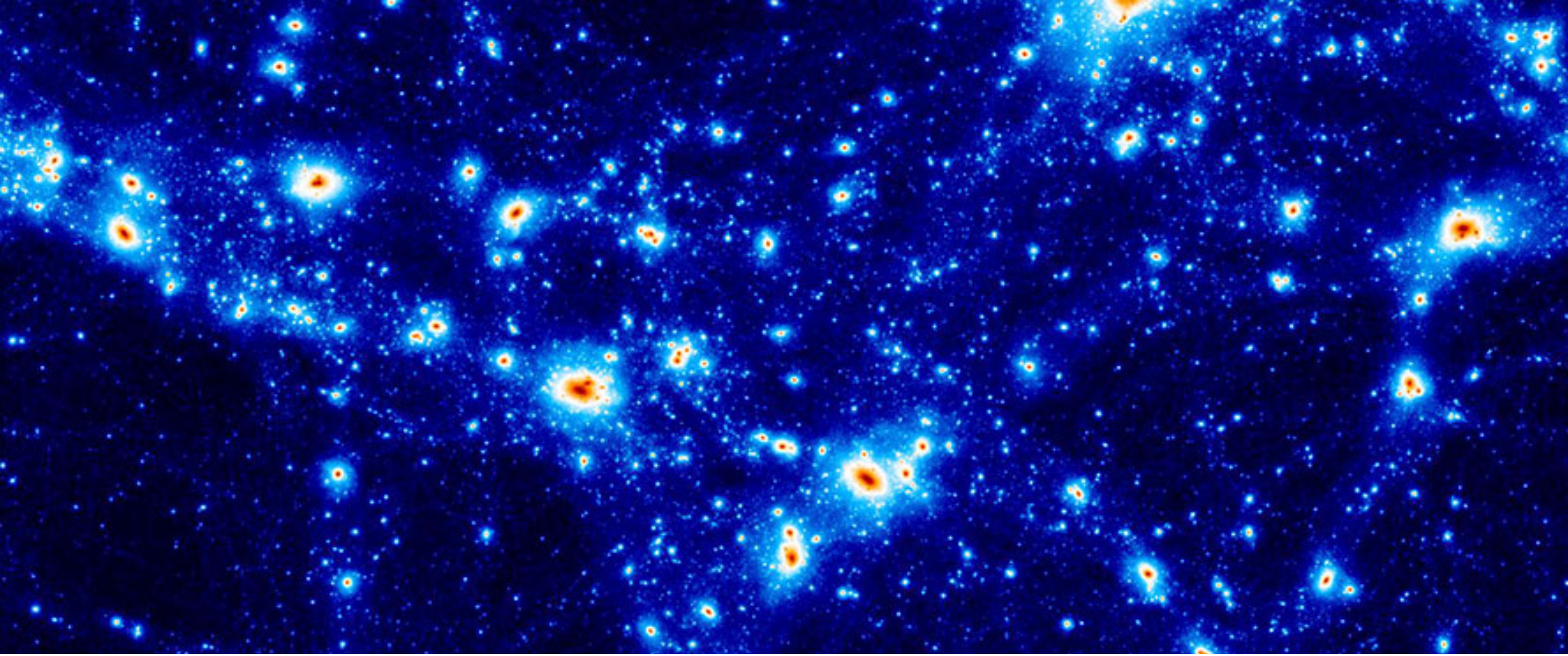
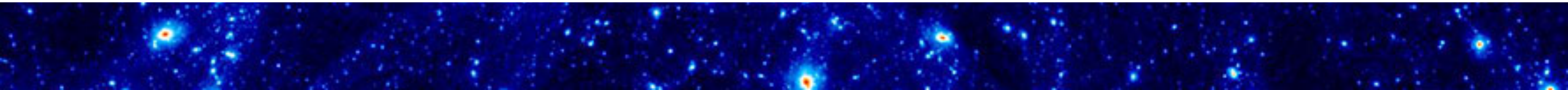
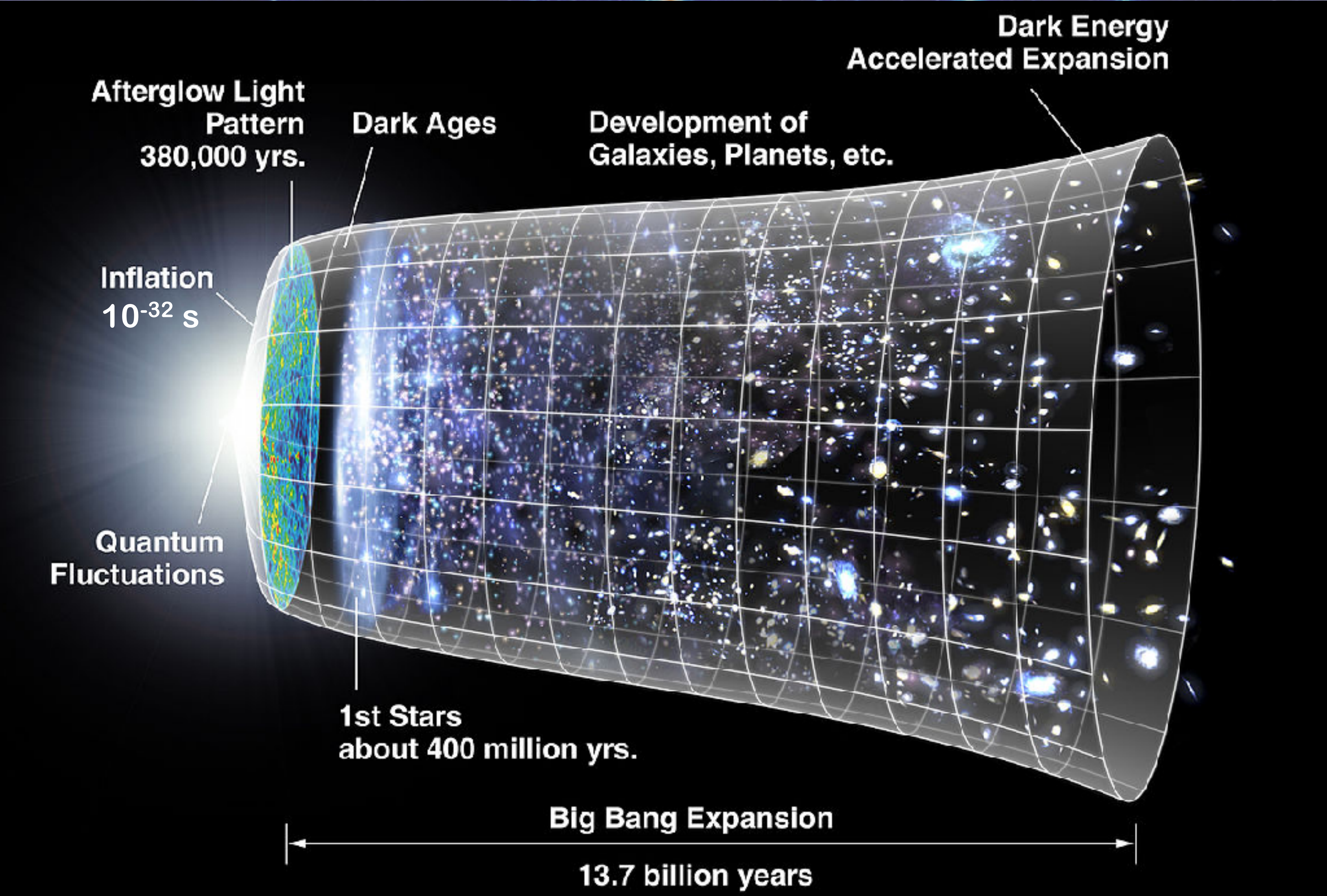


