Realistic shell-model calculations for astrophysically relevant Gamow-Teller distributions

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Structure and Reactions for Nuclear Astrophysics
IPHC Strasbourg

Nuclear weak-interaction processes in stars

Nuclear weak-interaction processes play a key role in many astrophysical scenarios.

- solar nuclear reaction network
- *r*- and *s*-process nucleosynthesis
- core-collpase (type-II) supernovae
- thermonuclear (type-Ia) supernovae

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weak interaction rates

$\Downarrow$

astrophysical simulations

Stellar weak interaction rates

- $(Z, A) + e^- \rightarrow (Z-1, A) + \nu$ electron capture
- $(Z, A) \rightarrow (Z-1, A) + e^+ + \nu$ β^+ decay
- $(Z, A) + e^+ \rightarrow (Z+1, A) + \bar{\nu}$ positron capture
- $(Z, A) \rightarrow (Z+1, A) + e^- + \bar{\nu}$ β^- decay

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Decay rate

$$\lambda^\alpha = \frac{\ln 2}{K} \sum_i \frac{(2J_i + 1)e^{-E_i/(kT)}}{G(Z, A, T)} \sum_j B_{ij} \Phi_{ij}^\alpha$$

$G(Z, A, T)$: partition function
Φ_{ij}^α: phase space integral $\Phi_{ij}^\alpha = \Phi_{ij}^\alpha(\rho Y_e, T)$
B_{ij}: reduced nuclear transition probability

- Gamow-Teller

$$B_{ij} = B_{ij}(GT) = \frac{\langle j || \sum_k \sigma^k t_{\pm}^k || i \rangle^2}{2J_i + 1}$$

Nuclear models

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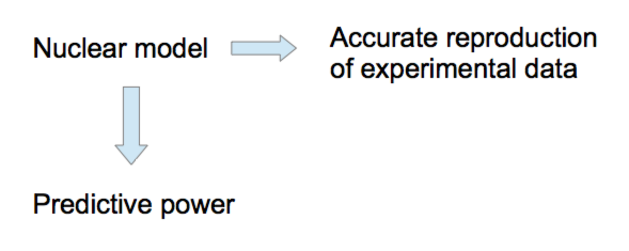

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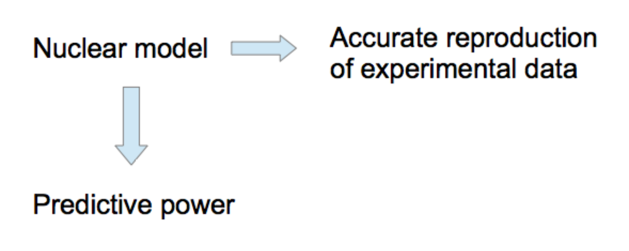

Fuller, Fowler and Newmann estimated stellar EC and β-decay rates $\Rightarrow$ independent particle model + experimental data

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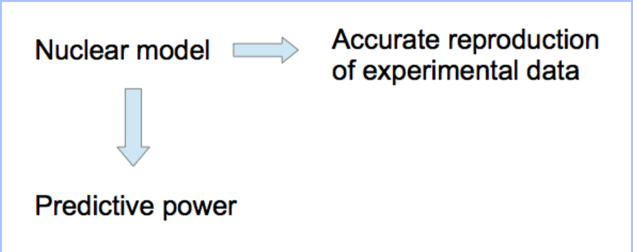

At the beginnning of the 90s Aufderheide, Mathews and collaborators pointed out that shell model is the method of choice for the calculation of stellar weak-interaction rates.

Nuclear models

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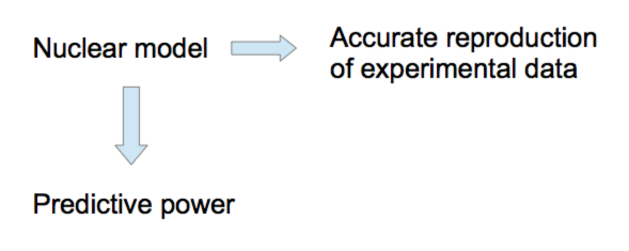
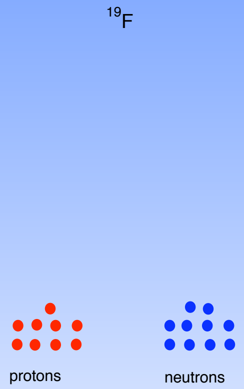


Langanke & Martinez Pinedo, 2000 $\Rightarrow$ *pf*-shell nuclei

An example: ^{19}F



- 9 protons & 10 neutrons interacting
- spherically symmetric mean field (e.g. harmonic oscillator)
- 1 valence proton & 2 valence neutrons interacting in a truncated model space

The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.

