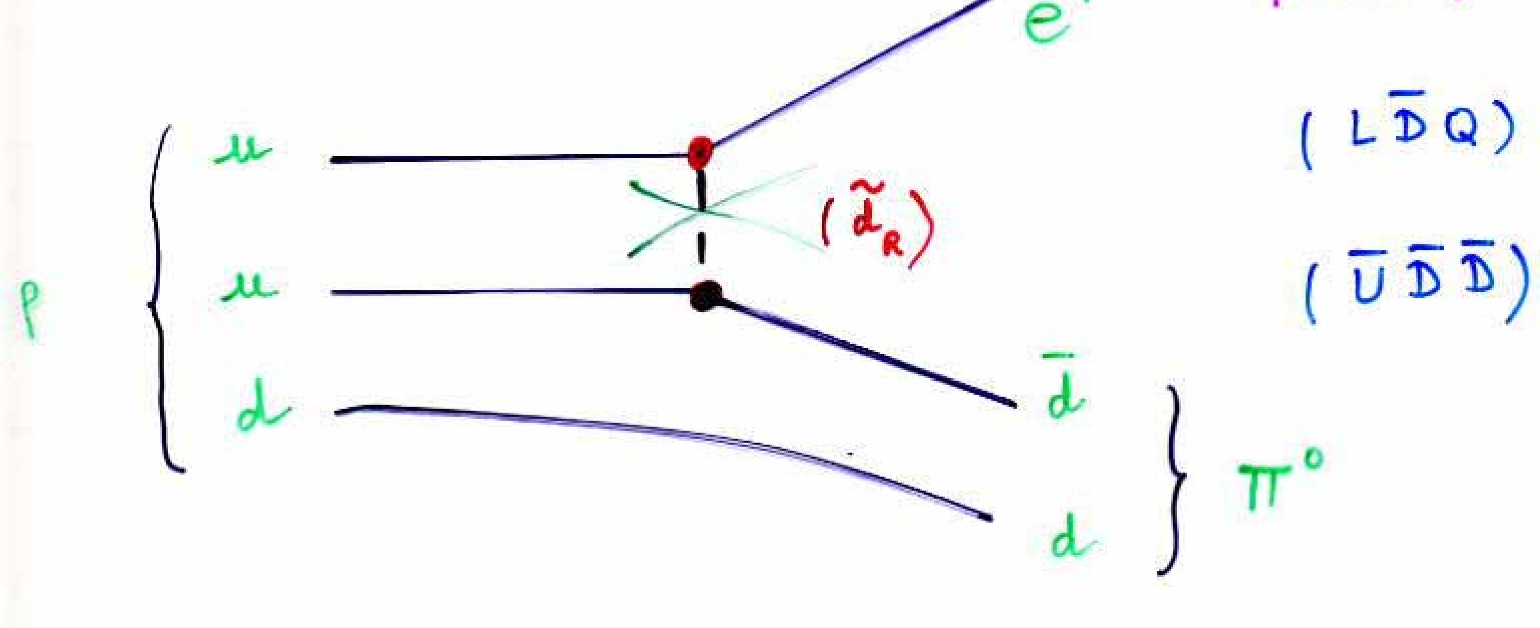
$W^{\pm}$'s, Z's, photons and gluons,

avoiding

unwanted exchanges of spin-0 particles ?

[Example of unwanted interactions due to spin-0 exchanges:

$\not{B}$ $\not{L}$ p decays!



should not appear!]

... related with introduction of

R-symmetry and R-parity,

continuous   discrete

essential in **searches for**

"supersymmetric particles"

(pair production of SUSY particles)

and on the nature of

non-baryonic dark matter

of the Universe

LSP stable

usually a neutralino
with annihilation cross sections roughly ≈
weak-interaction cross sections

neutralino = natural WIMP candidate

Which bosons and fermions may be related ?

* * *

Could one relate the Fermions

constituants of matter

to Bosons, messengers of interactions ?

and arrive to some sort of

Unification between Forces and Matter ??

would be very attractive, but ...