INTRODUCTION TO COSMOLOGY

针对两个无穷的物理研究 2017



A BRIEF HISTORY OF THE UNIVERS



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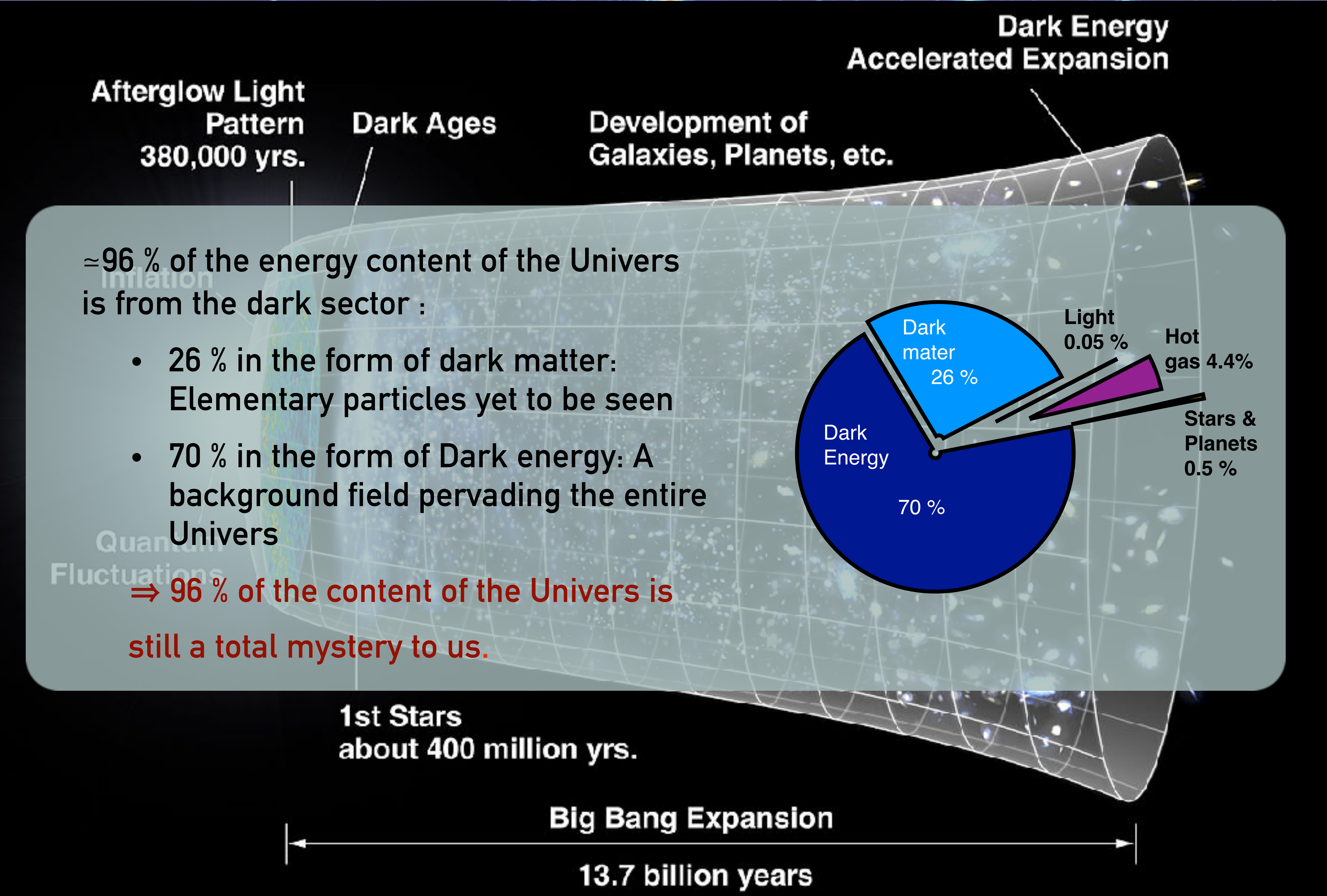