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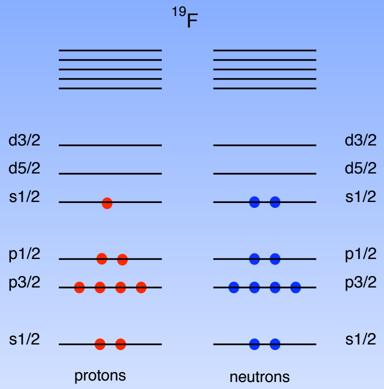

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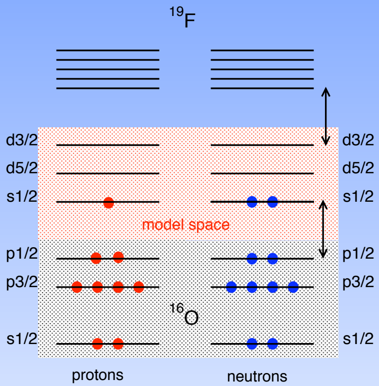

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Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} \quad (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian

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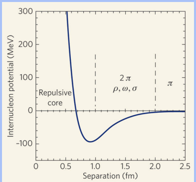
Workflow for a realistic shell-model calculation

1. Choose a realistic *NN* potential (*NNN*)
2. Determine the model space better tailored to study the system under investigation
3. Derive the effective shell-model hamiltonian and operators by way of a many-body theory
4. Calculate the physical observables (energies, e.m. transition probabilities, ...)

Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



How to handle the short-range repulsion ?

- Brueckner G matrix
- low-momentum NN potentials
  - V_{low-k} (Lee-Suzuki or SRG)
    chiral potentials rooted in EFT

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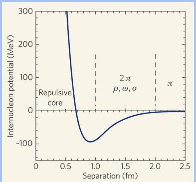

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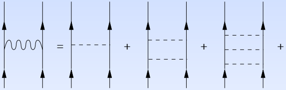

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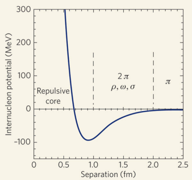

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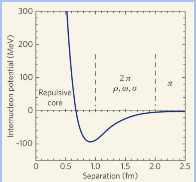

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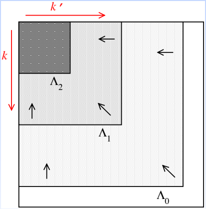

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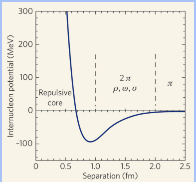

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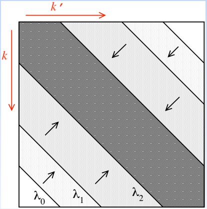

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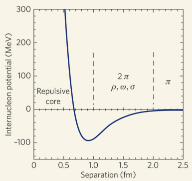

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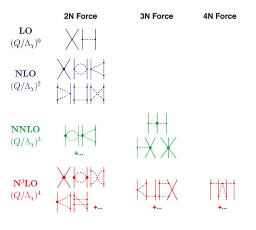

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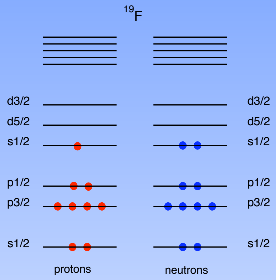
Effective shell-model hamiltonian

A-nucleon system Schrödinger equation

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i)$$



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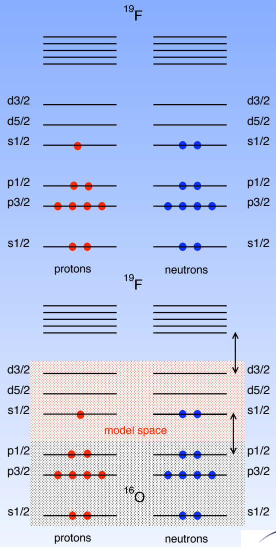
Model space

$$|\Phi_i\rangle = [a_1^\dagger a_2^\dagger \dots a_n^\dagger]_i|c\rangle \Rightarrow P = \sum_{i=1}^{d}|\Phi_i\rangle\langle\Phi_i|$$