!! ...

Which BOSONS and FERMIONS may be <u>related</u> ?

Are they fundamentally "of the same nature" ?

May be:

$$photon \overset{?}{\longleftrightarrow} neutrino$$

but which one: ν_e, ν_μ, ν_τ !!! ??

$$W^- \overset{?}{\longleftrightarrow} electron$$

but what about μ and τ ??

$$gluons \overset{?}{\longleftrightarrow} quarks$$

(8) $\qquad$ $6 \times 3 \times 2_{(L+R)}$

(neutral) $\qquad$ charged

...

none of these direct associations suitable!

For example:

- **Not one neutrino, but three ...**

difficult to select one (even a combination)

to associate to photon ...

nevertheless, led to developing extended

$\underline{N=2}$, then $\underline{N=4}$, supersymmetric theories ...

more symmetry ! great ! (?)

but: → new (bigger) multiplets; more constraints

$N=2$ hypermultiplets, requiring central charges in algebra

possibility of "mirror particles", etc.

connection with | **extra spacetime dimensions ...** |

e.g. photon associated with two spin-$\frac{1}{2}$ fermions

(but not neutrinos! — now to be called "photinos")

and 2 "spin-0 photons",

now to be viewed as extra degrees of freedom for

the photon in a 6-dimensional spacetime.

Not the easiest way to go to a complete, and realistic, theory ...

Which degrees of freedom should actually be present in the low-energy theory ?

We leave this for the time being, and return to

→ *"simple" ($\boldsymbol{N=1}$) supersymmetry ...*

$\gamma \leftrightarrow$ "ν"

↳ would have to carry "something"

- **If one neutrino could be associated with photon:**

SUSY generator would have to carry

one unit of the corresponding <u>lepton number</u>.

But it is a <u>self-conjugate</u> Majorana spinor...

How could a real object carry a "charge", i.e. one unit of a conserved additive quantum number ?

possible solution, and subsequent byproducts:

Led to introduce a **<u>continuous R-symmetry</u>**

acting "chirally" on the supersymmetry generator as:

$$Q \to e^{-\gamma_5 \alpha}\, Q$$

(Continuous) R-symmetry <u>($U(1)_R$)</u> leads to further restrictions on possible (superpotential) interactions

→ *<u>Not all possible superpotentials are admissible</u> ...*

Further restrictions to be taken into account,

in addition to supersymmetry and gauge invariance...

Continuous R-symmetry the progenitor of R-parity ...

→ *a stable dark matter candidate*

Furthermore:

If $\underline{\nu \longleftrightarrow \gamma}$,

ν interactions would be fixed by gauge invariance + SUSY,

This "ν" would interact with charged particles only,

proportionally to their electric charge

(not the case for a normal neutrino (ν_e, ν_μ, ν_τ), which interacts with Z)

Such a "neutrino" could not have weak-neutral-current interactions ...

cannot be identified with ν_e (nor ν_μ or ν_τ),

but should be identified as a new particle,

a "photonic neutrino"

called in 1977 **"photino"**

mixes with zino, higgsinos ... → **neutralinos**

in 2- Higgs

SUSY extensions of the Standard Model

Basic ingredients of

Supersymmetric Standard Model

(1976-77)

1) $SU(3) \times SU(2) \times U(1)$ gauge superfields

2) chiral quark and lepton superfields

3) two doublet Higgs superfields H_1 and H_2
   for electroweak breaking

4) trilinear superpotential for q and l masses

• Superpotential constrained to be

even function of quark and lepton superfields!

includes

$$h_e\, H_1 . \bar{E} L \; + \; h_d\, H_1 . \bar{D} Q \; - \; h_u\, H_2 . \bar{U} Q$$

+ $\mu\, H_1 H_2$
or
$\lambda\, H_1 H_2 N$ ...

• Otherwise: introduces unwanted (and dangerous)

B and/or L violations!

associated with unwanted exchanges of new spin-0 sparticles!