≃96 % of the energy content of the Univers is from the dark sector :

- 26 % in the form of dark matter: Elementary particles yet to be seen
- 70 % in the form of Dark energy: A background field pervading the entire Univers

⇒ 96 % of the content of the Univers is still a total mystery to us.

ORDER OF MAGNITUDE

Cosmology also goes down to the Planck scale …

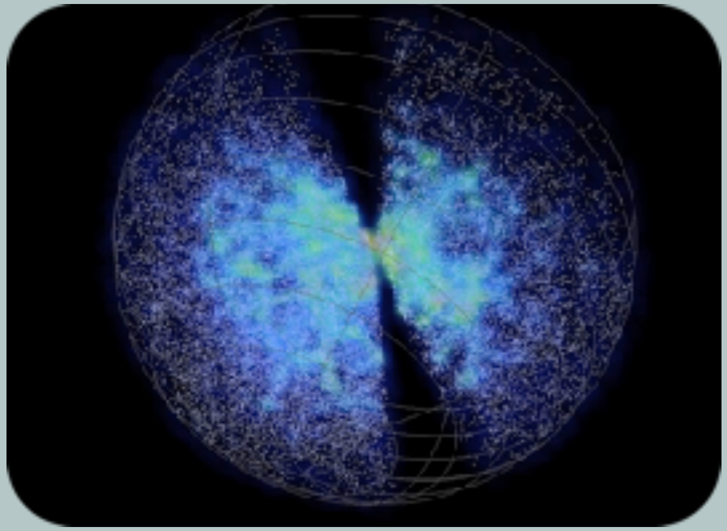
… but for now we are more interested in large scale !

Solar system:

- size: Billion of km (10^9 km)
- 1 Astronomical Unit (AU): 1.5×10^8 km
- Voyager reaches 128 AU

Galaxies:

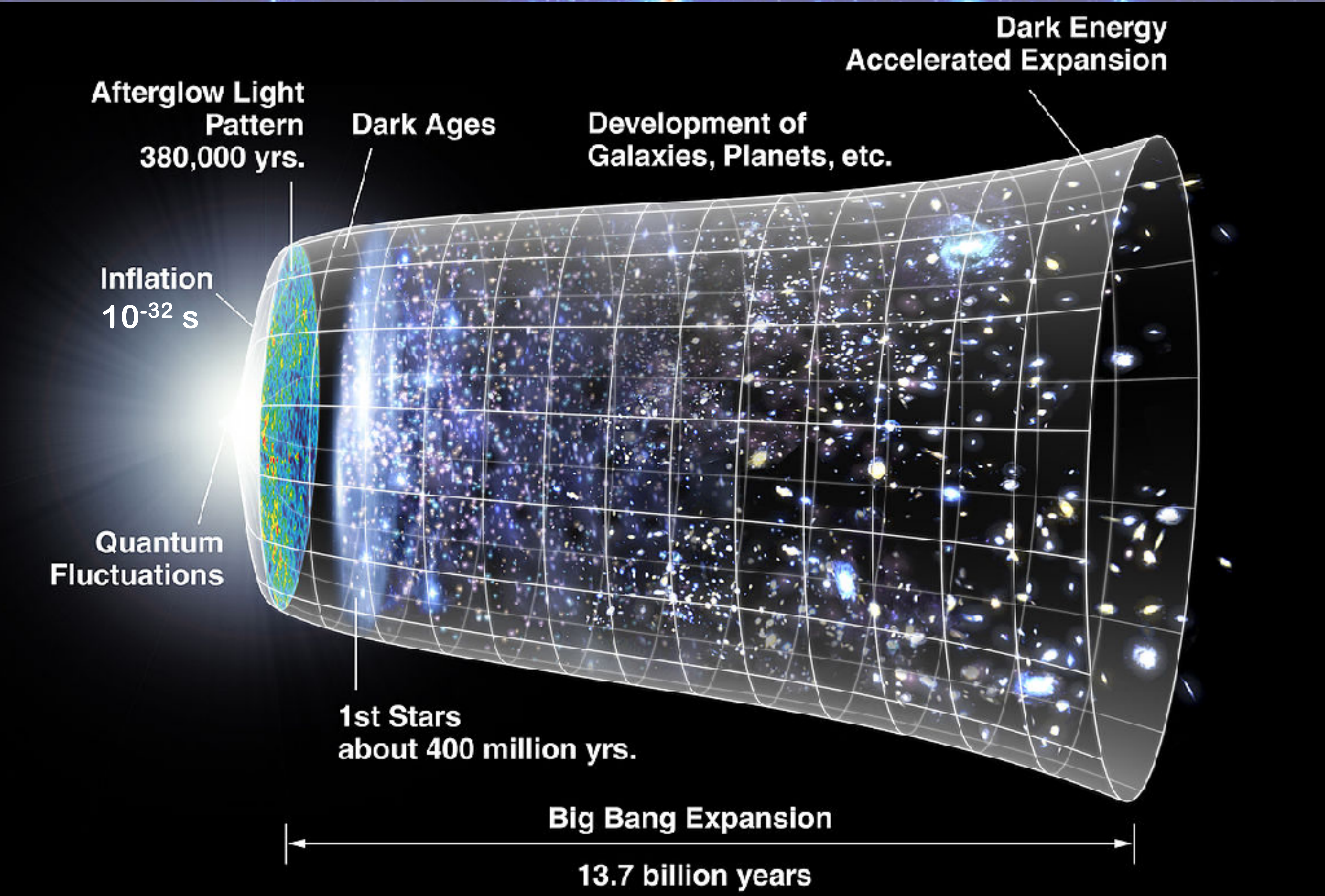
- size: Few 10 of kpc
- 1 parsec (pc) $\simeq$ 3 lyrs $\simeq 3\times10^{13}$ km
- Contains billions of stars



