Continuum shell model
Towards the unified theory of structure and reactions

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Workshop "Structure and Reactions for Nuclear Astrophysics"
IPHC, Strasbourg, November 22-24, 2017

Outline
- Introduction
- Shell Model finally complete! – the Gamow Shell Model
- Interplay of hermitian and anti-hermitian couplings
- Unified description of nuclear structure and reactions
- Origin of nuclear clustering
- Multineutron clustering: can four neutrons form a nucleus?
- Outlook

Collaborators

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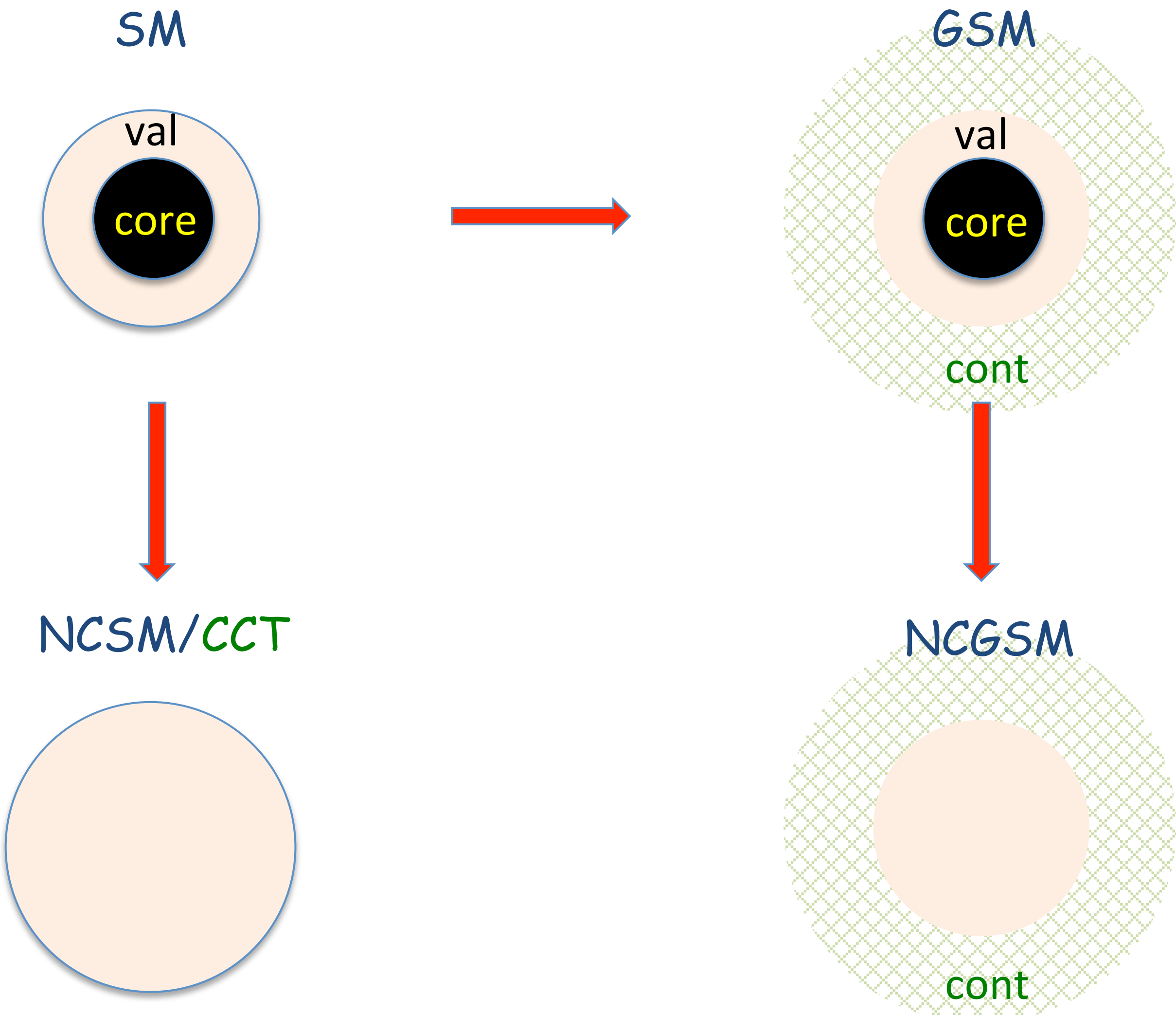
K. Fossez, Y. Jaganathen, N. Michel, J. Rotureau - NSCL-FRIB MSU, East Lansing
J. Okolowicz - Institute of Nuclear Physics, Krakow
A. Mercenne – GANIL
G. Dong – Huzhou Univ.
G. Papadimitriou – LLNL
B. Barrett – Univ. of Arizona, Tucson

Evolution of paradigms



Ab initio approach: GFMC, NCSM, CC

Ab initio approach: NCGSM, CC(B)

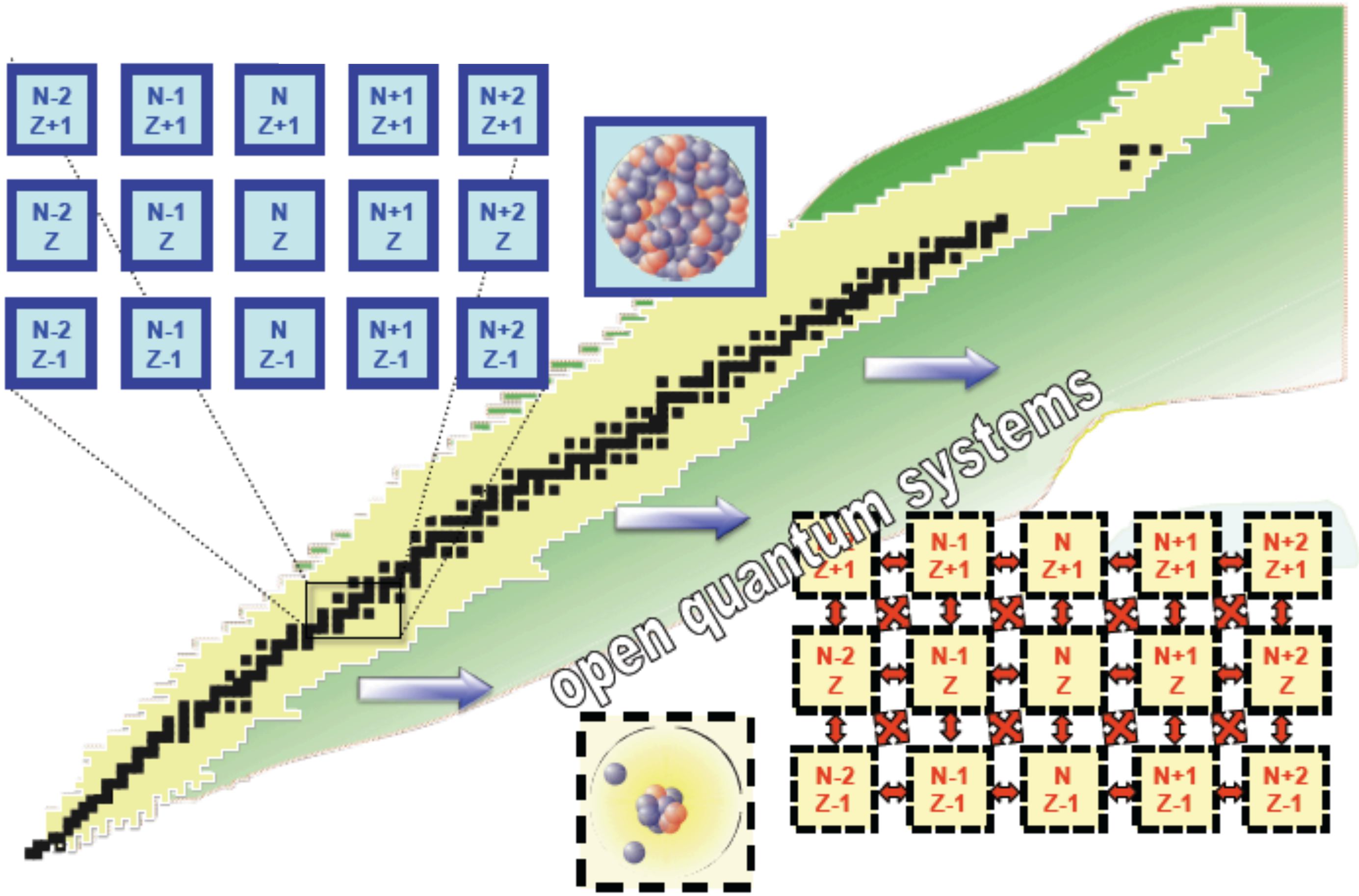
Challenges for the theory

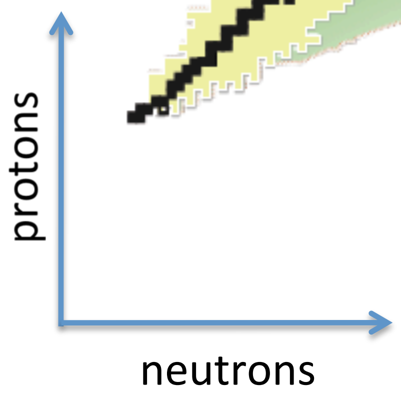
- How to reconcile shell model with reaction models?

