

**LAPP Seminar
16 January 2009**



Redefinition of the kilogram based on a physical constant

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with thanks to BIPM colleagues

- Organized under terms of the Metre Convention (1875).
- Supported by 51 Member States and 28 Associates.
- Carries out a scientific programme overseen by the International Committee for Weights and Measures (CIPM).
- Scientific Sections include Mass, Electricity, Time/Frequency/Gravimetry, Chemistry, Ionizing Radiation.
- Publishes the [SI Brochure](#), produced by the Consultative Committee for Units (CCU) of the CIPM.
- Changes to the SI units are recommended by the CCU in cooperation with other relevant technical CCs, for ultimate approval by the General Conference on Weights and Measures (CGPM), which is at the diplomatic level and meets every 4 years [next in 2011].

Topics to be presented

- Reminder of the SI units and quantities.
- Why in particular the kilogram must be redefined.
- The constraints on a new definition.
- The relation between the SI and the fundamental constants of physics (as listed by CODATA); the role of the fine-structure constant.
- When could/should the kilogram be redefined?

In an ideal world, units would be defined by fundamental constants

“Yet, after all, the dimensions of our earth and its time of rotation, though, relative to our present means of comparison, very permanent, are not so by physical necessity. The earth might contract by cooling, or it might be enlarged by a layer of meteorites falling on it, or its rate of revolution might slowly slacken, and yet it would continue to be as much a planet as before.

But a molecule, say of hydrogen, if either its mass or its time of vibration were to be altered in the least, would no longer be a molecule of hydrogen.

If, then, we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wavelength, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules.”

James Clerk Maxwell, 1870

6/7 of the SI base units at present

- **second (s)**: defined by fixing value of ν_{hfs} , the hyperfine-transition frequency of the caesium atom. [atomic clock]
- **metre (m)**: defined by fixing value of speed of light, c , with additional reference to s. [laser interferometry]
- **kilogram (kg)**: defined by assigning 1 kg to the mass of the international prototype of the kilogram. [artifact definition!]
- **ampere (A)**: defined by fixing the value of μ_0 , the magnetic constant, with additional reference to kg, m, s
- **kelvin (K)**: defined by assigning a value of 273.16 K to the t_{tpw} . [technology is in place to redefine in terms of a fixed value for k_B .]
- **mole (mol)**: defined by the number of ^{12}C atoms in 12 g. This constant, N_A , cannot be fixed without redefining the kilogram.

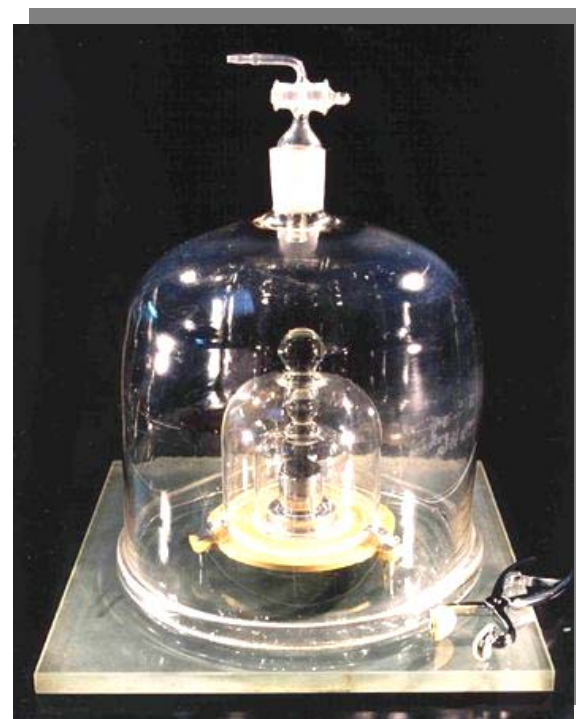
Definition of the kilogram

3rd CGPM, 1901 :

“Le kilogramme est l'unité de masse;
il est égal à la masse du prototype
international du kilogramme.”