Observable Univers:

- size: 10 Gpc $\simeq 10^{23}$ km
- Contains $\simeq 10^{11}$ galaxies

HOW TO DESCRIBE THE UNIVERS

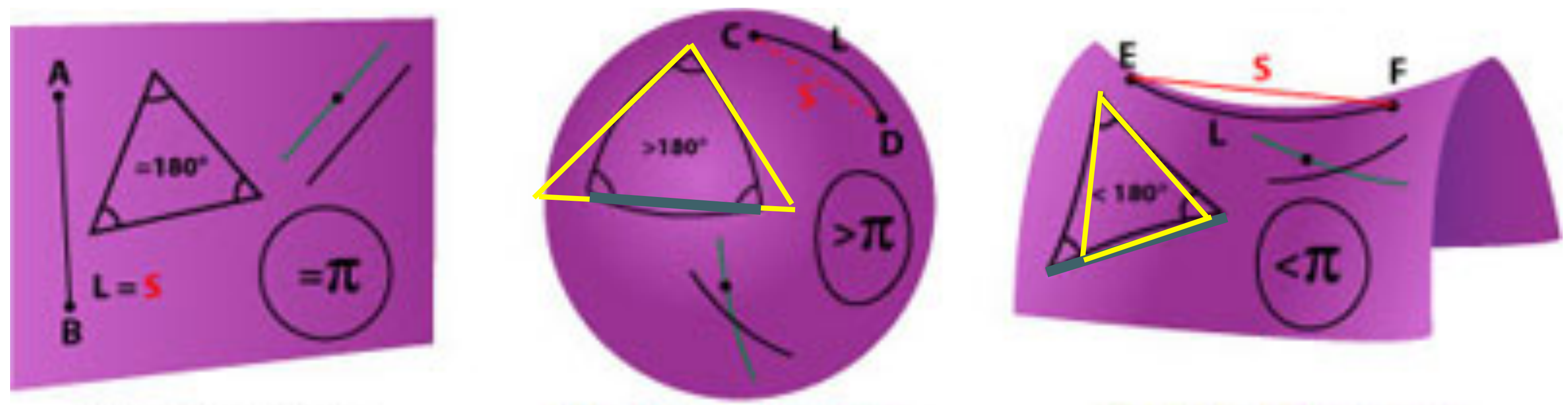


HOW TO DESCRIBE THE UNIVERS

The FLRW metric:

$$ds^2 = c^2dt^2 - a(t)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega\right)$$

k=0 : Plan *k>0 : Spheric* *k<0 : Hyperbolic*



Zero Curvature *Euclidian geometry* **Positive Curvature** *Elliptic geometry* **Negative Curvature** *Hyperbolic geometry*

HOW TO DESCRIBE THE UNIVERS

The FLRW metric:

$$ds^2 = c^2 dt^2 - a(t)^2 \left[d\chi^2 + S_k^2(\chi) d\Omega \right]$$

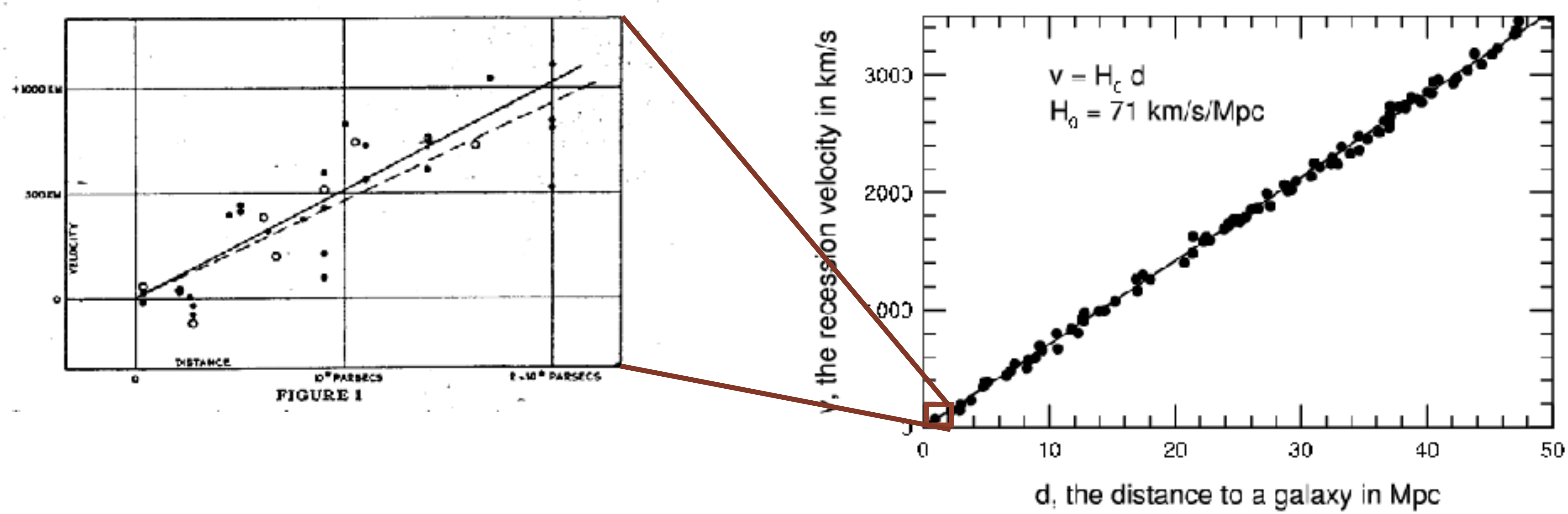
$$\begin{cases} S_k(\chi) = \sin\chi \,, k = +1 \\ S_k(\chi) = \chi \quad\quad\;\, , k = 0 \\ S_k(\chi) = Sh\,\chi \,, k = -1 \end{cases}$$