- Weakly bound systems; role of continuum…
  How to deal with non-localities due to the coupling to decay channels
  and the antisymmetrization

- How to handle multi-configuration effects in reaction theory

- How to understand (optical) potentials from microscopic interactions
- …



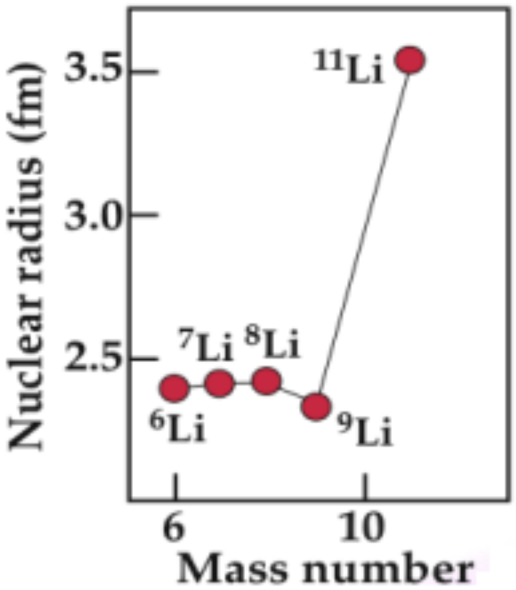


- Network of many-body states coupled via the continuum
- Emergence of energy scale(s) related to the threshold(s)
- S.p. field and pairing field are equally important:

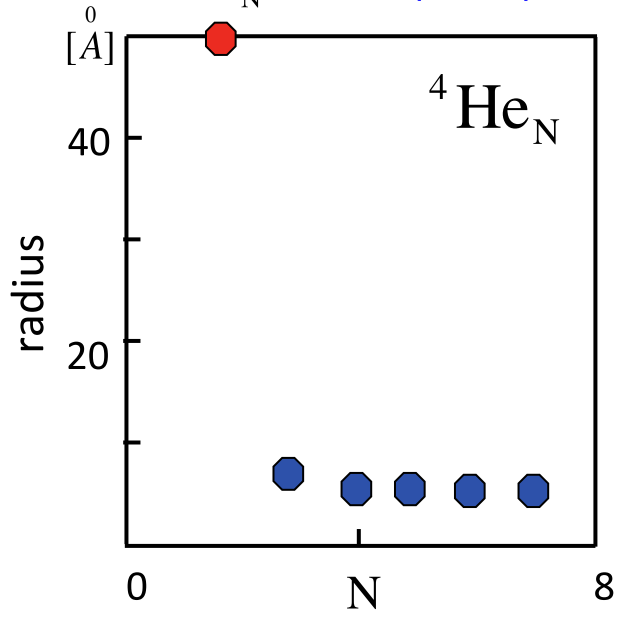
$$S_{1n} \approx -\lambda - \Delta$$

openness mean-field correlations

Borromean nuclei (1985)



^4He_N-dimer (2000)



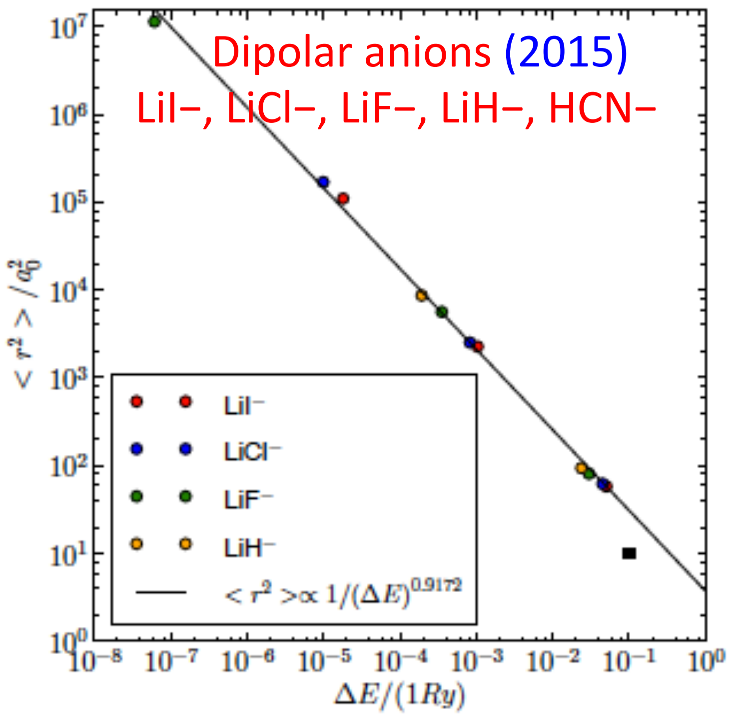
$\ell = 0: \ \langle r^2 \rangle$ diverges as $(-\varepsilon_\nu)^{-1}$

$\ell = 1: \ \langle r^2 \rangle$ diverges as $(-\varepsilon_\nu)^{-1/2}$

$\ell > 1: \ \langle r^2 \rangle$ remains finite

Dipolar anions (2015)

LiI−, LiCl−, LiF−, LiH−, HCN−



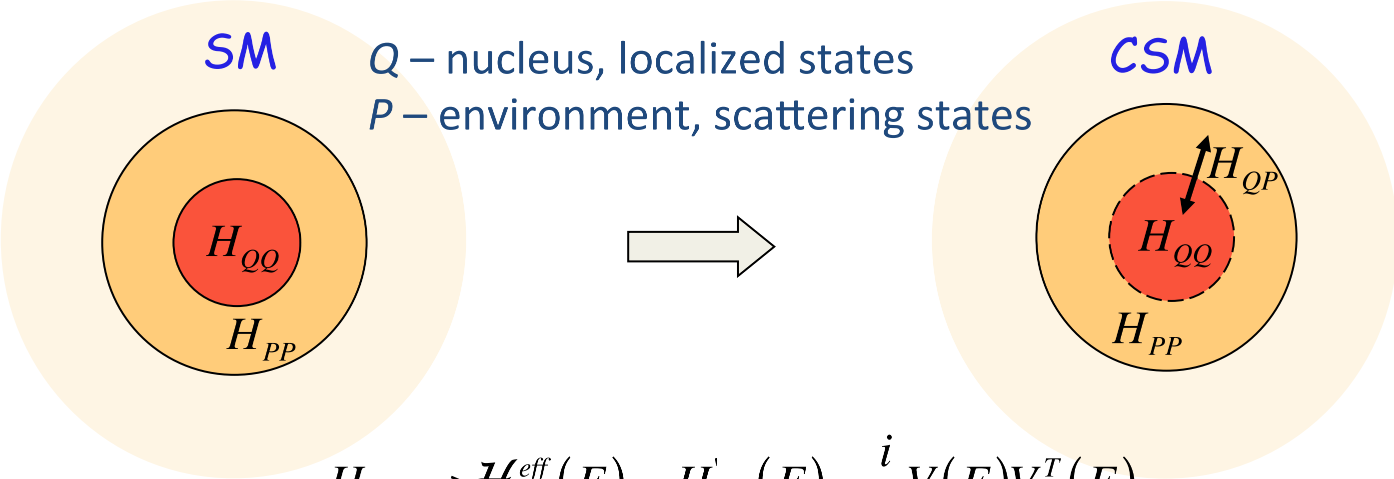
Paired densities decrease faster than unpaired densities ⇨ pairing anti-halo effect

K. Bennaceur, et al., PLB 496, 154 (2000)

Continuum shell model with the real-energy continuum

Shell Model Embedded in the Continuum (SMEC)

Q – nucleus, localized states
P – environment, scattering states



$$H_{QQ} \rightarrow \mathcal{H}_{QQ}^{eff}(E) = H'_{QQ}(E) - \frac{i}{2}V(E)V^T(E)$$

closed quantum system

hermitian

anti-hermitian

open quantum system

For bound states: $\mathcal{E}_\alpha(E)$ is real and $\mathcal{E}_\alpha(E) = E$
For unbound states: physical resonances = poles of S-matrix

C. Mahaux, H.A. Weidenmüller, *Shell Model Approach to Nuclear Reactions (1969)*
H.W.Bartz et al, Nucl. Phys. A275 (1977) 111
R.J. Philpott, Nucl. Phys. A289 (1977) 109
K. Bennaceur et al, Nucl. Phys. A651 (1999) 289
J. Rotureau et al, Nucl. Phys. A767 (2006) 13

Coupling of 'internal' (in Q) and 'external' (in P) states induces effective A-particle correlations and determines the structure of many-body states

Gamow shell model: Quasi-stationary formulation of Shell Model in complex k-plane

$$i\hbar\frac{\partial}{\partial t}\Phi(r,t) = \hat{H}\Phi(r,t) \qquad \Phi(r,t) = \tau(t)\Psi(r)$$

$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi \quad \leftarrow \quad \begin{cases}\Psi(0,k) = 0 \\ \Psi(\vec{r},k) \underset{r\to\infty}{\longrightarrow} O_l(kr)\end{cases}$$

Discrete energy solutions are poles of the $\mathfrak{S}$-matrix

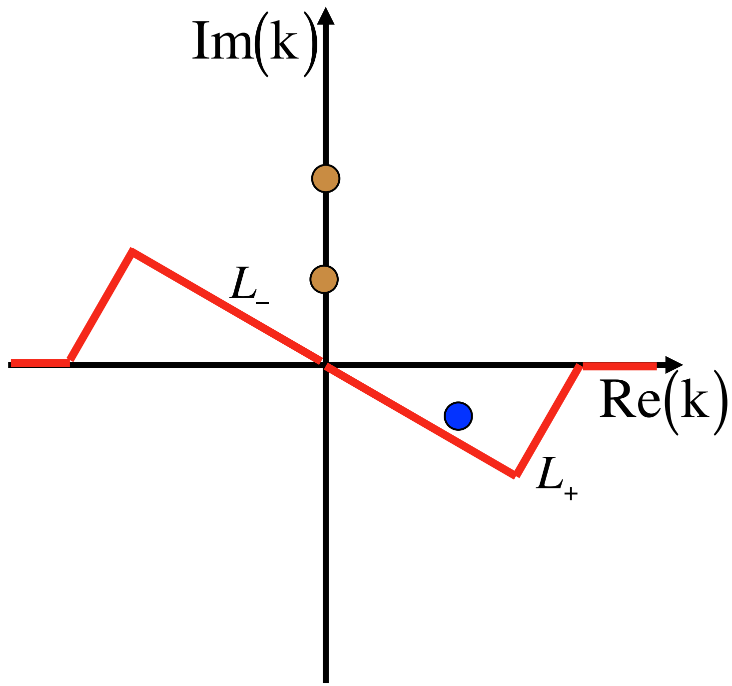
$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k|dk = 1 \quad ; \quad \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

$$|SD_i\rangle = |u_{i_1}\ldots u_{i_A}\rangle \longrightarrow \sum_k |SD_k\rangle\langle\widetilde{SD}_k| \cong 1$$