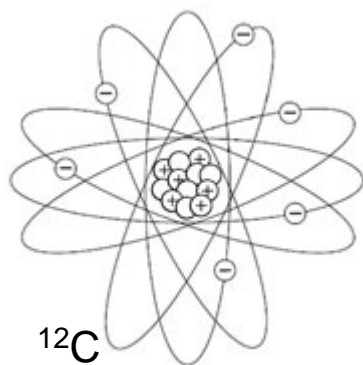
“The kilogram is the unit of mass;
it is equal to the mass of the international
prototype of the kilogram.”

(international prototype manufactured in
1880s, put into service in 1889)



What the definition means

The mass in kilograms of any object X is given by :



$\div 10^{26}$



$\times 10^1$

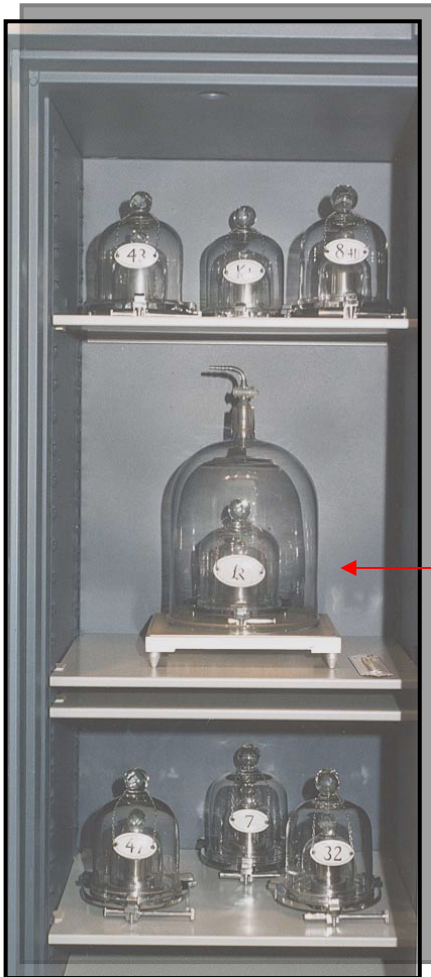
This ratio represents a measurement having an experimental uncertainty

$$\{m_X\} [\text{kg}] = \left\{ \frac{m_X}{m_K} \right\} [\text{kg}]$$



$$\{m_X\} [\text{kg}] = \left\{ \frac{m_X}{m_n} \right\} \cdot \left\{ \frac{m_n}{m_{n-1}} \right\} \cdot \dots \cdot \left\{ \frac{m_2}{m_1} \right\} \cdot \left\{ \frac{m_1}{m_K} \right\} \cdot [\text{kg}]$$

Conservation of the international prototype



official copies (3 of 6)

international prototype

official copies (3 of 6)

What's new in physics since 1889

- atomic view of matter
- special and general relativity
- quantum electrodynamics
- quantum chromodynamics

Leading to kilogram involvement in:

- fundamental physical constants whose SI values are manifestly measured in terms of the kilogram: m_e , h , $m(^{12}\text{C})$ etc.
- other fundamental constants whose SI values are measured less obviously in terms of the kilogram : e , K_J , R_K etc.
- Conversion factors between the microscopic and macroscopic worlds: N_A , F , etc.
- Conversion factors to other unit systems: eV in J, u in kg, etc.

Consequences of 1901 definition and 2007 physics

Even though m_X may represent something fundamental--
 m_e , m_p , $m(^{12}\text{C})$, $(\hbar c/G)^{1/2}$ etc., nevertheless

$$\{m_X\} [\text{kg}] = \left\{ \frac{m_X}{m_{\mathcal{K}}} \right\} [\text{kg}]$$

This curious situation persists largely because:

Experimental uncertainties of $\{m_X/m_{\mathcal{K}}\}$ are still much larger than the precision of the best commercial balances;

Inconclusive evidence that $\{m_X/m_{\mathcal{K}}\}$ is changing, where m_X is "something" that can reasonably be considered more stable than the mass of \mathcal{K} ;

When comparing molecular, atomic, and subatomic masses amongst themselves, it is **traditional** to use the dalton, Da, a non-SI unit which avoids correlations to $m_{\mathcal{K}}$.

1927

from

***La Création du BIPM
et son œuvre***

«Il semble donc que l'unité de masse soit garantie
au cent-millionième près pour plus de 10 000 ans,
et cette durée est à peine commencée.

