

## Nuclear clustering in the energy density functional approach

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# Outline

## Brief outline of the talk

**EDF framework**



**Nuclear clustering**

How atomic nuclei cluster

Cluster structures in Ne isotopes

Towards unified framework



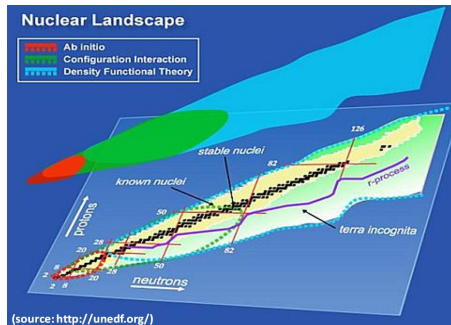
**Conclusion**

# EDF framework



# EDF framework

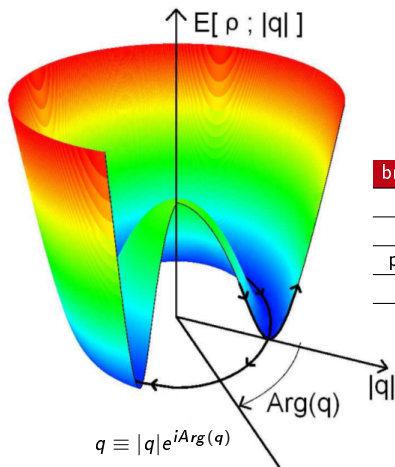
## Basic properties of the theory



- the nuclear many-body problem is mapped onto a one-body problem
- covariant functionals: meson-exchange or contact interaction
- the coupling parameters are fine-tuned to experimental data
- self-consistent calculation of bulk properties

# EDF framework

## Spontaneous symmetry breaking



broken symmetry	nuclei	cause
translational	all	localization
rotational	non-spherical	deformation
particle number	superfluid	pairing
parity	pear-shaped	octupole def.

T. Duguet, Lectures in Theoretical Nuclear Structure (Leuven, 2015).

# EDF framework

## Symmetry restoration and configuration mixing

- linear combination of symmetry-projected RHB states

$$\underbrace{|JM\pi; \alpha\rangle}_{\text{collective state}} = \sum_j \sum_K \underbrace{f_\alpha^{JK\pi}(q_j)}_{\text{weight function}} \underbrace{\hat{P}_{MK}^J \hat{P}^\pi}_{\text{projectors}} \underbrace{|\phi(q_j)\rangle}_{\text{RHB state}}$$

- variational principle leads to HWG equation

$$\sum_j \left[ \underbrace{\mathcal{H}^{J\pi}(q_i, q_j)}_{\text{Hamiltonian kernel}} - E_\alpha^{J\pi} \underbrace{\mathcal{N}^{J\pi}(q_i, q_j)}_{\text{norm kernel}} \right] f_\alpha^{J\pi}(q_j) = 0$$

- after a *bit* of maths, we obtain modified HWG equation:

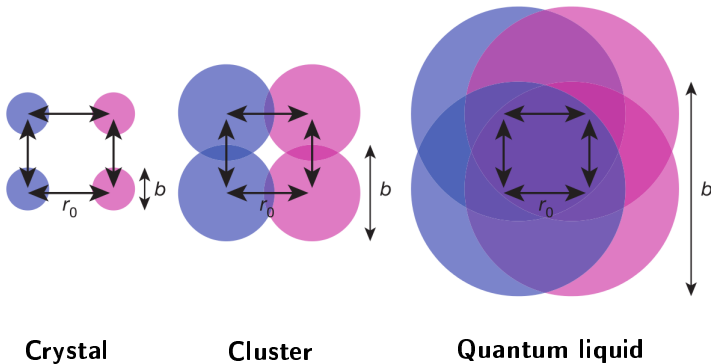
$$\sum_l \mathcal{H}_{kl}^{J\pi c} g_l^{J\pi\alpha} = E_\alpha^{J\pi} g_k^{J\pi\alpha}$$

