

# Magnetic cycles of the Sun and solar-type stars



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*Université de Montréal*

*With P. Beaudoin, P. Charbonneau, A.S. Brun, J.D. do Nascimento Jr.*

# Outline

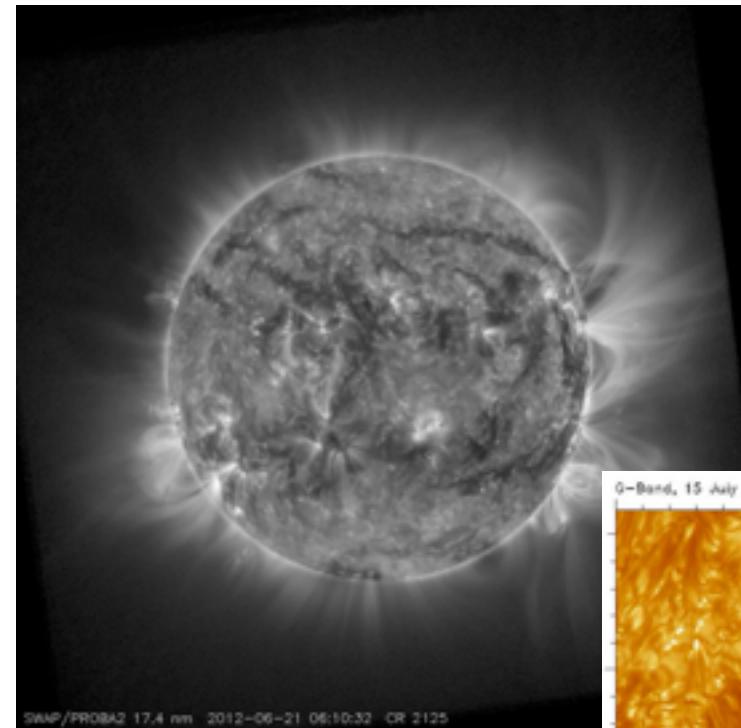
## I. Cycles in magnetized stars: observations and theories

- From the solar magnetic cycle to stellar magnetic cycles
- Dynamo theory for stellar magnetic cycles

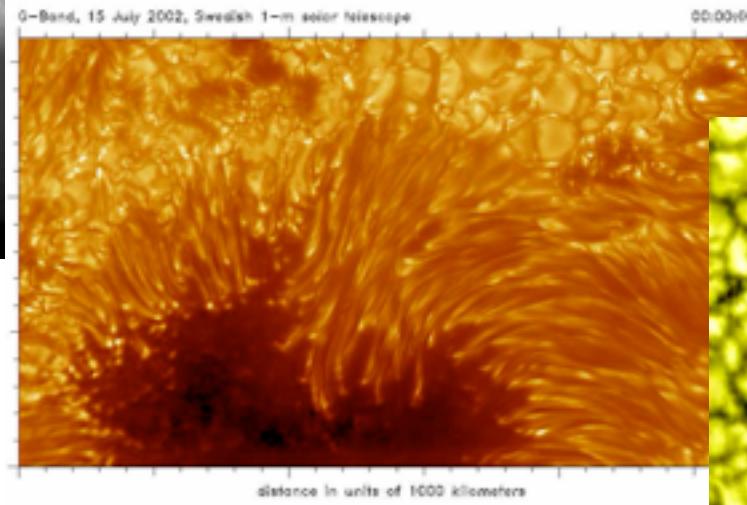
## II. Ab-initio modelling of stellar magnetic cycles

- Understanding the basic ingredients of stellar dynamo:
  - Large-scale flows and differential rotation
  - Turbulent electro-motive force
- Cyclic dynamo in 3D models: recent progress

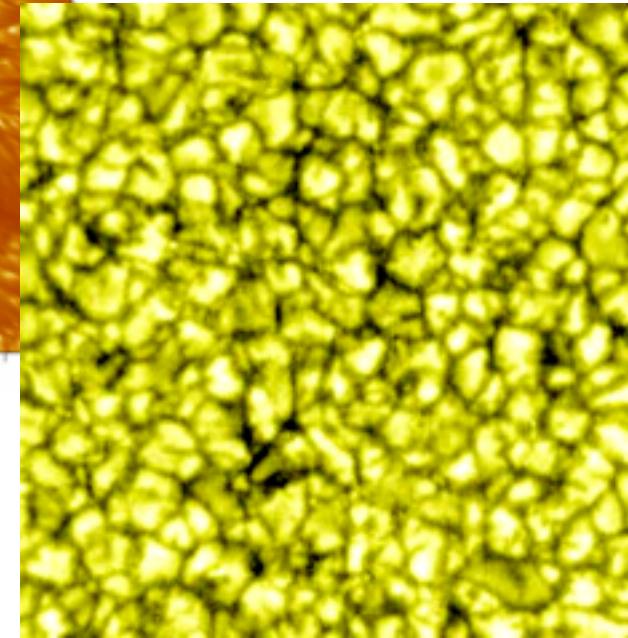
# The many scales of solar magnetism



Spots  
Size  $\sim$  10 Mm  
Life  $\sim$  days

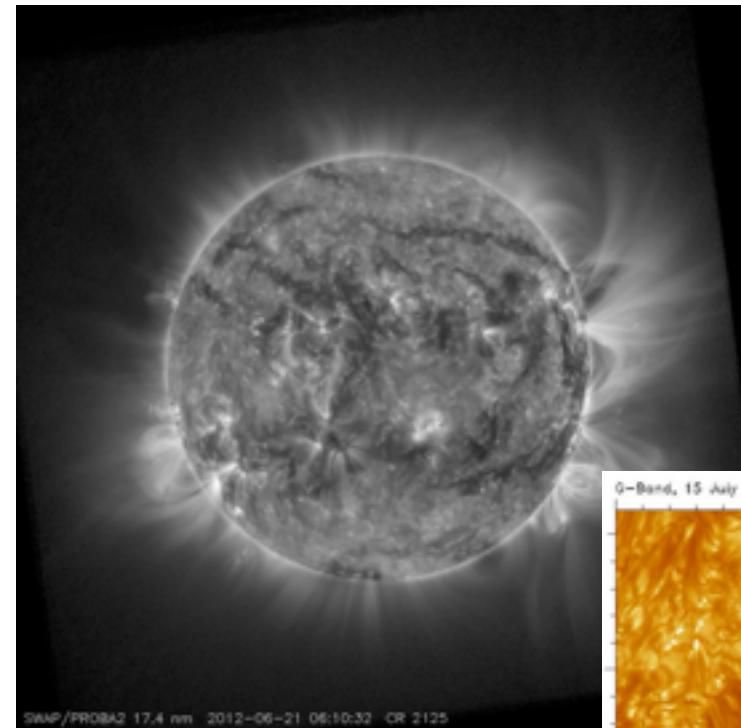


Sun  
Size  $\sim$  700 Mm  
Rotation  $\sim$  month

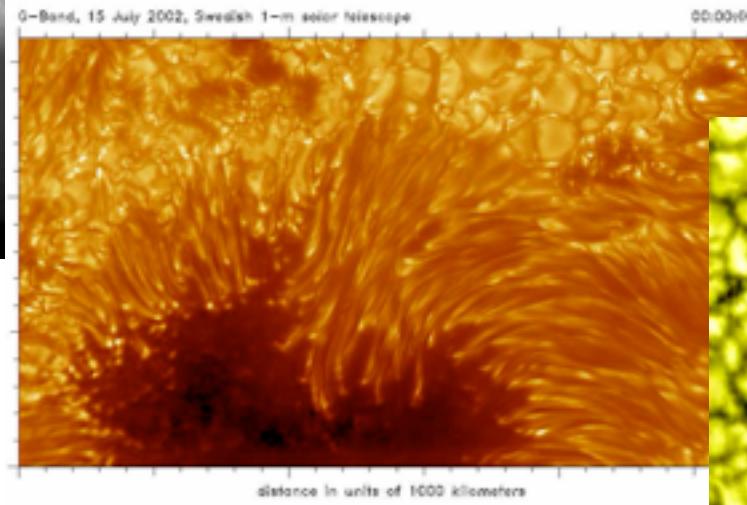


Granules  
Size  $\sim$  1 Mm  
Life  $\sim$  10 minutes

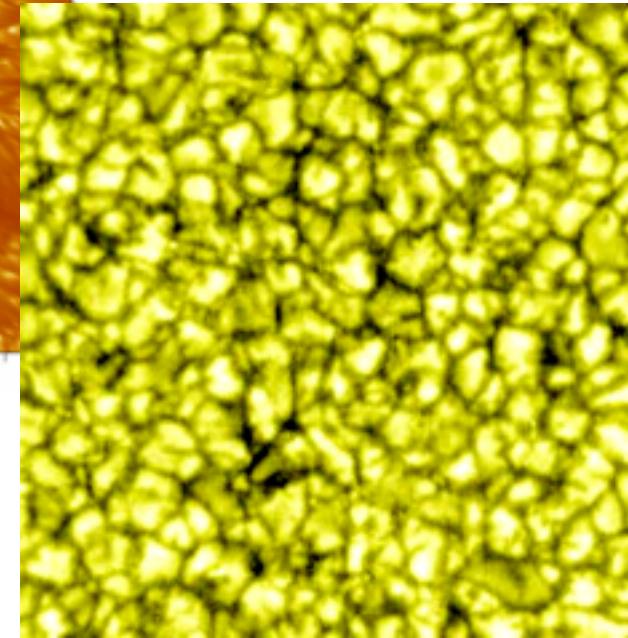
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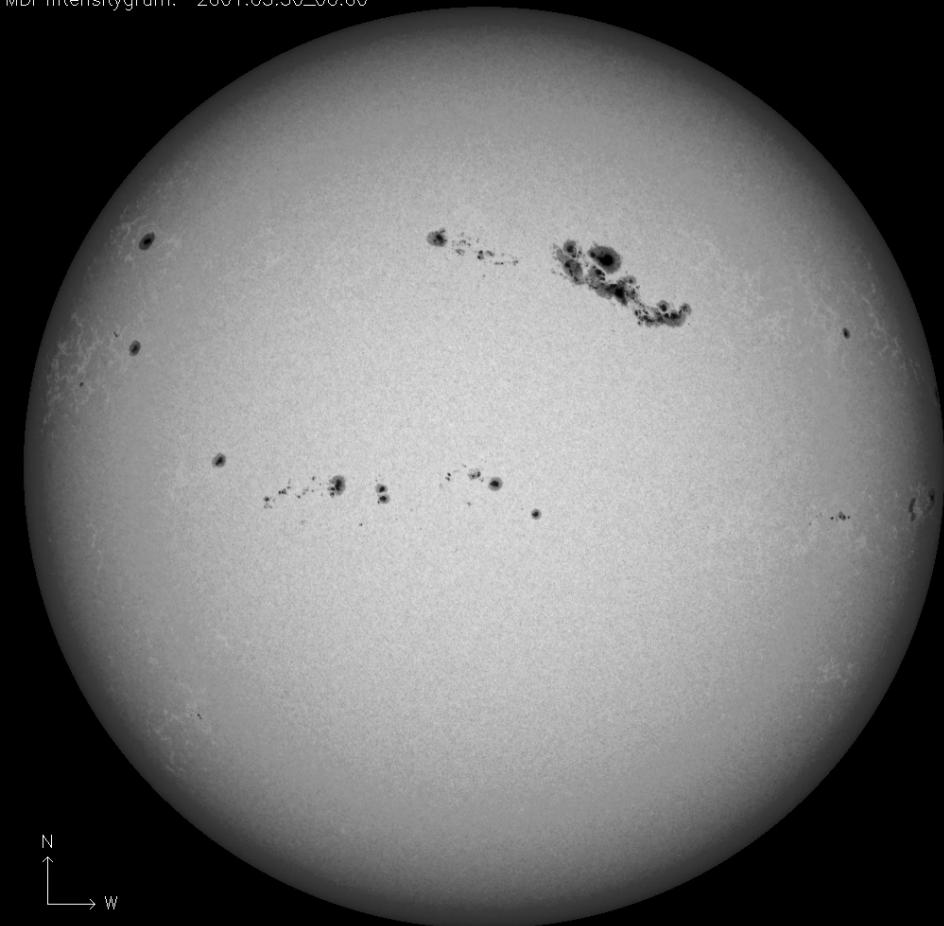
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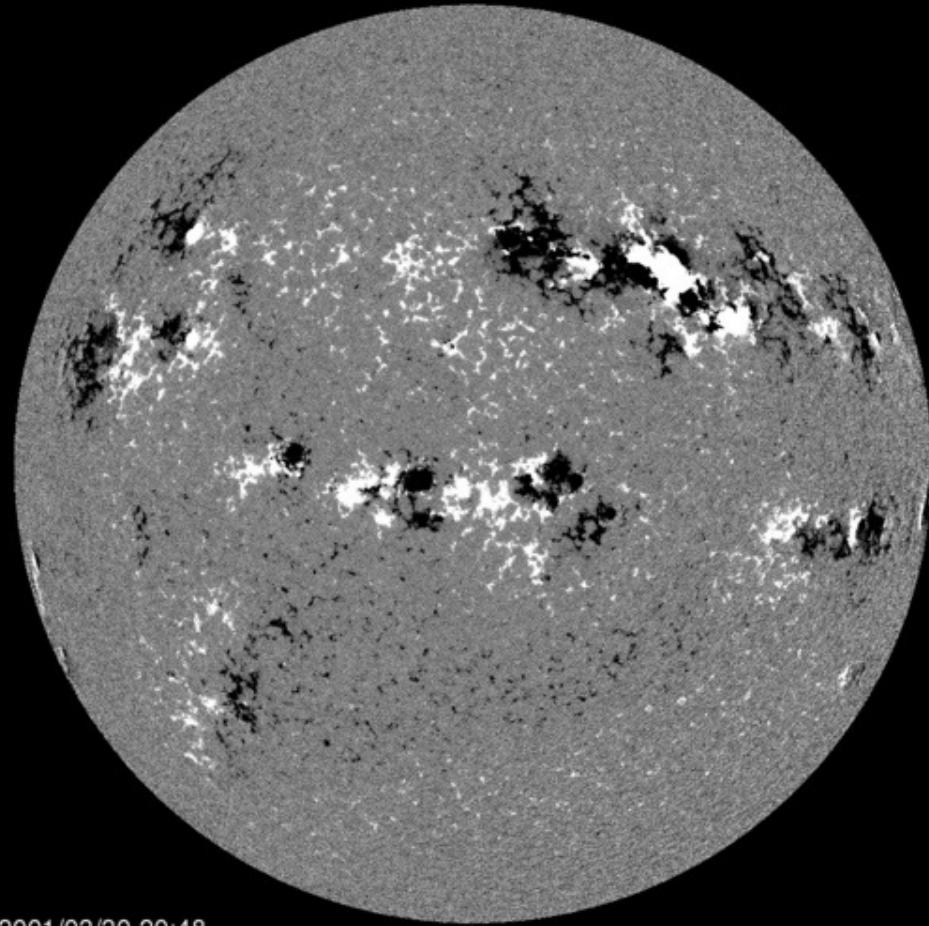
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# Sunspots and their magnetic origin

MDI Intensitygram: 2001.03.30\_00:00



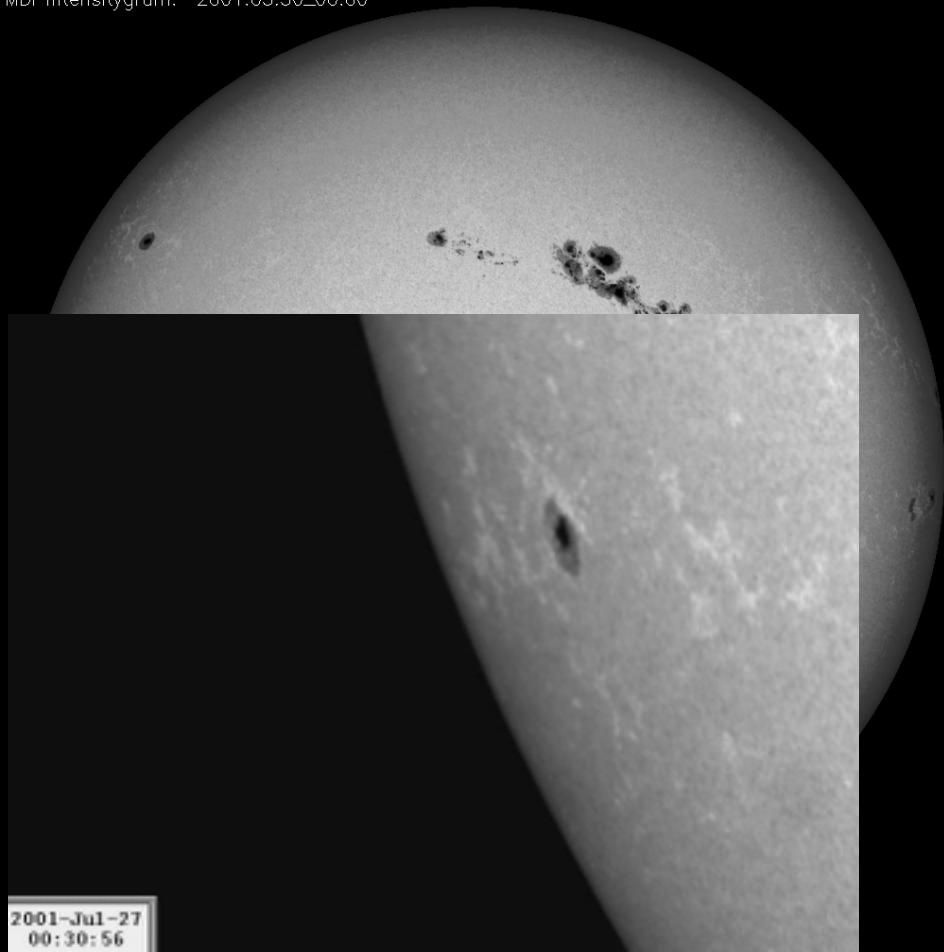
2001, maximum activity



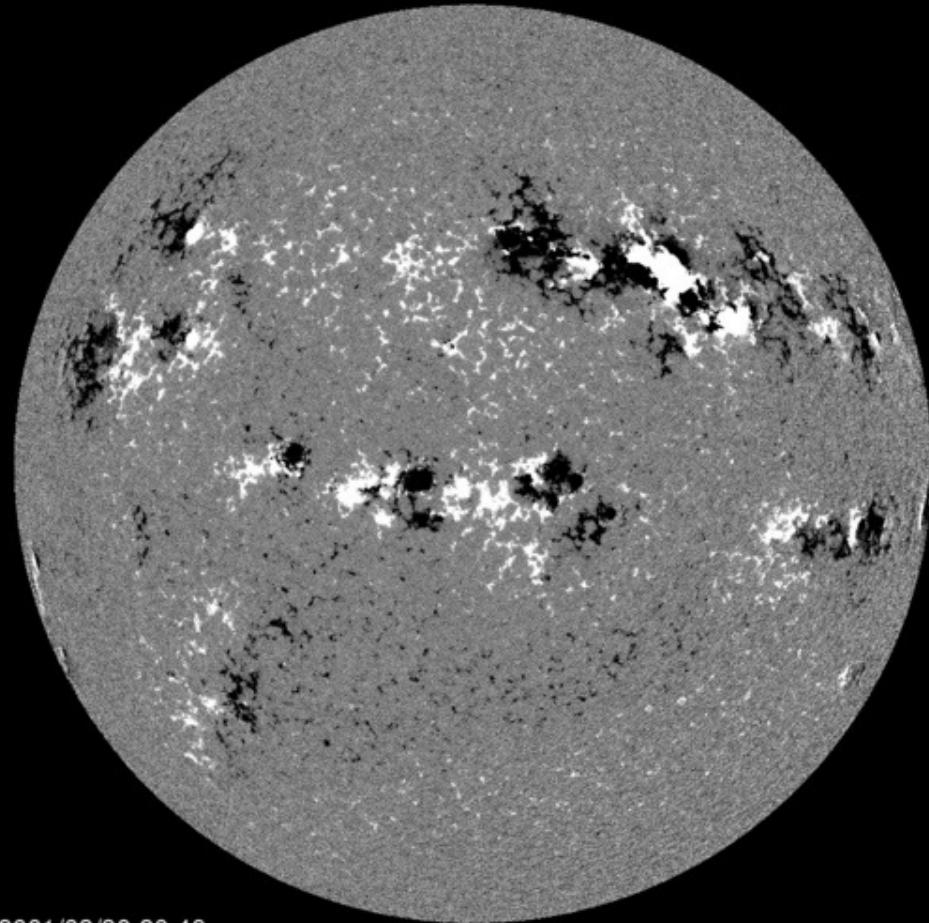
Magnetogram

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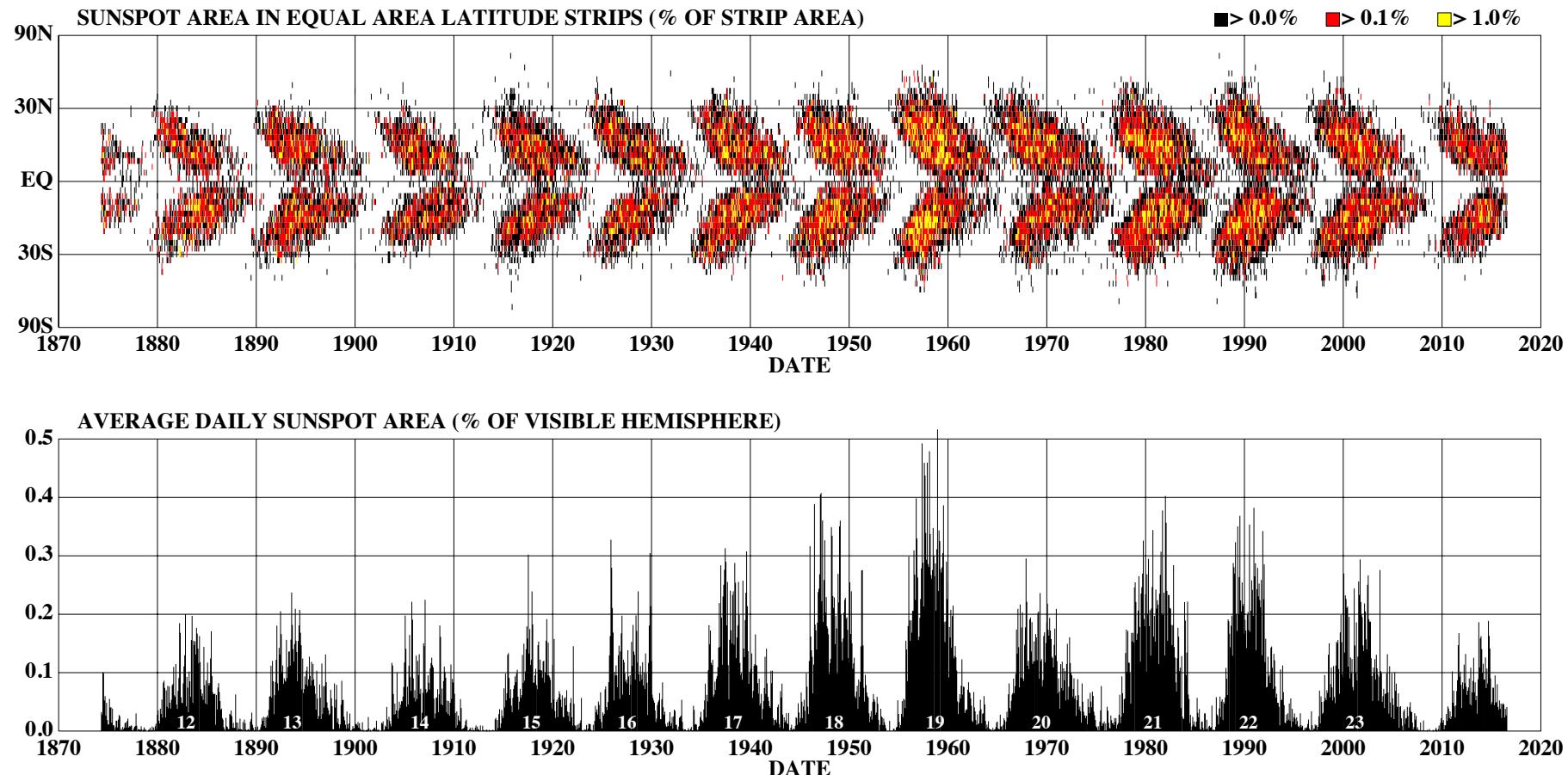
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2001/03/30 20:48

Magnetogram

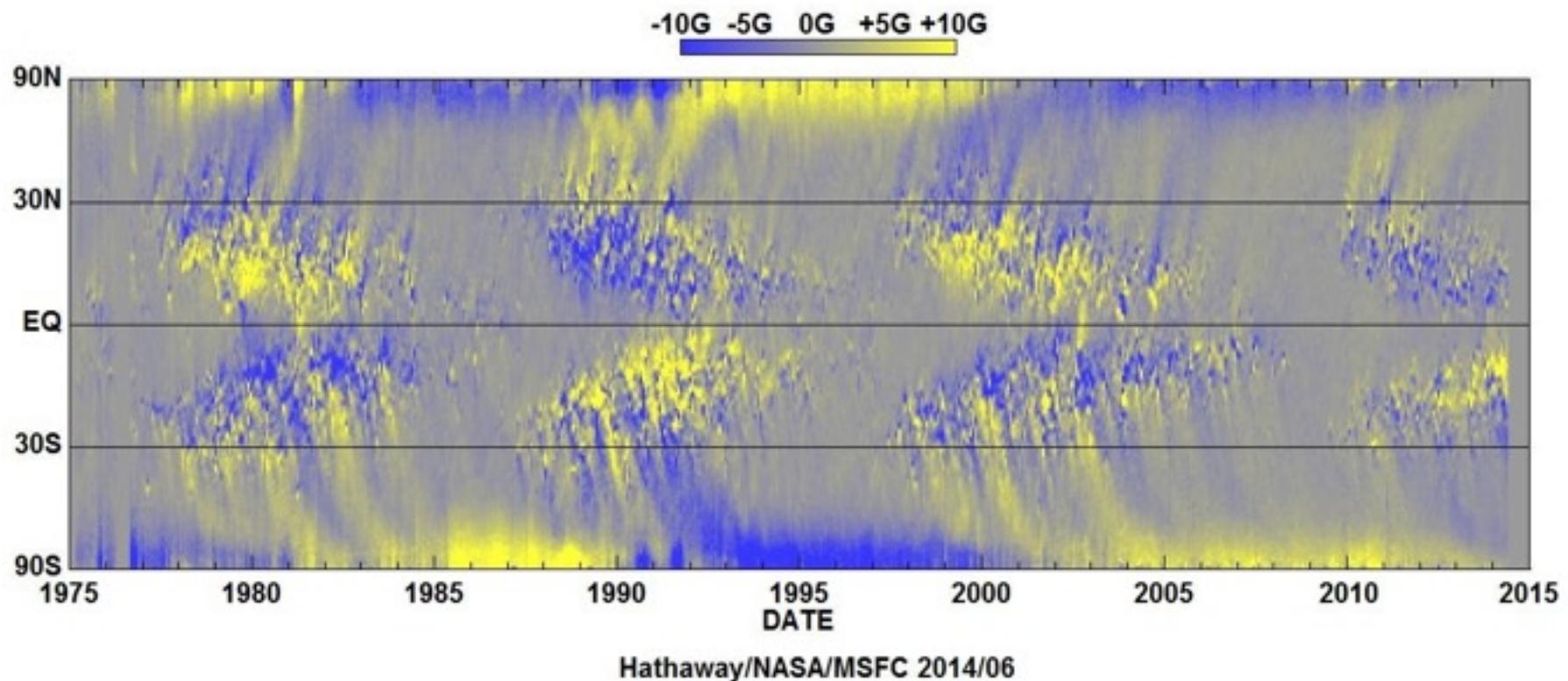
# The solar magnetic cycle: butterfly diagram