Sans doute, bien avant qu'elle soit écoulée, les
travaux exécutés par les métrologistes des siècles
futurs auront conduit à des solutions encore plus
parfaites.»

Ch.-Ed. Guillaume

**BIPM Director
Nobel laureate**

2003

from

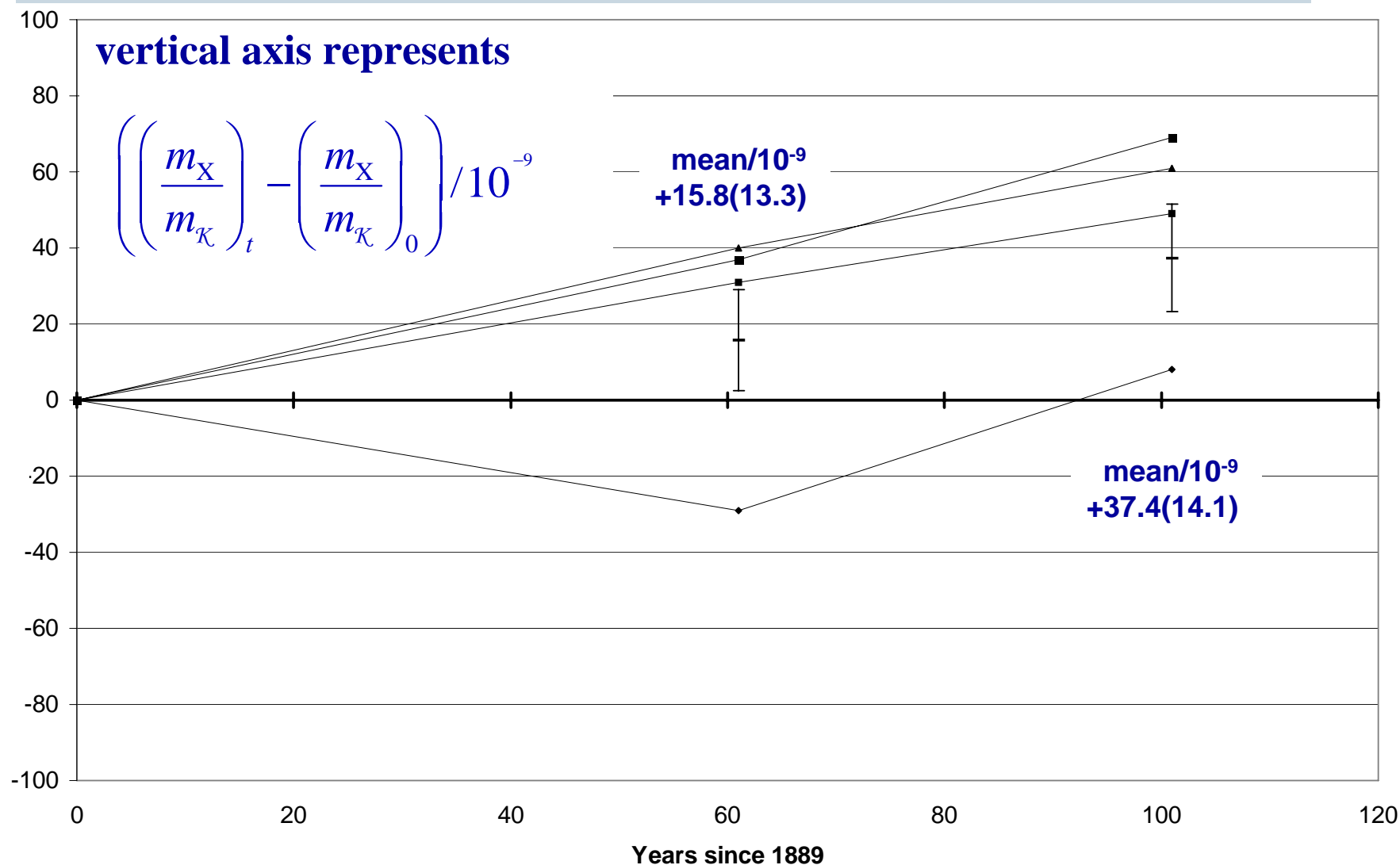
New York Times

(27 May 2003)