$\downarrow$

Gamow Shell Model

N. Michel et al, PRL 89 (2002) 042502
R. Id Betan et al, PRL 89 (2002) 042501
N. Michel et al, PRC 70 (2004) 064311



$$H \rightarrow [H]_{ij} = [H]_{ji}$$

Complex-symmetric eigenvalue problem for hermitian Hamiltonian

Ab initio no-core GSM (A<8)

G. Papadimitriou et al, PRC 88 (2013) 044318
K. Fossez et al., PRL 119, 032501 (2017)

Gamow particle + rotor model

K. Fossez et al., Phys. Rev. C93, 011305® (2016)

Coupled channel formulation of the Gamow shell model

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1 \quad ; \ \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

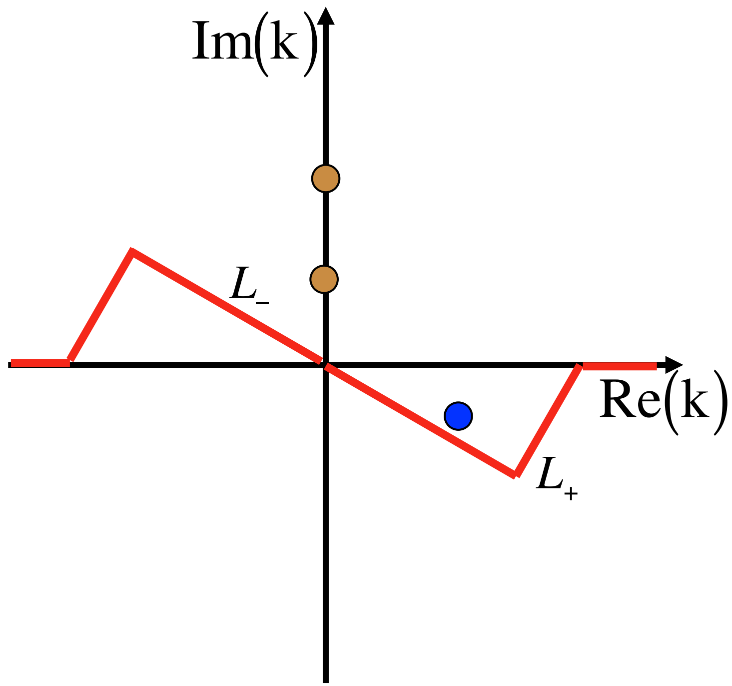
$$|\Psi\rangle = \sum_c \int_0^\infty dr \frac{u_c(r)}{r} r^2 \hat{A}|CS\rangle_c$$

GSM channel state

Channel basis: $\{c\} = \{A_T, J_T; a_P, \ell_P, J_{\text{int}}, J_P\}$

$$\hat{A}|CS\rangle_c \equiv |(c,r)\rangle = \hat{A}\left[|\Psi_T^{J_T}\rangle \otimes |r, \ell_P, J_{\text{int}}, J_P\rangle\right]_{M_A}^{J_A}$$

Y. Jaganathen et al, PRC 88, 044318 (2014)
K. Fossez et al., PRC 91, 034609 (2015)



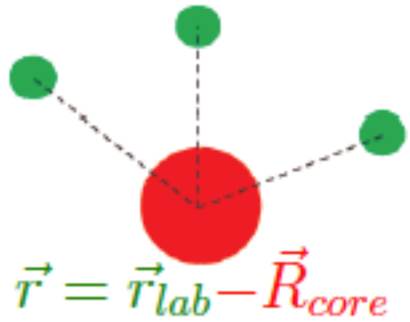
- Entrance and exit channels defined : GSM in this representation is a tool *par excellence* for nuclear reaction studies
- Diagonalizations of the Hamiltonian matrix in both representations are equivalent if the channel basis is complete ⟶ stringent test for reaction studies

- **Center of mass treatment:** Cluster Orbital Shell Model relative coordinates

Y. Suzuki, K. Ikeda, PRC 38 (1998) 410

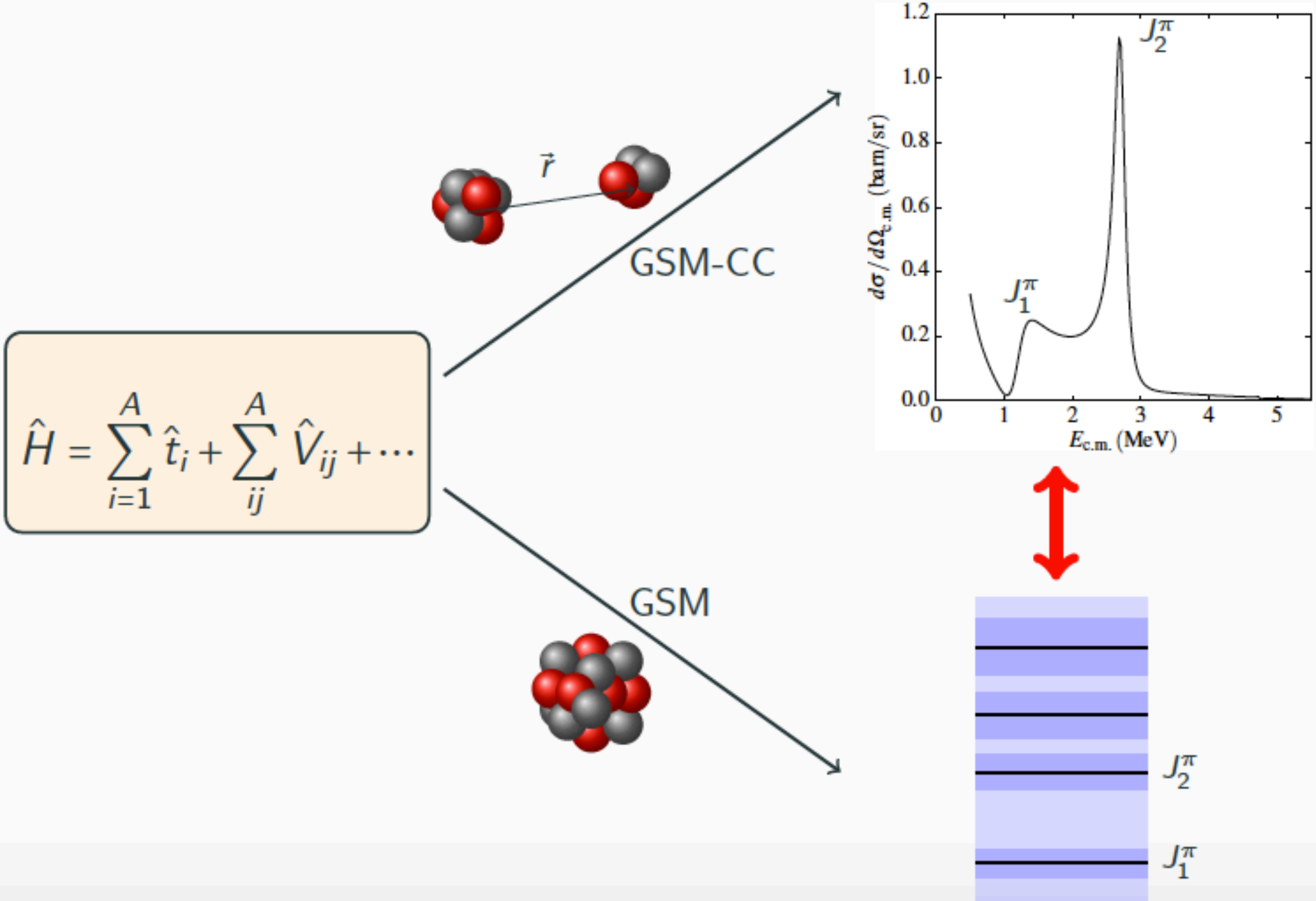
$$H = \sum_{i=1}^{A_v}\left(\frac{\mathbf{p}_i^2}{2\mu} + U_i\right) + \sum_{i<j}^{A_v}\left(V_{ij} + \frac{p_i p_j}{A_c}\right)$$

"Recoil" term coming from the expression of H in the COSM coordinates. No spurious states



- **Scattering wave functions** $\left|\Psi_{\mathrm{GSM}}(A-p)\otimes\Phi_{\mathrm{proj}}(p)\right\rangle$ **are the many-body states**
- **Antisymmetry exactly handled**
- **Core arbitrary**