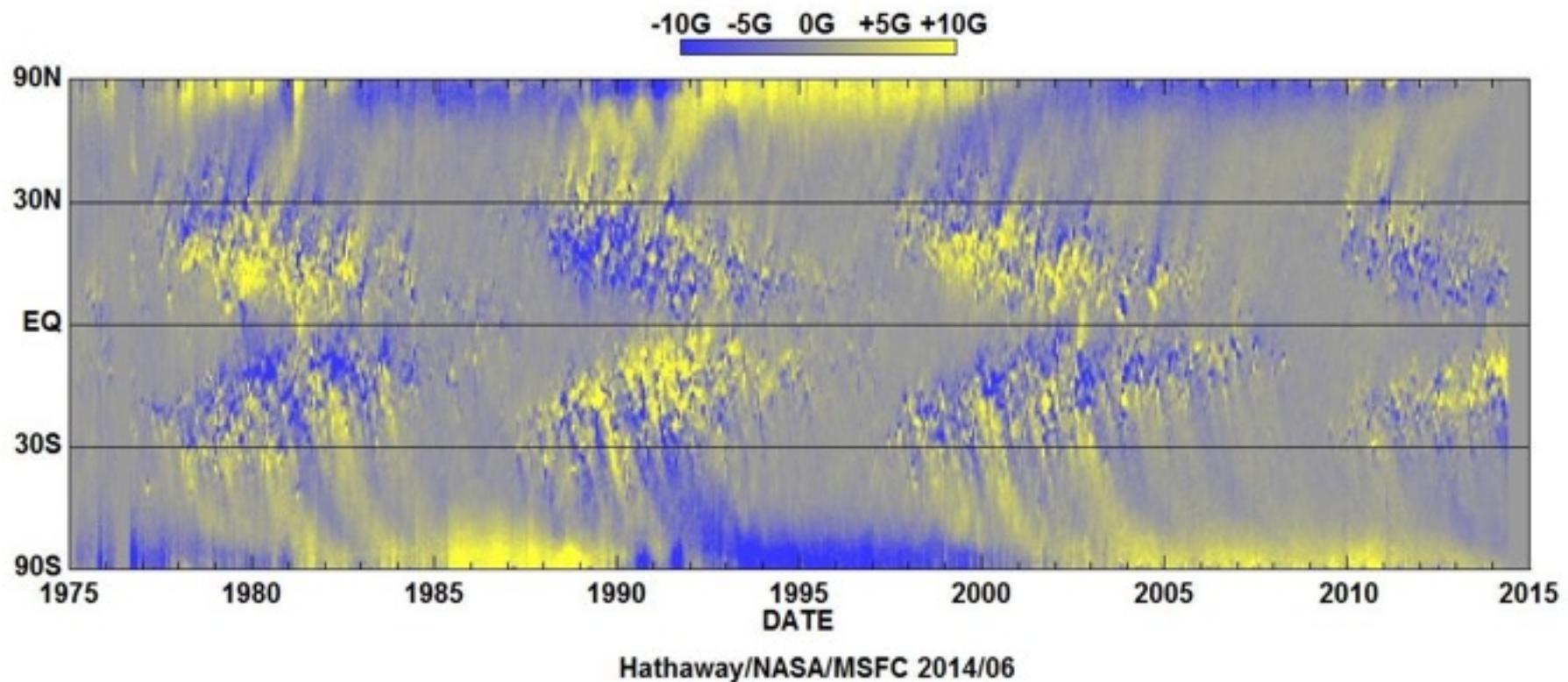
<http://solarscience.msfc.nasa.gov/images/BFLY.PDF>

HATHAWAY NASA/ARC 2016/10

# The solar magnetic cycle as seen from the surface mag. field



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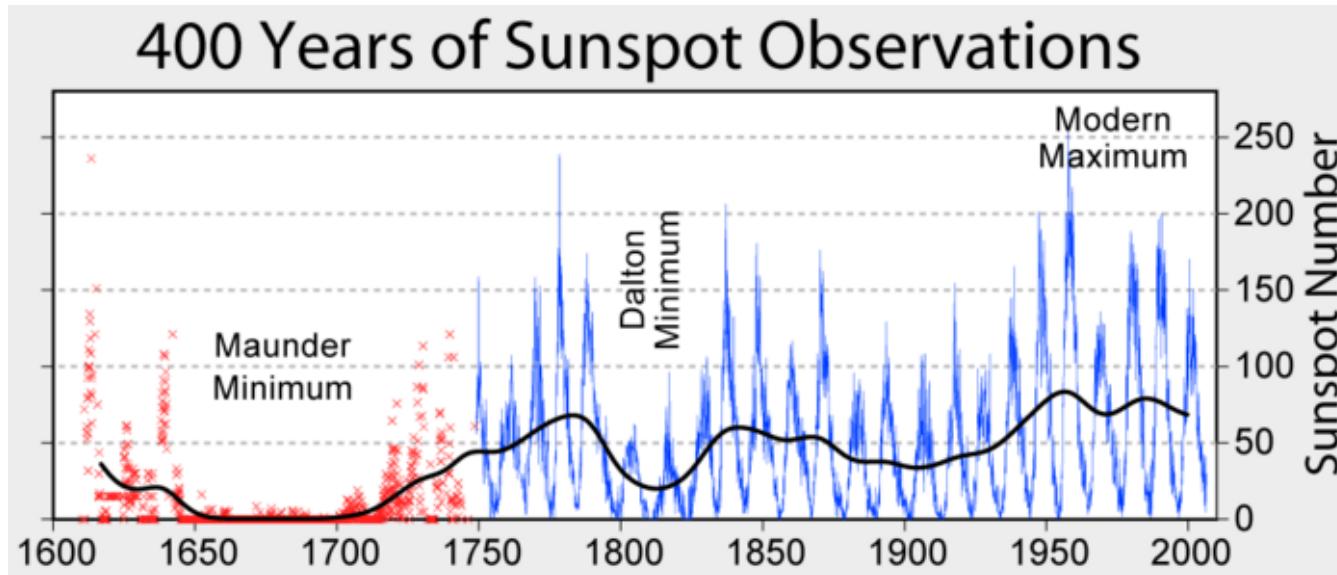


Why 11/22 years?

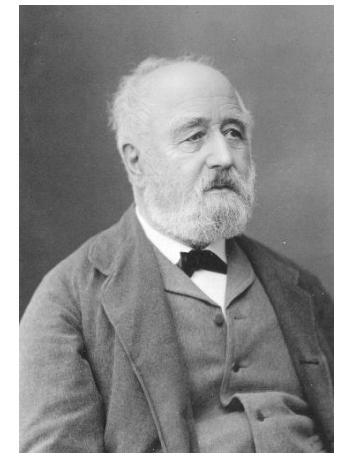
# Historical records of the solar cycle

Discovered by H. Schwabe in 1843 after 17 years of observations of the Sun looking for planet Vulcan

Heinrich Schwabe

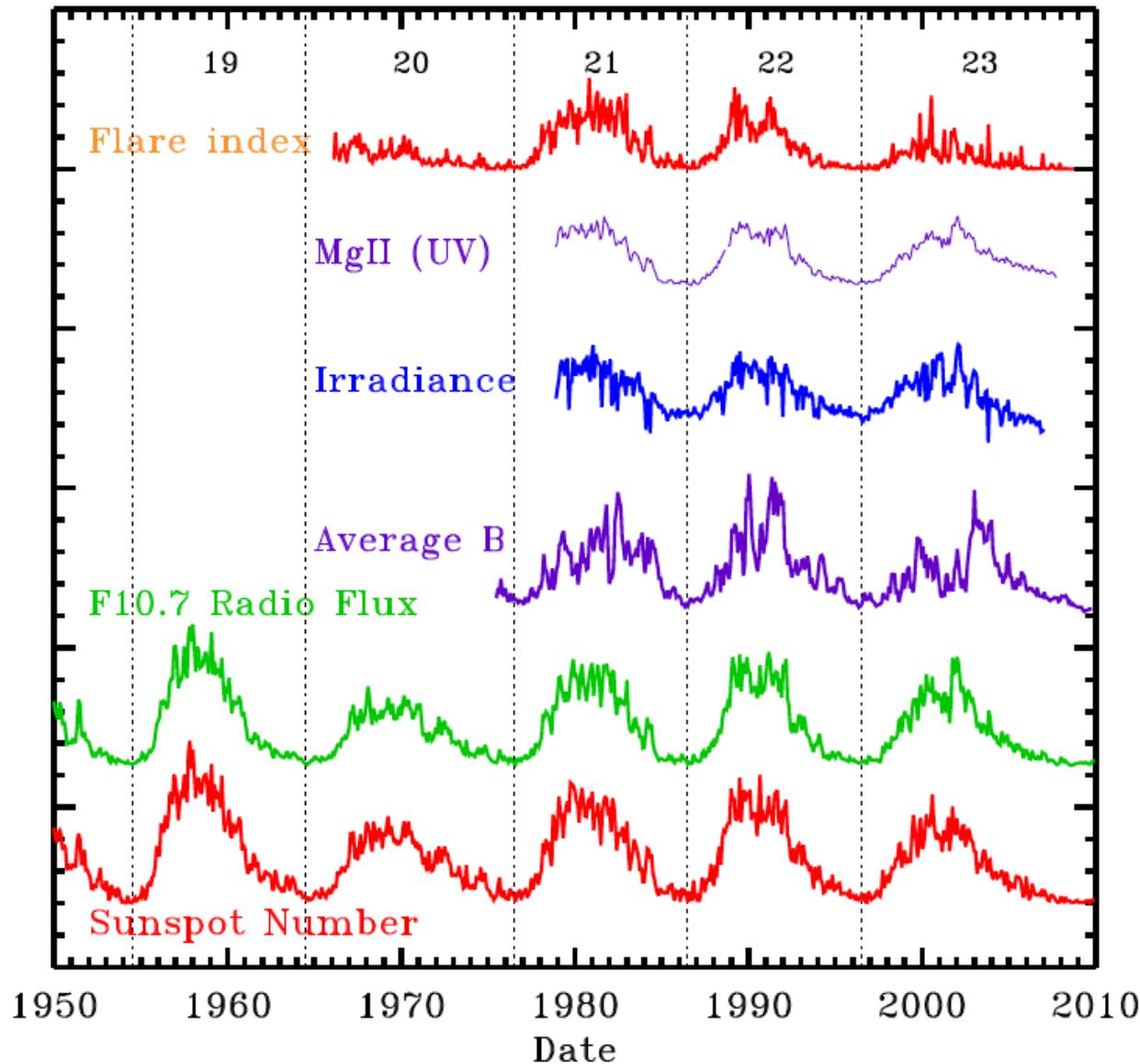


The period of the sunspot cycle is  $\sim 11$  years, but its amplitude fluctuates significantly from one cycle to the other



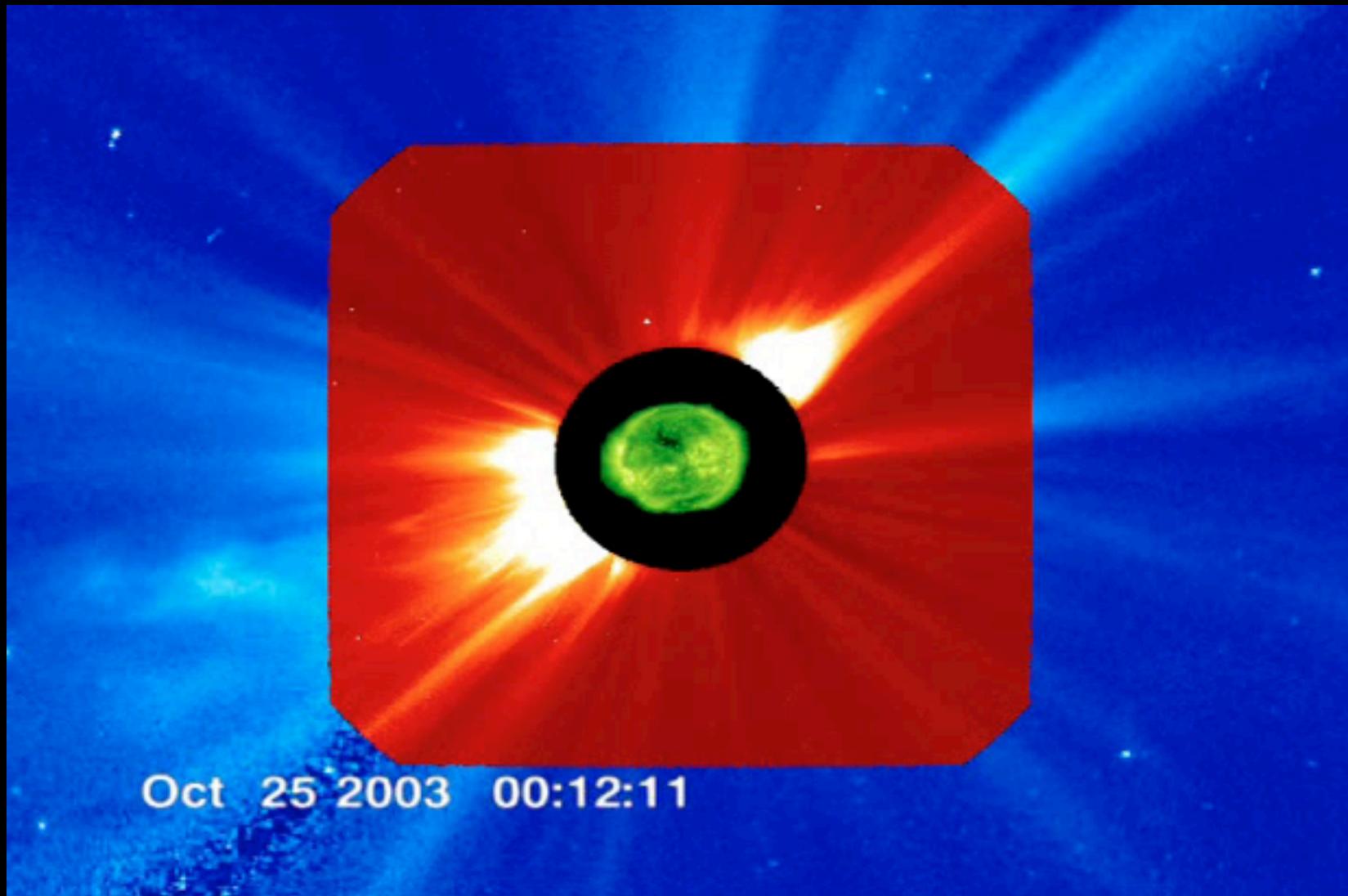
Rudolf Wolf

# One cycle to rule them all



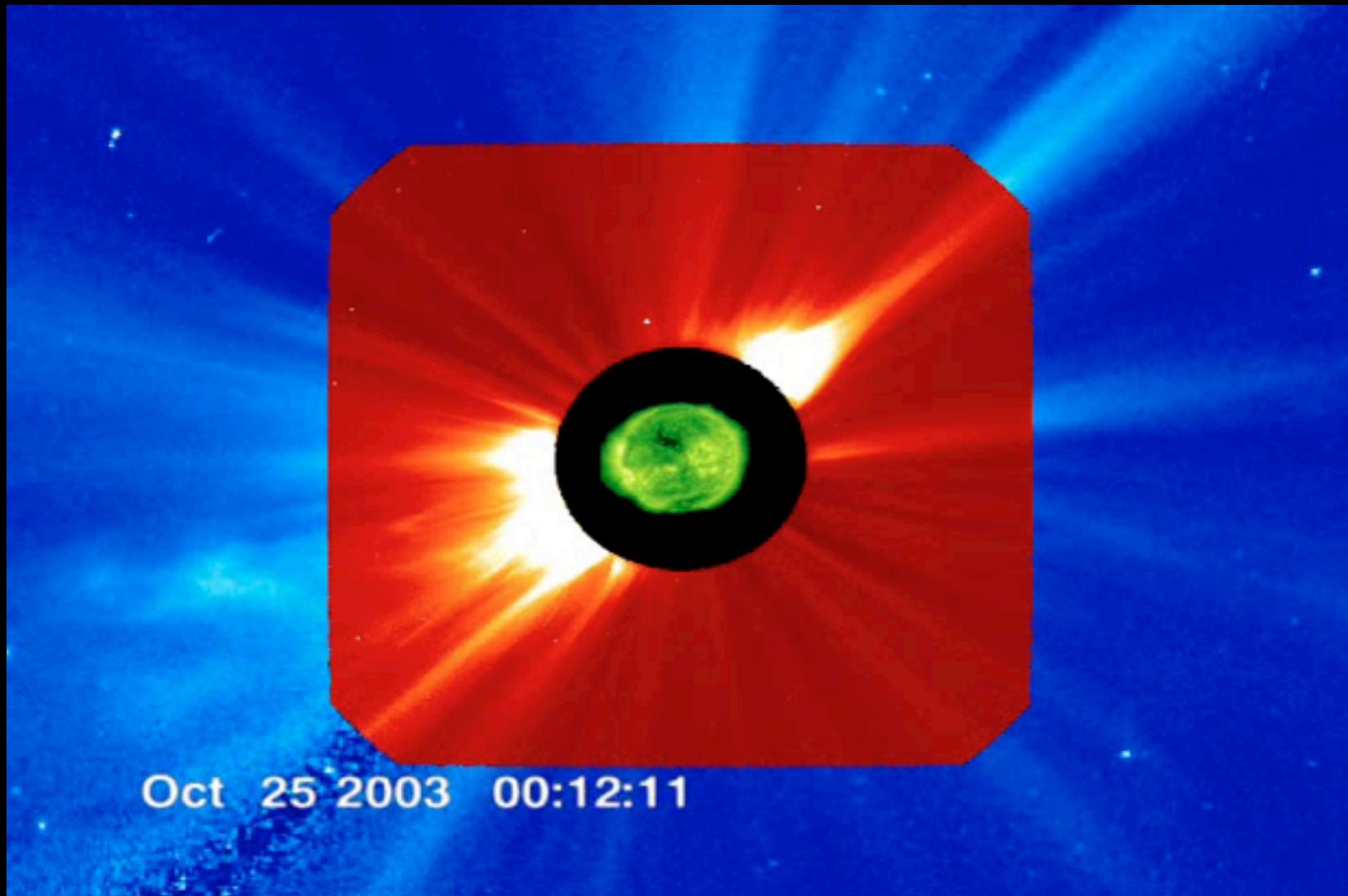
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## Eruptive events from magnetic active regions



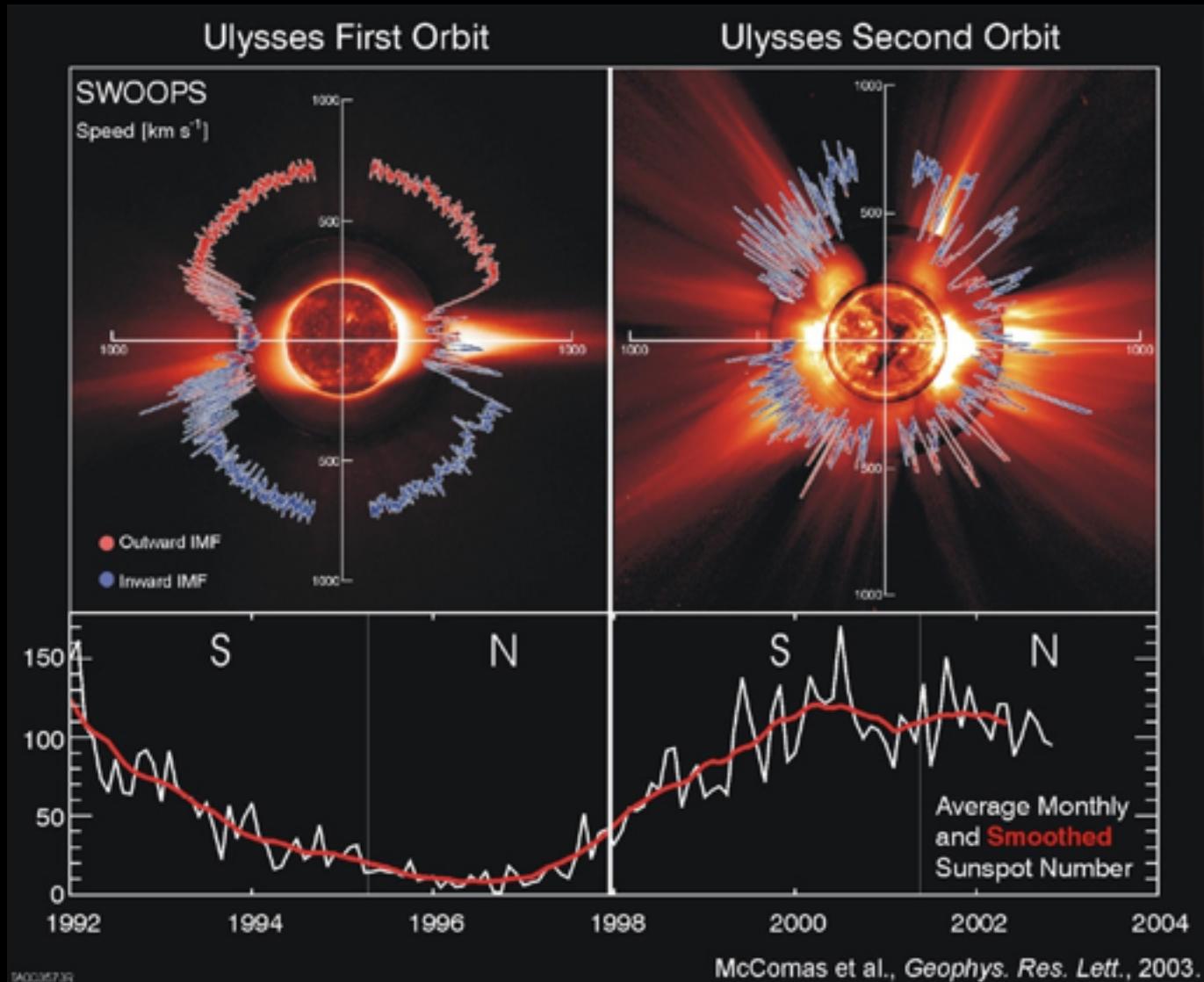
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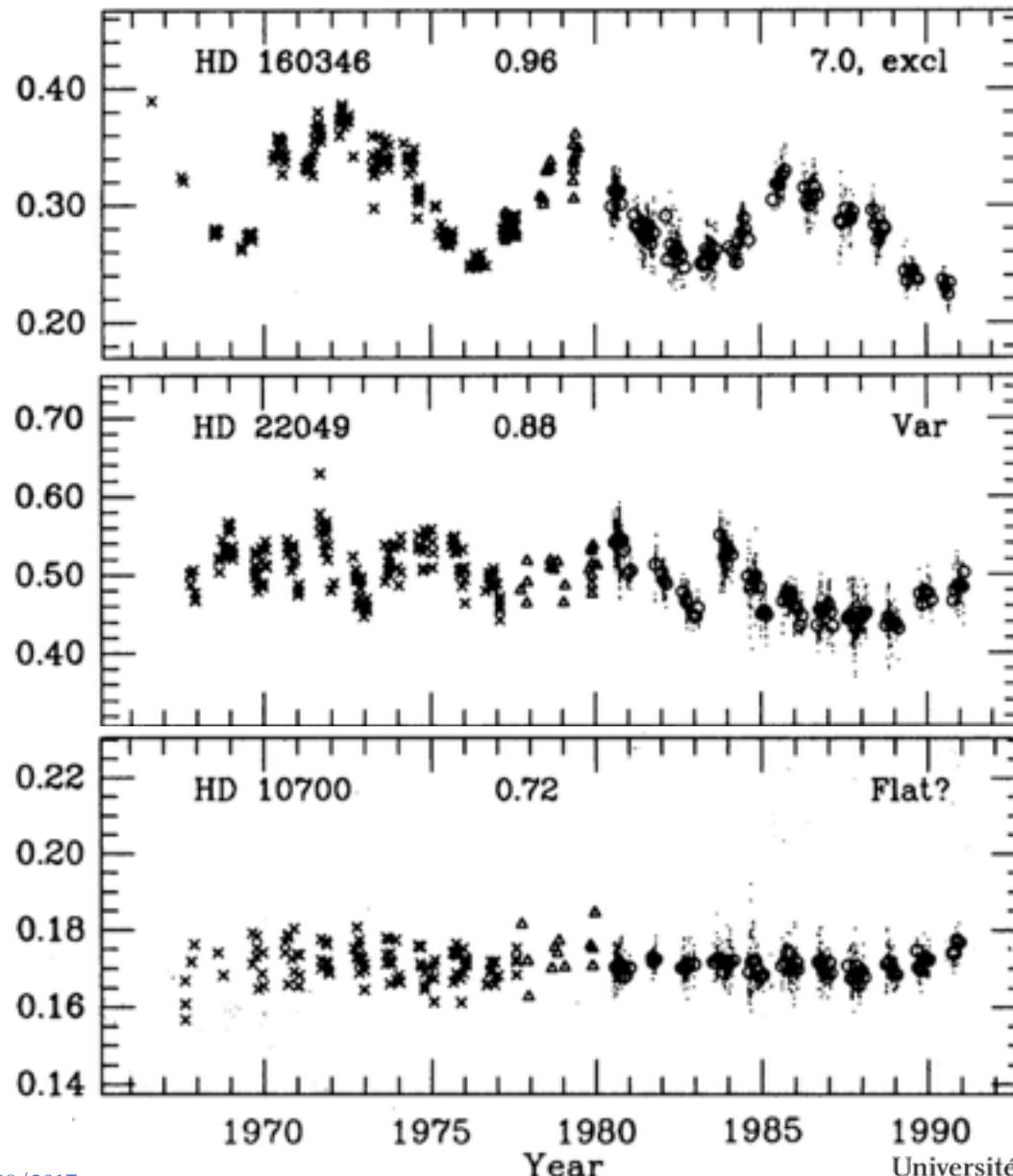


# Solar cycle and the solar wind

## Long term modulations

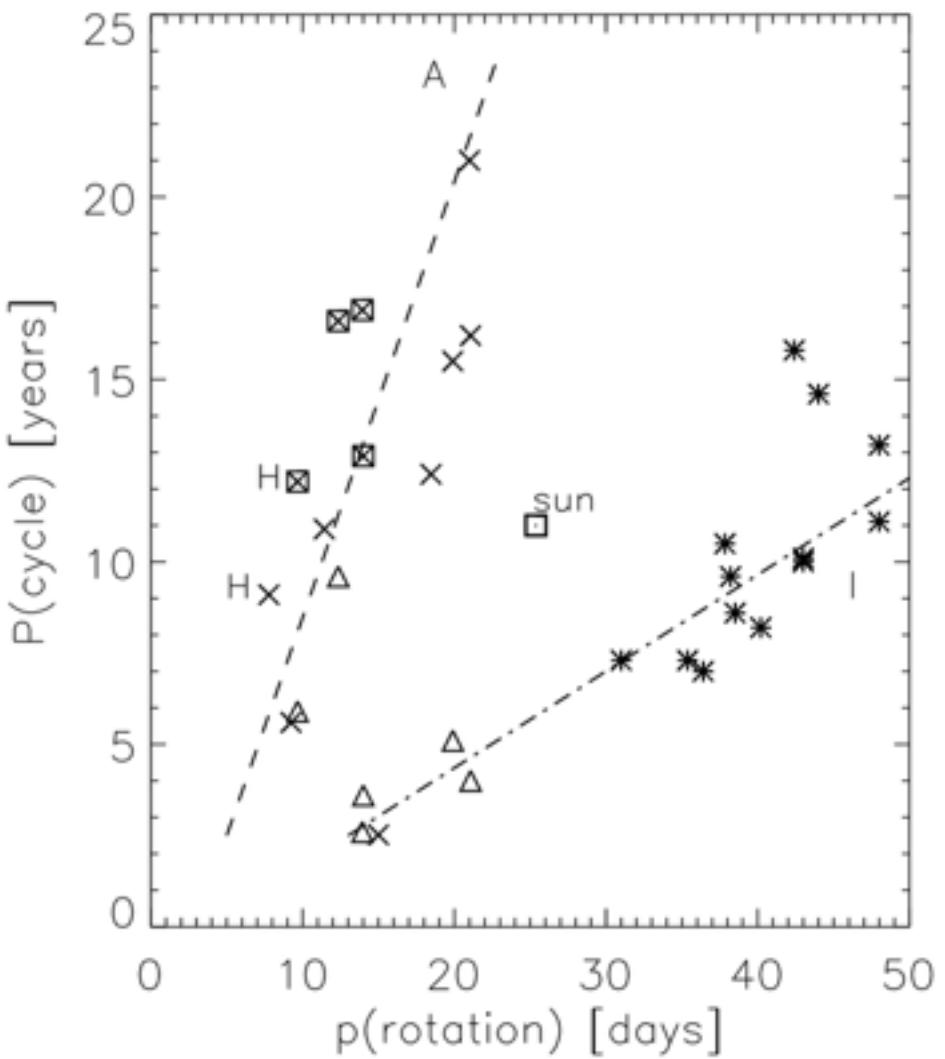


# Magnetic activity cycles on other stars from Call H&K



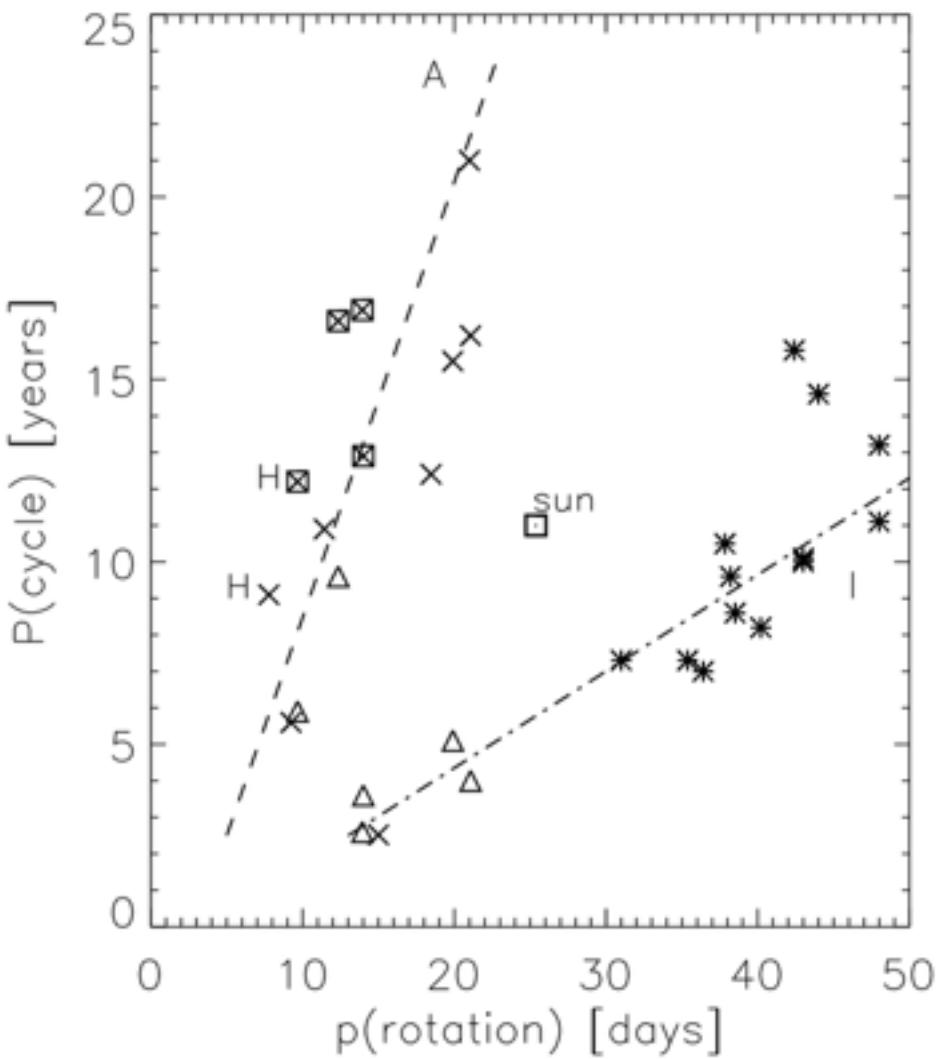
[Baliunas+, ApJ 1995]