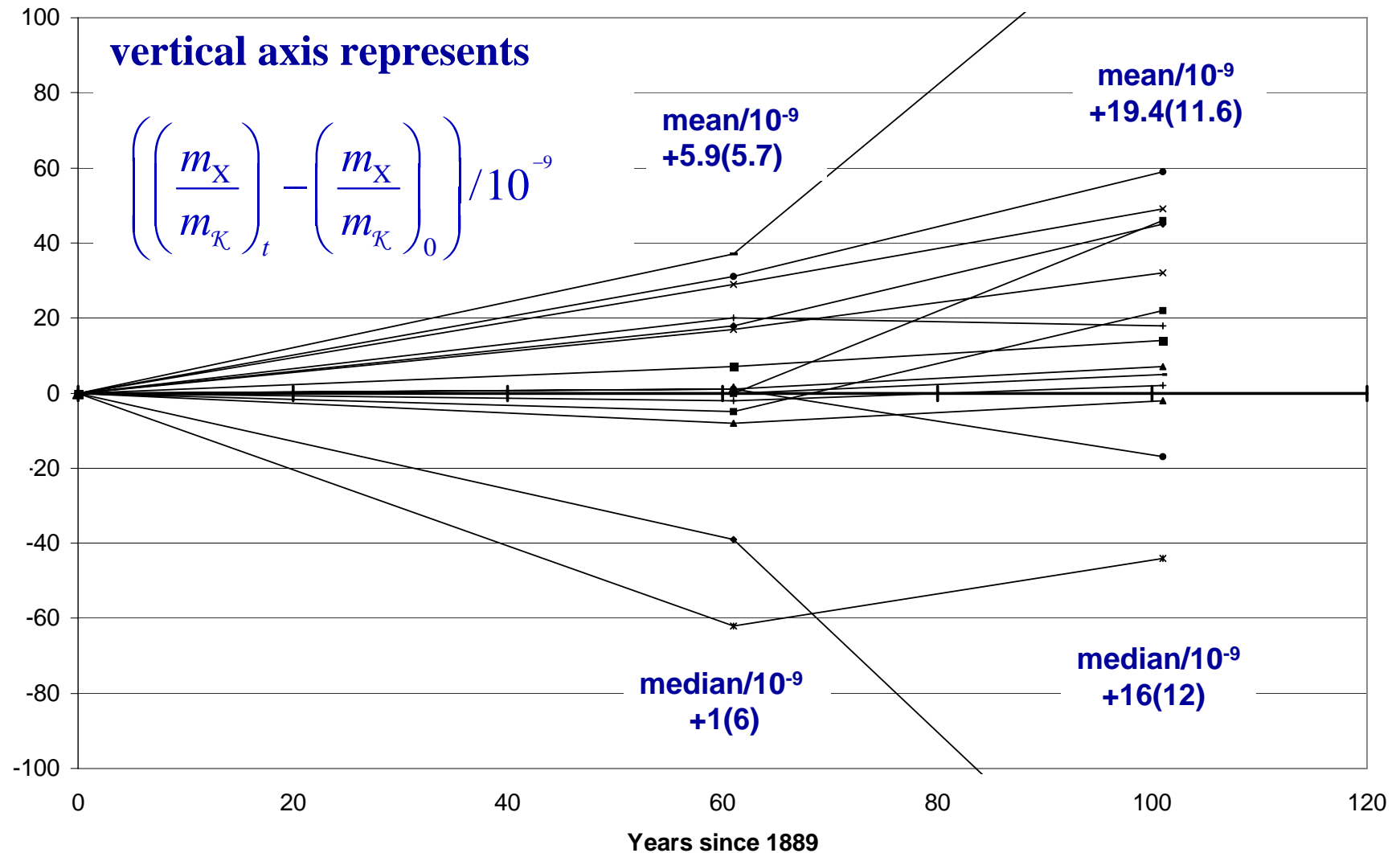
***Scientists Struggling
to Make the Kilogram Right Again***

...The **kilogram is getting lighter**, scientists say,
sowing potential **confusion** over a range of scientific
endeavor...