"R-parity-violating" superpotential in general excluded

No real utility, only a source of problems,
+ philosophy is to restrict $\mathcal{L}$ by symmetries,
not write as many contributions as possible ...

Minimal particle content of

Supersymmetric Standard Model

Spin 1	Spin 1/2	Spin 0
gluons g photon γ	gluinos $\tilde{g}$ photino $\tilde{\gamma}$	
$W^\pm$ Z	winos $\widetilde{W}^\pm_{1,2}$ zinos $\widetilde{Z}_{1,2}$ higgsino $\tilde{h}^0$	$H^\pm$, H, h, A } Higgs bosons
	leptons l quarks q	sleptons $\tilde{l}$ squarks $\tilde{q}$

2 neutral gauginos + 2 neutral higgsinos mix into 4 neutralinos

+ possible additional ingredients

additional singlet chiral superfield ("NMSSM"),
with trilinear $\lambda\, H_1\, H_2\, N\; + \;...$ superpotential

and/or, possibly, extra $U(1)$ gauge superfield

"hidden sector" associated with supersymmetry-breaking

new "vectorlike" families of quarks and leptons ...

2 Higgs doublets $\Longrightarrow$

$$\left\{\begin{array}{l}\underline{\text{charged Higgses } H^{\pm}}, \\ \\ \underline{\text{several neutral ones}}, \text{ now known as } h\,,\ H\,,\ A\,,\ \dots\end{array}\right.$$

Higgs potential:

$$\begin{aligned} V_{\rm Higgs} &= \frac{\vec{D}^2}{2} + \frac{D'^2}{2} + \dots \\ &= \frac{g^2}{8}\,(h_1^\dagger\,\vec{\tau}\,h_1 + h_2^\dagger\,\vec{\tau}\,h_2)^2 + \frac{g'^2}{8}\,(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \dots \\ &= \frac{g^2+g'^2}{8}\,(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + \frac{g^2}{2}\,|h_1^\dagger h_2|^2 + \dots\ . \end{aligned}$$

$$\boxed{V_{\rm Higgs} \;=\; \frac{g^2}{8}\,(\dots)^2 + \frac{g^2+g'^2}{8}\,(\dots)^2 + \dots}$$

$\rightarrow$ quartic Higgs potential of “MSSM”

Unknown λ of SM replaced by:

quartic Higgs couplings fixed by

$$\frac{g^2+g'^2}{8} \quad \text{and} \quad \frac{g^2}{2}\ .$$

Success!

At this point: worth to return to question:

WHY 2 HIGGS DOUBLETS

in SUSY extensions of S.M. ?

3 reasons

② H_1 gives mass to d, e
H_2 " " u
) m_q, m_ℓ

① Avoid massless chargino!

③ Avoid new anomalies that would be introduced by 1 chiral higgsino doublet:

$$\begin{pmatrix} h^0 \\ h^- \end{pmatrix}_L$$

---

* and discuss more implications of SUSY in Higgs / chargino / neutralino sector

Early attempt [*Nucl. Phys.* B 90, 104 (1975)]

at toy-model relating

the photon with would-be "neutrino",

reinterpreted as a "photino"

the W^- with would-be "electron",

reinterpreted as "wino" or "chargino"

led to a

$SU(2) \times U(1)$ electroweak theory

with electroweak symmetry spont. broken by

a pair of chiral doublet Higgs superfields,

now known as H_1 and H_2.

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \quad , \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

both left-handed

with $< h_1^0 > = \frac{v_1}{\sqrt{2}}$, $< h_2^0 > = \frac{v_2}{\sqrt{2}}$

and a mixing angle β

$$\tan\beta = \frac{v_2}{v_1} .$$

Why **two** doublet Higgs superfields ?