# What sets the cycle period of star?

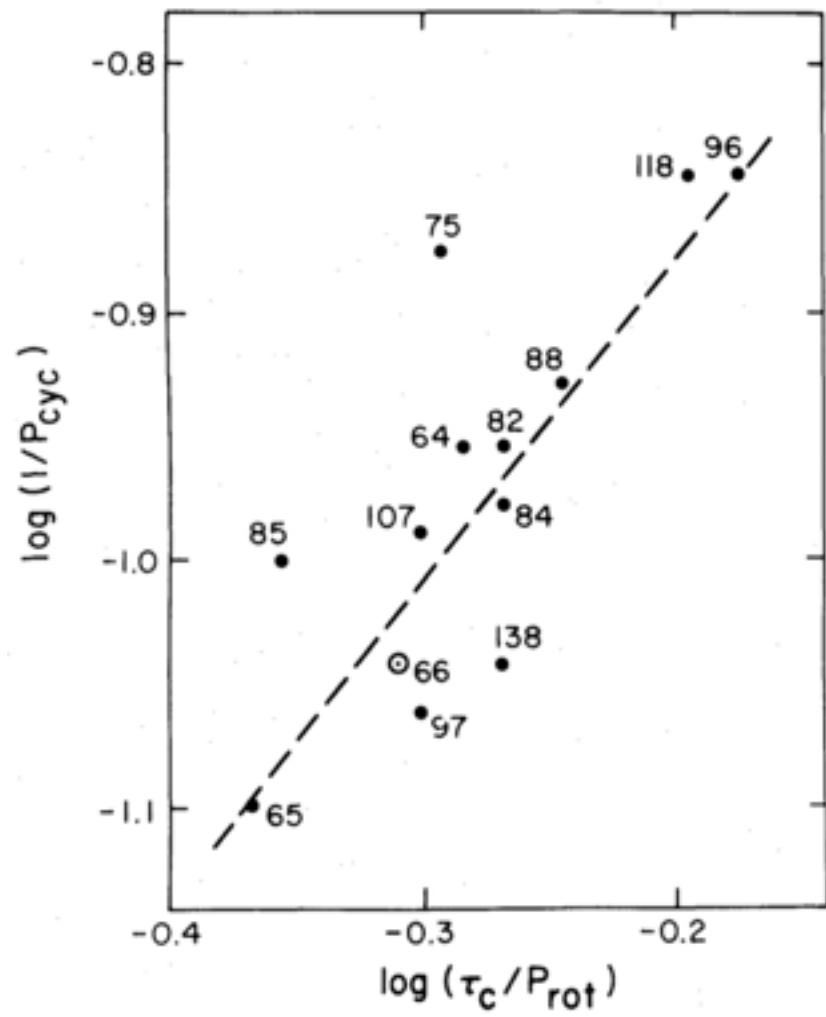


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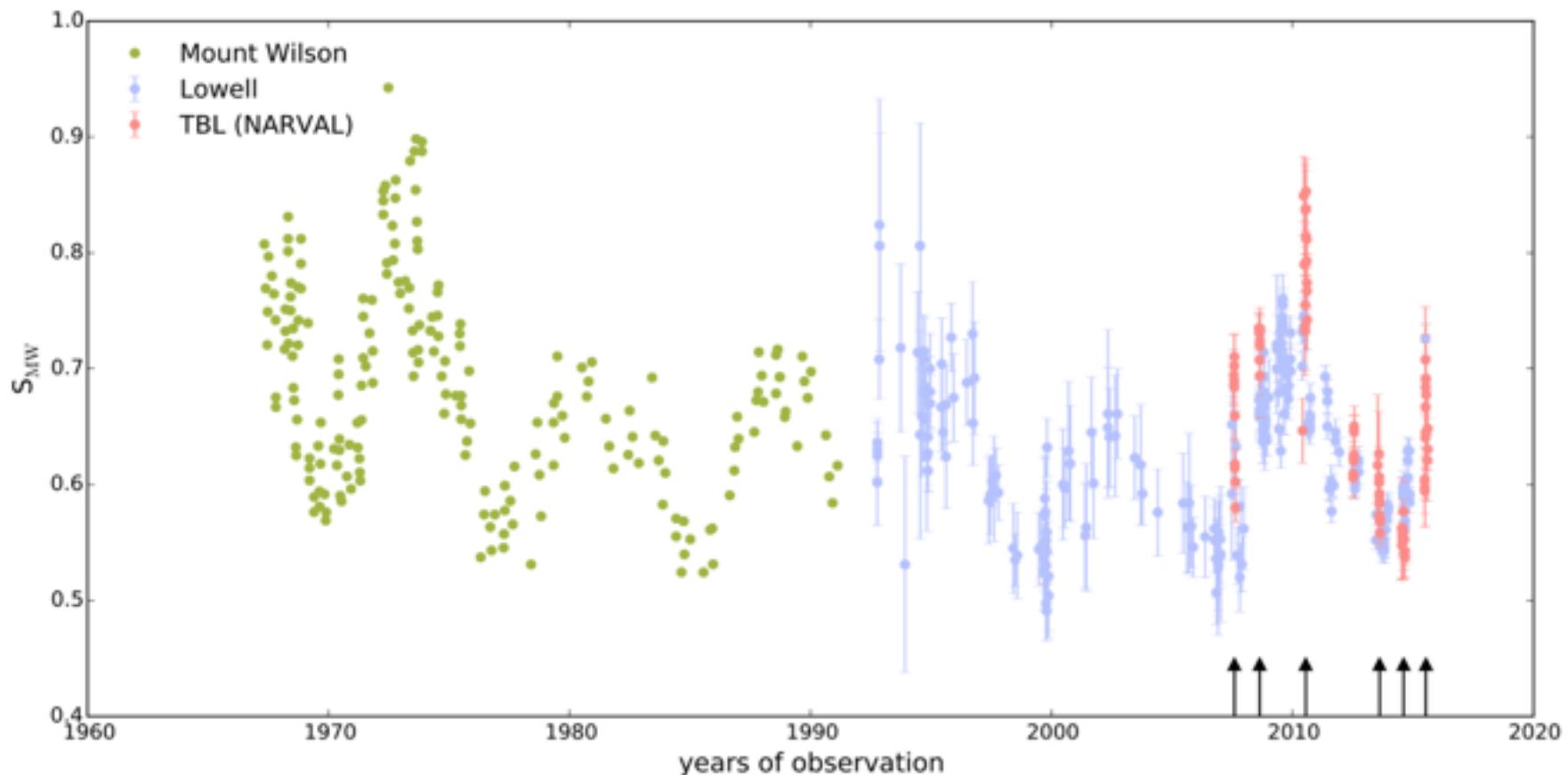


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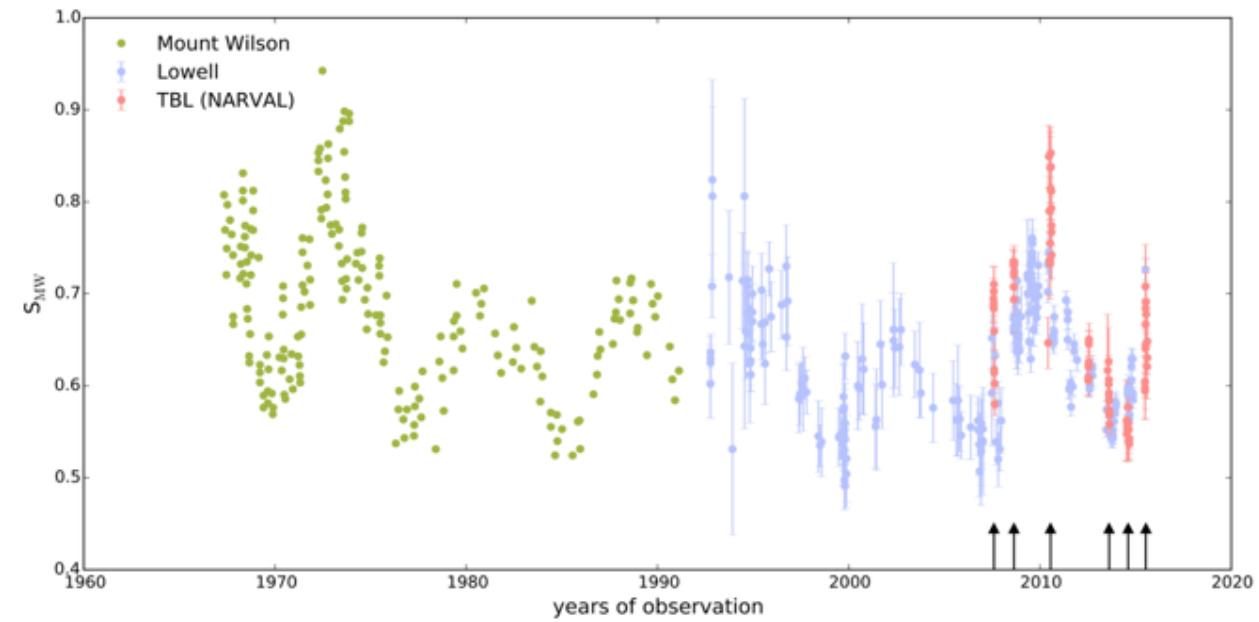


[Noyes+, ApJ 1984]

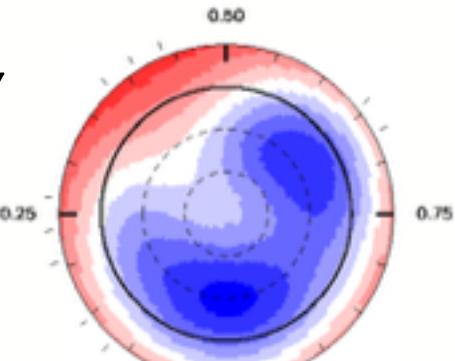
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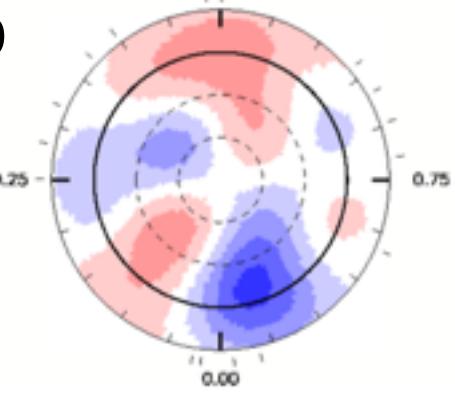
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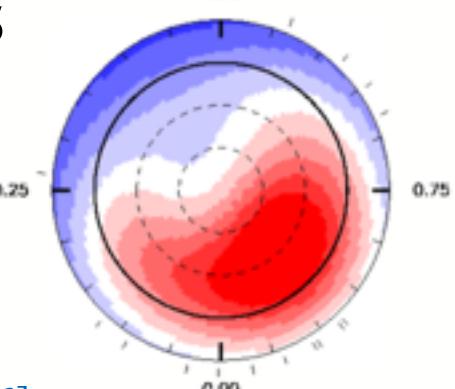
2007



2010



2015



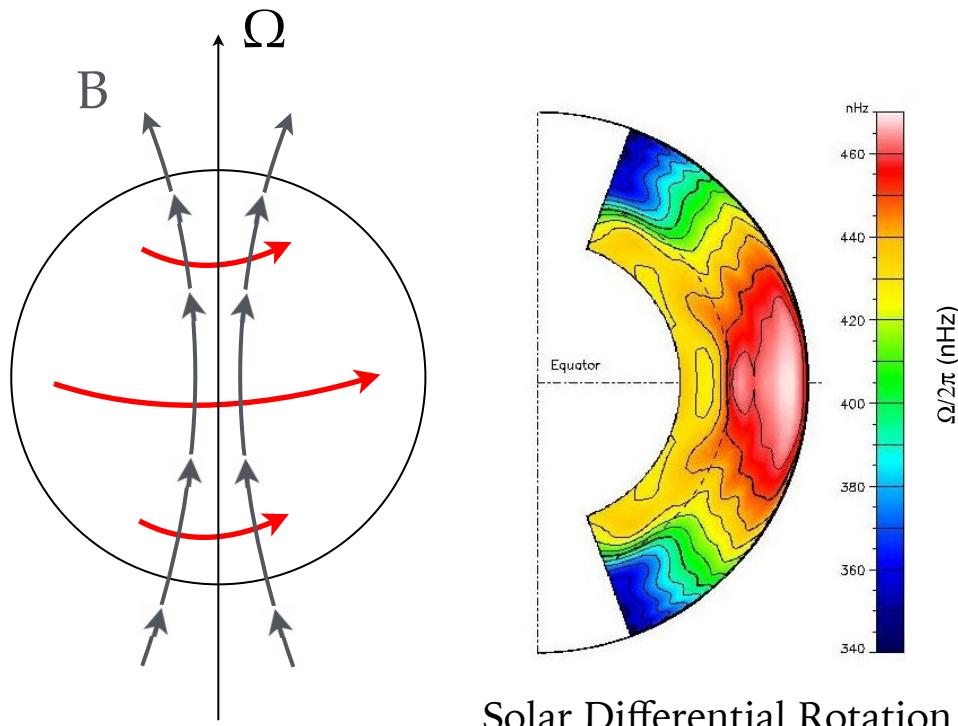
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Cowling's theorem: 3D axisymmetric flows cannot sustain a dynamo action

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« Omega » effect: differential axisymmetric rotation easily convert poloidal field to toroidal field

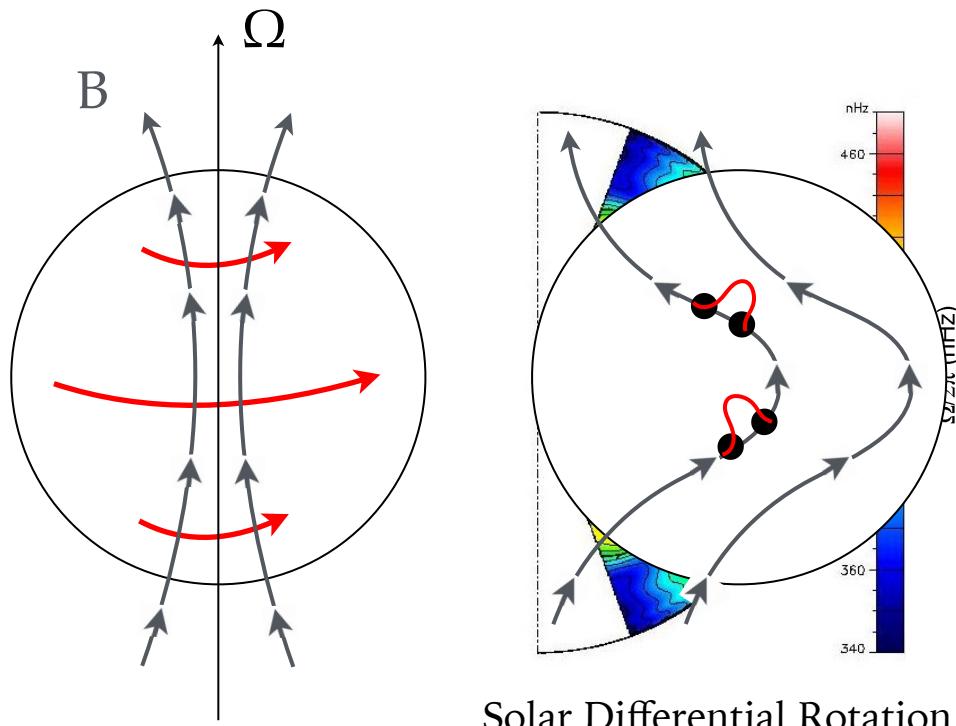


Solar Differential Rotation

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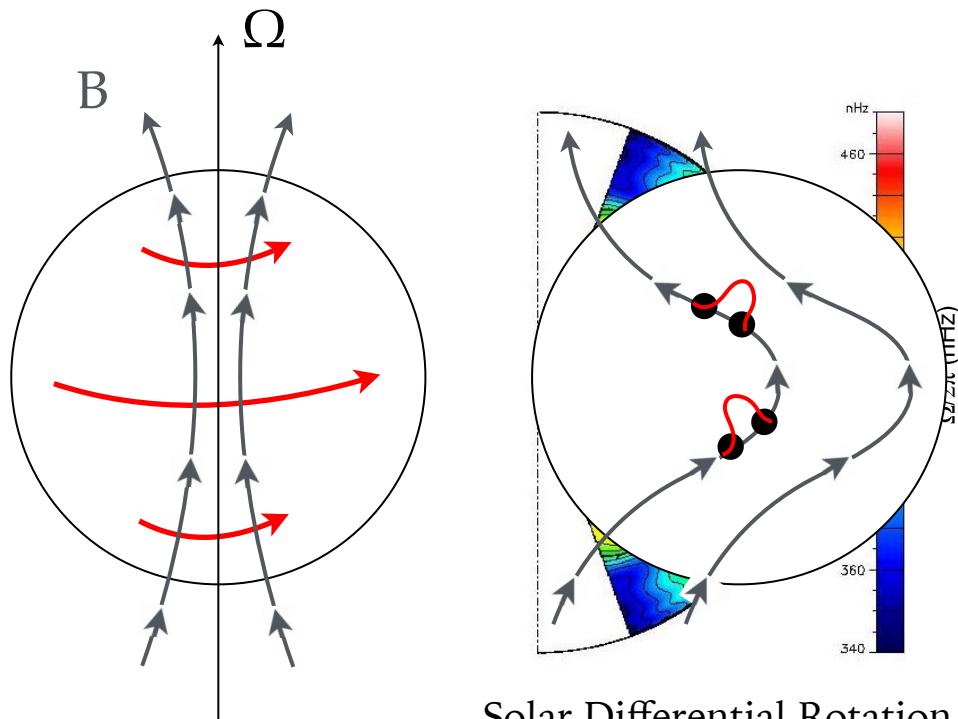
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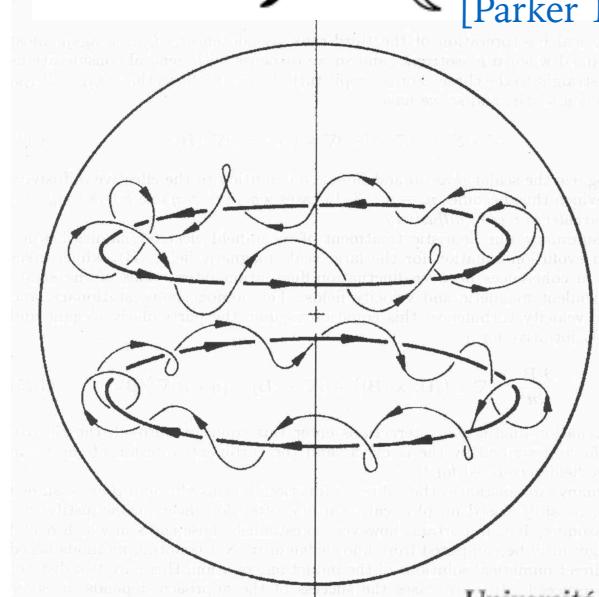
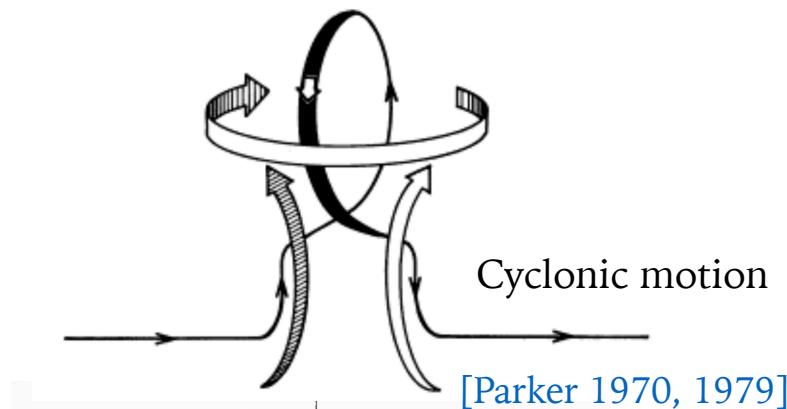
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Solar Differential Rotation

Toroidal to poloidal conversion



# Elements of dynamo theory for stars: mean fields models

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) - \nabla \times (\eta_m \nabla \times \mathbf{B}) \quad \textit{Induction equation}$$

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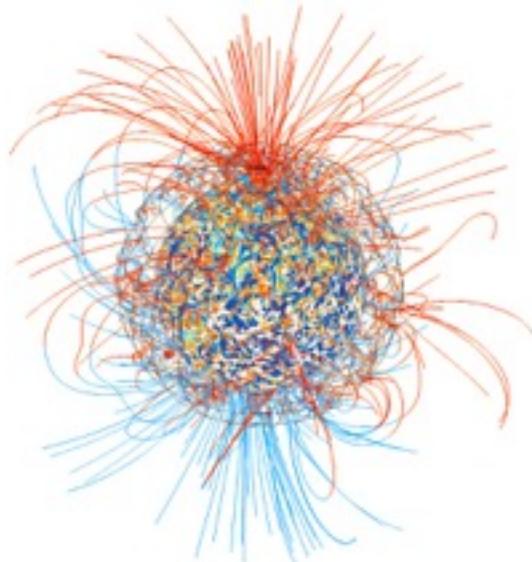
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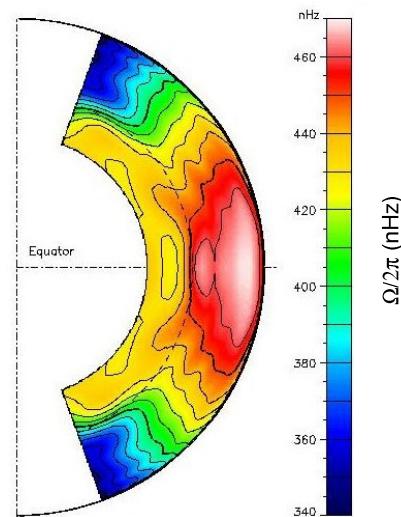
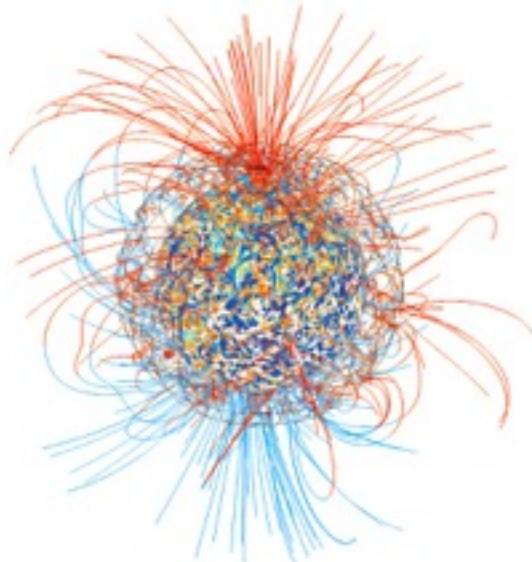