Ensemble average of \mathcal{K} and four oldest official copies



Ensemble average of \mathcal{K} and the oldest national prototypes



Summary: real problems with the artefact definition

- mass ratios of "identical" prototypes have clearly changed during a century;
- as experiments improve, we will find evidence of $\{m_{\text{FC}}/m_{\mathcal{K}}\}$ change with time;
- mass artefacts suffer surface contamination over long periods of time;

in 1989, the International Committee for Weights and Measures (CIPM) decided to interpret the 1901 definition of the kilogram as referring to the mass of \mathcal{K} just after cleaning and washing using “the BIPM method”



Mass metrologists are not alone

The definition of the kilogram has an impact on other areas of science and metrology:

- Electrical metrology
- Chemistry
- Physics (especially the fundamental constants)

Why electrical metrologists care about the kilogram definition-1

“The **ampere** is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} **newton** per metre of length.”

The definition ensures the consistency of SI units.
For instance, thanks to this definition, the SI unit
“watt” is the same for

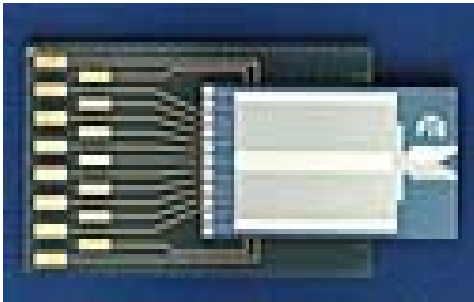
$I^2 \cdot R$ (electrical power) and $m \cdot a \cdot v$ (mechanical power)

The definition implicitly fixes a value for the permeability of vacuum, μ_0 , (sometimes called the magnetic constant) and the impedance of vacuum

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \quad (1 \text{ N/A}^2 = 1 \text{ H/m}) ; \quad \mu_0 c \equiv Z_0 = 376.7... \Omega$$

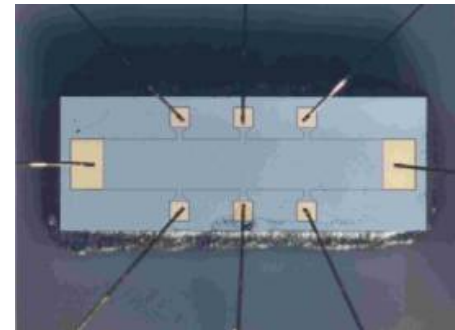
Why electrical metrologists care about the kilogram definition-2

Josephson effect



$$U_J(n) = \frac{n f}{K_J}, \quad K_J = \frac{2e}{h}$$

quantized-Hall effect



$$R_H(i) = \frac{R_K}{i}, \quad R_K = \frac{h}{e^2}$$

Conventional (non SI values)

$$K_{J-90} \equiv 483\,597.9 \text{ GHz/V}$$

$$R_{K-90} \equiv 25\,812.807 \, \Omega$$

Resumé

Present voltage and resistance metrology rely on quantum-mechanical phenomena and two 'fundamental' constants of physics.

Josephson constant, K_J ,
and
von Klitzing constant, R_K .

According to current knowledge, $K_J = 2e/h$ and $R_K = h/e^2$.

Therefore, fixing the values of h and of e

- fixes the value of the Josephson constant,
- fixes the value of the von Klitzing constant,

thereby eliminating the need for conventional (non SI) values, K_{J-90} and R_{K-90} that are used today.

But...

c : speed of light in vacuum. This already has a fixed value in the SI.

μ_0 already has a fixed value in the SI due to the definition of the ampere

Therefore, it is impossible for e , h to be fixed as well:

$$\alpha = \frac{\mu_0 c e^2}{2h} = \frac{Z_0}{2R_K}$$

Choices must be made.

Many interesting proposals, including those of the *Académie des Sciences*

A new SI?

<http://www.bipm.org/en/committees/cc/ccu/>
Presentation by Prof. Ian Mills (president of CC Units)

Present SI

second: $\nu_{\text{hfs}}(\text{Cs})$

metre: second and c

kilogram: international prototype

ampere: second, metre, kilogram and μ_0

kelvin: triple point of water (V-SMOW)

mole: number of atoms in 0.012 kg of ^{12}C

New SI ?

second: $\nu_{\text{hfs}}(\text{Cs})$

metre: second and c

kilogram: second, metre, h

ampere: second, e

kelvin: second, metre, kilogram, k_{B}

mole: a fixed number of entities

Constraint

$$\alpha = \frac{\mu_0 c e^2}{2 h}$$

where α is measured
and dimensionless
(independent of unit
systems)

Why chemists care about the kilogram definition

“The **mole** is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 **kilogram** of carbon 12; its symbol is ‘mol’.