- With only one Higgs doublet (H_1):

$$\left\{ \begin{array}{ll} & \text{one charged Dirac "gaugino"} \quad \lambda^- = \lambda^-_L + \lambda^-_R \\ + & \text{one chiral charged "higgsino" e.g.} \quad \psi^-_L \end{array} \right.$$

$\Longrightarrow$ one massive charged Dirac fermion $(\psi^-_L + \lambda^-_R)$

one charged chiral fermion (λ^-_L) *would stay massless*

- With **two** Higgs doublets, we get

two charged Dirac fermions,

which may be defined as

$\tilde{h}^-_{1L}$

$$\left\{ \begin{array}{lcl} \tilde{W}^-_1 & = & \psi^-_L + \lambda^-_R \\ \tilde{W}^-_2 & = & \lambda^-_L + \psi^-_R \end{array} \right.$$

$\leftarrow$ from $\begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}_L$

now known as **"charginos"**

Also: higgsino sector vectorlike $\longrightarrow$ **no anomalies ...**

- (initially massless) Majorana fermion

$$\lambda_\gamma = \sin\theta \; \lambda_3 + \cos\theta \; \lambda'$$

partner of photon under supersymmetry

$\rightarrow$ **"photino"**

Charged fermions:

gaugino/higgsino mixings with mass matrix

$$\mathcal{M} = \begin{pmatrix} (m_2 = 0) & \frac{g\,v_2}{\sqrt{2}} = m_W\sqrt{2}\,\sin\beta \\ \frac{g\,v_1}{\sqrt{2}} = m_W\sqrt{2}\,\cos\beta & \mu = 0 \end{pmatrix}$$

"winos" or "charginos".

- for the time being:
  - no gaugino mass term: $m_2 = 0$

[absent in global supersymmetry (unless radiatively generated)]

  - no "μ term"

[$\mu\, H_1 H_2$ superpotential (violates continuous $U(1)_R$ symmetry)

may be replaced by trilinear coupling

$\lambda\, H_1 H_2 N$ with extra singlet chiral superfield N]

- Translation of singlet allows for regeneration of the μ term:

$$\lambda\, H_1 H_2 N \;\rightarrow\; \underbrace{\lambda < N >}_{\mu}\, H_1 H_2$$

- Chiral singlet allows for the gauging of an extra-$U(1)$ symmety

$\rightarrow$ new Z' boson

or U boson light (useful for Light Dark Matter annihilations)

- $\mu\, H_1 H_2$ **becomes allowed once we drop continuous** $\boxed{U(1)_R}$ **reducing it to a discrete subgroup of**

  $\boxed{\underline{R\text{-parity}}}$ **transformations.**

we shall come back to this later

Continuous R-invariance acts as

$$\begin{cases} V(x,\theta,\bar\theta) \longrightarrow V(x,\theta\, e^{-i\alpha},\bar\theta\, e^{i\alpha}) \\ H_{1,2}(x,\theta) \longrightarrow H_{1,2}(x,\theta\, e^{-i\alpha}) \end{cases}$$

for $SU(2)\times U(1)$ gauge superfields $\vec V$ and V'
and chiral Higgs superfields H_1 and H_2

associated with (additive) **"R-number"** conservation

- Superpotential $\mathcal{W}(x,\theta)$ transforms according to:

$$\mathcal{W}(x,\theta) \longrightarrow e^{2i\alpha}\ \mathcal{W}(x,\theta\, e^{-i\alpha})$$

so that its "F-component" $\Re \int \mathcal{W}\, d^2\theta$ is R-invariant.

$\Longrightarrow$ no $\mu\, H_1 H_2$ superpotential term at this stage

as long as continuos R-invariance is present

Will be broken anyway, and reduced to R-parity, by the gravitino mass term ...

As a result, continuous R-invariance may be used to control the size of the (supersymmetric) μ term, which may remain naturally "small", i.e. of the order of the gaugino mass parameters $m_{1/2}$ ...

No μ problem here!

The question of <u>gluino masses</u>

What about our initial

continuous $U(1)$ <u>R-symmetry</u> ?