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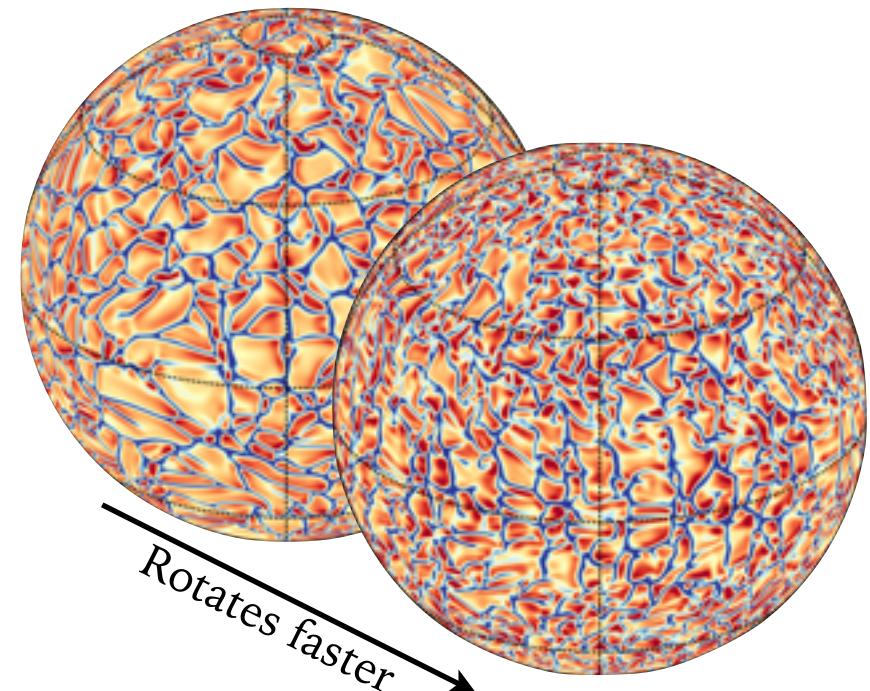
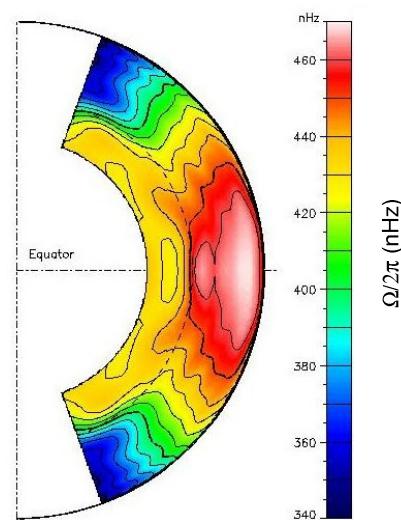
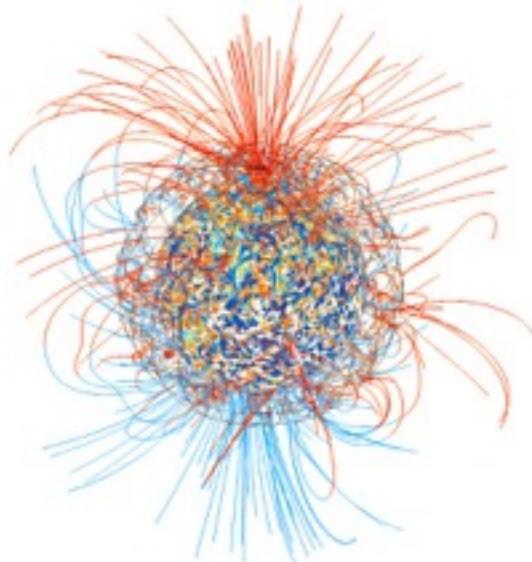
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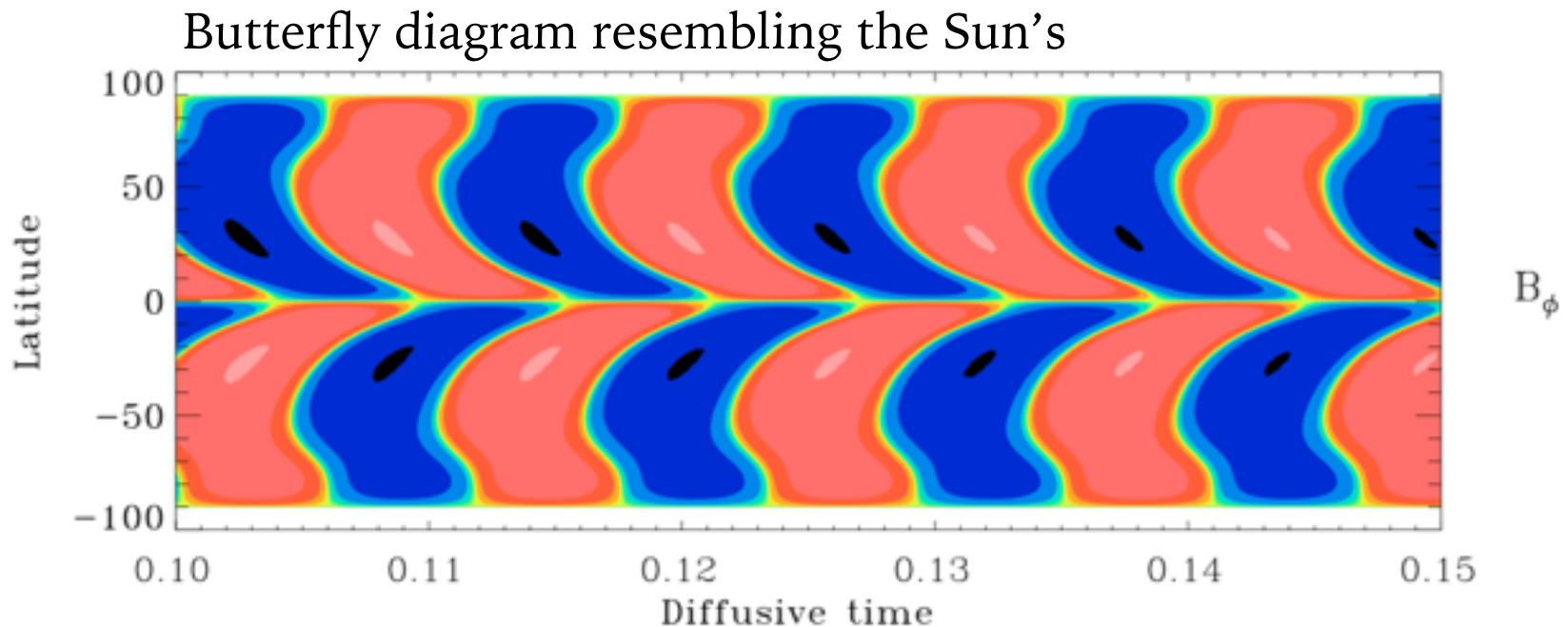
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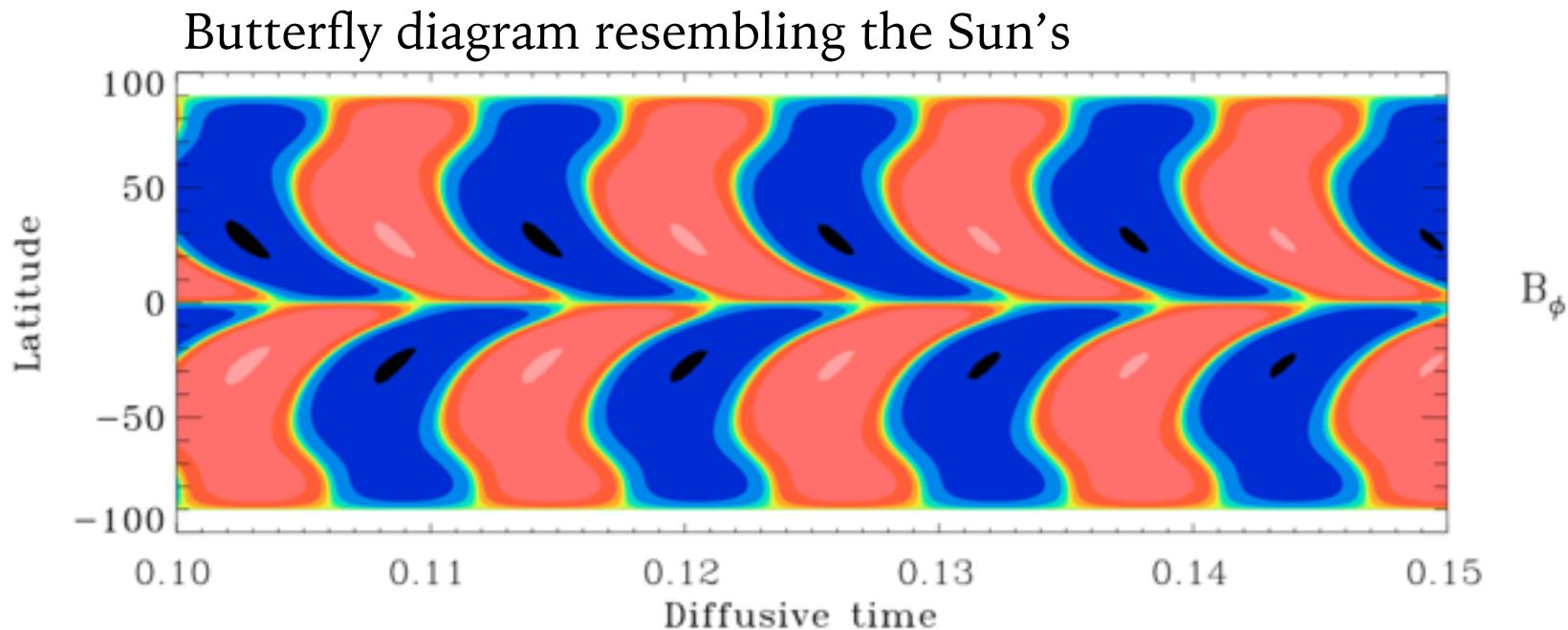
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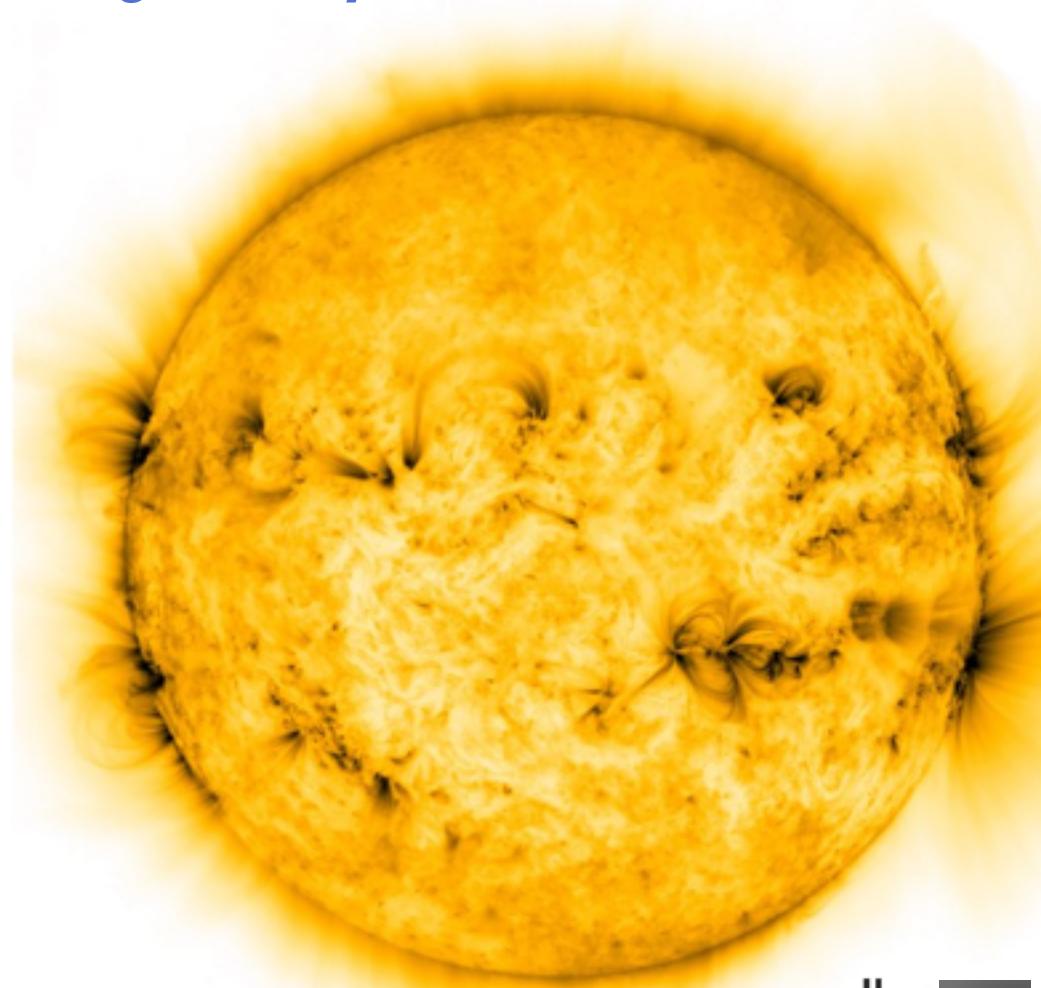
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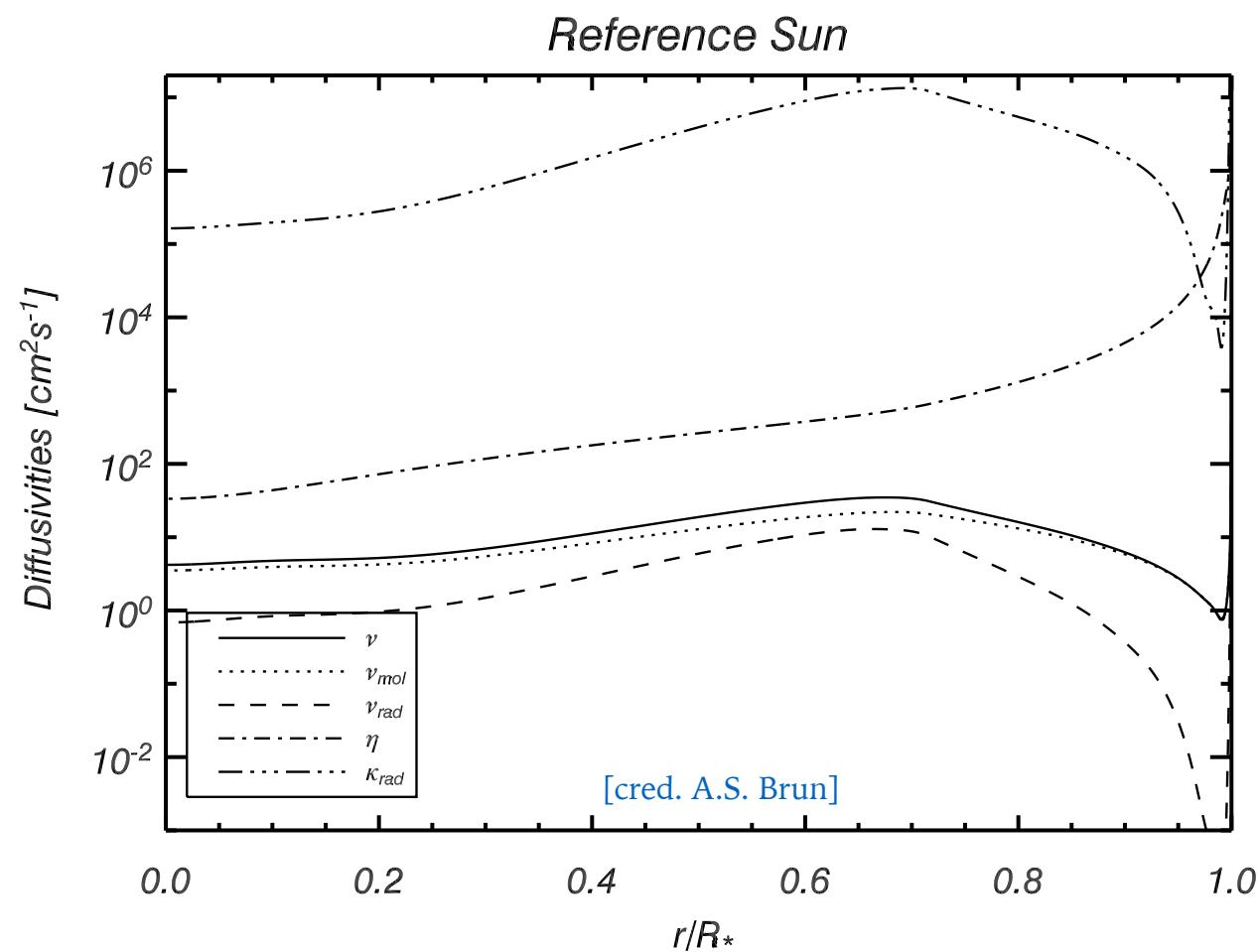


BUT still several ad-hoc parameters tweaked to reproduce the Sun

# Ab-initio modelling of stellar magnetic cycles

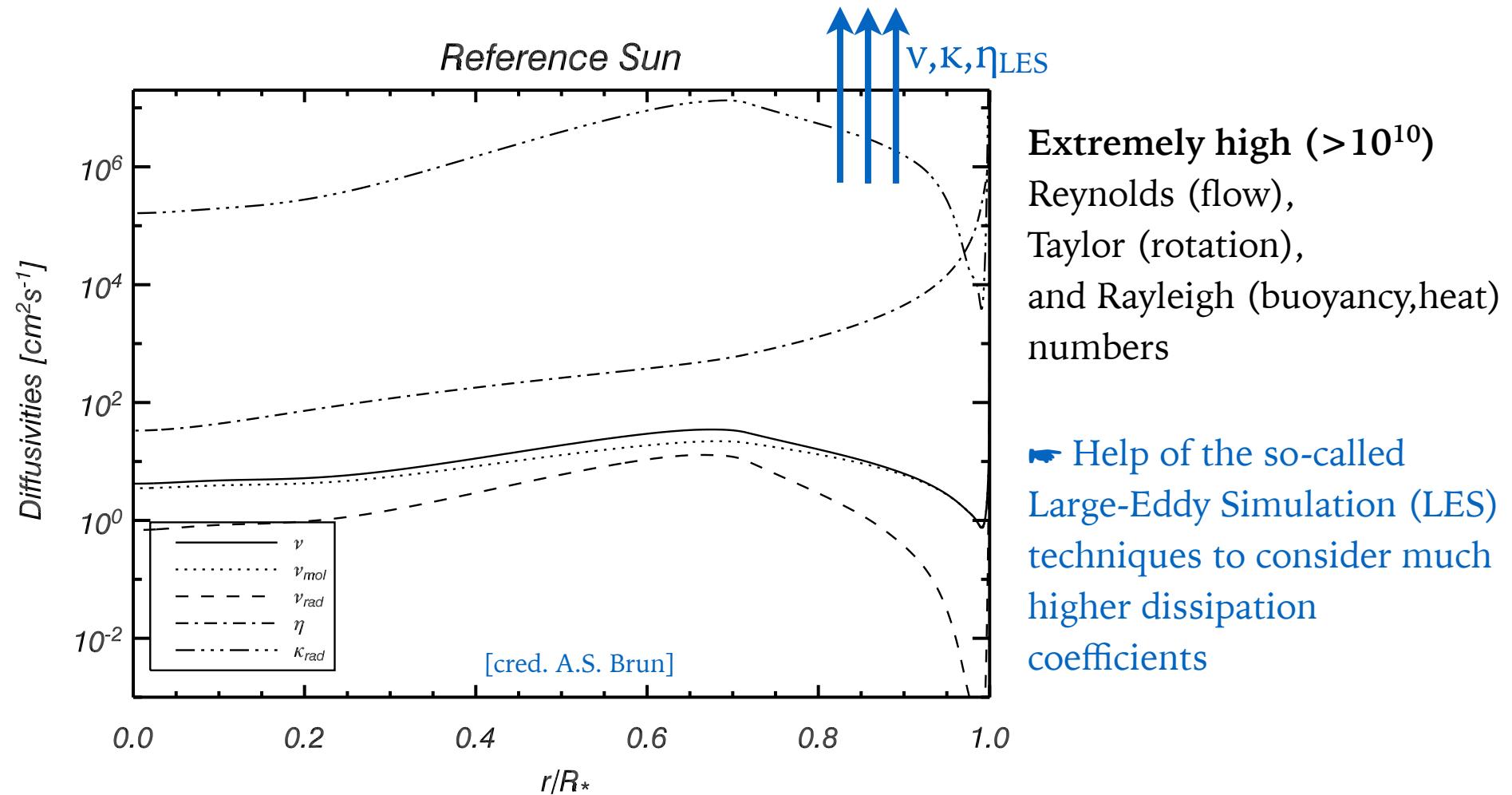


# Challenge: ab-initio models of stellar convective dynamos



Extremely high ( $>10^{10}$ )  
Reynolds (flow),  
Taylor (rotation),  
and Rayleigh (buoyancy,heat)  
numbers

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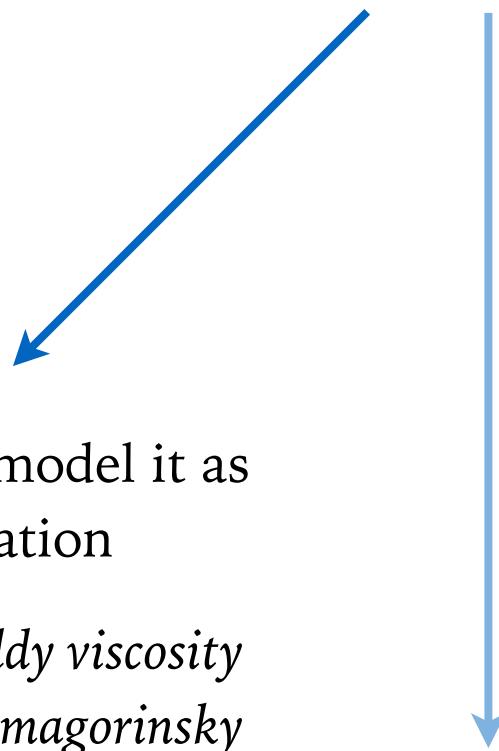
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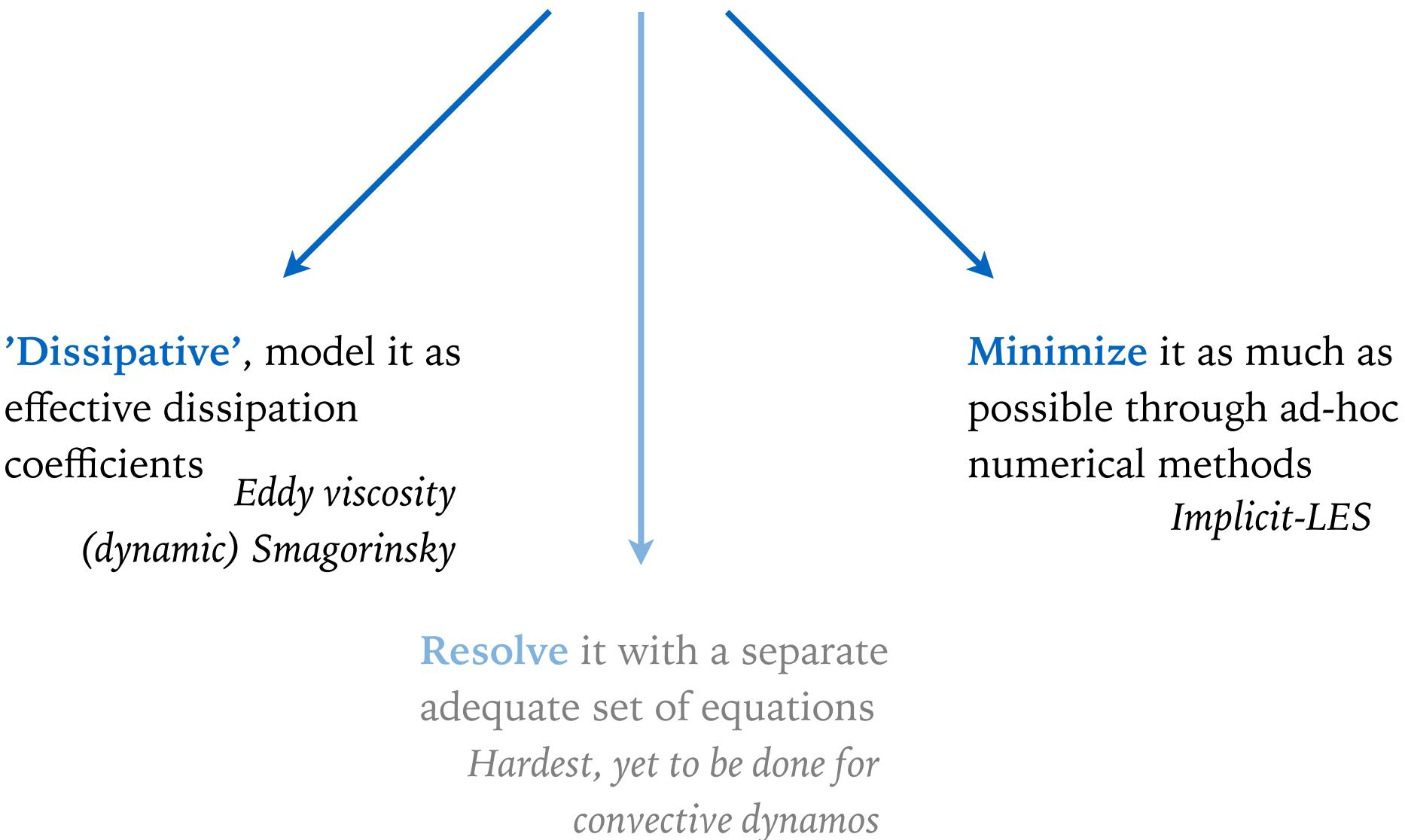
**Resolve** it with a separate  
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*Hardest, yet to be done for  
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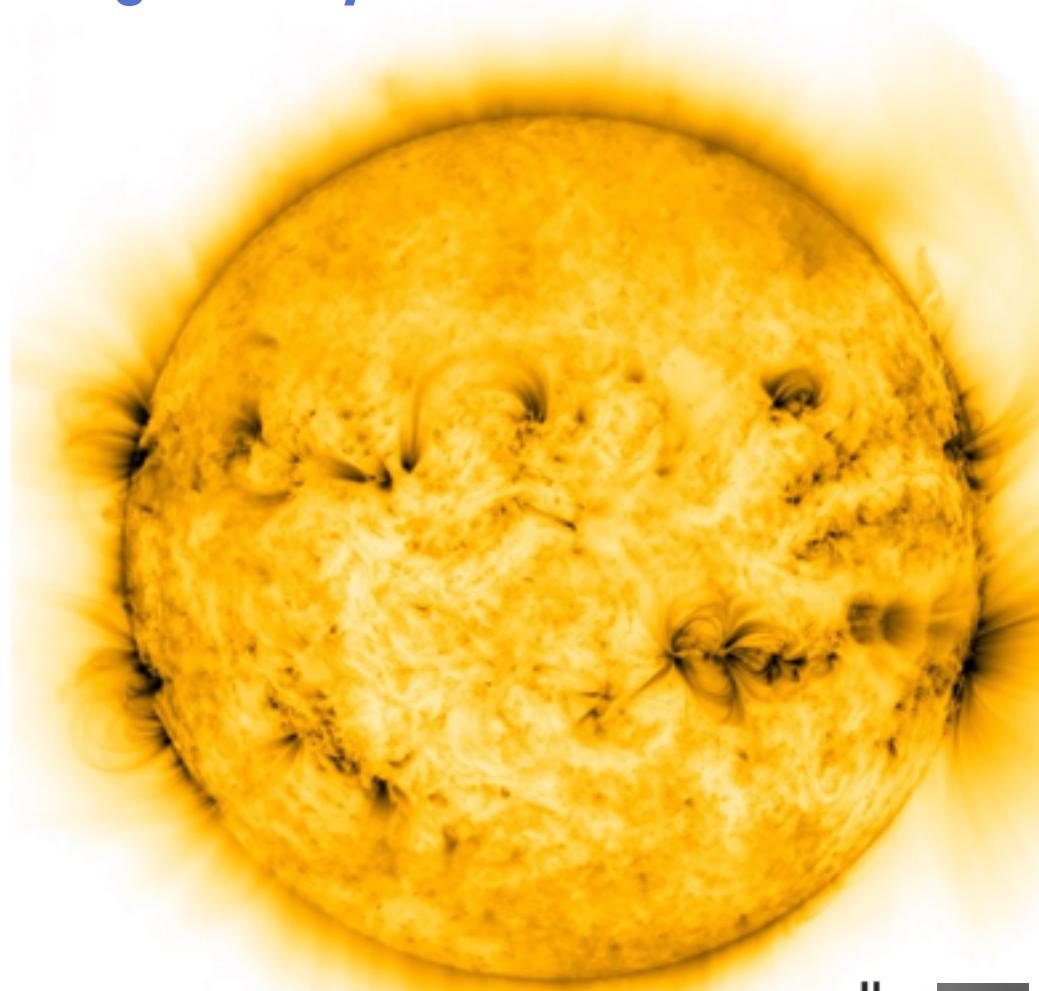
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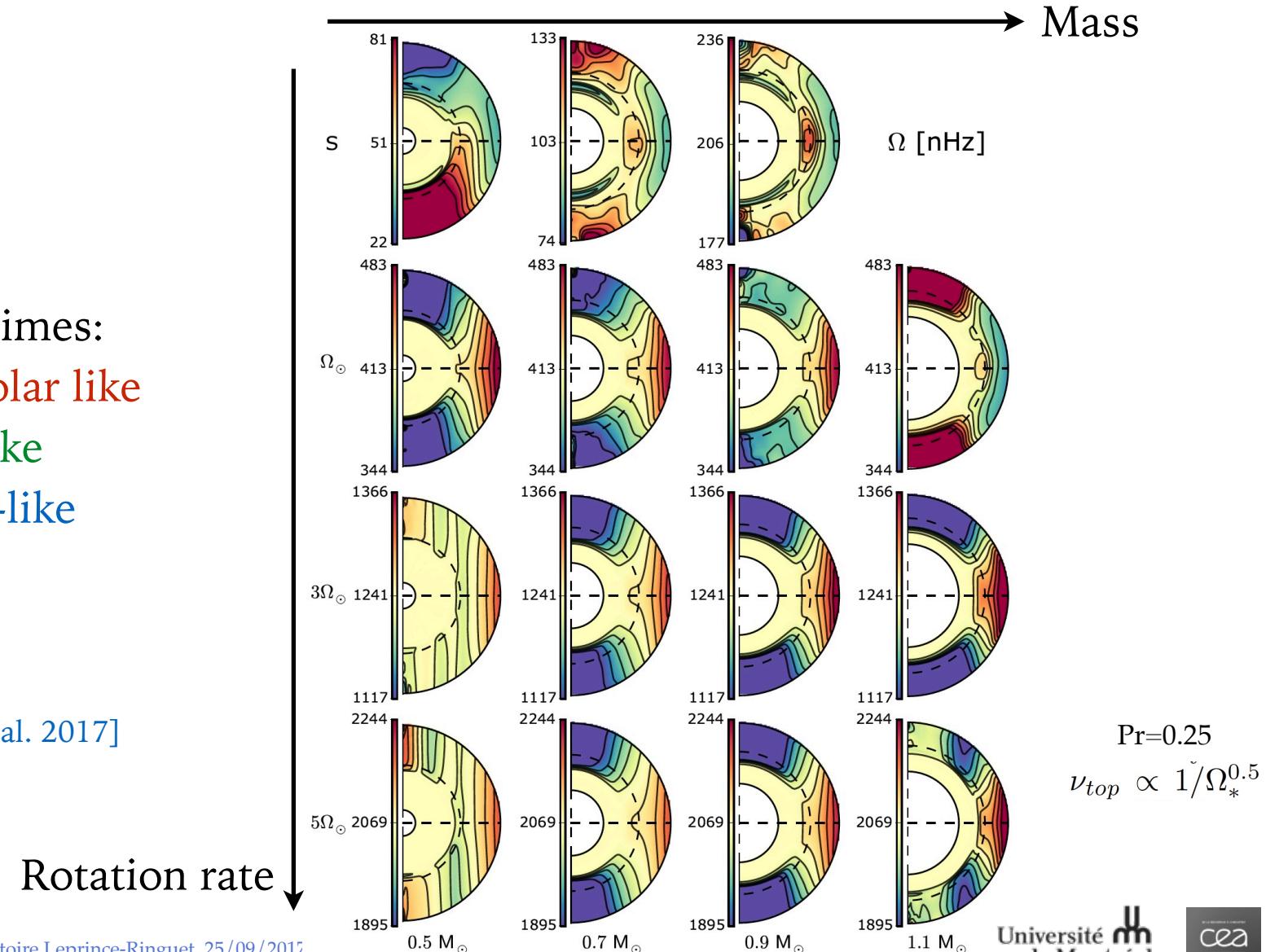
# Ab-initio modelling of stellar magnetic cycles

## Basic ingredients of stellar dynamos



# Understanding the basic ingredients of stellar dynamos

What determines the shape and amplitude of the differential rotation?