- calculation of excitation spectra and various observables

# Nuclear clustering

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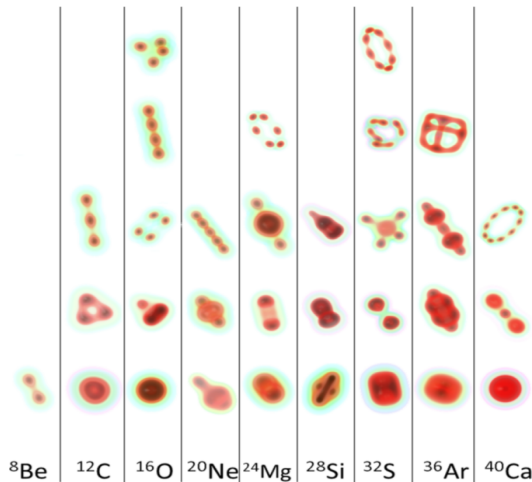
## Clustering as transitional phenomenon



J.-P. Ebran *et al.*, Nature 487, 341 (2012).

# Nuclear clustering

## Variety of shapes

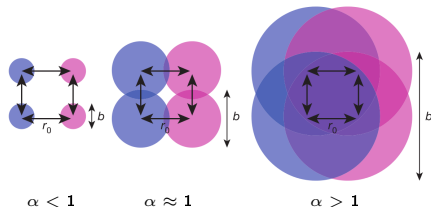


J.-P. Ebran *et al.*, PRC 90, 054329 (2014).

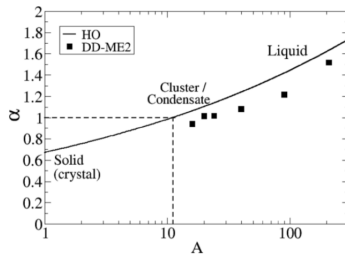
# Nuclear clustering

## Localisation parameter

$$\alpha = \frac{b}{r_0} = \frac{\sqrt{\hbar} A^{1/6}}{(2mV_0r_0^2)^{1/4}}$$



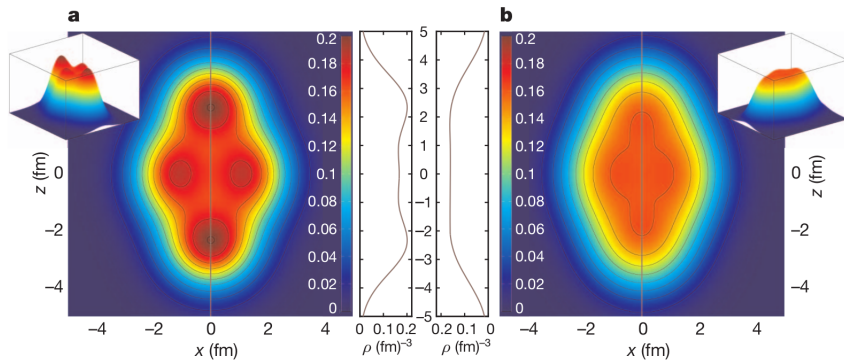
	Self-consistent	
	SLy4	DDME2
$^{20}\text{Ne}$	0.99	0.97
$^{24}\text{Mg}$	1.00	0.95
$^{28}\text{Si}$	0.99	0.96
$^{32}\text{S}$	0.99	0.96
$^{208}\text{Pb}$	1.28	1.31



J.-P. Ebran *et al.*, PRC 87, 044307 (2013).

# Nuclear clustering

## Relativistic vs. non-relativistic functionals



**DD-ME2**

$$V_0 = -82.4 \text{ MeV}$$

**Sly4**

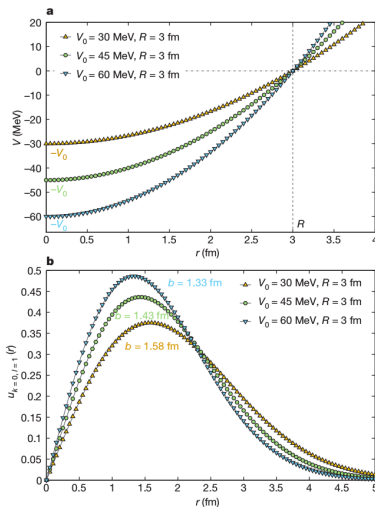
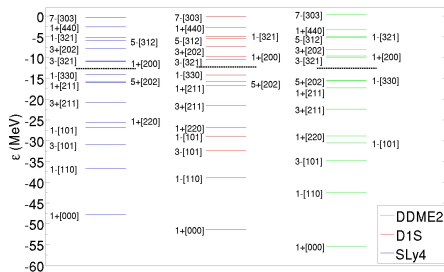
$$V_0 = -72.4 \text{ MeV}$$

J.-P. Ebran *et al.*, *Nature* 487, 341 (2012).