Model-space eigenvalue problem

$$H_{\text{eff}} P|\Psi_\alpha\rangle = E_\alpha P|\Psi_\alpha\rangle$$



The shell-model effective hamiltonian

$$
\left(\begin{array}{c|c}
PHP & PHQ \\
\hline
QHP & QHQ
\end{array}\right)
\quad
\begin{array}{c}
\mathcal{H} = X^{-1}HX \\
\Longrightarrow
\end{array}
\quad
\left(\begin{array}{c|c}
P\mathcal{H}P & P\mathcal{H}Q \\
\hline
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The shell-model effective hamiltonian

Folded-diagram expansion

$\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

$\Rightarrow$ Recursive equation for H_{eff} $\Rightarrow$ iterative techniques (Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\mathrm{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots ,$$

generalized folding

The perturbative approach to the shell-model H^{eff}

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

The $\hat{Q}$-box can be calculated perturbatively

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

The diagrammatic expansion of the $\hat{Q}$-box

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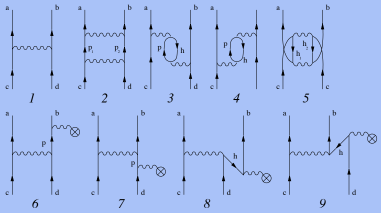

Effective operators

$\Phi_\alpha =$ eigenvectors obtained diagonalizing $H_{\rm eff}$ in the reduced model space $\Rightarrow |\Phi_\alpha\rangle = P|\Psi_\alpha\rangle$

$$\langle\Phi_\alpha|\hat{O}|\Phi_\beta\rangle \neq \langle\Psi_\alpha|\hat{O}|\Psi_\beta\rangle$$

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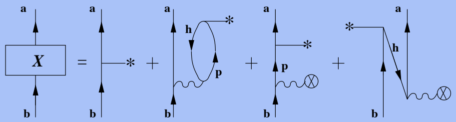
$\hat{O}_{\rm eff}$ can be derived consistently in the MBPT framework

One-body operator



Realistic shell-model calculations

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- H^{eff} for systems with one and two valence nucleons
- $\hat{Q}$-box $\Rightarrow$ Goldstone diagrams up to third order in V_{NN} (up to 2p-2h core excitations)
- Effective operators consistently derived by way of MBPT

pf-shell nuclei

Astrophysical interest

- electron capture rates relevant for late stellar evolution
- GT-strength distributions
  - β-decay experiments
  - charge-exchange reactions: (n,p), (d,^{2}He), (t,^{3}He)

$$\left[\frac{d\sigma}{d\Omega}(q=0)\right] = \hat{\sigma} B(GT)$$

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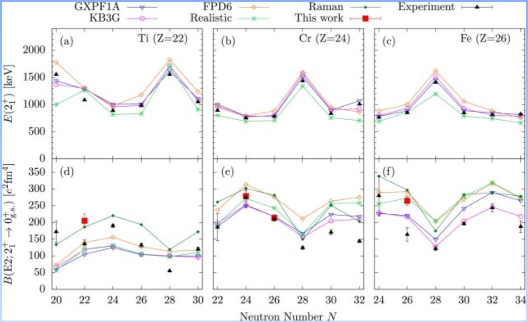
Realistic shell-model calculation

- Realistic potential $V_{NN} \Rightarrow$ high-precision *NN* CD-Bonn potential
- $\Rightarrow H_{\rm eff}$ & $GT_{\rm eff}$ consistently derived in the MBPT frame

Results: N=Z light nuclei



K. Arnswald, *et al*, Phys. Lett. B 772 (2017), 599

- recoil distance Doppler-shift method → 2^+ lifetimes in ^{44}Ti, ^{48}Cr, and ^{52}Fe
- RSM results in overall good agreement with the experiment

Results: GT matrix elements from individual β decays

$$M(GT) = [(2J_i + 1)B(GT)]^{1/2} \rightarrow R(GT) = M(GT)/W$$

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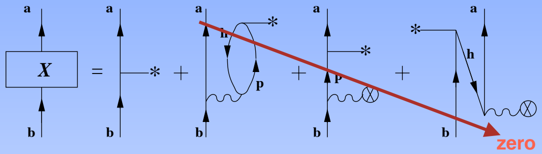