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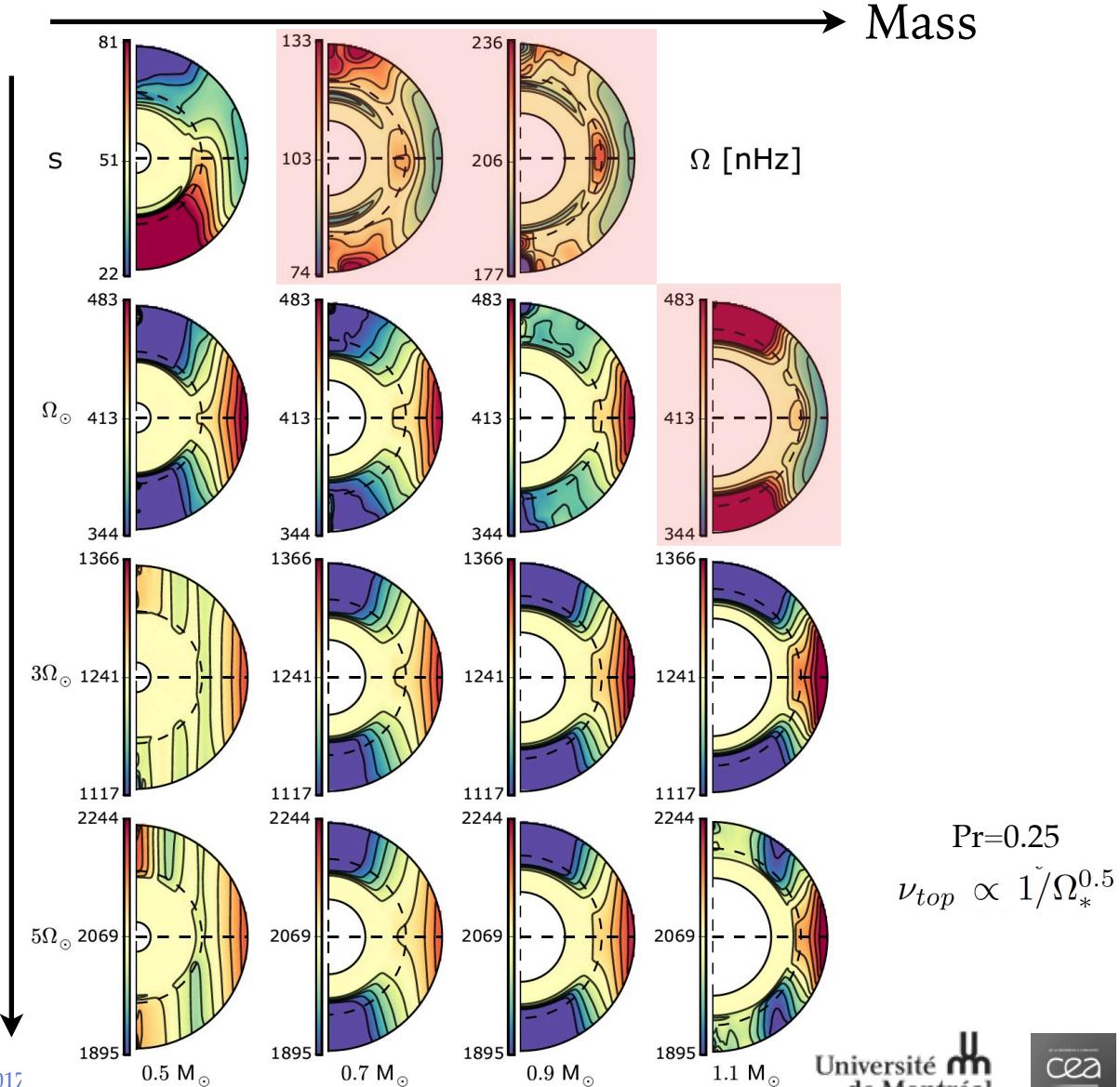
What determines the shape and amplitude of the differential rotation?

Three regimes:

- Anti-Solar like
- Solar-like
- Jupiter-like

[Brun, Strugarek et al. 2017]

Rotation rate



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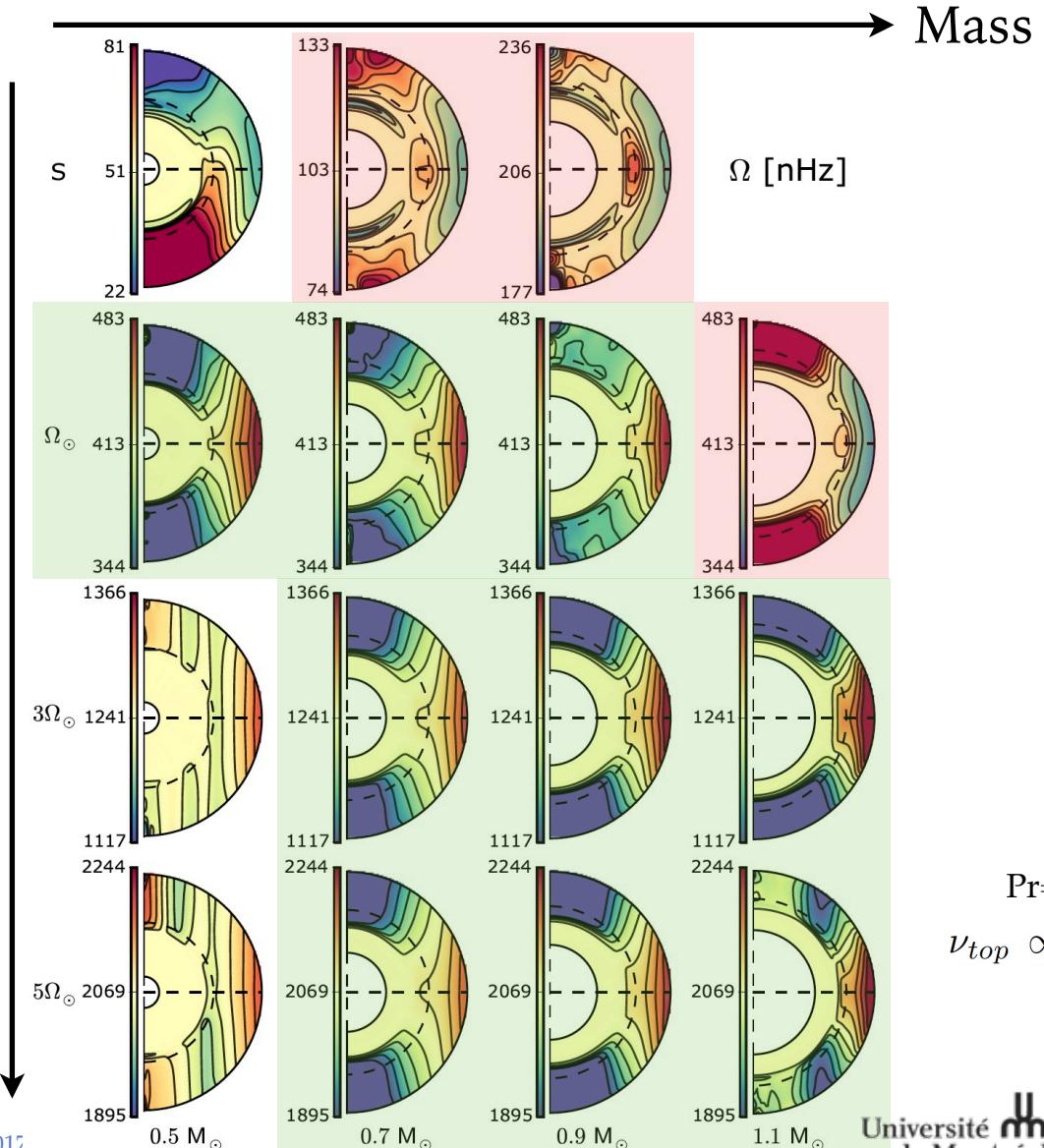
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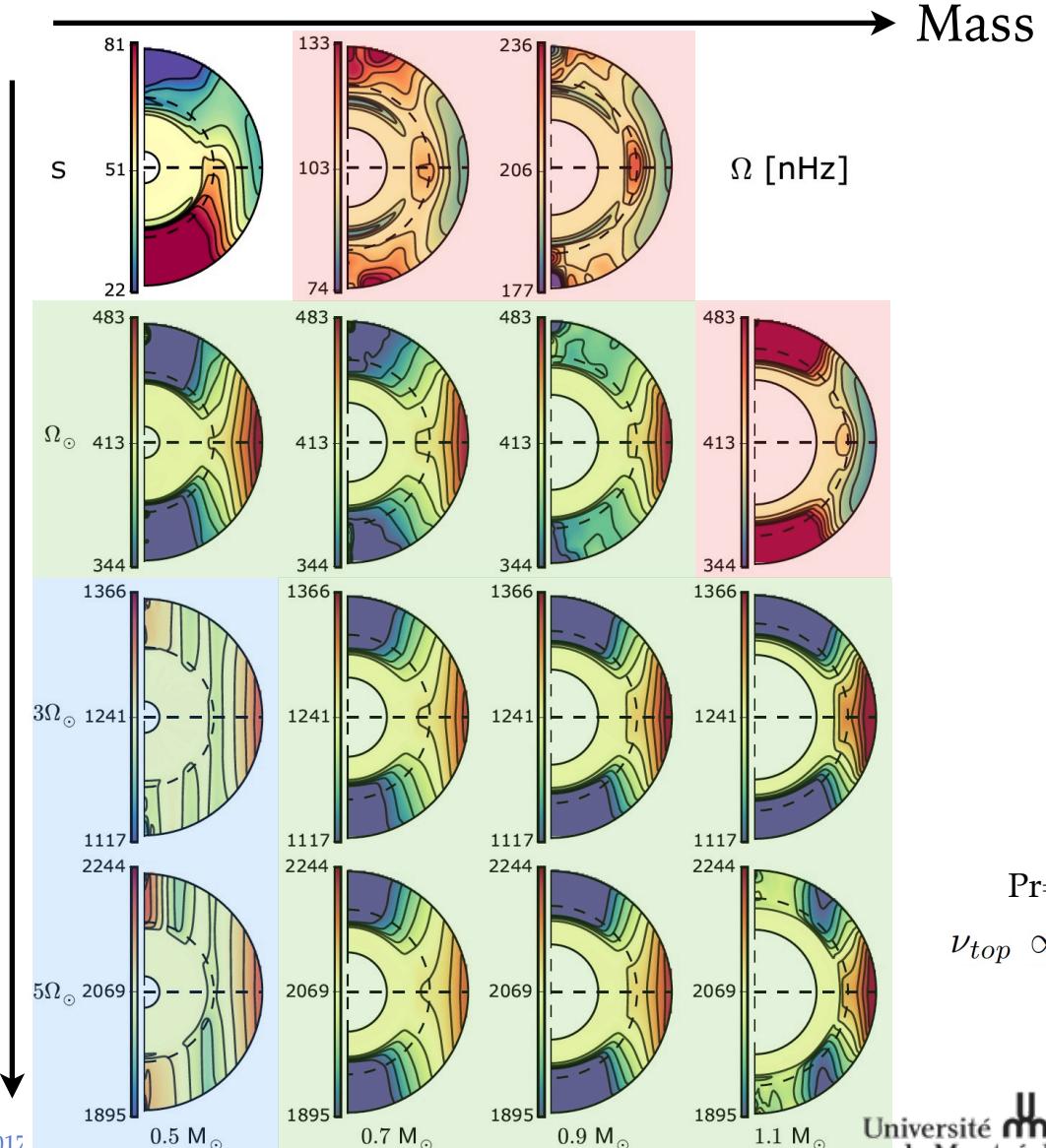
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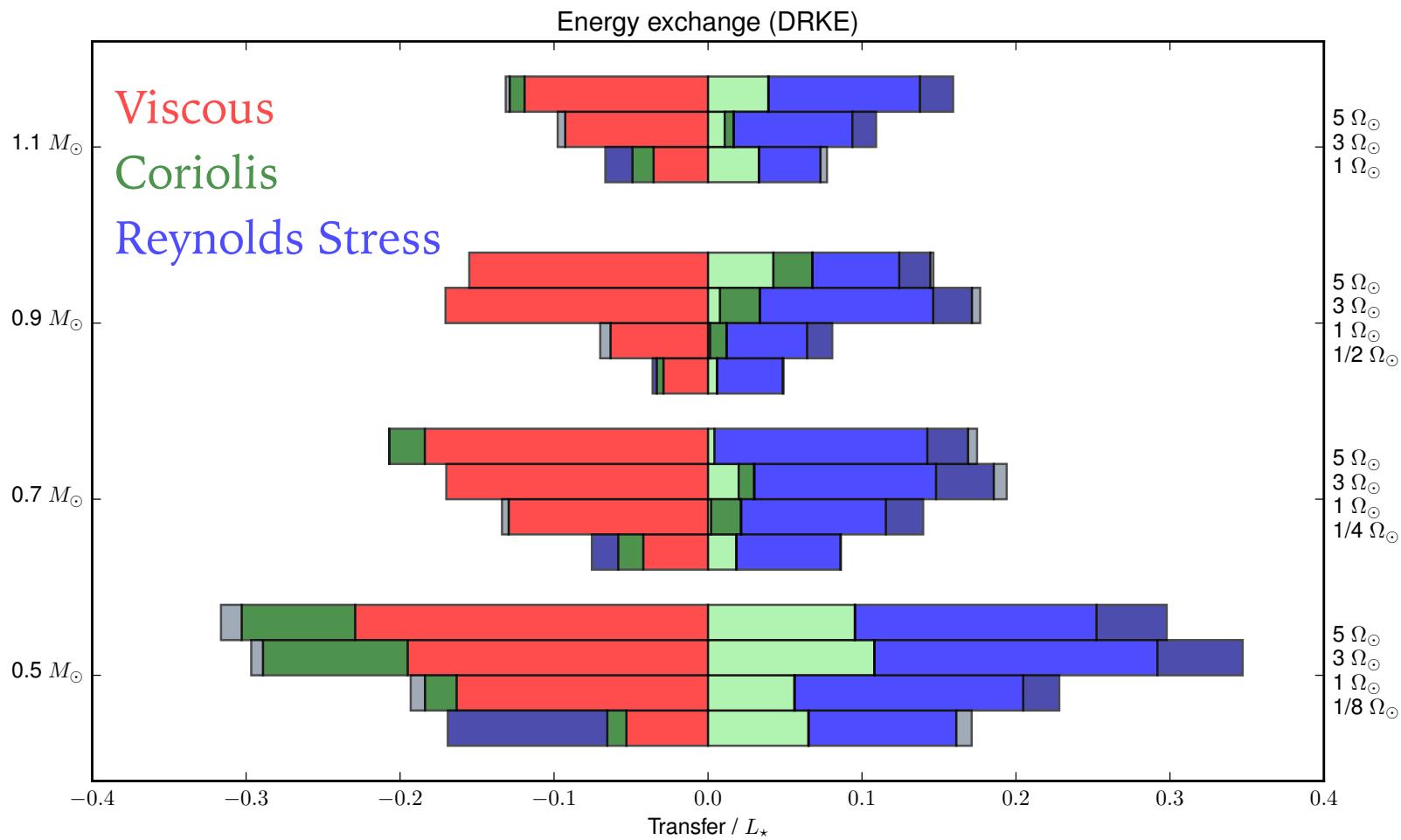
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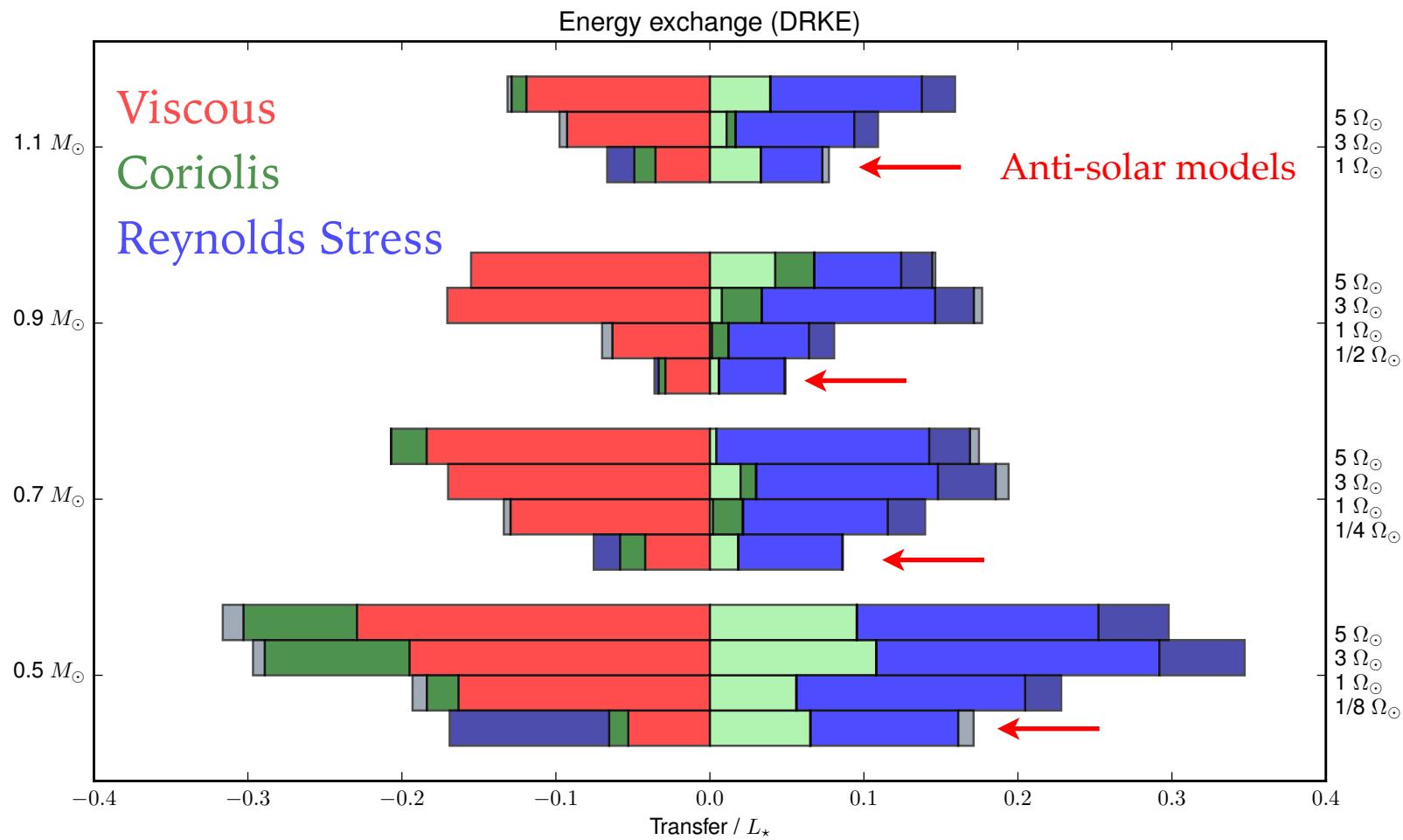
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Stellar evolution

$$\begin{aligned} L_* &\sim M_*^{4.6} \\ R_* &\sim M_*^{1.3} \\ \rho_{bcz} &\sim M_*^{-6.9} \end{aligned}$$

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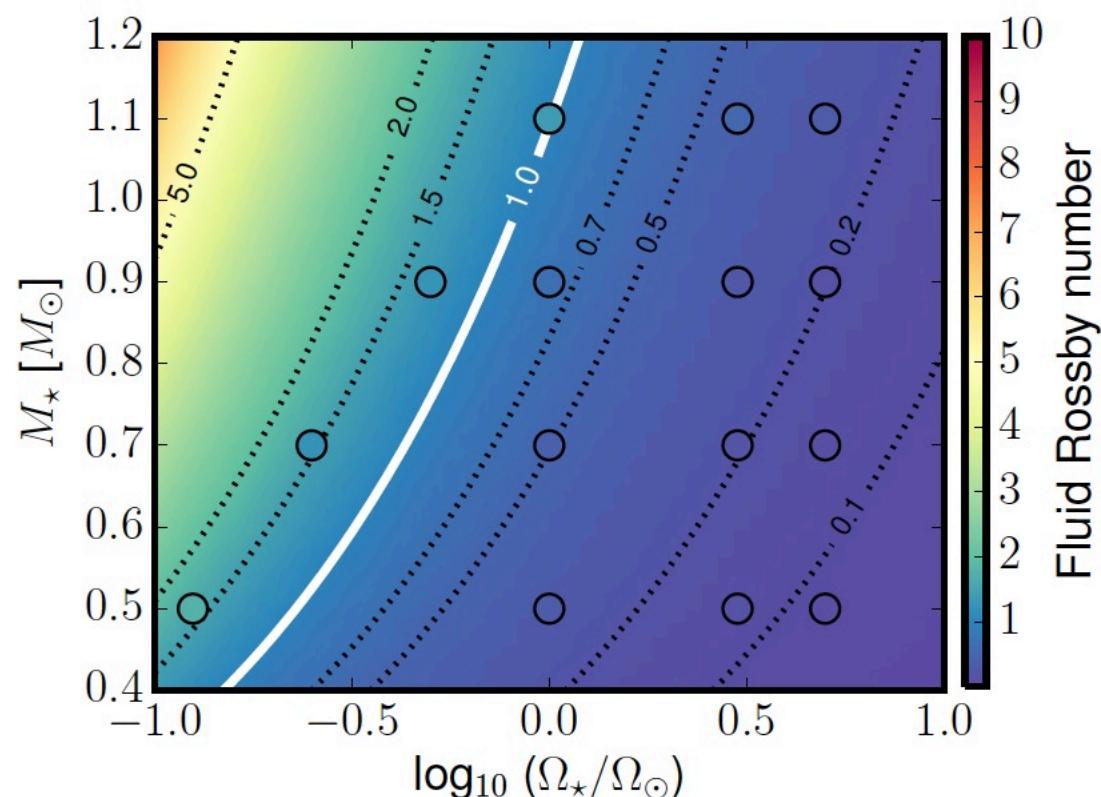
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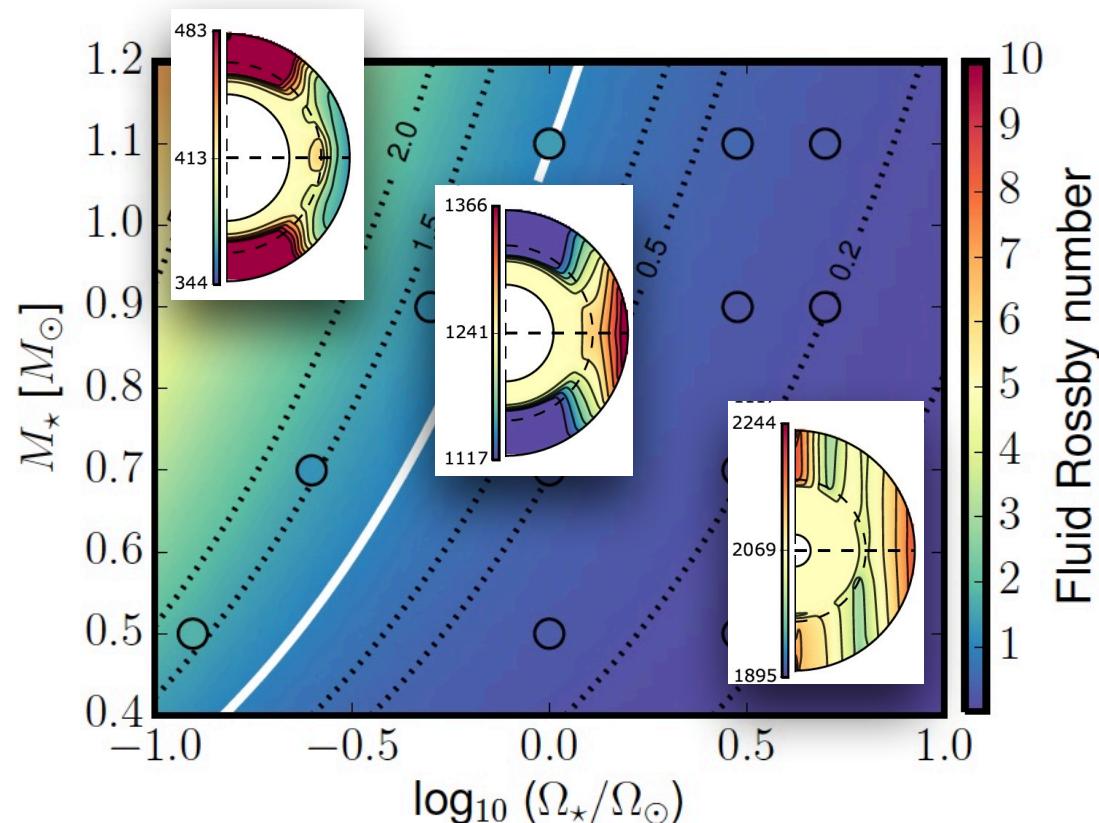
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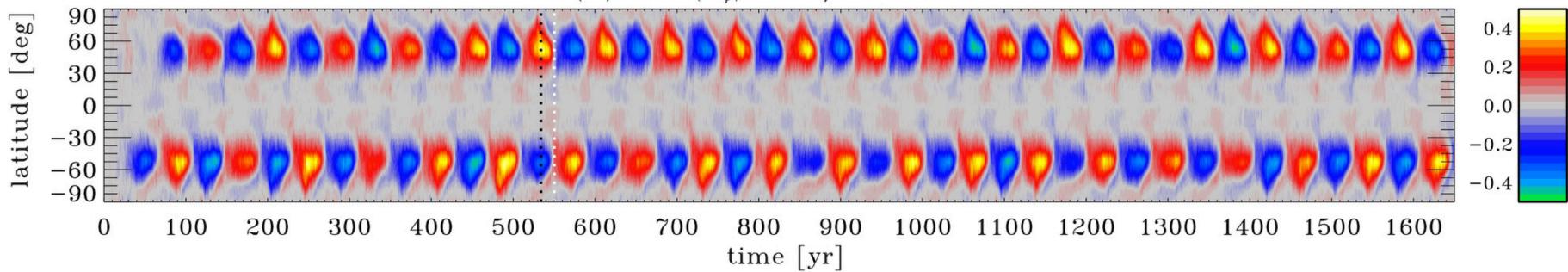
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# Understanding the basic ingredients of stellar dynamos

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$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{U} \rangle \times \langle \mathbf{B} \rangle) + \nabla \times (\alpha \langle \mathbf{B} \rangle) - \nabla \times (\eta \nabla \times \langle \mathbf{B} \rangle)$$

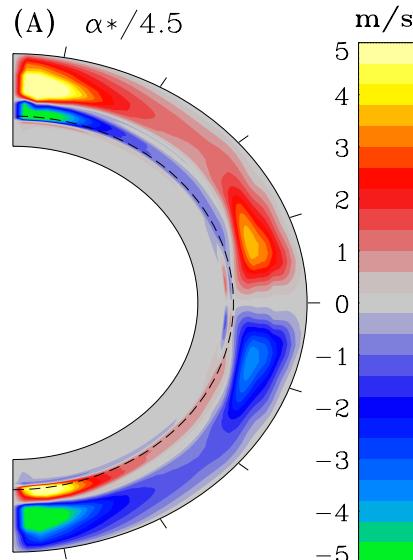
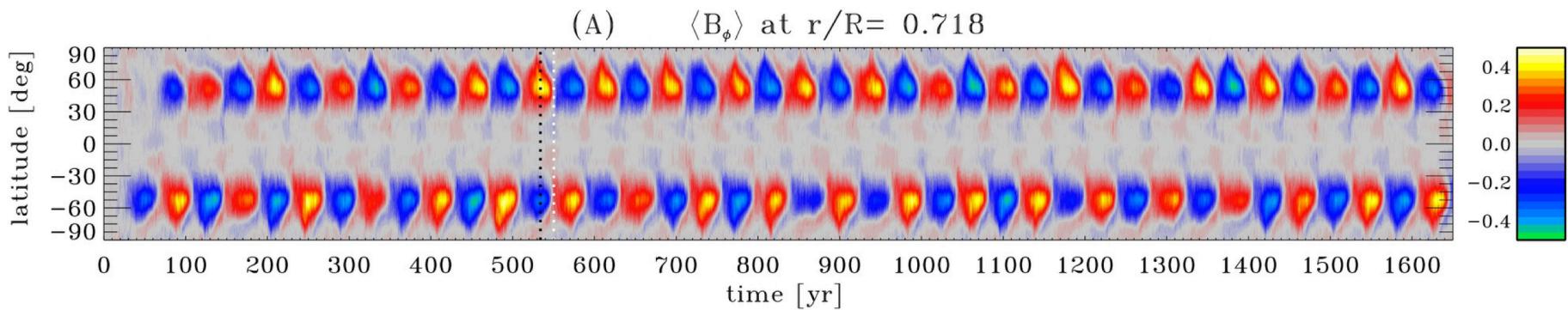
(A)  $\langle B_\phi \rangle$  at  $r/R = 0.718$