# Nuclear clustering

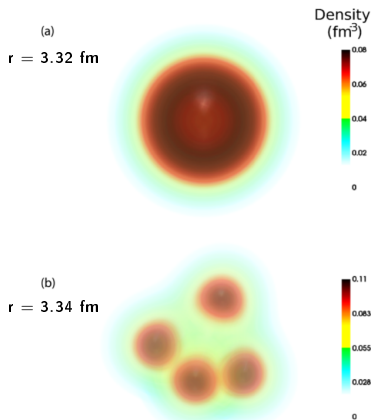
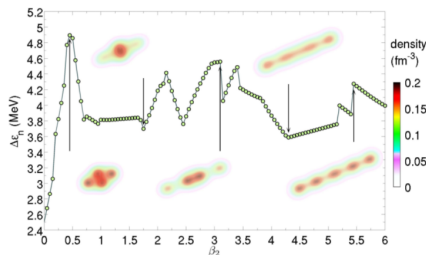
## Depth of the confining potential



J.-P. Ebran *et al.*, Nature 487, 341 (2012)., PRC 90, 054329 (2014).

# Nuclear clustering

## Role of deformation and density



J.-P. Ebran *et al.*, PRC 90, 031303(R) (2014)., PRC 90, 054329 (2014).

# Structure of Neon isotopes

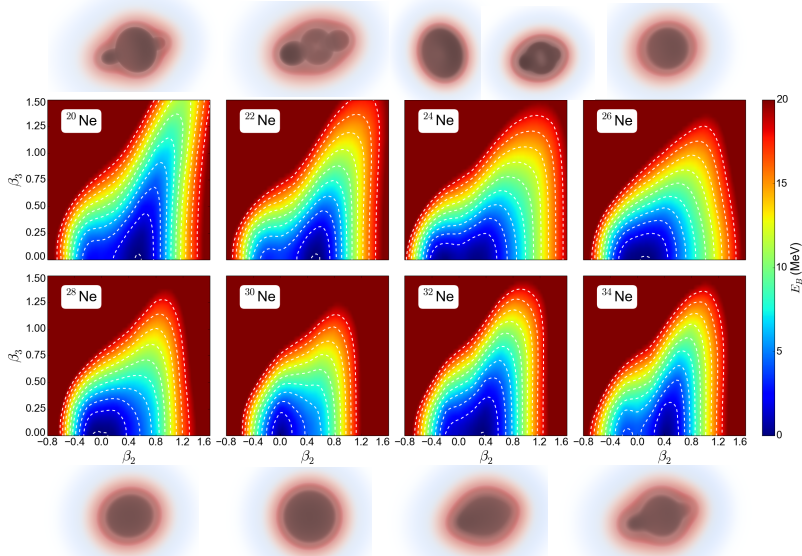
# Structure of Neon isotopes

## Parameters of the calculation

- RHB wave functions expanded in HO basis with  $N_{sh}^{max} = 10(11)$
- DD-PC1 functional and TMR pairing
- angular momentum and parity projection
- mixing of 130 – 150 configurations with  $\beta_2 \in [-0.8, 1.6]$  and  $\beta_3 \in [-2.0, 2.0]$