$$\frac{dR}{dt} = H(t) \times R(t) \text{ with } H(t) = \frac{\dot{a}(t)}{a(t)}$$

The univers is expending at a rate of H(t)

In real space, co-moving bodies move away from each other due to the expansion.

Hubble's Law



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$$c \int_0^\tau \frac{dt}{a(t)} = \chi$$

Distance and time are interchangeable

The horizon always grows with time

HOW TO DESCRIBE THE UNIVERS

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Distance and time are interchangeable

The horizon always grows with time

$$1 + z = \frac{a(t_o)}{a(t)} = \frac{\lambda_o}{\lambda_e}$$

In an expanding (collapsing) Univers, photon wavelength is shifted toward the red (blue)

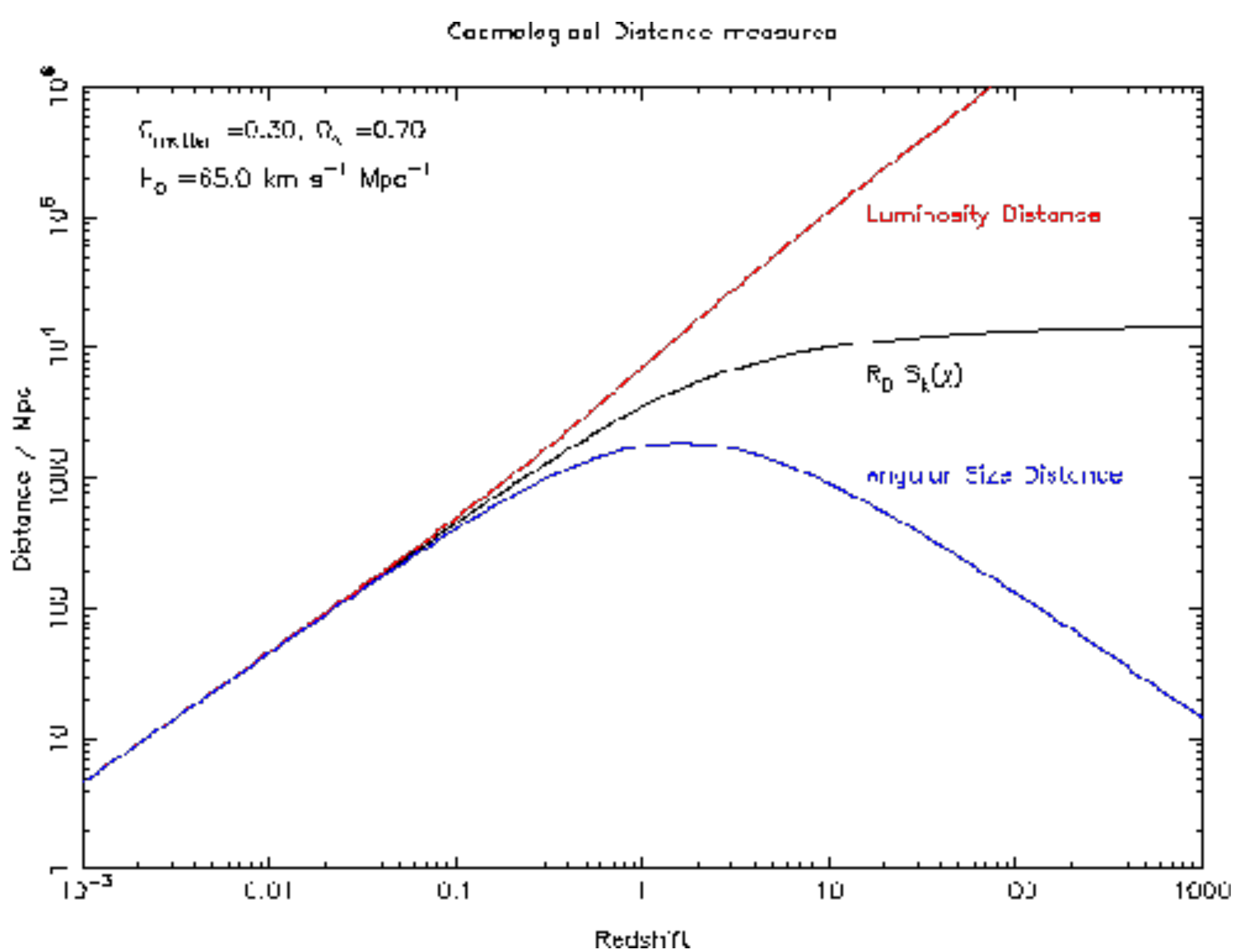
COSMOLOGICAL DISTANCES

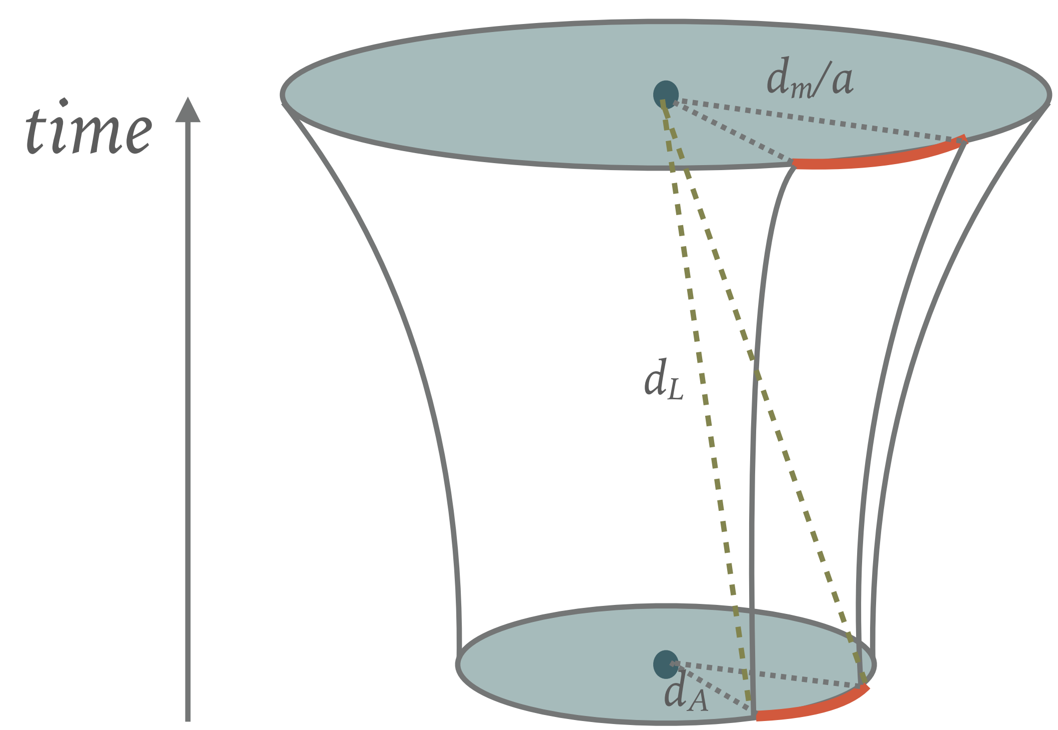
$$\text{Comoving distance:} \quad d_m = a(t) S_k(\chi)$$

$$\text{Luminous distance:} \quad d_L = (1+z) d_m$$

$$\text{Angular distance:} \quad d_A = \frac{d_m}{1+z}$$

$$d_L = (1+z)^2 d_A$$





SHAPING THE UNIVERS

So fare, we only considered geometry …

… and haven't yet use general relativity which states :

1- Gravitation can be described by a metric: $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$

2- General relativity connect the metric to the matter/energy

$$G_{\mu\nu} = \underbrace{\boxed{R_{\mu\nu} - \frac{1}{2} g^{\mu\nu} R}}_{\textit{Geometry}} = \underbrace{\boxed{8\pi G T_{\mu\nu}}}_{\textit{Energy}}$$

- $G_{\mu\nu}$: The Einstein Tensor
- $R_{\mu\nu}$: The Ricci tensor
- $R = g^{\mu\nu} R_{\mu\nu}$: The Ricci Scalar
- $T_{\mu\nu}$: The Energy momentum tensor

THE FRIEDMAN EQUATION

After …

- Calculating the 64 partial derivatives of $g_{\mu\nu}$,
- Using those partial derivatives to calculate the 64 Christoffel symbols,
- Calculating the 256 partial derivatives of those Christoffel symbols,