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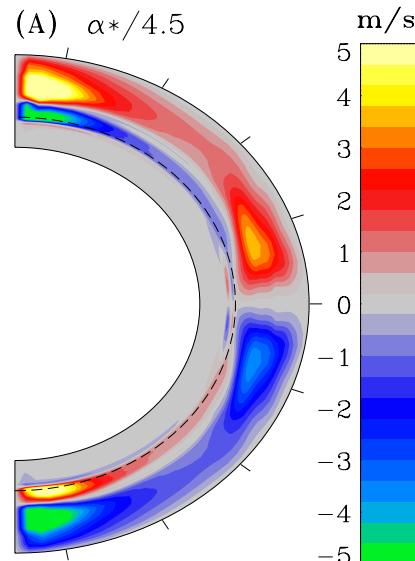
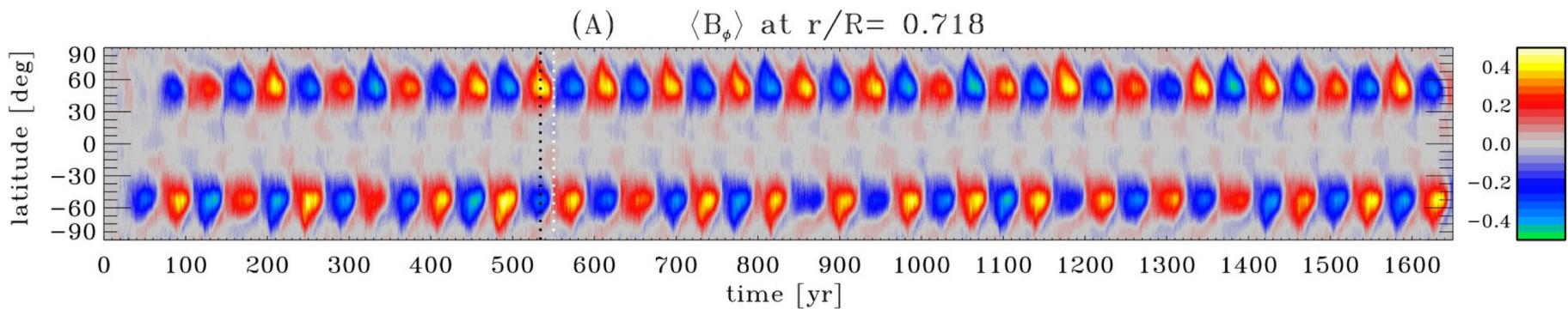
$\alpha$

Singular Value  
Decomposition

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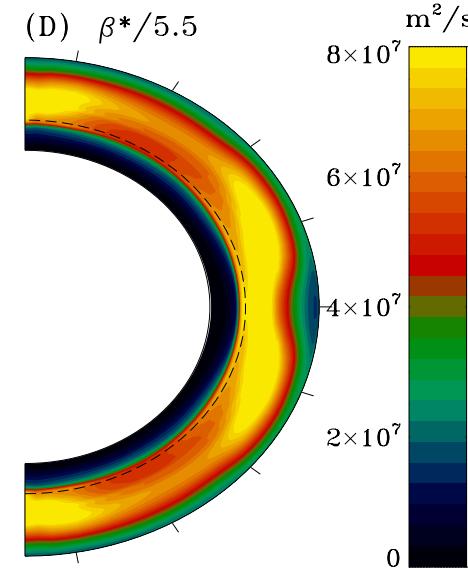
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$\alpha$

$\eta$

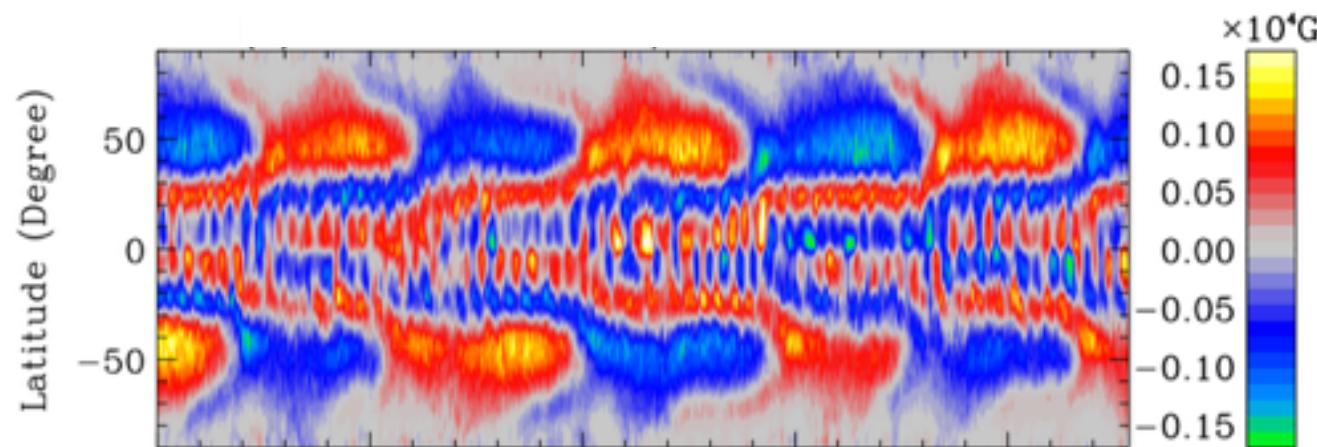
Singular Value Decomposition



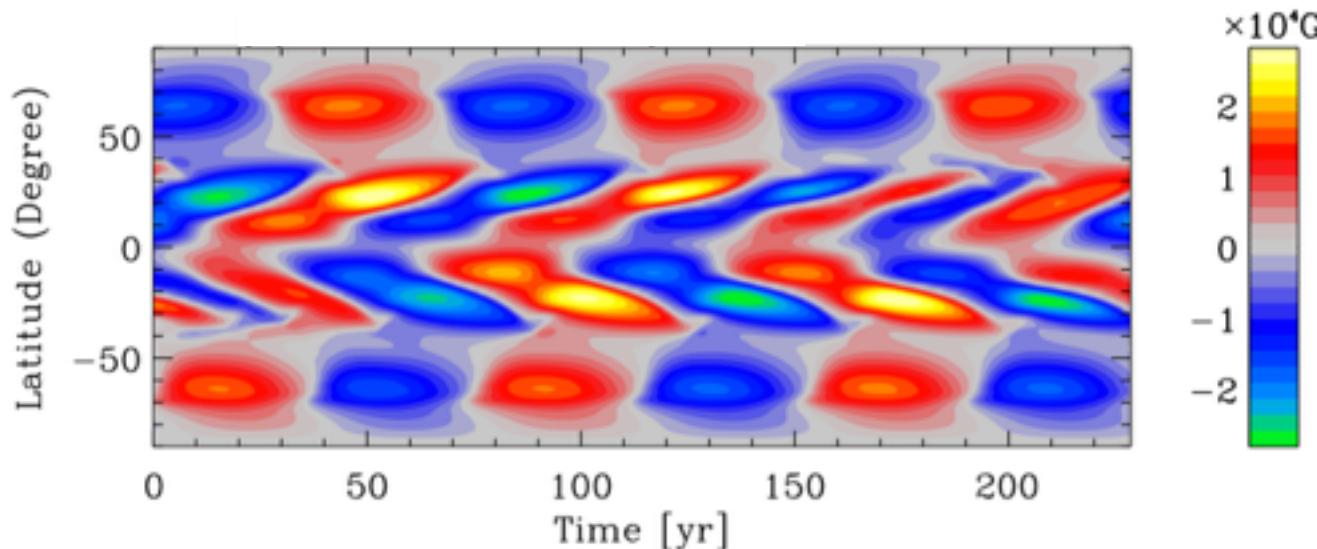
[Schrinner+ 2007; Simard+ 2016]

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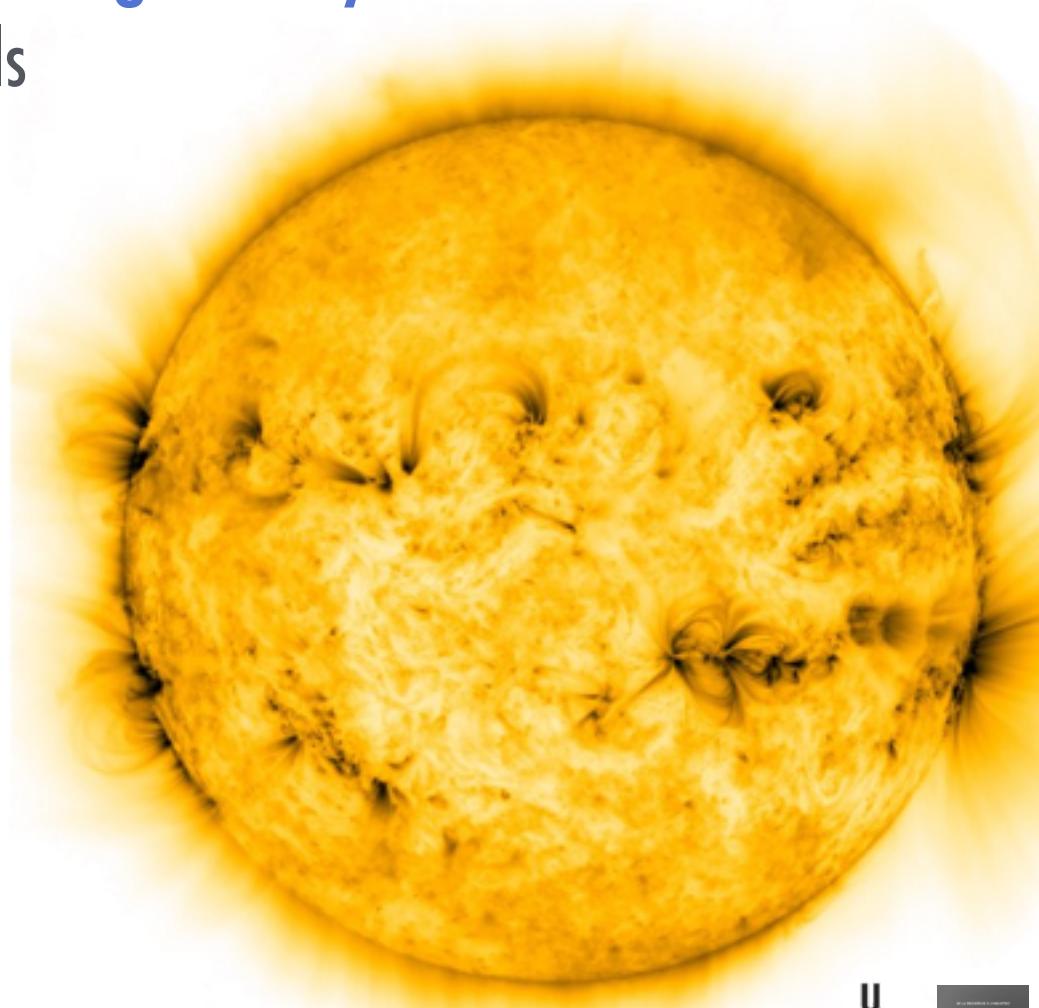
3D Model



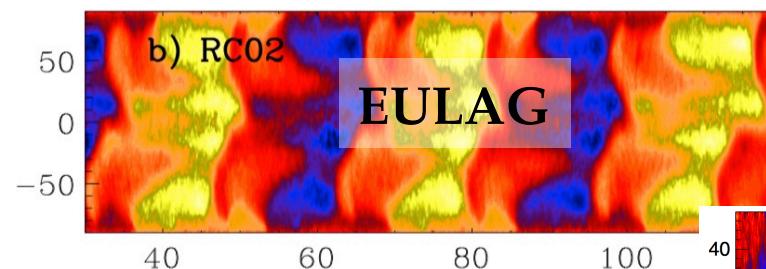
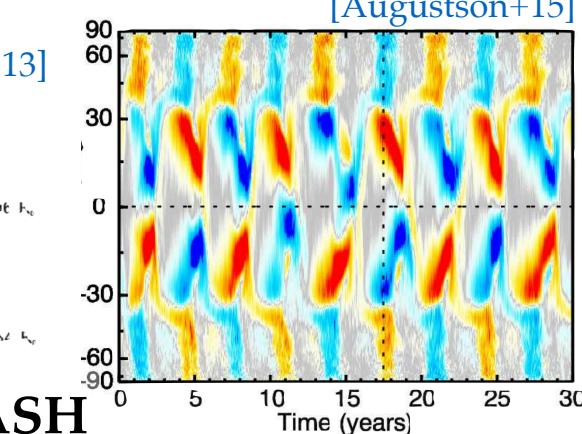
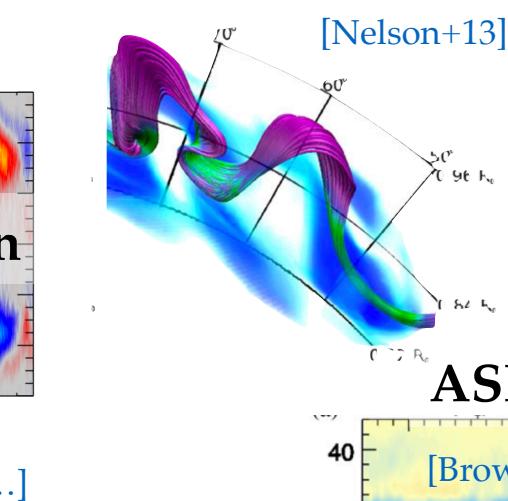
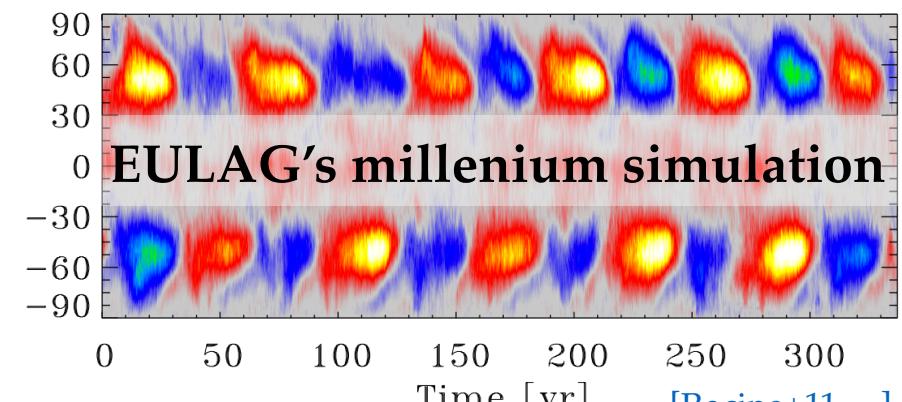
Mean field  
model using  
extracted  $\alpha, \eta$

# Ab-initio modelling of stellar magnetic cycles

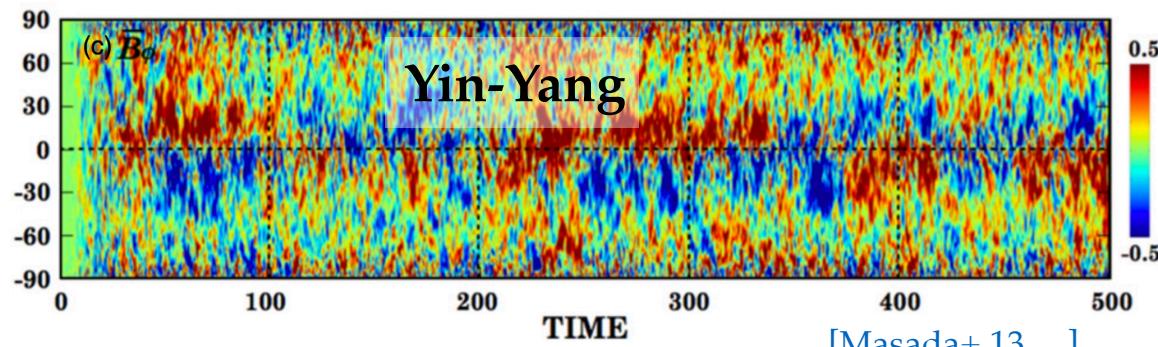
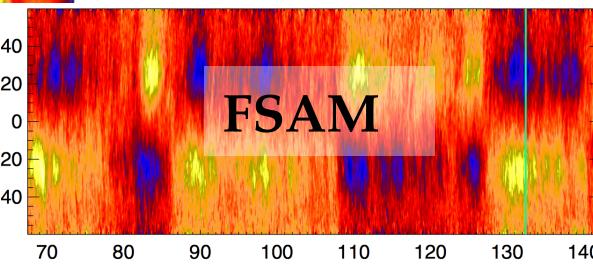
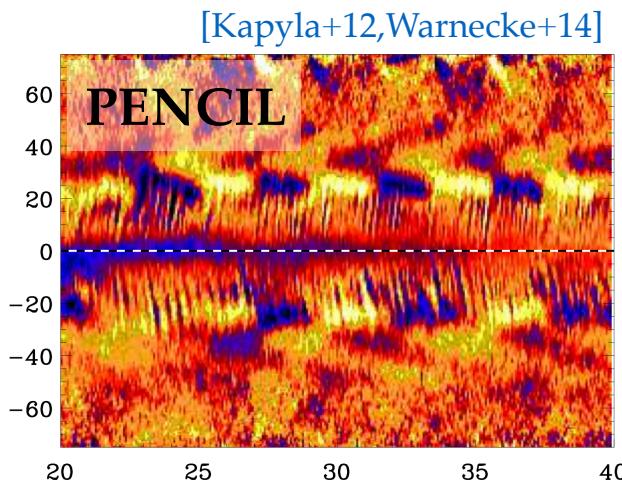
## Non-linear, 3D stellar dynamo models



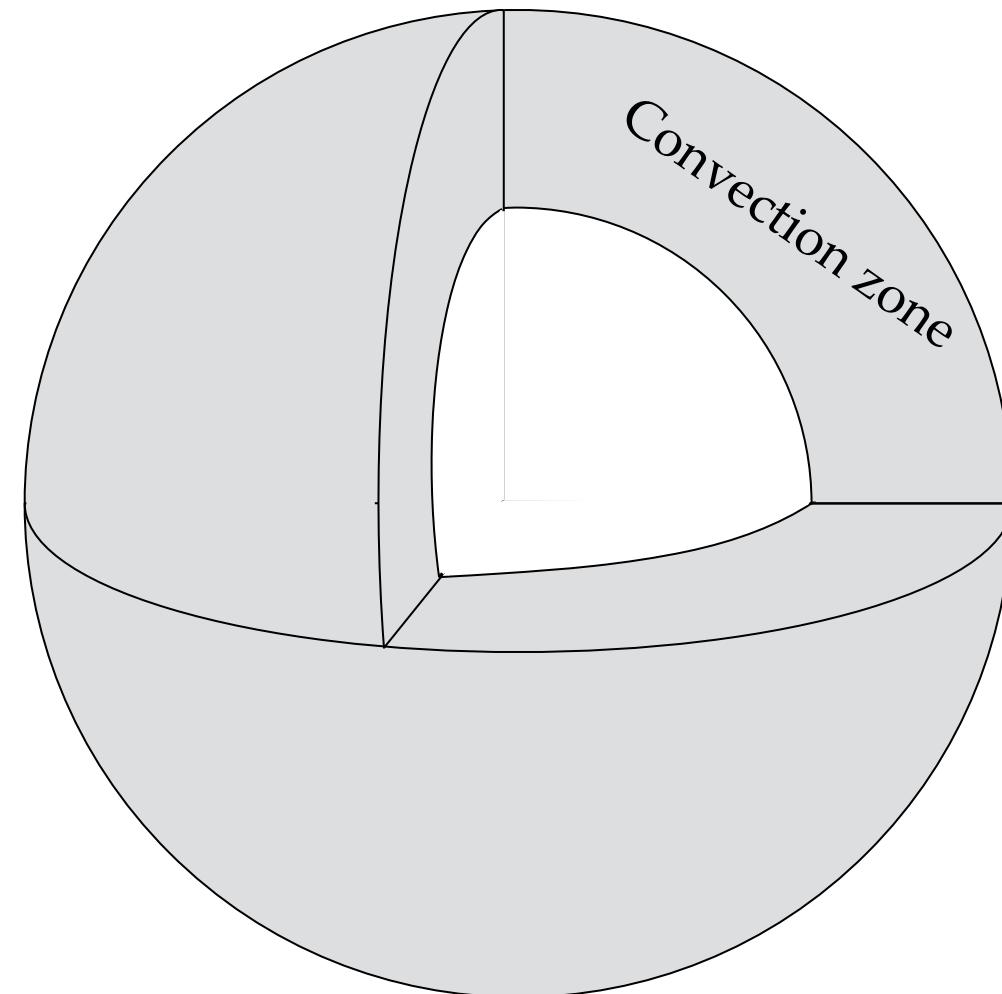
# The zoo of 3D models



and much  
more...



# Tool: the EULAG-MHD CODE



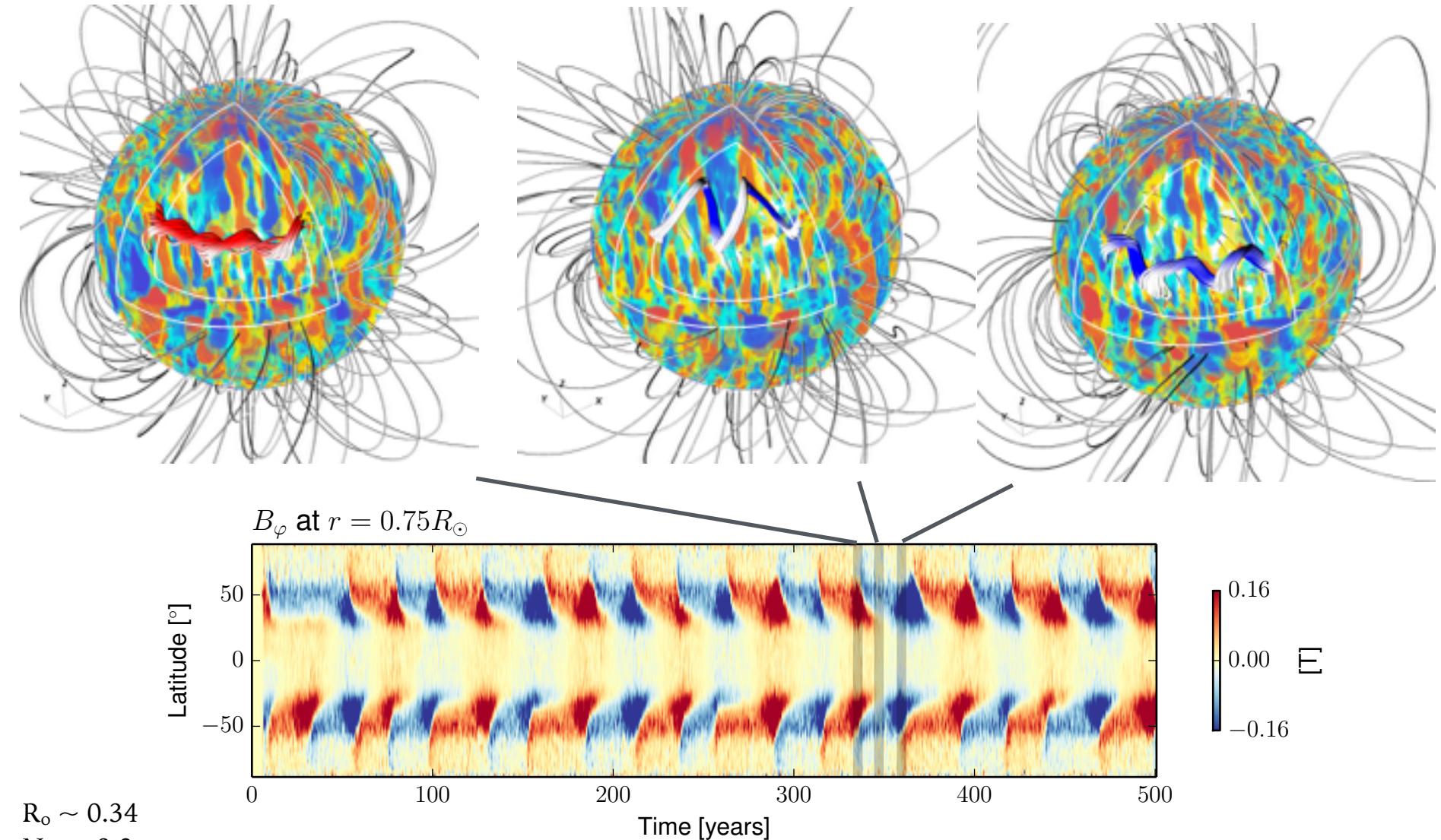
*Core advection scheme:*

MPDATA, a minimally dissipative iterative upwind NFT scheme; equivalent to a dynamical, adaptive subgrid-scale model

*Convective instability,*

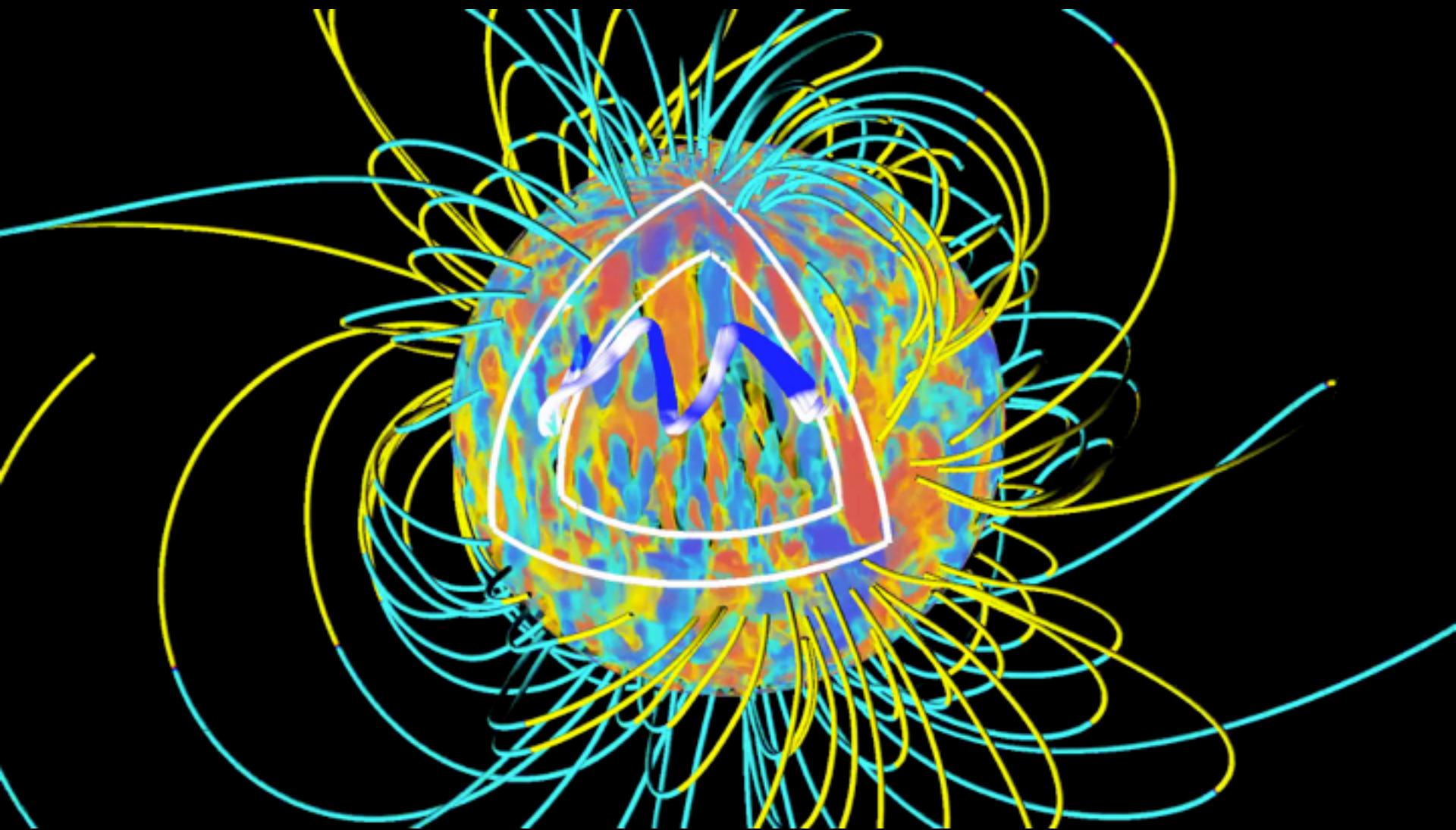
superadiabatic ambient profile combined with Newtonian cooling in energy equation