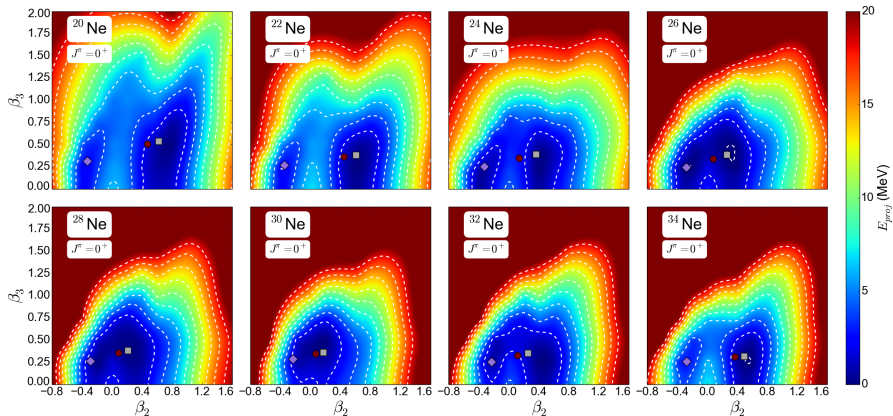
# Structure of Neon isotopes

## Mean-field potential energy surfaces



# Structure of Neon isotopes

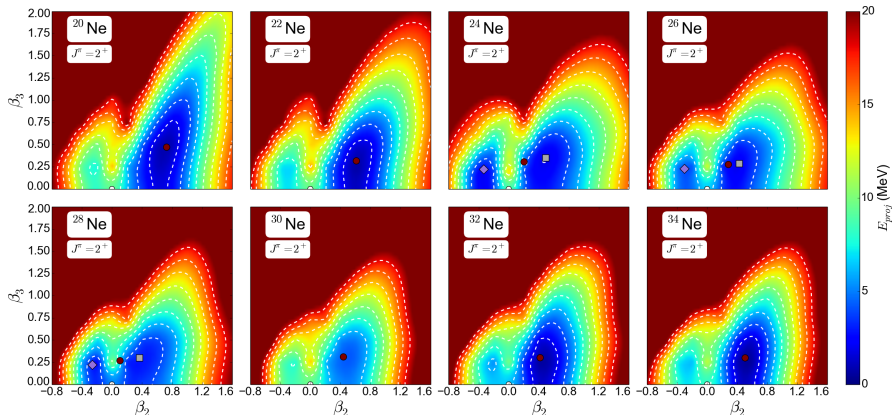
## Symmetry-projected potential energy surfaces



P.M. *et al.*, submitted to PRC

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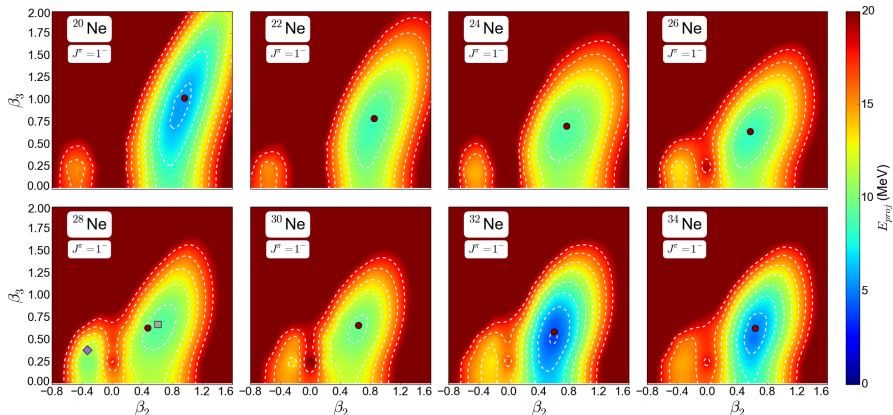
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P.M. *et al.*, submitted to PRC

# Structure of Neon isotopes

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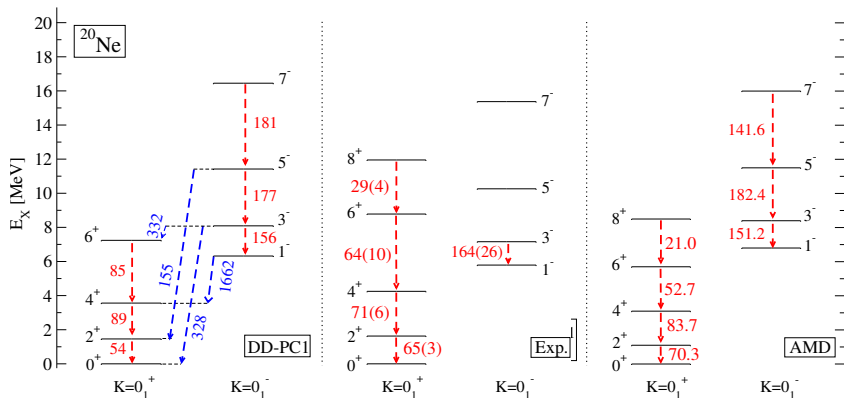