Unified desciption of structure and reactions in the Gamow Shell Model



p+^{18}Ne excitation function

Y. Jaganathen, N. Michel, M.P., PRC 89 (2014) 034624

18Ne	EXP	GSM	GSM-CC
0+	0.00	0.00	
2+	1.89	1.56	

S_p=3.921 MeV
S_n=19.237 MeV

19Na	EXP	GSM	GSM-CC
5/2+	0.32	0.28	0.29
3/2+	0.44	0.25	0.27
1/2+	1.07	1.08	1.13

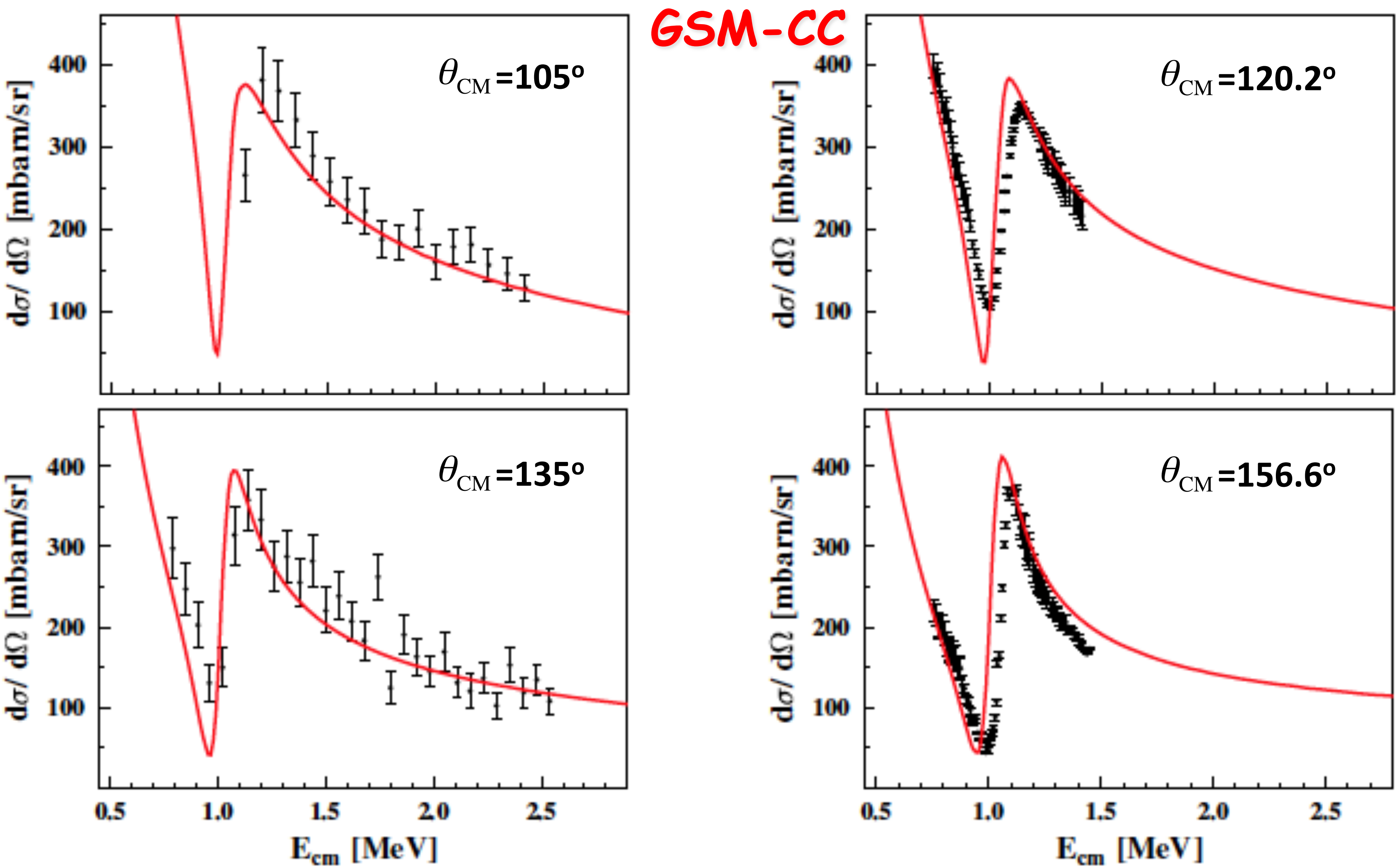
S_p=-0.32 MeV
S_n=20.18 MeV

Interaction: FHT finite-range interaction: $V(ij)=V^C + V^{SO} + V^T + V^{Coul}$

H. Furutani, H. Horiuchi, R. Tamagaki, PTP 60 (1978) 307; 62 (1979) 981

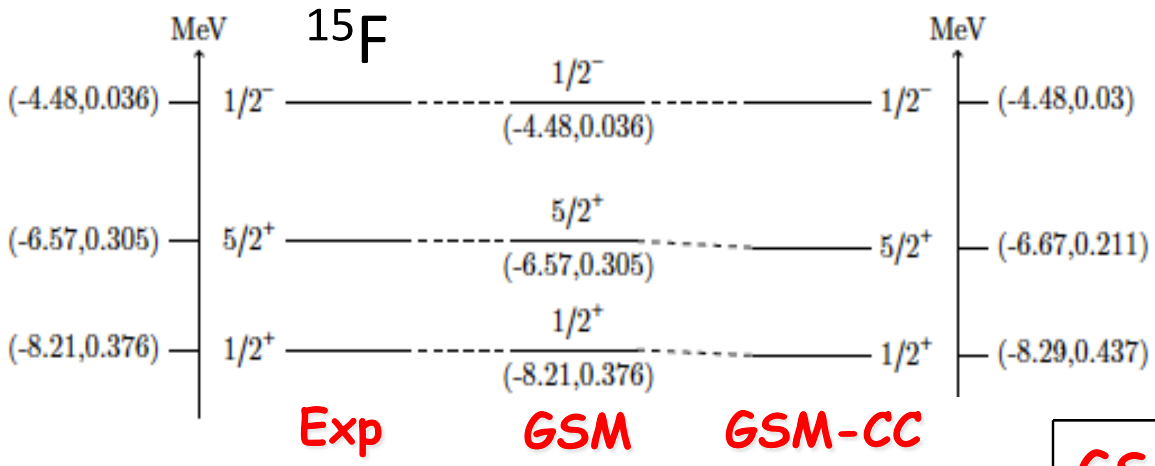
GSM and GSM-CC results (almost) identical → Scattering states $J=0^+,1^+,2^+,\ldots$ and higher lying (bound) states in ^{18}Ne are unimportant

p+^{18}Ne excitation function at different angles

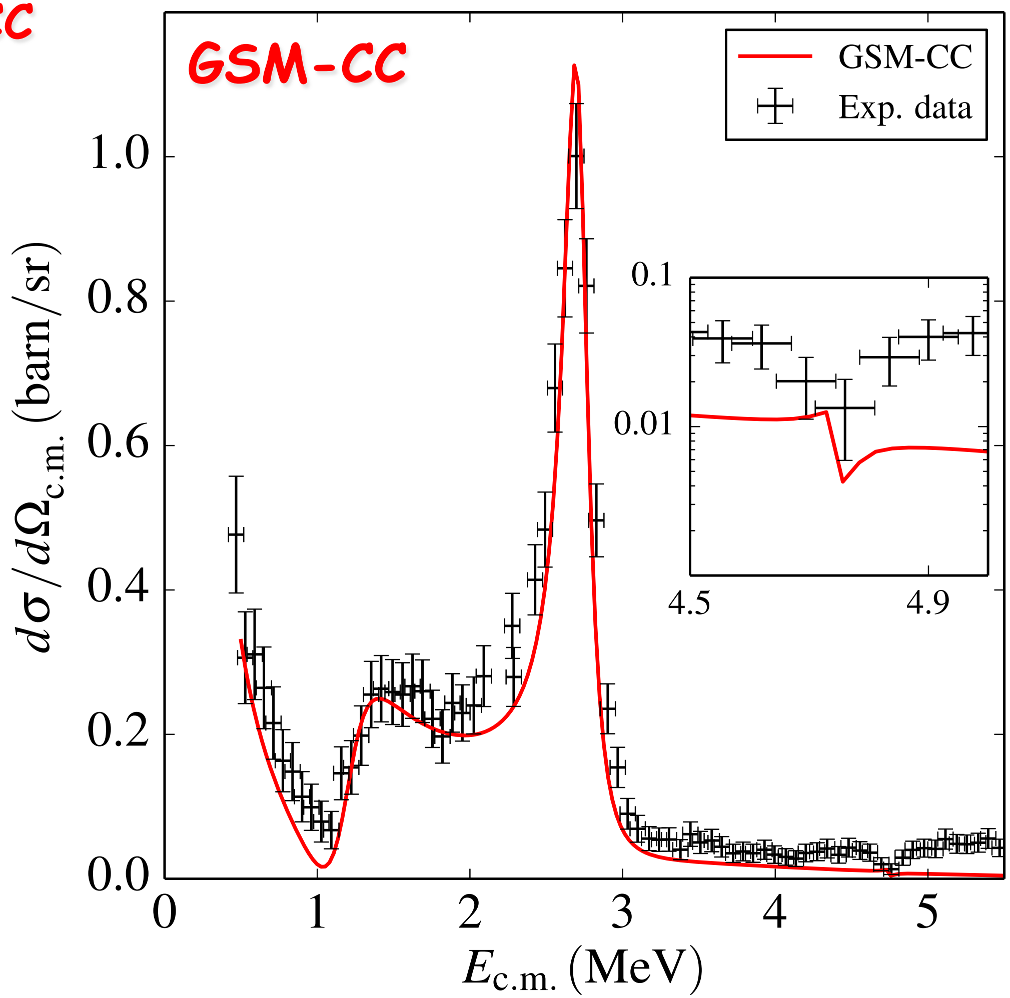


Exp: F. de Oliveira Santos et al., Eur. Phys. J. A24, 237 (2005)
B. Skorodumov et al., Phys. Atom. Nucl. 69, 1979 (2006)
C. Angulo et al., PRC 67, 014308 (2003)