# Prototype cyclic dynamo in a convective enveloppe

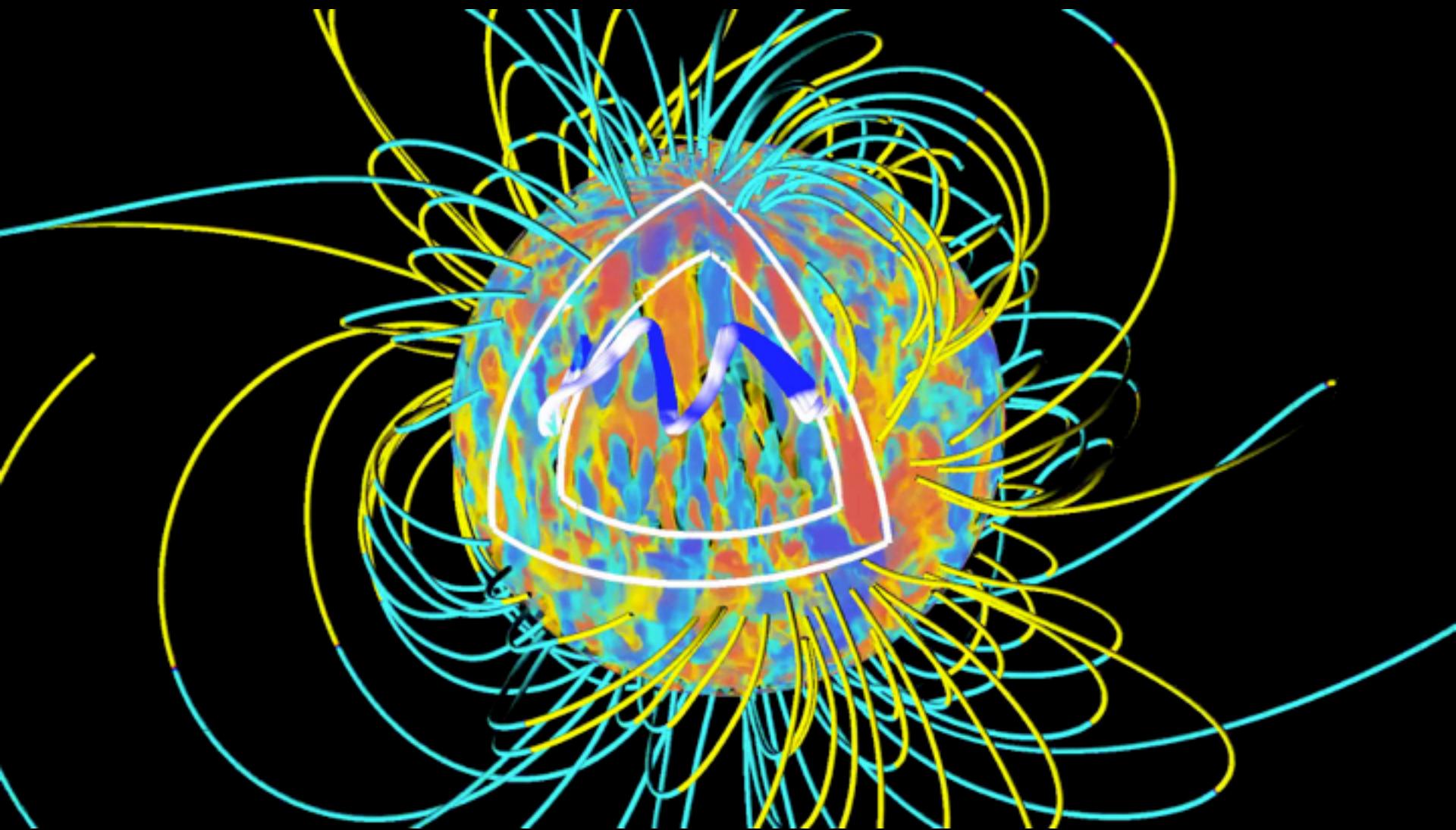


No stable radiative zone

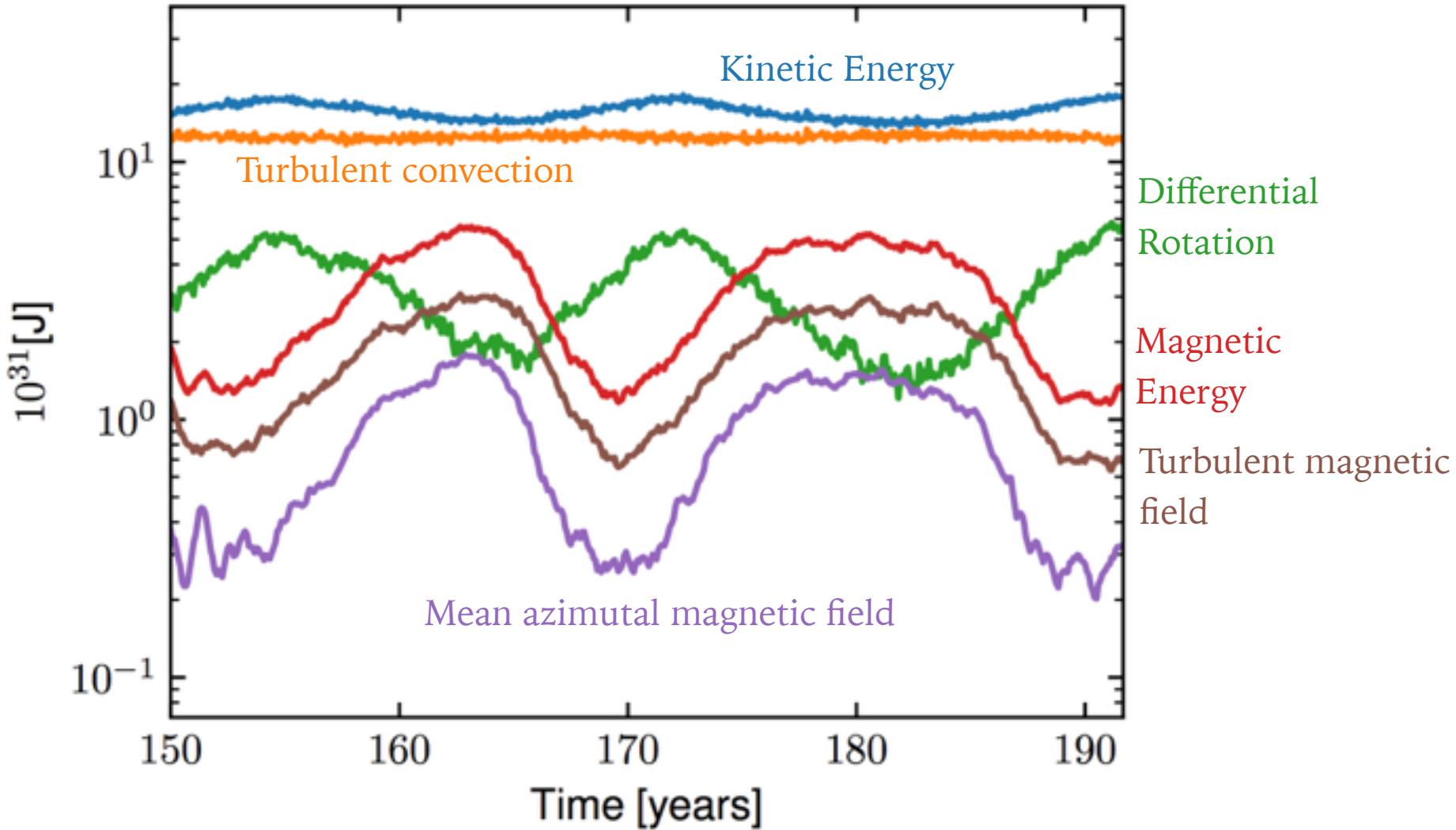
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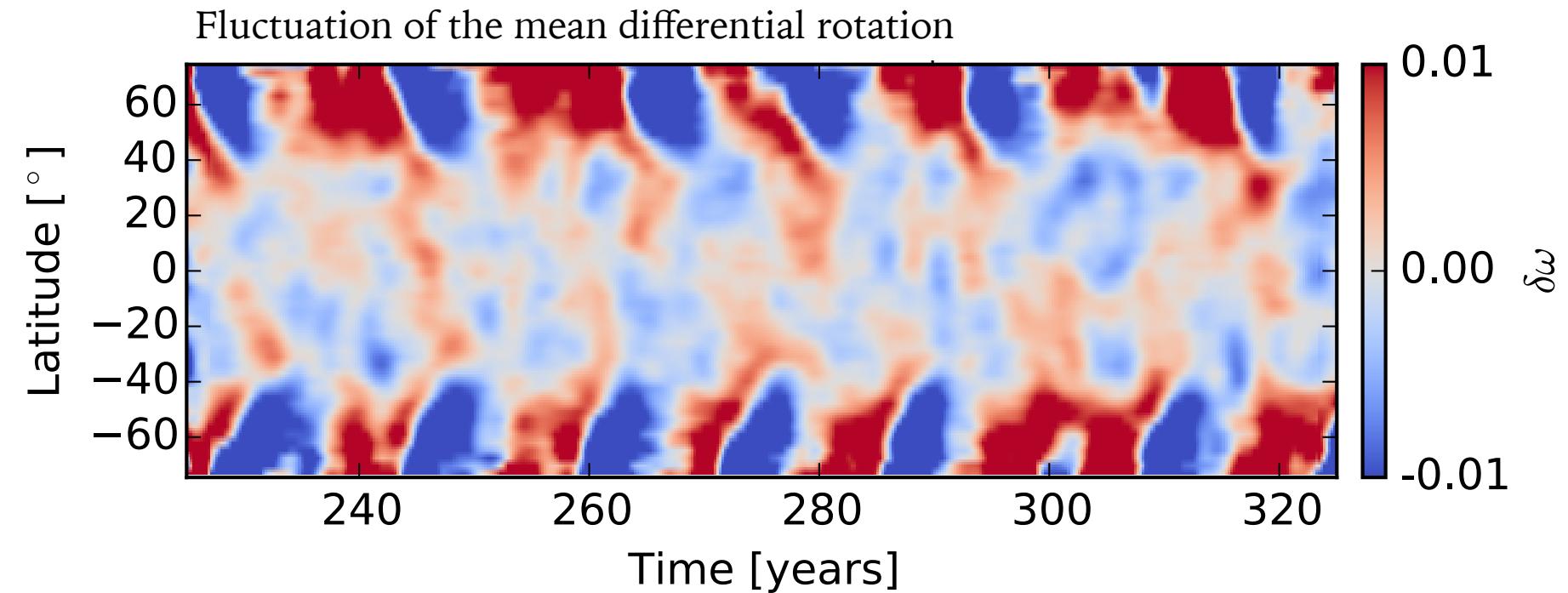
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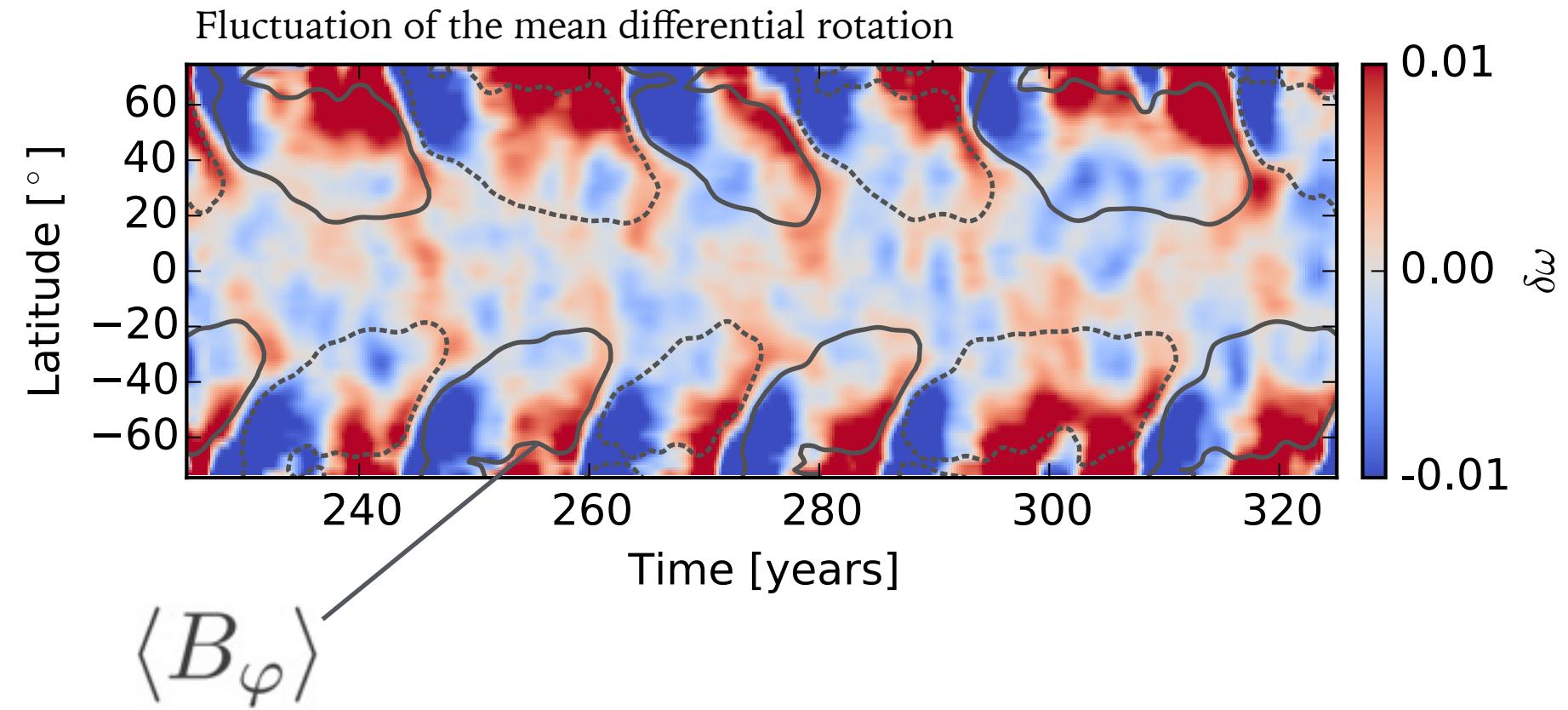
# Energetics of the prototype cyclic dynamo



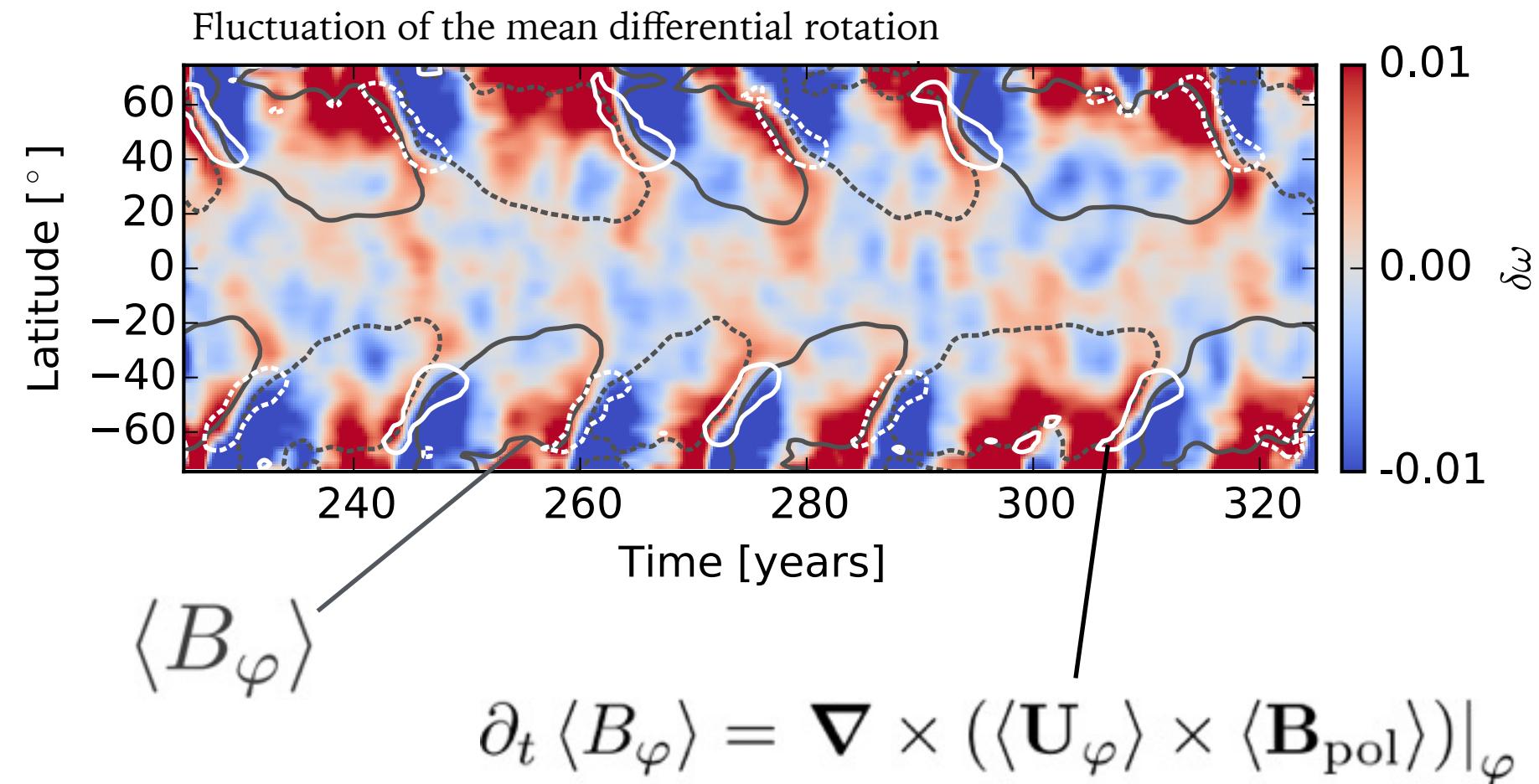
# Fluctuations of the differential rotation drive the reversals



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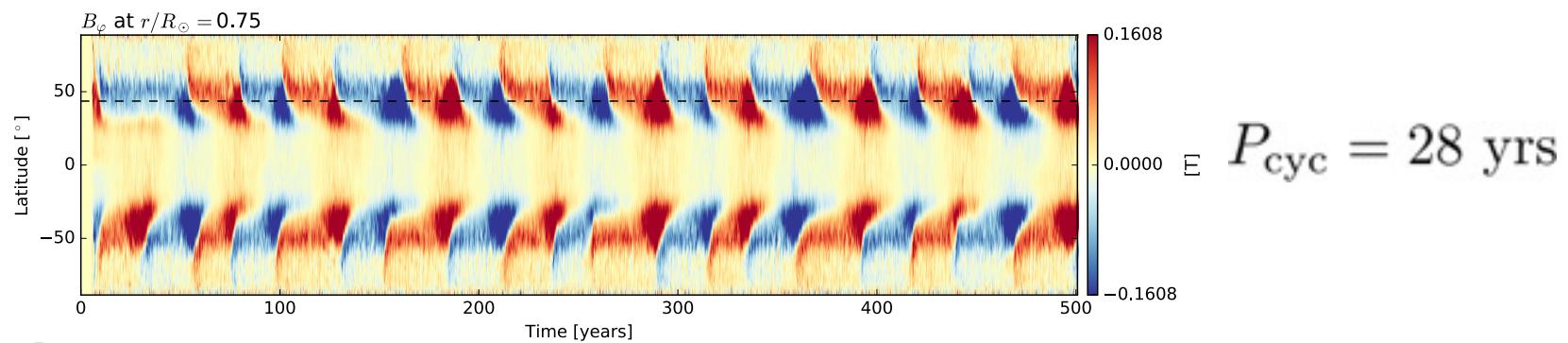


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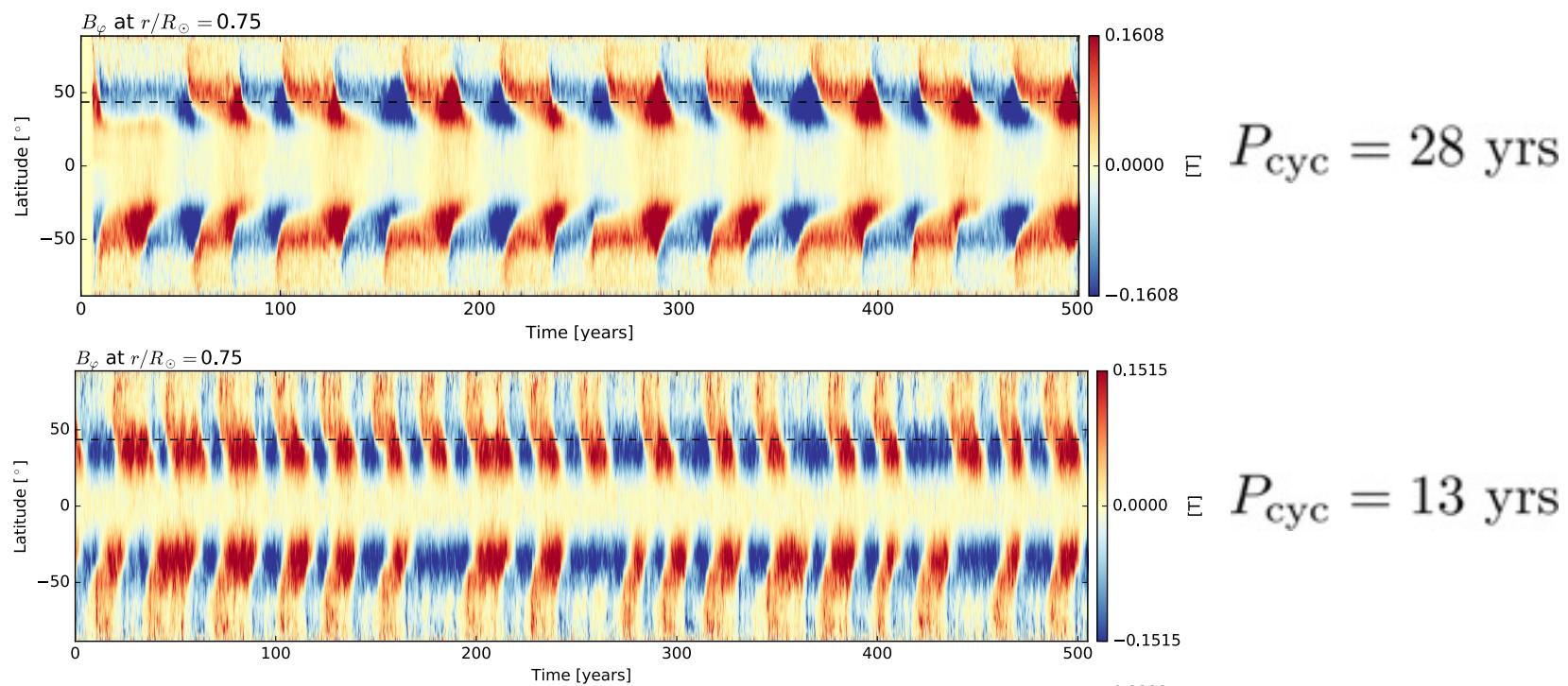
# Systematic modulation of the cycle period

Ref.



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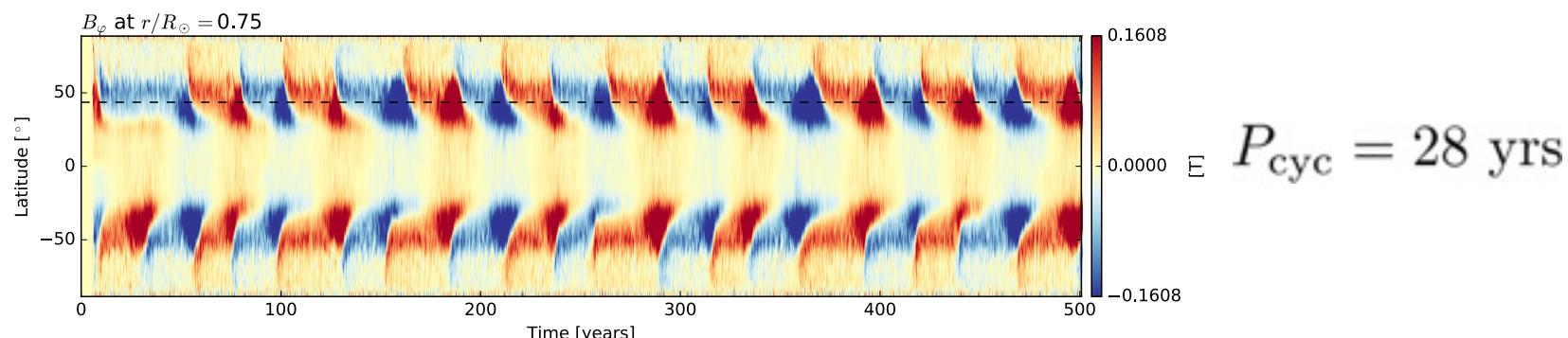
Ref.



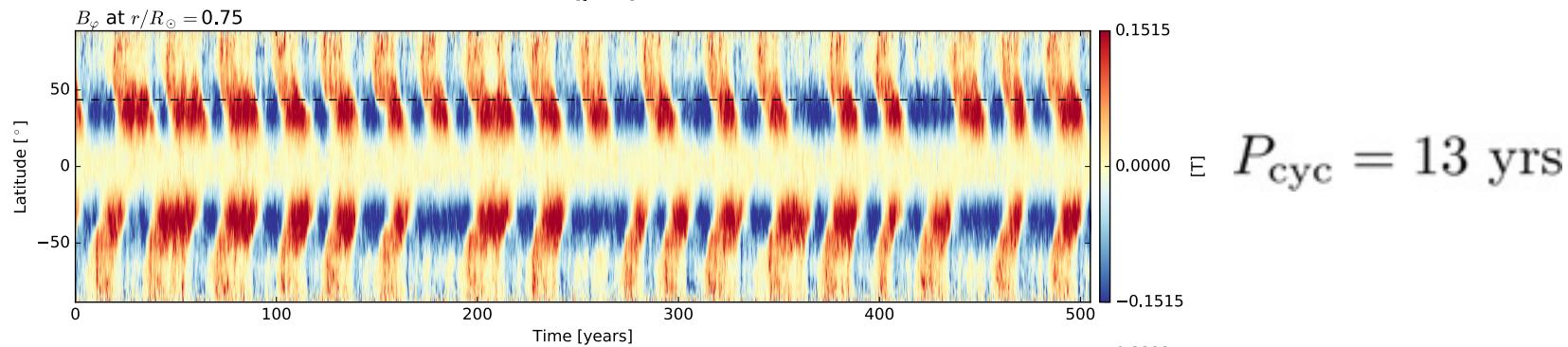
$\Omega/2$

# Systematic modulation of the cycle period

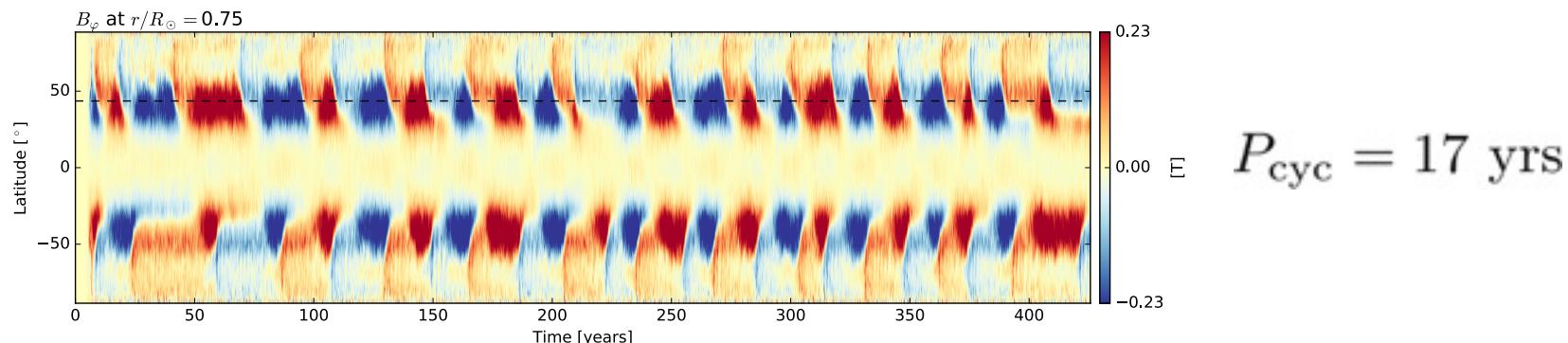
Ref.