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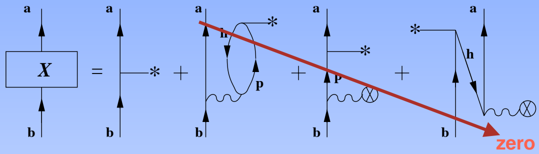
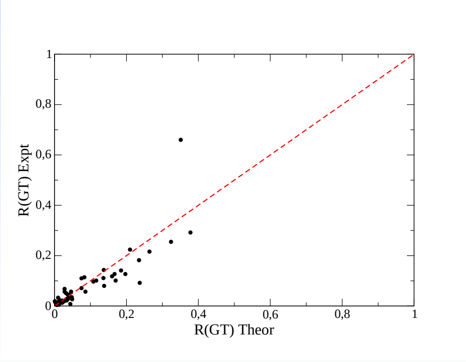




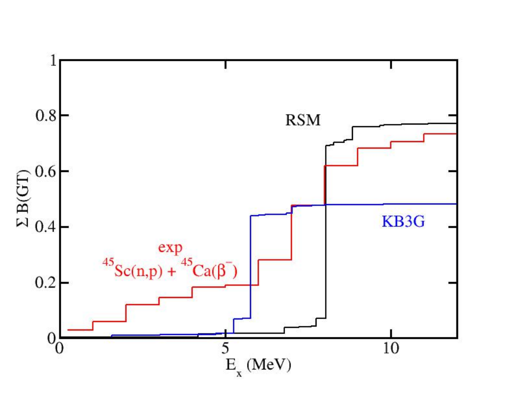
Results: GT strength distributions from charge-exchange reactions

- GT-strength distribution
  - $^{45}Sc \rightarrow ^{45}Ca$
  - $^{48}Ti \rightarrow ^{48}Sc$
  - $^{54}Fe \rightarrow ^{54}Mn$

- Comparison with experiment and with phenomenological shell-model results (KB3G with quenching factor = $(0.74)^2$)

$^{45}Sc \rightarrow {}^{45}Ca$

$^{45}Sc(\frac{7}{2}^-)_{gs} \rightarrow {}^{45}Ca(\frac{5}{2}, \frac{7}{2}, \frac{9}{2})^-$



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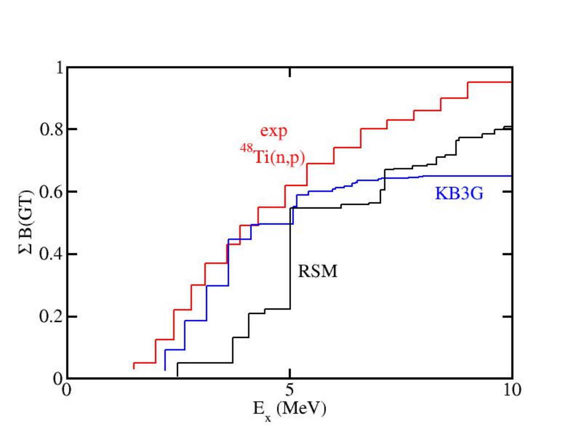
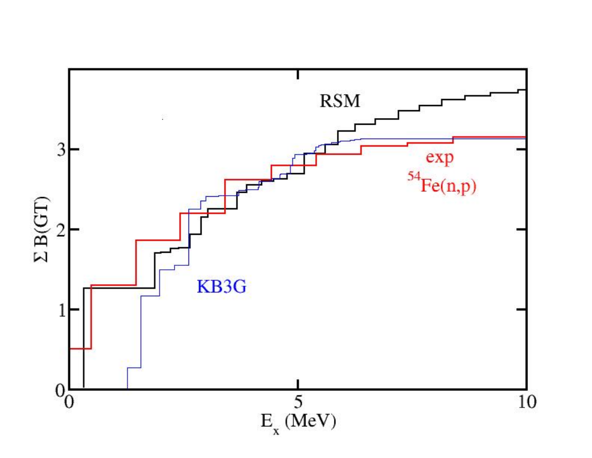
$^{48}Ti \rightarrow {}^{48}Sc$

$^{48}Ti(0^+)_{gs} \rightarrow {}^{48}Sc(1^+)$



$^{54}Fe \rightarrow ^{54}Mn$

$^{54}Fe(0^+)_{gs} \rightarrow ^{54}Mn(1^+)$



Conclusions and perspectives

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- RSM calculations provide a satisfactory description of observed GT-strength distributions in *pf* nuclei without resorting to any "ad hoc" GT-operator quenching
- investigate other nuclei of astrophysical interest (e.g. *sd* nuclei)
- calculation of *EC* rates at relevant stellar temperatures and densities ($T \simeq 1 - 10 \times 10^9 K$, $\rho Y_e \simeq 10^7 - 10^9 g/cm^3$)