- Using those partial derivatives to calculate the 16 components of the Ricci tensor,
- Calculating the Ricci scalar,
- Plug the results of those calculations into Einstein's field equations,
- Apply some straightforward calculus and algebra to those equations …

one can derived the generalised Friedman equation (taking c=1)

$$\frac{\dot{a}^2}{a^2} = \sum_i \frac{8\pi G\rho_i}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{1}$$

$$\frac{\ddot{a}}{a} = -\sum_i \frac{4\pi G}{3}(\rho_i + 3P_i) + \frac{\Lambda}{3} \tag{2}$$

$$(1) \leftrightarrow (2) \quad \rightarrow \quad -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \Lambda - \frac{k}{a^2} = \sum_i 8\pi G P_i \tag{3}$$

$$\frac{d(1)}{dt} \quad \rightarrow \quad \sum_i \dot{\rho}_i = 3\frac{\dot{a}}{a}\sum_i (\rho_i + P_i) \tag{4}$$

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We could also consider the Dark Energy as a fluid of equation of state $\omega = P/\rho = -1$.
For cosmological constant, we verify: $\Lambda = 8\pi G\rho_\Lambda$ & $-4\pi G\rho_\Lambda/3(1+3\omega) = 8\pi G\rho_\Lambda/3 = \Lambda/3$

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$$\boxed{\frac{\dot{a}^2}{a^2} = \sum_i \frac{8\pi G\rho_i}{3} - \frac{k}{a^2} \qquad (1)}$$

This equation gives hints on how the Univers expend with respect to its content.

$$\frac{\ddot{a}}{a} = -\sum_i \frac{4\pi G}{3}(\rho_i + 3P_i) \qquad (2)$$

$$(1) \leftrightarrow (2) \rightarrow -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2} = \sum_i 8\pi G P_i \qquad (3)$$

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This tell us how the expansion is accelerating or decelerating depending on the content of the Univers.
The expansion of the Univers accelerate if $\omega < -1/3$, with $\omega = P/\rho$ the equation of state of the fluid.