$\Omega/2$



Lum.  $\times 2$



# Cycle period is inversely prop. to the Rossby number

Basic ingredients of stellar dynamos

- Differential rotation
- Cyclonic turbulence

‘Go to’ parameter is the  
**Rossby number**

$$R_o = \frac{\text{NL Advection}}{\text{Coriolis}} \sim \frac{|\nabla \times \mathbf{U}|}{2\Omega_*}$$

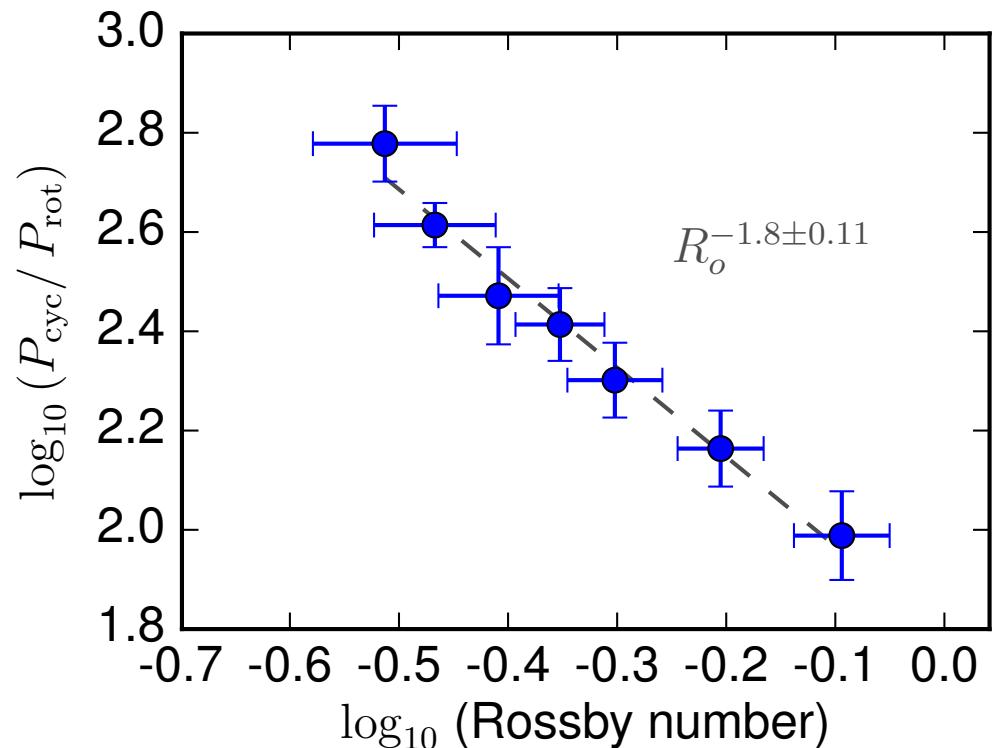
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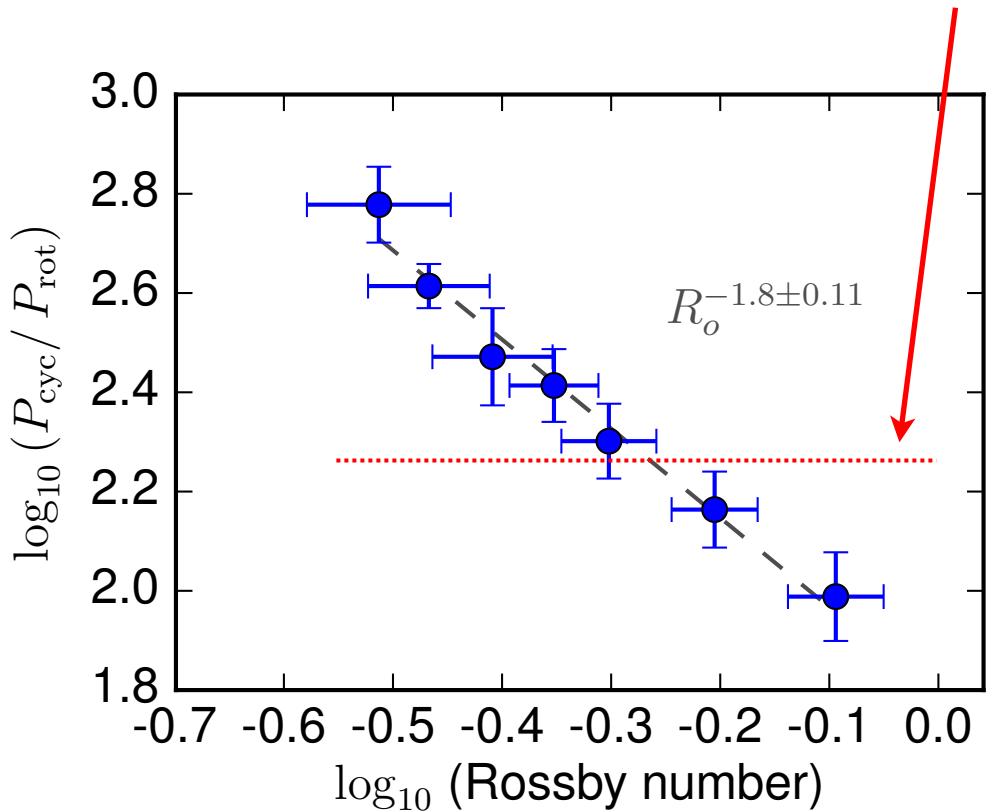
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Basic linear  $a\Omega$  kinematic dynamo theory



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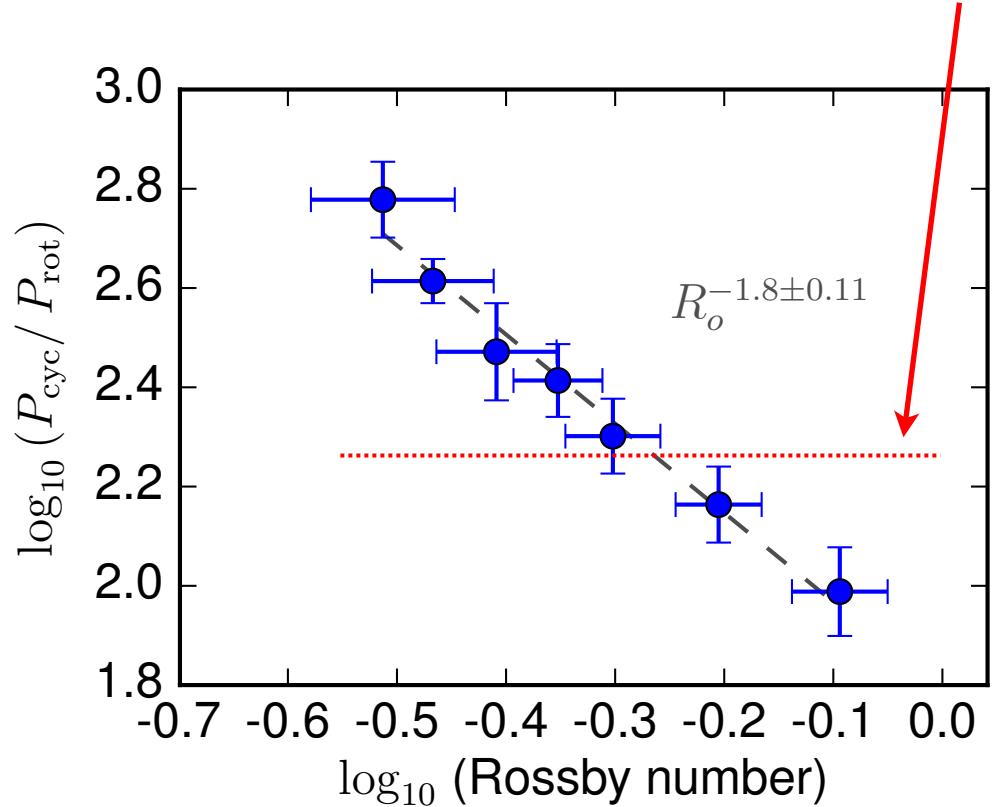
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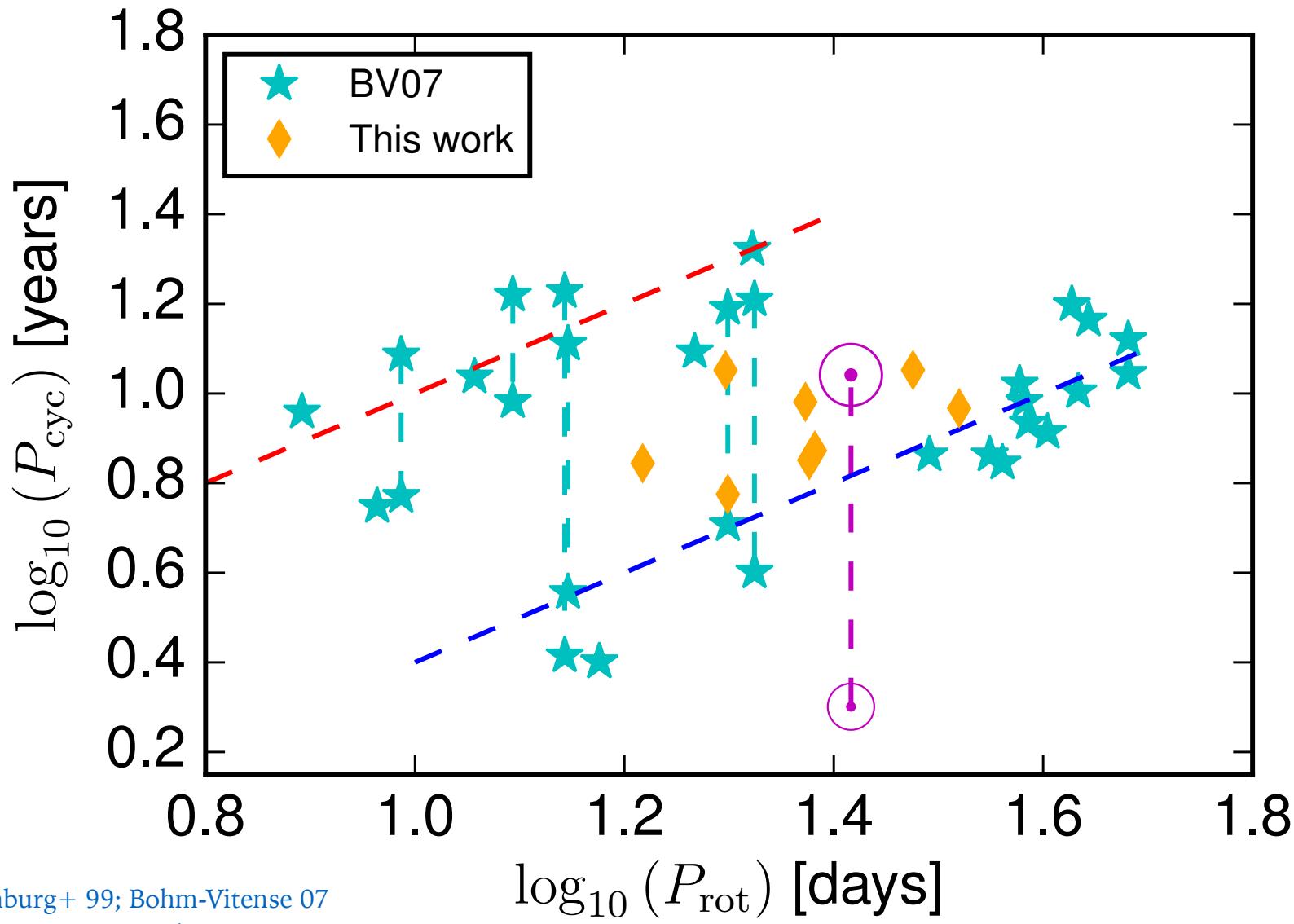
Basic linear  $a\Omega$  kinematic dynamo theory



Fundamentally non-linear convective dynamo:  
not a classical dynamo wave

Could this be realistic?

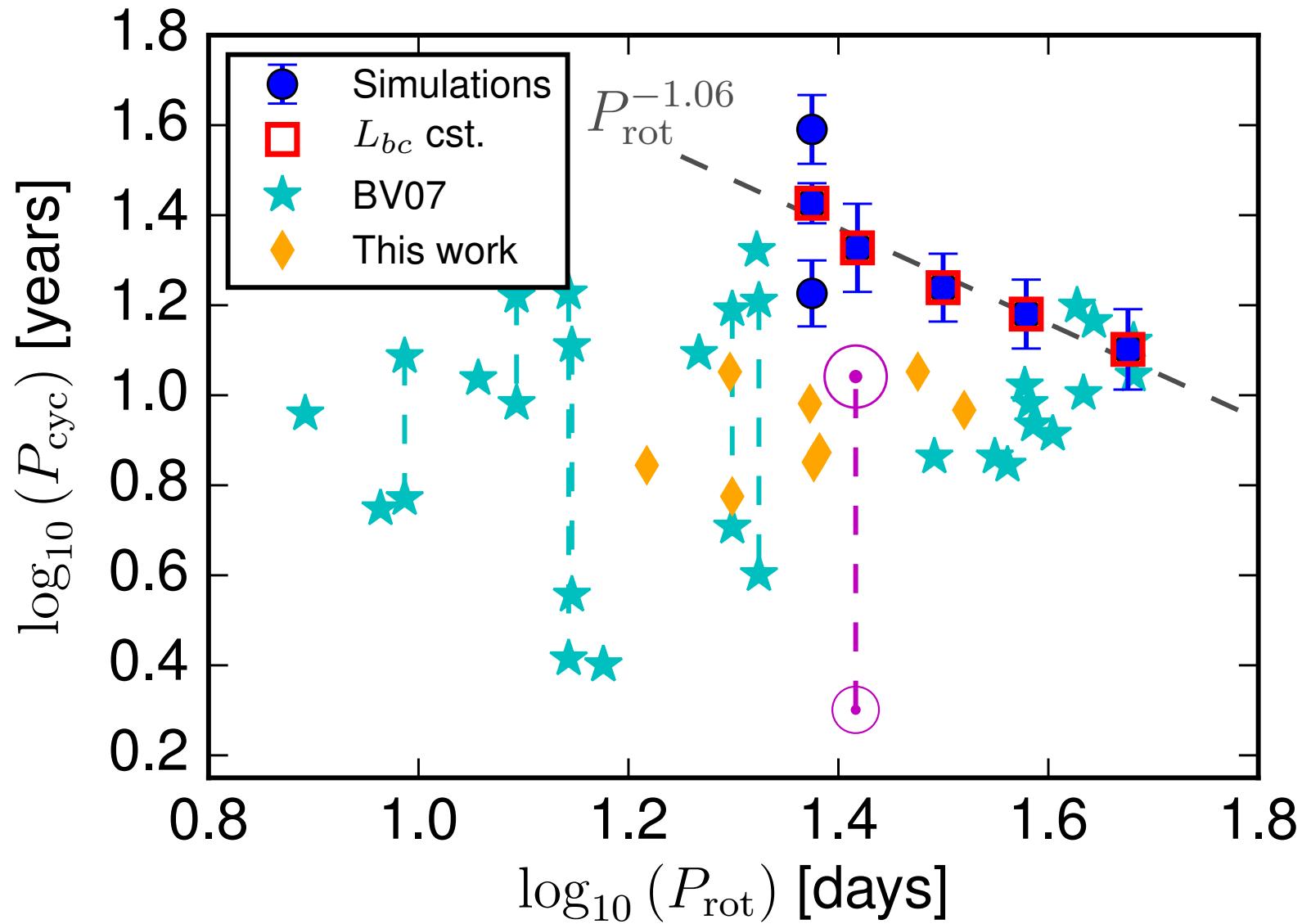
# Cycle period – Rotation period diagram



Brandenburg+ 99; Bohm-Vitense 07

Metcalfe+ 16; Strugarek+ 17

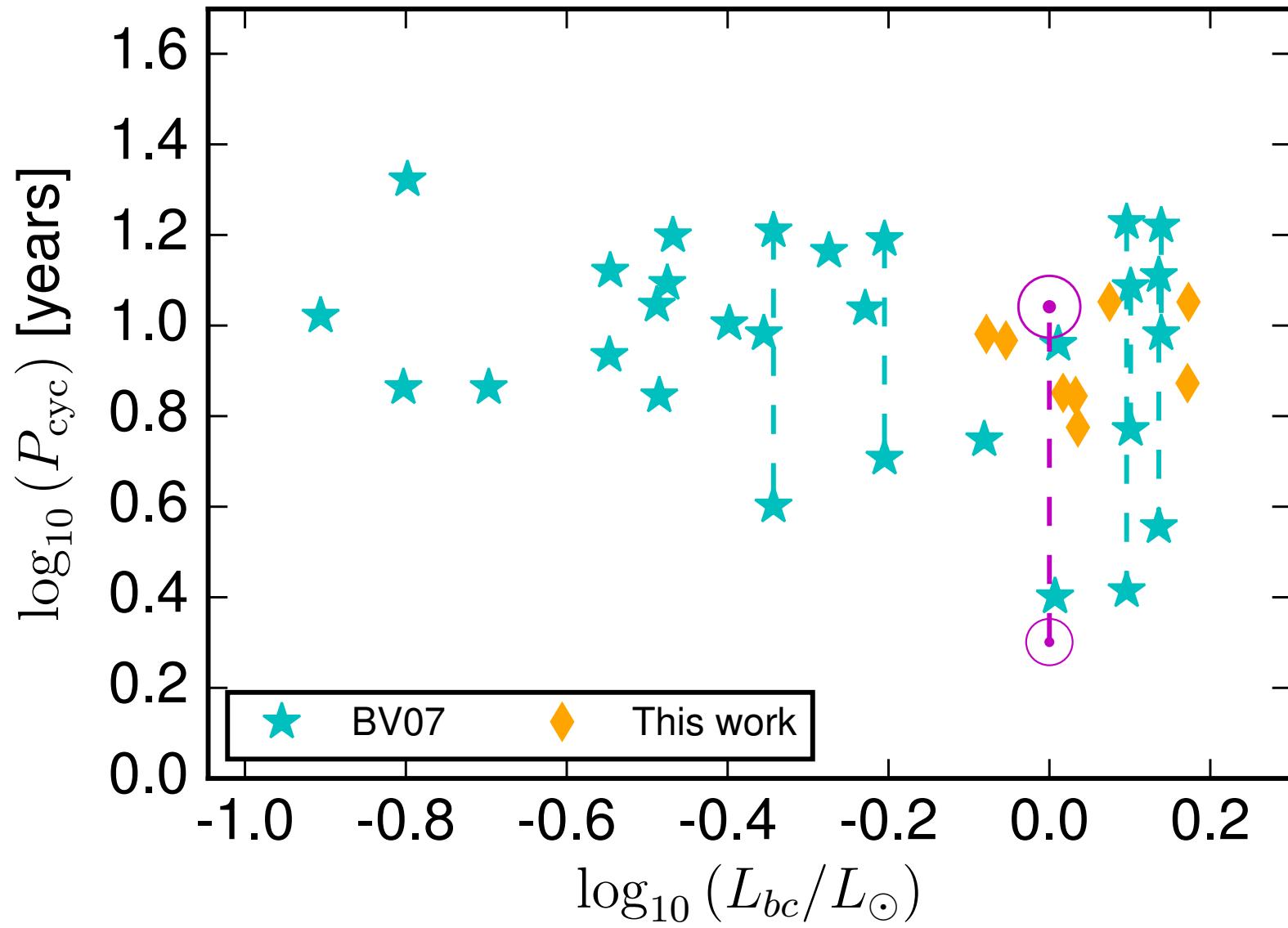
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[Strugarek+ 17]

A. Strugarek, Seminar at Laboratoire Leprince-Ringuet, 25/09/2017

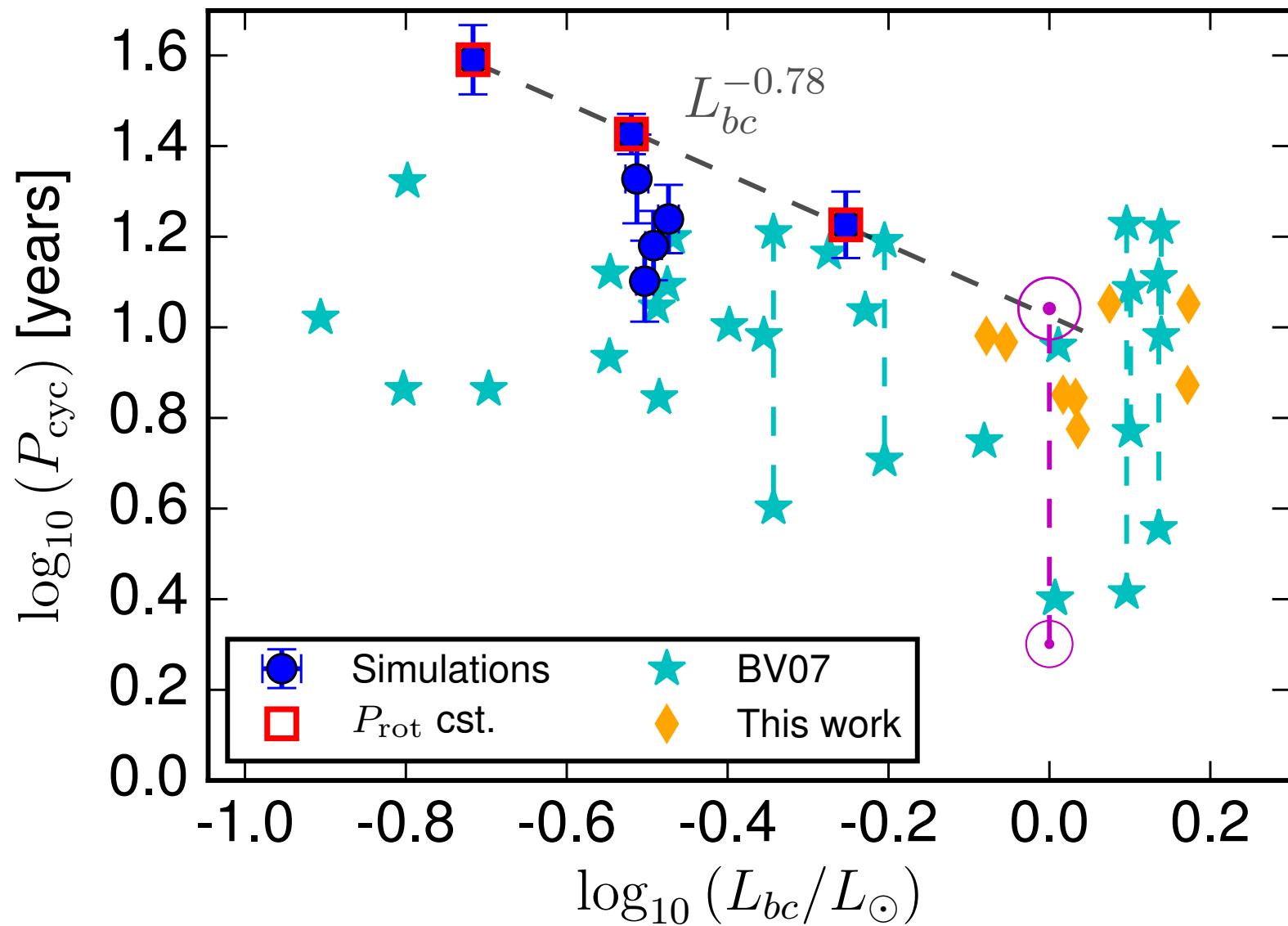
# Cycle period – Luminosity diagram



[Strugarek+ 17]

A. Strugarek, Seminar at Laboratoire Leprince-Ringuet, 25/09/2017

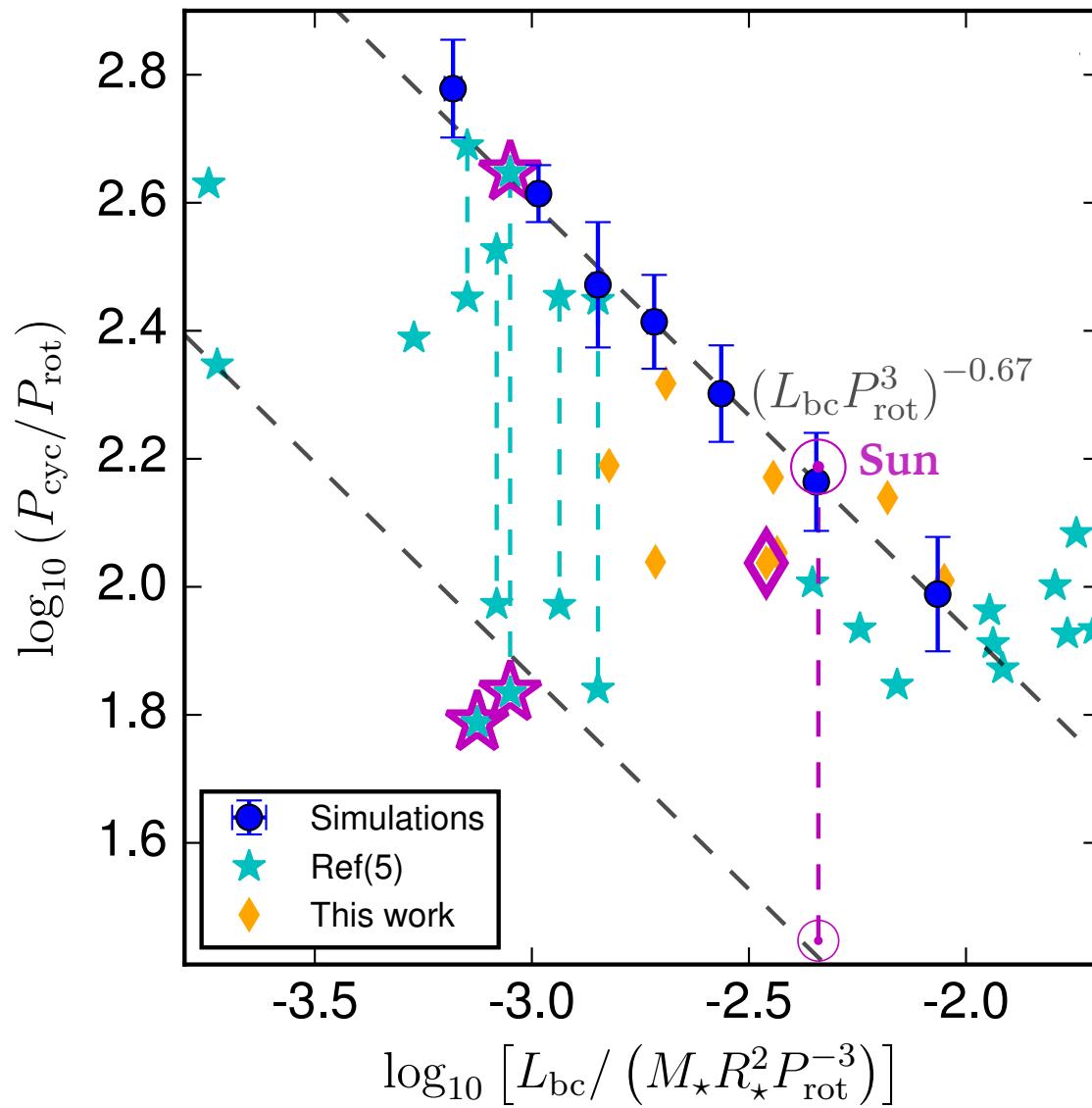
# Cycle period parametrization in the stellar context



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A. Strugarek, Seminar at Laboratoire Leprince-Ringuet, 25/09/2017

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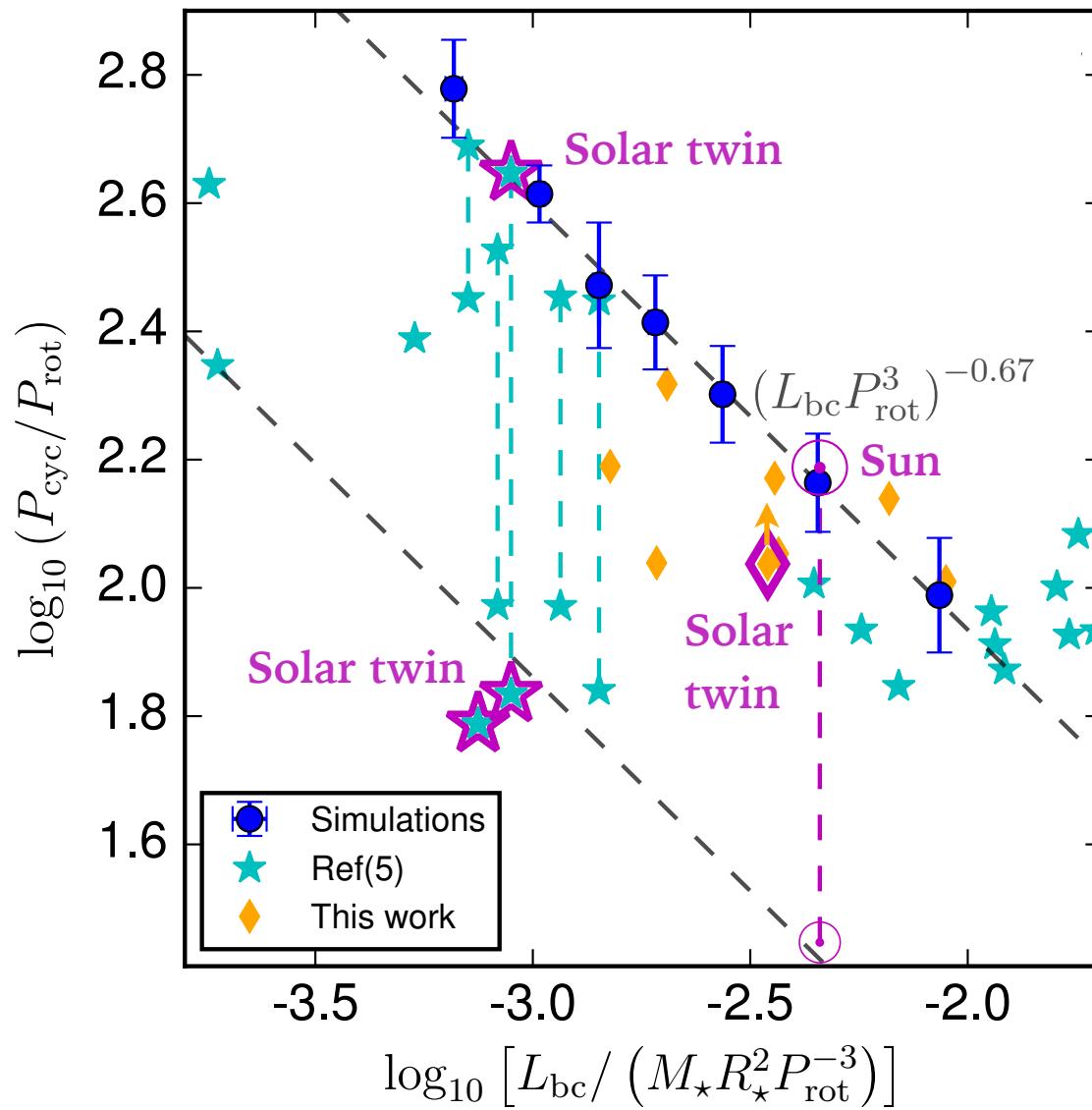


A non-linear  
dynamo mechanism  
relying on the  
temporal fluctuation  
of the large-scale  
differential rotation,  
reconciling solar  
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# Conclusions

**Global, non-linear 3D turbulent simulations** have been very useful to refine our understanding of the dynamics of magnetized stellar convection zones

Such simulations produced a **large variety of solutions** over the past decade: large-scale field self-organization, reversals, and magnetic cycles.

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Exciting times as fundamental aspects of stellar dynamos  
are being revised from both observations and theories