“When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.”

This definition is closely linked to the Avogadro Constant

$$N_A = \frac{0.012 \left[\text{kg} \cdot \text{mol}^{-1} \right]}{m \left({}^{12}\text{C} \right) [\text{kg}]} = \frac{\text{molar mass of } x}{\text{mass of } x}$$

The dalton (example)

$$\{m_{^{12}\text{C}}\} [\text{kg}] = \left\{ \frac{m_{^{12}\text{C}}}{m_{\mathcal{K}}} \right\} [\text{kg}] = 12 [\text{u}]$$

$$\{m_{\text{e}}\} [\text{kg}] = \left\{ \frac{m_{\text{e}}}{m_{^{12}\text{C}}} \right\} \left\{ \frac{m_{^{12}\text{C}}}{m_{\mathcal{K}}} \right\} [\text{kg}] = 12 \left\{ \frac{m_{\text{e}}}{m_{^{12}\text{C}}} \right\} [\text{u}]$$

From 2006 CODATA adjustment

quantity relative uncertainty (ppm)

(1 u = 1 Da)

$m_{^{12}\text{C}}$	0.05
m_{e}	0.05
$m_{\text{e}} / m_{^{12}\text{C}}$	0.000 42

$$[\text{u}] = \frac{1}{12} \left\{ \frac{m_{^{12}\text{C}}}{m_{\mathcal{K}}} \right\} [\text{kg}]$$

Summary of why physicists care about the kg definition

- Conceptually peculiar to measure fundamental physical constants in terms of an artefact manufactured in the 19th century.
- Comparison to the international prototype is the dominant uncertainty in the SI values for many fundamental constants of physics; therefore: either large covariances must be taken into account or non-SI units like the dalton must be used.

Corollary: SI values of many fundamental constants change whenever there is a newly-measured link to \mathcal{K} , and yet the link to \mathcal{K} is not “fundamental”.

Large correlations due to links to a macroscopic kg

TABLE L The variances, covariances, and correlation coefficients of the values of a selected group of constants based on the 2006 CODATA adjustment. The numbers in bold above the main diagonal are 10^{16} times the numerical values of the relative covariances; the numbers in bold on the main diagonal are 10^{16} times the numerical values of the relative variances; and the numbers in italics below the main diagonal are the correlation coefficients.^a

	α	h	e	m_e	N_A	m_e/m_μ	F
α	0.0047	0.0002	0.0024	-0.0092	0.0092	-0.0092	0.0116
h	<i>0.0005</i>	24.8614	12.4308	24.8611	-24.8610	-0.0003	-12.4302
e	<i>0.0142</i>	<i>0.9999</i>	6.2166	12.4259	-12.4259	-0.0048	-6.2093
m_e	<i>-0.0269</i>	<i>0.9996</i>	<i>0.9992</i>	24.8795	-24.8794	0.0180	-12.4535
N_A	<i>0.0269</i>	<i>-0.9996</i>	<i>-0.9991</i>	-1.0000	24.8811	-0.0180	12.4552
m_e/m_μ	<i>-0.0528</i>	<i>0.0000</i>	<i>-0.0008</i>	<i>0.0014</i>	<i>-0.0014</i>	6.4296	-0.0227
F	<i>0.0679</i>	<i>-0.9975</i>	<i>-0.9965</i>	<i>-0.9990</i>	<i>0.9991</i>	<i>-0.0036</i>	6.2459

^aThe relative covariance is $u_r(x_i, x_j) = u(x_i, x_j)/(x_i x_j)$, where $u(x_i, x_j)$ is the covariance of x_i and x_j ; the relative variance is $u_r^2(x_i) = u_r(x_i, x_i)$; and the correlation coefficient is $r(x_i, x_j) = u(x_i, x_j)/[u(x_i)u(x_j)]$.