THE FRIEDMAN EQUATION

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<u>The fluid equation:</u> Tell us how the energy density does vary with the expanding Univers.

$$\rho(a) \propto a^{-3(1+\omega)}$$

$$\boxed{\frac{d(1)}{dt} \rightarrow \sum_i \dot{\rho}_i = 3\frac{\dot{a}}{a}\sum_i (\rho_i + P_i) \qquad (4)}$$

THE COSMOLOGICAL PARAMETERS

Remember H(t) = ?

How does the Friedman equations depends on H ?

$$\frac{\dot{a}^2}{a^2} = \sum_i \frac{8\pi G\rho_i}{3} - \frac{k}{a^2} \quad (1)$$

$$\frac{\ddot{a}}{a} = -\sum_i \frac{4\pi G}{3}(\rho_i + 3P_i) \quad (2)$$

THE COSMOLOGICAL PARAMETERS

Remember H(t) = ?

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

The Friedman equation become:

$$\begin{aligned} H^2 &= \sum_i \frac{8\pi G \rho_i}{3} - \frac{k}{a^2} && (1) \\ \dot{H} + H^2 &= -\sum_i \frac{4\pi G}{3}(\rho_i + 3P_i) && (2) \end{aligned}$$

$$\dot{\rho}_i = -3H(\rho_i + P_i) \quad (4)$$

And we define the critical density: the density of a matter dominated Univers

$$\rho_c = \frac{3H^2}{8\pi G}$$

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$$\rho_c = \frac{3H^2}{8\pi G}$$

The 1st Friedman equation becames:

$$1 = \sum_i \Omega_i + \Omega_k \qquad (1)$$

THE COSMOLOGICAL PARAMETERS

Radiation energy density parameter:

- $\Omega_\gamma = \rho_\gamma/\rho_c$ ← the fraction of radiation

Matter energy density parameter:

- $\Omega_m = \rho_m/\rho_c$ ← the fraction of matter

Curvature parameter:

- $\Omega_k = -k/(Ha)^2$

Dark Energy density parameter:

- $\Omega_{DE} = \rho_\Lambda/\rho_c = \Lambda/(3H^2)$ with $\Lambda = 8\pi G\rho\Lambda$

Radiation energy density parameter:

- $\Omega_\gamma{}^0$ ← the fraction radiation today

Matter energy density parameter:

- $\Omega_m{}^0$ ← the fraction of matter today

Curvature parameter:

- $\Omega_k{}^0$ ← the curvature today

Dark Energy density parameter:

- $\Omega_{DE}{}^0$ ← the fraction dark energy today

Lets assume we normalise the scale factor today at 1. The Friedman equation become:

$$H^2 = H_0^2\left[\Omega_\gamma^0(1+z)^4 + \Omega_m^0(1+z)^3 + \Omega_{DE} + \Omega_k(1+z)^2\right] \qquad (1)$$

Do you remember the cosmological distances ?

$$d_m = a(t)S_k(\chi) \qquad d_A = \frac{d_m}{1+z} \qquad d_L = (1+z)d_m$$

THE COSMOLOGICAL PARAMETERS

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Cosmological distances becomes

$$d_A = \frac{1}{(1+z)^2} S_k\left[\int_0^z \frac{dz}{H}\right] = \frac{1}{(1+z)^2} S_k\left[\int_0^z \frac{dz}{H_0\sqrt{\Omega_\gamma^0(1+z)^4 + \Omega_m^0(1+z)^3 + \Omega_k^0(1+z)^2 + \Omega_\Lambda^0}}\right]$$

$$d_L = \frac{1}{(1+z)^2} d_A = S_k\left[\int_0^z \frac{dz}{H_0\sqrt{\Omega_\gamma^0(1+z)^4 + \Omega_m^0(1+z)^3 + \Omega_k^0(1+z)^2 + \Omega_\Lambda^0}}\right]$$

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$$
\begin{aligned}
d_A &= \frac{1}{(1+z)^2} S_k\left[\int_0^z \frac{dz}{H}\right] = \frac{1}{(1+z)^2} S_k\left[\int_0^z \frac{dz}{H_0\sqrt{\Omega_\gamma^0(1+z)^4+\Omega_m^0(1+z)^3+\Omega_k^0(1+z)^2+\Omega_\Lambda^0}}\right] \\
d_L &= \frac{1}{(1+z)^2} d_A = S_k\left[\int_0^z \frac{dz}{H_0\sqrt{\Omega_\gamma^0(1+z)^4+\Omega_m^0(1+z)^3+\Omega_k^0(1+z)^2+\Omega_\Lambda^0}}\right]
\end{aligned}
$$

Measuring distances allows to infer constraints on the cosmological parameters.