p+^{14}O excitation function and spectroscopy of ^{15}F



F. De Grancey et al, PLB 758, 26 (2016)

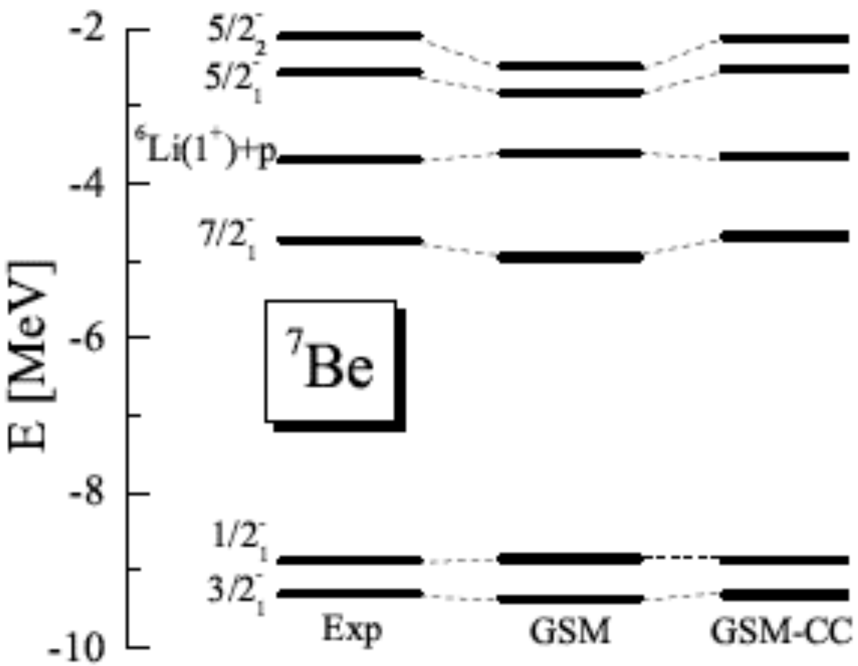


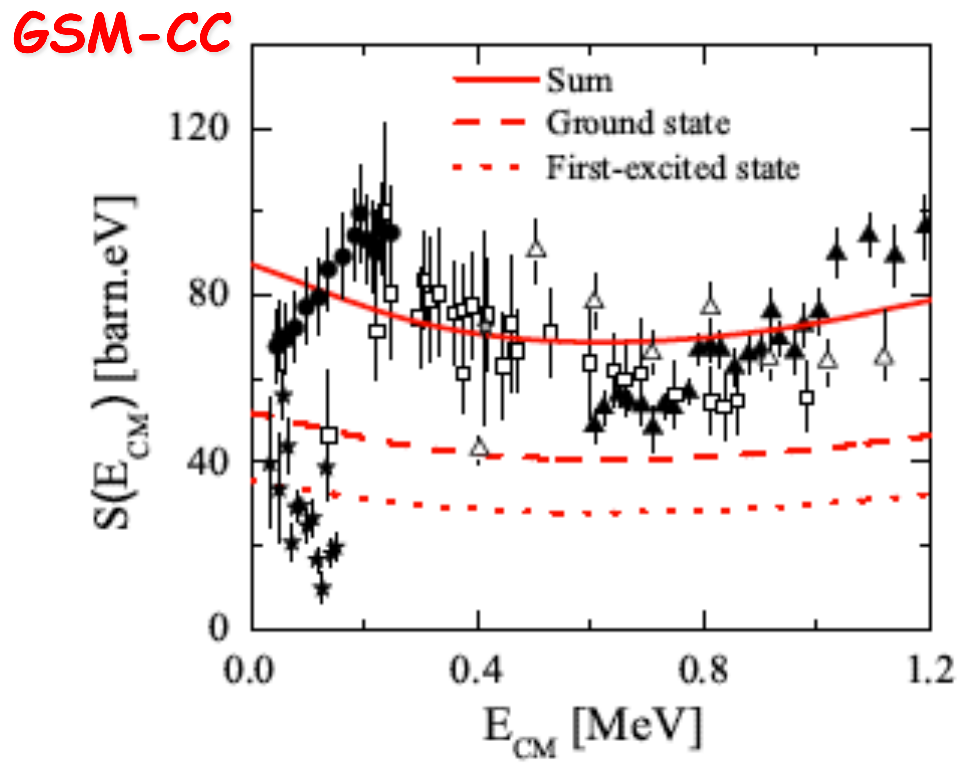
- Model space: $0p_{1/2}$, $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$
- FHT finite-range interaction:

  $V(ij)=V^C + V^{SO} + V^T + V^{Coul}$

  H. Furutani, et al, PTP 60, 307 (1978)

^{6}Li(p,γ) radiative capture cross sections





- Model space: $0p_{3/2}$, $0p_{1/2}$, $0s_{1/2}$, $0d_{5/2}$, $0d_{3/2}$

- Interaction: FHT finite-range interaction

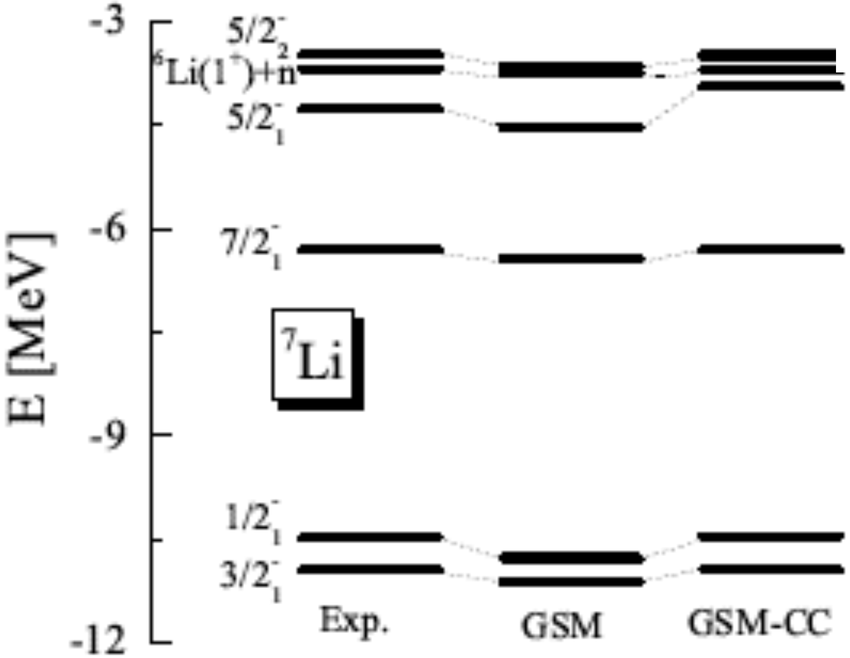
E1, E2, M1 components included

$S^{GSM\text{-}CC}(0)$=88.34 b eV

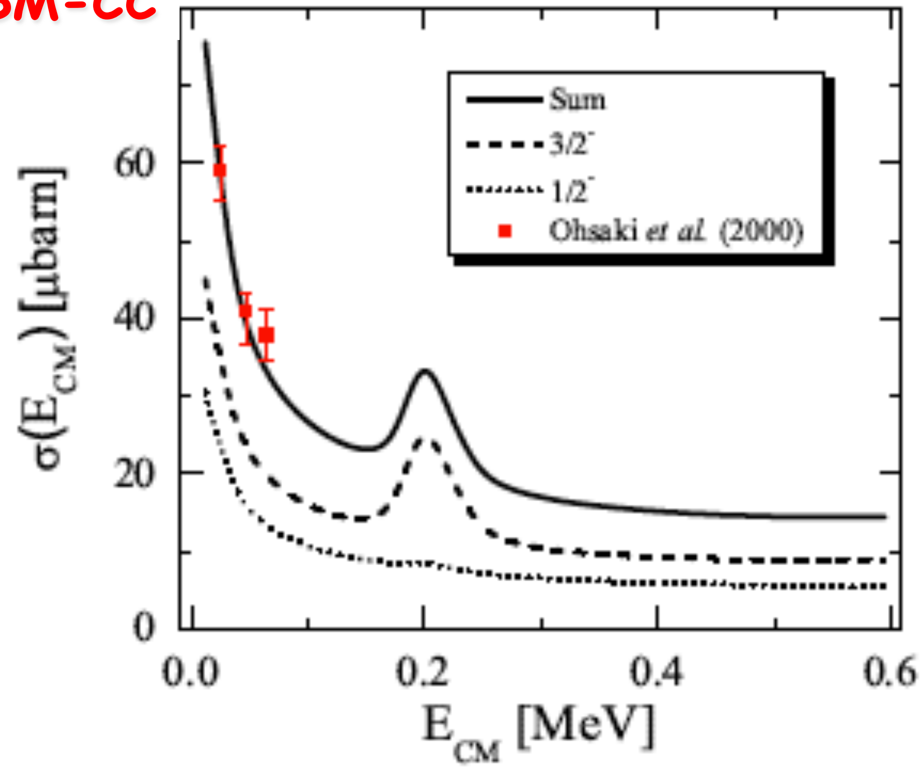
$S^{exp\text{-}acc}(0)$=79+/-18 b eV

G. Dong, et al., J. Phys. G: Nucl. Part Phys. 44. 045201 (2017)

Mirror reaction: $^6Li(n,\gamma)$



GSM-CC



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G. Dong, et al., J. Phys. G: Nucl. Part Phys. 44. 045201 (2017)

Origin of near-threshold clustering

"…α-cluster states can be found in the proximity of α-particle decay threshold…"

K. Ikeda, N. Takigawa, H. Horiuchi (1968)

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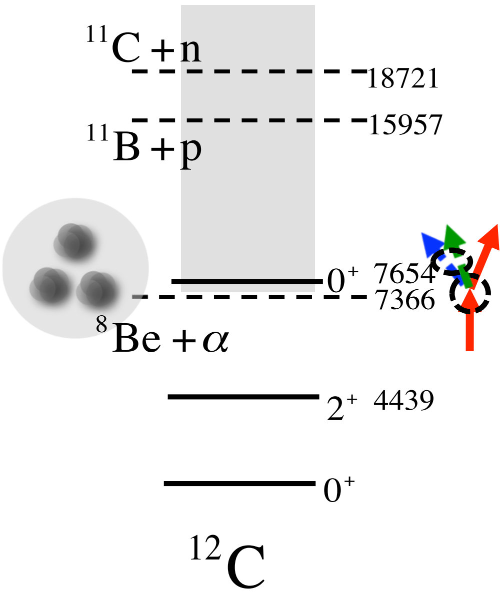
Why cluster (correlated) states appear in the proximity of cluster thresholds?