It acts chirally on gluinos

$$g \longrightarrow e^{\gamma_5 \alpha}\, g$$

and would force them to be massless ...

Unbroken continous R-invariance

$$\Longrightarrow$$

massless GLUINOS !!!

which don't seem to exist !!

We have to get rid of the

unwanted **R - SYMMETRY**

so that

GLUINOS can be **MASSIVE**

How ?

Fortunately :

GRAVITY does it for us !

Spin $\frac{3}{2}$ gravitino is Majorana

(or use : radiative corrections
transmitted by vectorlike messenger quarks)

gravitino mass term $m_{3/2}$

breaks R- invariance P.L.B 70 (1977) 461

$\Rightarrow$ U(1) reduced to R- PARITY

$$R_p = (-1)^R$$

identified as $(-1)^{2S} (-1)^{3B+L}$

No continuous R- invariance $\Rightarrow$

- $\mu H_1 H_2$ term

gets reallowed in Superpotential.

- gauginos may have direct gravity-induced masses (C.F.G.)

$m_{1/2}$ or m_1, m_2, m_3

$\Rightarrow$ "Complete" Chargino mass matrix :

$$\begin{pmatrix} m_2 & m_W \sqrt{2} \sin\beta \\ m_W \sqrt{2} \cos\beta & \mu \end{pmatrix}$$

Both charginos may now be heavier than W. $\Downarrow$

thanks to μ and m_2

The natural mass scale of

SUSY particles

is normally expected to be :

$\lesssim$ TeV scale

(independently of GUTs)

otherwise the corresponding new scale

would create a hierarchy problem

in the electroweak theory .

The new particles of the

SUSY standard model

(esp. higgs bosons + higgsinos)

make possible a

high-energy unification of the gauge couplings g_s, g_1, g_2

as required in the GUT framework

(mostly thanks to

2 Higgs doublets + higgsinos)

("Minimal SU(5) GUT")