At low redshift ($z<<1$) cosmological distance only depends on H_0

PECULIAR CASE: MATTER DOMINATED FLAT UNIVERS

$$H^2 = H_0 \frac{\Omega_m}{a^3} = \frac{H_0}{a^3} \qquad (1)$$

This equation has a strait-forward solution:

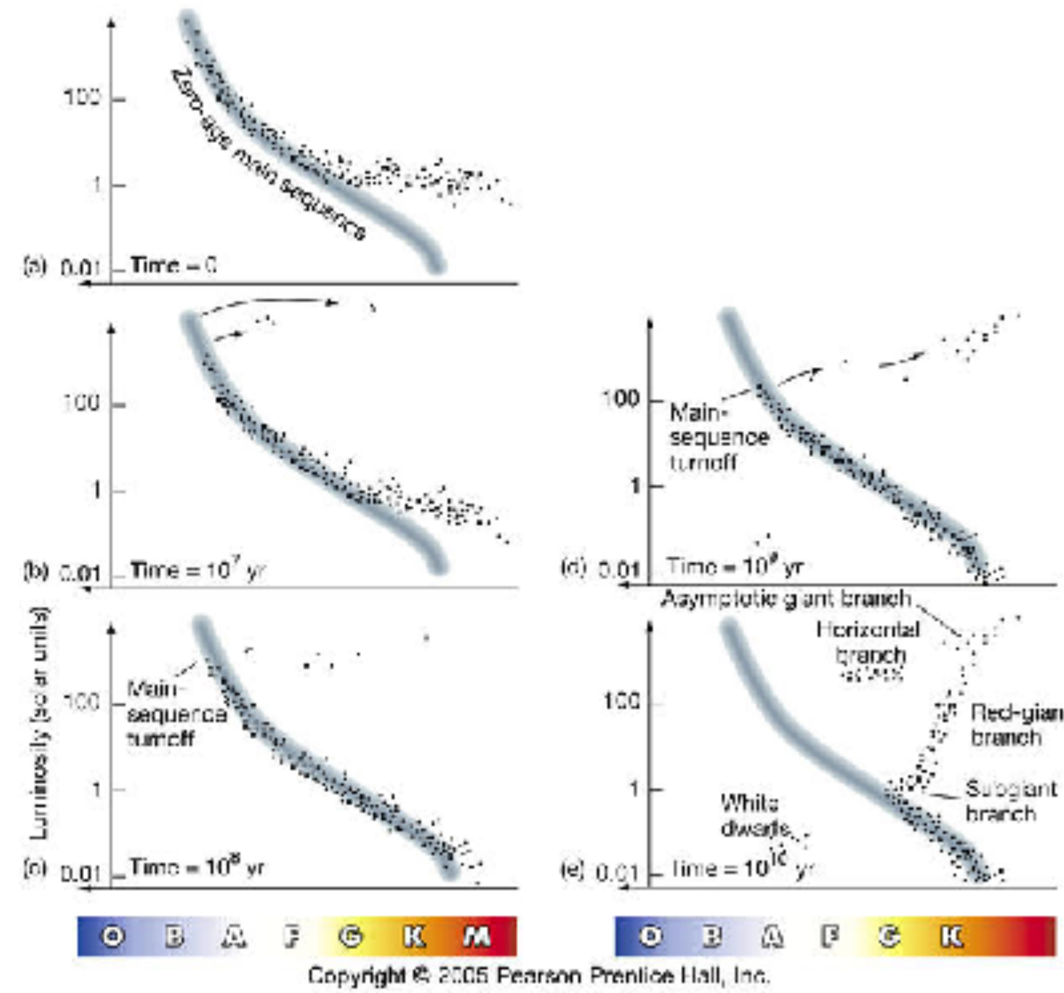
$$\frac{\dot a}{a} \propto a^{-3/2} \Rightarrow a(t) \propto t^{2/3} \Rightarrow \ddot a \propto \frac{3}{4\sqrt{t}}$$

(1) The Univers always expands itself.
(2) The expansion slow down with time.

The age of the Univers:

$$t_0 = \int_0^{t_0} dt = \int_0^1 \frac{da}{\dot a} = \int_0^\infty \frac{dz}{H(1+z)} = \frac{2}{3H_0}$$

$H_0 = 71.9$ (km/s)/Mpc$\simeq$(13.6 Gyr)$^{-1}$ $\Rightarrow$ $t_0 = 9.06$ Gyr



An age of 9 Gyr isn't compatible with the measured age of the oldest observed star which have been estimated to be more than 11 Gyr old ⇒ This simple calculation already implies that something is missing !

CONCLUSIONS

Modern cosmology is constructed upon cosmological principle and FLRW metrics …

- How strong are those assumptions ?

  This have to be tested at high precision level ← recent measurement verify those principle at 10^{-5} precision level

- We know at small scale the Univers isn't homogenous (but it still remains homogenous on average)

  Galaxy Clusters, galaxies and stars results from initial perturbations in the metric ant Energy-momentum tensor that have grown while the Universe expended. Thos fluctuation provide additional valuable information to test the cosmological models (see the lecture of J. Bell and A Pisani tomorrow)

Current observations of our Univers lead us to the so-called Λ-CDM model:

- So far no observations were able to rule-out this simplest model. However, many questions remain to be solved:
  - *96 % of the energy content of the Univers is still a mystery to us: What are made of the Dark Energy and the Dark Matter ?*
  - *Is Dark Energy a cosmological constant ? Why Cosmological constant so weak ?*
  - *Why does the Univers is so flat ?! $\Omega_K=0.000 \pm 0.005$!*
  - *What generated the primordial energy density fluctuation that produces the small scale structures of the Univers*