Anthropic principle?

J.D. Barrow, F.J. Tippler, *The Anthropic Cosmological Principle* Oxford University Press (1988)

E. Epelbaum et al., PRL 110, 112502 (2013)

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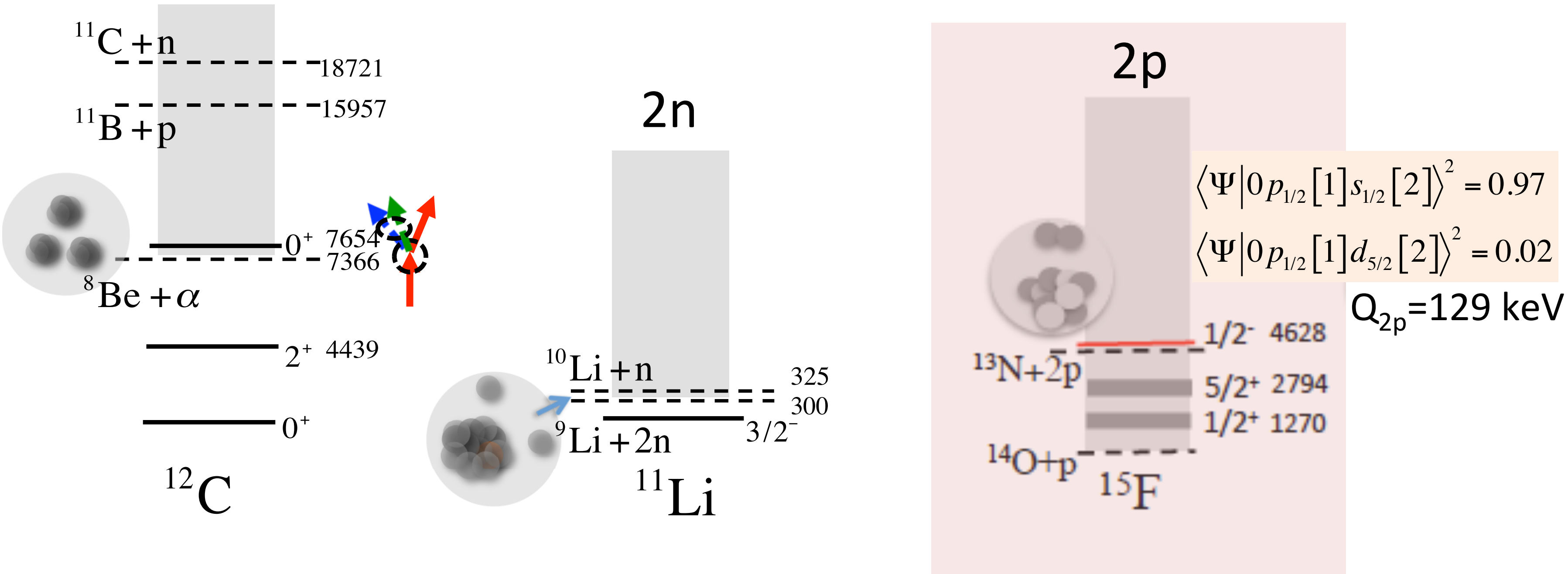

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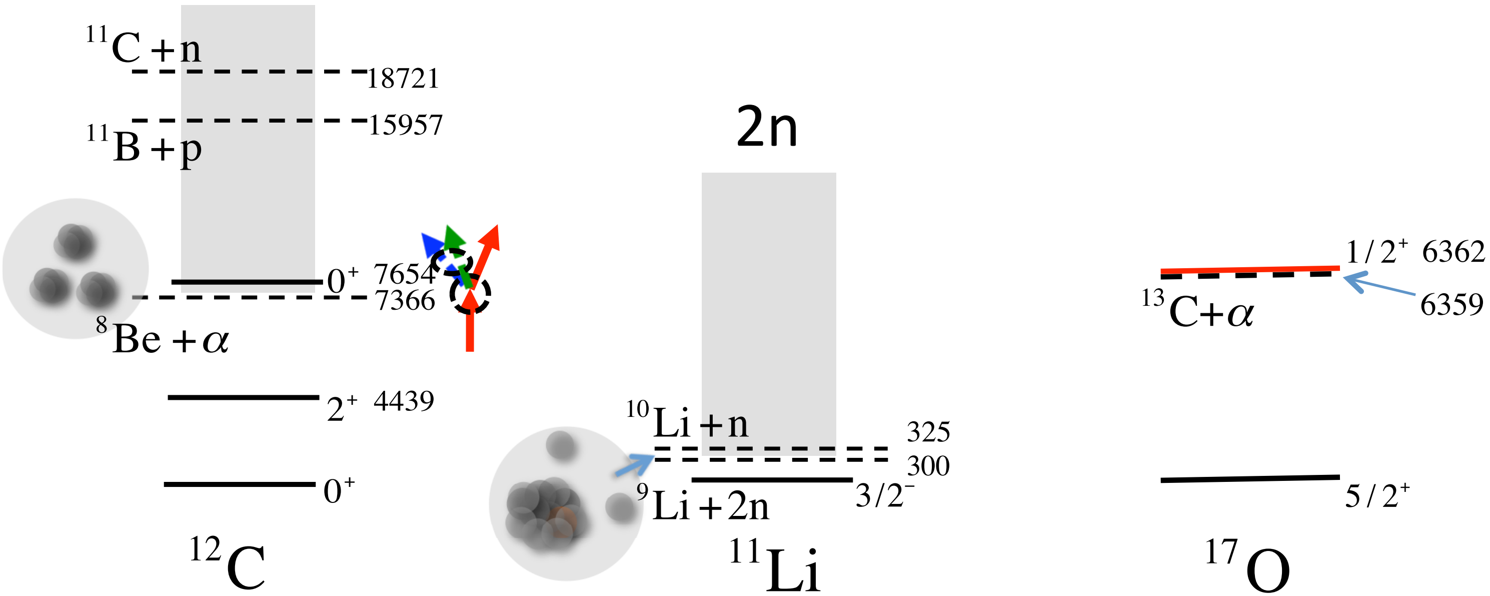

Why so many states, both on and off nucleosynthesis path exist "fortuitously" close to open channels?

Origin of near-threshold clustering

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K. Ikeda, N. Takigawa, H. Horiuchi (1968)

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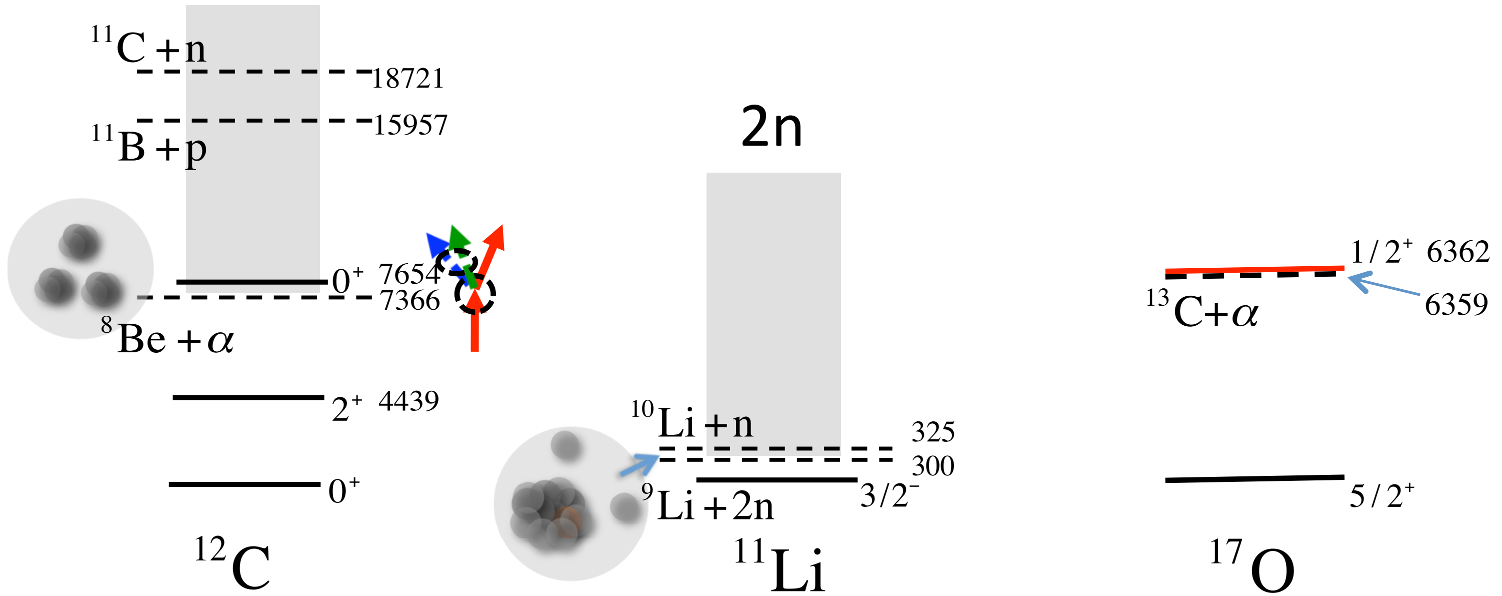