EQUIVALENTLY

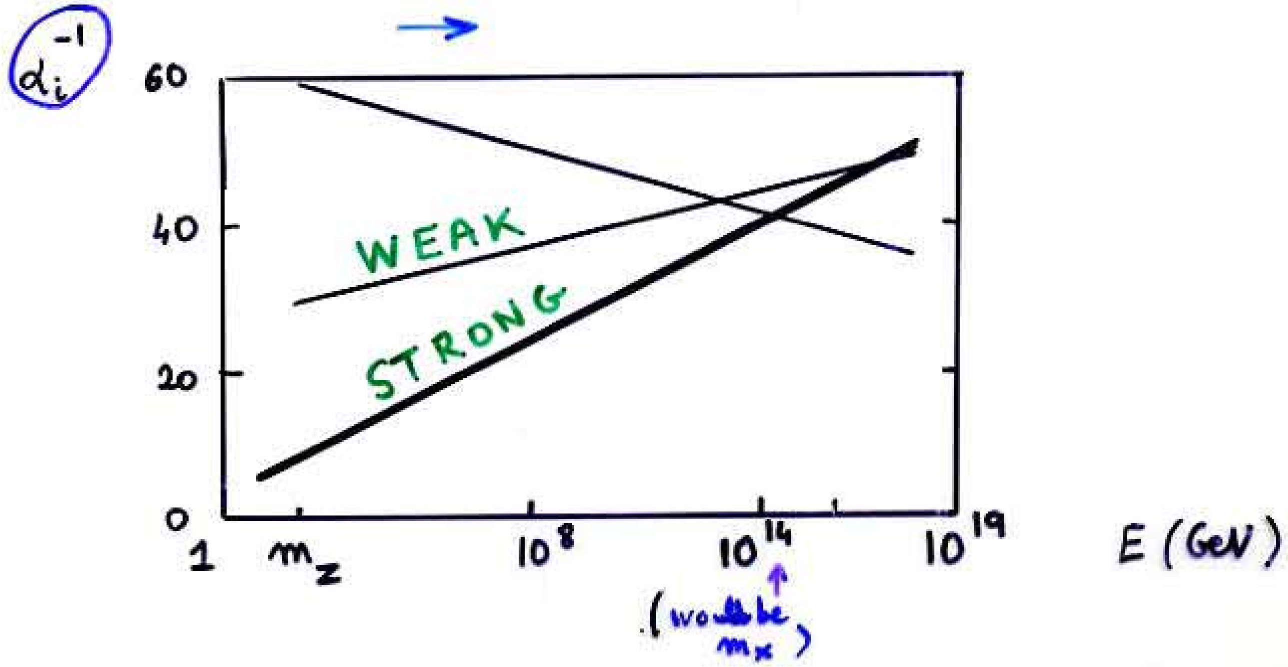
NOW, WE KNOW: $\sin^2\theta \simeq .23$

$\alpha_3(m_Z) \simeq .12$

⇒ COMPUTE THE EVOLUTION OF THE COUPLINGS

WITH THE FIELD CONTENT OF THE STANDARD MODEL:

(at low-energies)

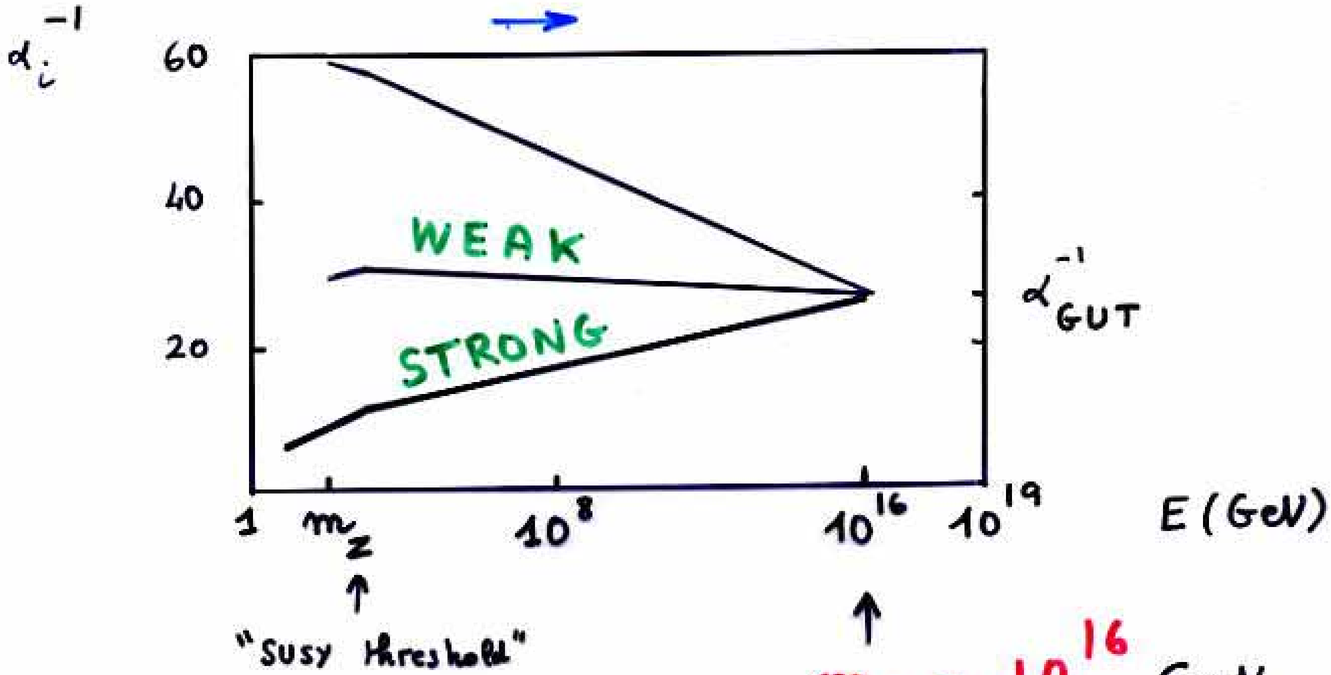


NO GRAND-UNIFICATION

OF THE GAUGE COUPLINGS

FOR THE STANDARD MODEL!