P.M. *et al.*, submitted to PRC



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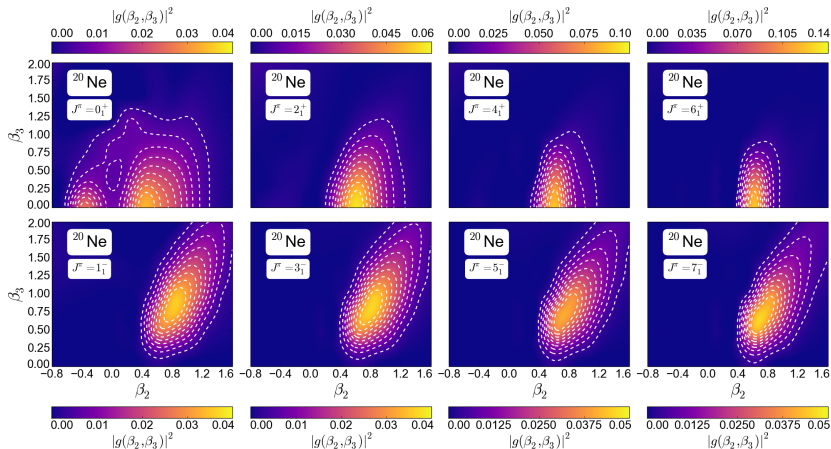
## Collective energy spectra of $^{20}\text{Ne}$



P.M. *et al.*, submitted to PRC

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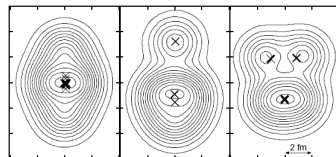
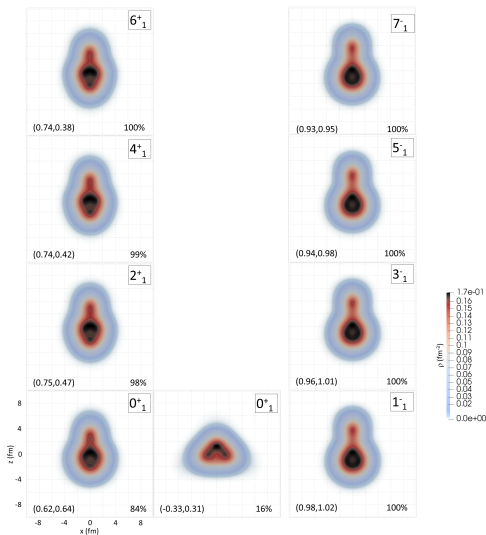
## Collective wave functions of $^{20}\text{Ne}$



P.M. *et al.*, submitted to PRC

# Structure of Neon isotopes

## Characteristic intrinsic nucleon densities



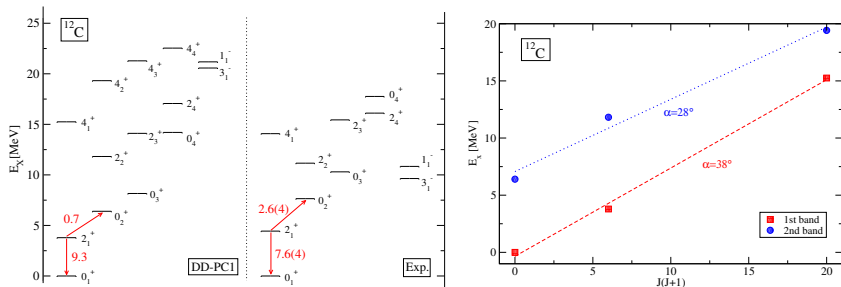
Y. Taniguchi *et al.* PTP (2004) 112 (3)

P.M. *et al.*, submitted to PRC

## Towards unified framework

# Towards unified framework

## Collective energy spectra of $^{12}\text{C}$ and the Hoyle state



- restoration of particle number symmetry
- calculation of (in)elastic form factors
- unified description of quantum-liquid and cluster states

P.M. *et al.*, preliminary

# Conclusion

## The wrap-up

- EDF as powerful tool for nuclear structure calculations
- description of clustering in atomic nuclei
  - depth of the confining potential
  - role of deformation and density
- going beyond mean-field: spectroscopic predictions
- application of the model: cluster structures in Neon isotopes
- towards unified description of quantum liquid and cluster states