Conjecture

Near-threshold clustering/correlations result from the interplay between internal mixing by interactions and external mixing via the decay channel(s)

J. Okolowicz, M.P., W. Nazarewicz, Prog. Theor. Phys. Suppl. 196 (2012) 230; Fortschr. Phys. 61 (2013) 66

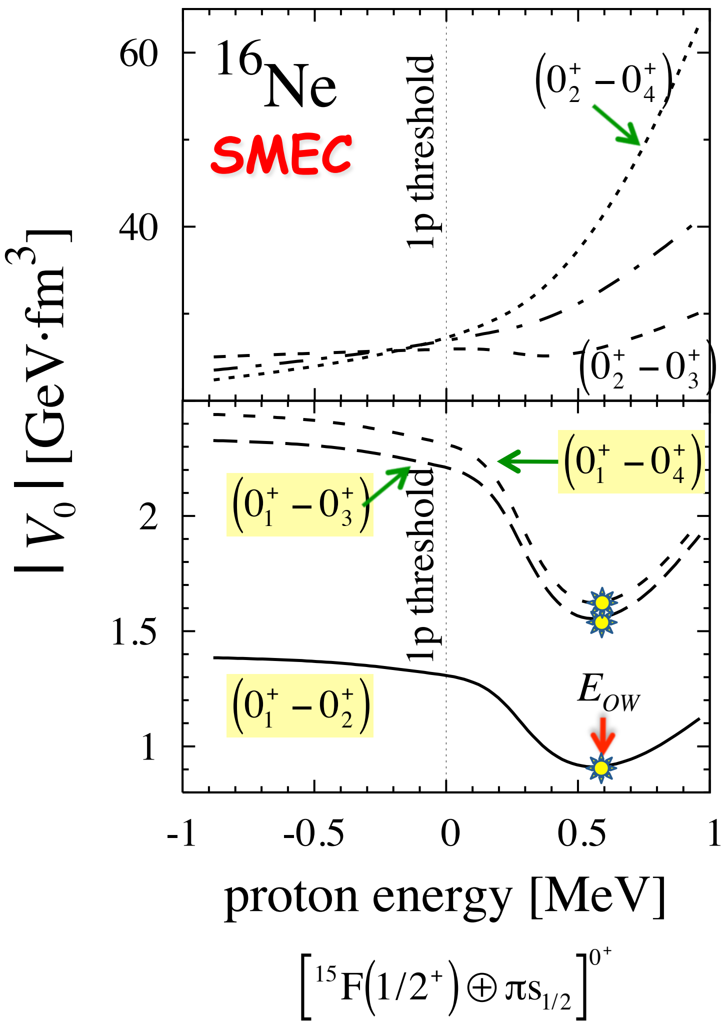
➔ Emergence of new energy scale!

How to study mixing of SM wave functions via the continuum?

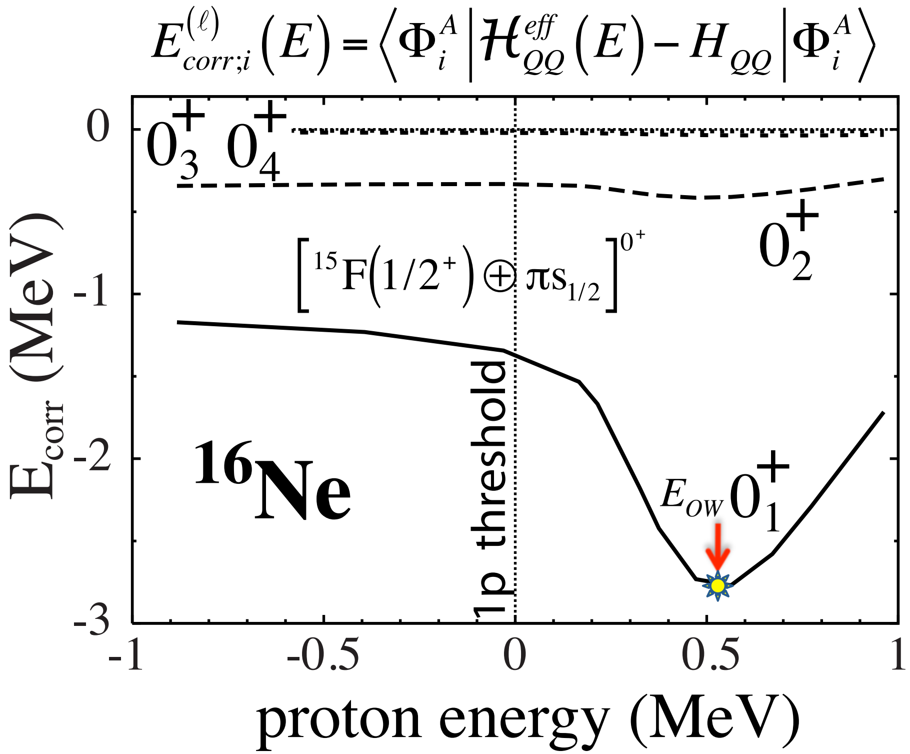
- The mixing of eigenfunctions (avoided crossing) is caused by a nearby exceptional point of the complex-extended Hamiltonian.

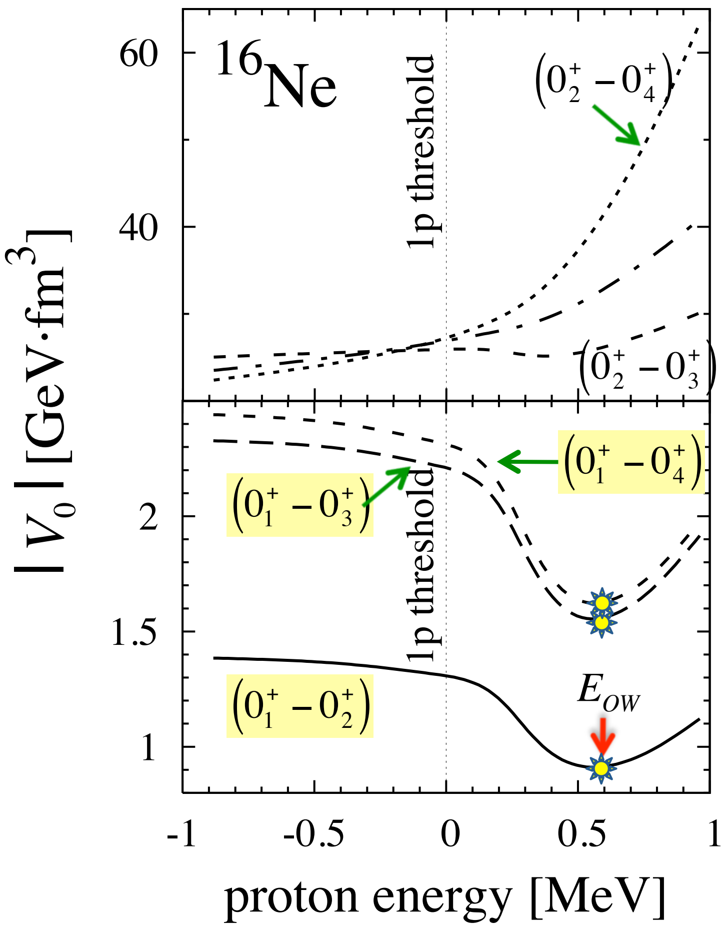
- The configuration mixing of resonances is characterized by lines $\mathcal{E}_{\alpha_1}(E) = \mathcal{E}_{\alpha_2}(E)$ of coalescing eigenvalues (exceptional threads) of the complex-extended CSM Hamiltonian (complex V_0)

→ the smaller is $|V_0|$, the stronger is the mixing of physical resonances



Continuum coupling correlation energy to SM eigenstates

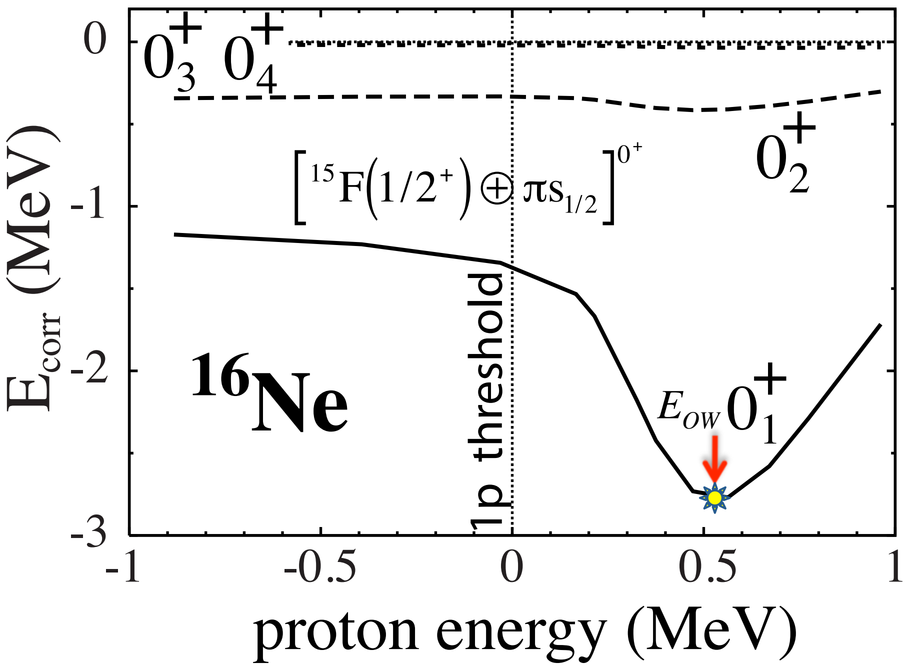




Okolowicz et al., Prog. Theor. Phys. Suppl. 196 (2012) 230
Fortschr. Phys. 61 (2013) 66