EVOLUTION OF THE COUPLINGS IN THE SUPERSYMMETRIC STANDARD MODEL:



$m_X \sim 10^{16}$ GeV

⇒

SU(5) GRAND-UNIFICATION

MAY OCCUR AT ↑

THIS MAY BE TAKEN AS AN INDIRECT INDICATION FOR THE EXISTENCE OF **SUPERPARTNERS** AT A NOT-TOO-HIGH SCALE $\lesssim$ TeV

Is there a μ- problem ?

Ok! But μ is a

supersymmetric parameter !

It could be very large !!

Should this be considered as a problem ?

No !

As the size of μ may be controlled by using

- either extra-$U(1)_A$ which acts in the same way on H_1 and H_2
- or continuous $U(1)_R$ symmetry.

μ breaks $U(1)_R$ (and $U(1)_A$)

$m_{3/2}$!! !!

μ may be naturally $\approx m_{3/2}$

$\approx m_{1/2}$

<u>no μ problem here</u>

Are we happy with MSSM ?

- rad. breaking of $SU(2) \times U(1)$ induced by SUSY breaking
- $\lambda \sim g^2$ or g'^2 $\rightarrow$

Lightest Higgs $<$ $\underbrace{m_Z + \text{rad. corr.}}_{\lesssim 130 \text{ GeV}}$

if Squarks heavy

($\sim$ TeV!)

The allowed parameter space is very seriously reduced.

"Tension" on the parameters.

A hierarchy pb tends to be "recreated"....

etc ....

→ Return to "NON-MINIMAL" extensions

as considered in the 70's

- $\mu H_1 H_2 \longrightarrow \lambda H_1 H_2 N$

  ↓

  extra chiral singlet

- Superpotential

$$\mathcal{W} = \lambda H_1 H_2 N + f(N)$$

→ $\underline{\mu_{eff} = \lambda \langle N \rangle}$

$$f(N) = \sigma N$$

$$f(N) = K N^3$$ (other popular choice)

$$\mathcal{W} = (\lambda H_1 H_2 + \sigma) N$$

allows for electroweak breaking

even before SUSY-breaking

→ Gauge-Higgs Unification

+ Lightest Higgs heavier than in MSSM.

$\mu H_1 H_2 \rightarrow \lambda H_1 H_2 N$

⇝ possibility of gauging

extra U(1) - factor in gauge group

⇝ "U(1) - NMSSM"

(dates back to 77)

→ New Z' boson

→ TeV scale ?

or very light, weakly coupled ?

U - boson, cf. Light Dark Matter hypothesis.

Extended supersymmetry :

$$N = 1 \longrightarrow N = 2 \longrightarrow N = 4$$

$\Longrightarrow$ Rôle of possible

extra (compact) dimensions of spacetime

- extremely small ??

$\sim L_{\text{Planck}} \simeq 10^{-33}$ cm (or fixed by the GUT scale ?) ???

- or significantly larger ?

$\sim 10^{-16}$ or 10^{-17} cm $\leftrightarrow$ TeV scale ?

Size of extra dimensions
may determine supersymmetry-breaking
and mass scale of superpartners

cf. $m_{3/2} = \frac{\pi \hbar}{L c}$ (or $\frac{1}{2R}$) using

discrete boundary conditions involving *R*-parity!

$\rightarrow$ relations like $m^2(\text{winos}) = m_W^2 + \frac{\pi^2}{L^2}$, etc. (85)

- "large" (non-universal) extra dimensions ?

optimistic point of view :

Both supersymmetry and extra dimensions could show up at particle colliders !!

many scenarios possible

we would already be happy with supersymmetry !

We are waiting for experimental data especially from LHC ...