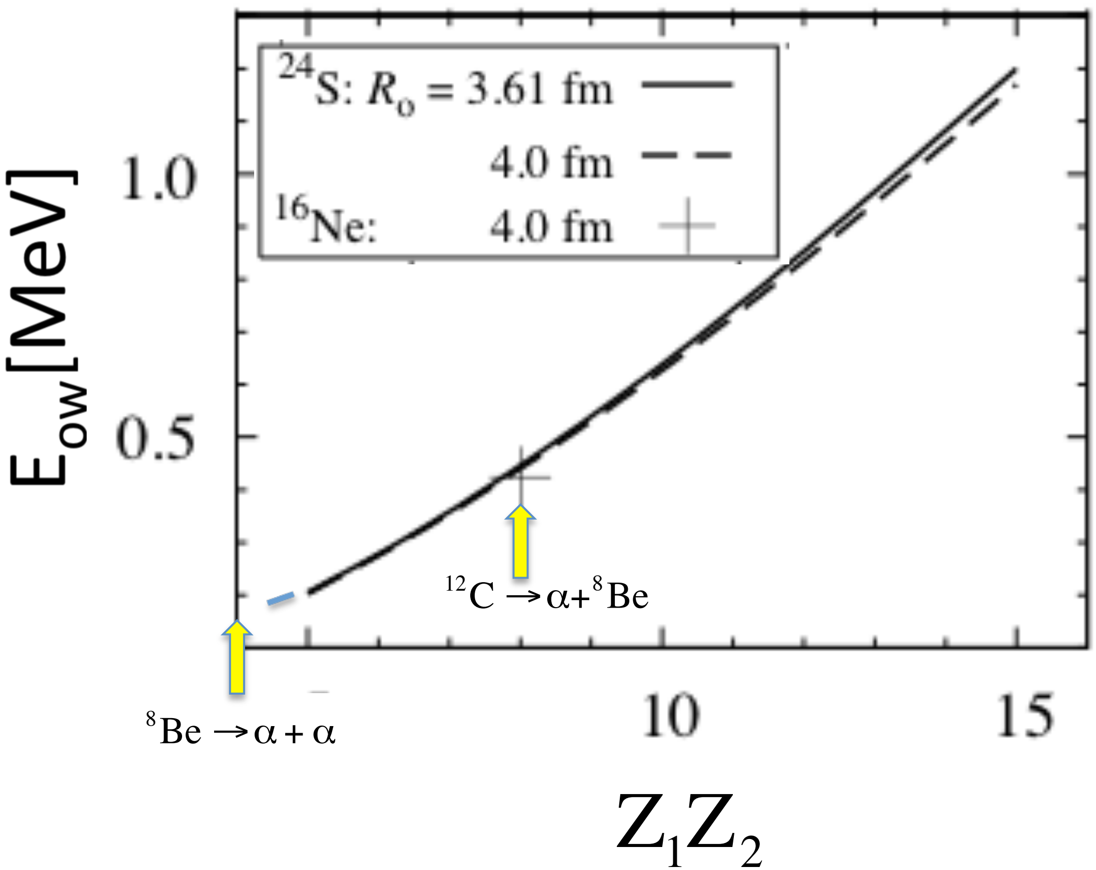
Continuum coupling correlation energy to SM eigenstates

$$E_{corr;i}^{(\ell)}(E) = \left\langle \Phi_i^A \middle| \mathcal{H}_{QQ}^{eff}(E) - H_{QQ} \middle| \Phi_i^A \right\rangle$$



Okolowicz et al., Prog. Theor. Phys. Suppl. 196 (2012) 230
Fortschr. Phys. 61 (2013) 66

- Interaction through the continuum leads to the formation of the collective eigenstate which couples strongly to the decay channel and carries many of its characteristics
- The point of strongest collectivity is determined by an interplay between the Coulomb/centrifugal interactions, and the continuum coupling
- The continuum-coupling correlation energy and collectivity of the aligned state gradually diminish with increasing Coulomb barrier



$S_{3\alpha}\left[{}^{12}C\left(0_2^+\right)\right] \cong -290\text{keV}$

$S_{\alpha}\left[{}^{8}Be\left(0_1^+\right)\right] = -97\text{keV}$

- For a given Z_1Z_2 , the centroid of the 'opportunity energy window' depends weakly on the charged particle decay channel and parameters of the potential

Conclusion

The mixing of SM eigenstates via the continuum has universal features which explain:

(i) the emergence of near-threshold clustering/correlations and optimal energies for its appearance,

(ii) a gradual disappearance of charged-particle clustering in heavier nuclei

J. Okolowicz, M.P., W. Nazarewicz, Prog. Theor. Phys. Suppl. 196 (2012) 230; Fortschr. Phys. 61 (2013) 66

What is specific and what is generic in this emerging phenomenon?

Specific	Generic
Energetic order of emission thresholds and absence of stable cluster entirely composed of like nucleons	Correlations in the near-threshold states depend on the nature of the nearby decay threshold

Multineutron clustering/correlations?

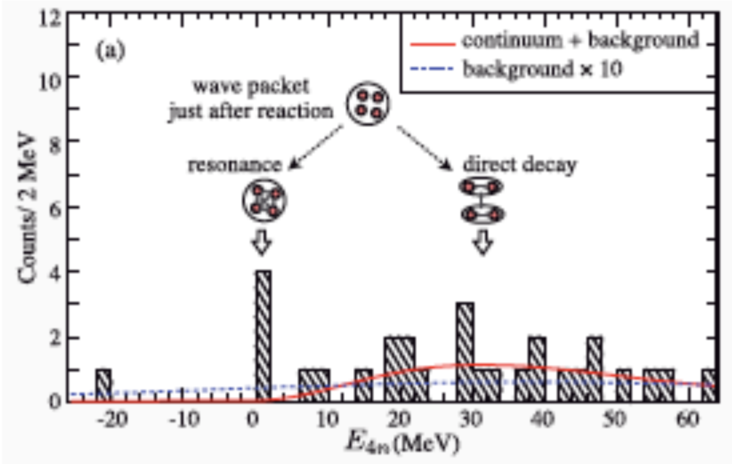
Exp. claim: $^{14}Be^{*} \rightarrow {}^{10}Be+4n$
F.M. Marquez et al, PRC 65, 044006 (2002)

- Bound tetraneutron is incompatible with the present understanding of nuclear forces
- Isospin structure of the nuclear force prevents that: $S_{4n} < S_{2n}/S_{1n}$

but…

The nature of nuclear forces does not preclude tetraneutron correlations in the vicinity of the 4n-emission threshold which can cause dynamical effects in the cross-sections

Exp. claim: $^{4}He(^{8}He,^{8}Be)$
K. Kisamori et al., PRL 116, 052501 (2016)



E=0.83±0.65(stat)±1.25(syst) MeV
Γ≤2.6 MeV

Can four neutrons form a nucleus?

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Subtle interplay between the many-body components of the nuclear interaction, the Pauli principle and the coupling to the multineutron continuum → need for a full continuum with chiral EFT interactions

Energy (width) of J=0$^+$ pole of the 4n system in *ab initio* no-core Gamow Shell Model

	$\lambda = 1.7\,\mathrm{fm}^{-1}$	$\lambda = 1.9\,\mathrm{fm}^{-1}$	$\lambda = 2.1\,\mathrm{fm}^{-1}$
N3LO	7.27 (3.69)	7.28 (3.67)	7.28 (3.69)
N2LO$_{opt}$	7.32 (3.74)	7.33 (3.78)	7.34 (3.95)
N2LO$_{sat}$ *	7.24 (3.48)	7.22 (3.58)	7.27 (3.55)

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$\Gamma \leq 2.6$ MeV

K. Fossez et al., PRL 119, 032501 (2017)

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$E=0.83\pm0.65(stat)\pm1.25(syst)$ MeV
$\Gamma\leq2.6$ MeV

Conclusion

- NCGSM results for 4n-system depend weakly on details of the chiral EFT interaction
- No dependence on the renormalization cutoff of the interaction → weak dependence on the 3-, 4-body interactions
- Lifetime of the 4n-system is too short to be a nucleus → it can be a feature in scattering experiments but not a genuine nuclear state

K. Fossez et al., PRL 119, 032501 (2017)

Outlook

1. Shell model treatment of (weakly) bound/unbound states ➔ unification of (nuclear) structure and reactions using Slater determinant and coupled channel representations of the Gamow shell model
2. Collectivization of wave functions can be results of:
   - 'internal mixing' (e.g. low-lying collective vibrations, rotational states, giant resonances, ...)
   - 'external mixing' via decay channel(s) (e.g. coherent enhancement/ suppression of radiation, multichannel effects in spectroscopic factors and reaction cross-sections, ...)
   - interplay of internal and external mixing (e.g. near-threshold clustering/ correlations, modification of spectral fluctuations, coalescence of eigenvalues, ...)
3. Exsitence of cluster (correlated) states in a proximity of the respective cluster/particle decay thresholds is the generic phenomenon which finds a natural explanation in the continuum shell model.
   The near-threshold state 'aligns' with the decay channel as a result of the collectivization among all SM states/resonances of the same quantum numbers coupled to the same decay channel(s)