From CODATA 2006 recommendation

<http://physics.nist.gov/cuu/Constants/JPCRDMTN2008.pdf>

Administrative and legal structure

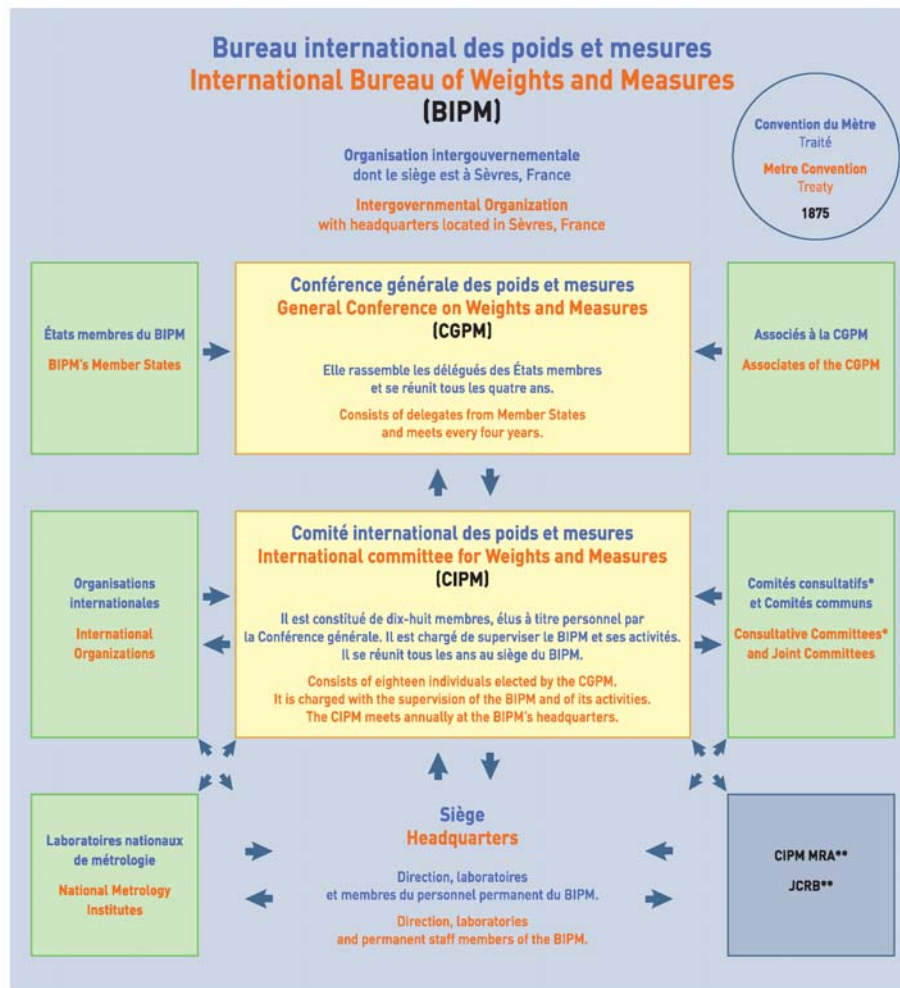
The **Metre Convention**. A diplomatic treaty signed in 1875. There are now 51 member states.

The treaty established the International Bureau of Weights and Measures (**BIPM**)



as well as a scientific oversight committee (**CIPM**) that meets yearly.

Every four years, the General Conference (**CGPM**) of member states meets to take of business at the diplomatic level. The CGPM must approve the budget of the BIPM for the following 4 years. It also must approve any changes to the units of the International System of units (SI).



CCU
CCM
CCEM
CCQM

- * Actuellement au nombre de dix, ils sont chargés de conseiller le CIPM et le Siège, notamment sur des questions techniques, et d'aider à l'administration du CIPM MRA.
- ▶ There are currently ten CCs which advise the CIPM and the Headquarters, e.g. on technical matters, and the administration of CIPM MRA.
- ** Le JCRB est le sigle utilisé pour désigner le Comité mixte des organisations régionales de métrologie et du BIPM. Le CIPM MRA fait référence à l'Arrangement de reconnaissance mutuelle du BIPM rédigé par le CIPM.
- ▶ The JCRB refers to the Joint Committee of the Regional Metrology Organizations and the BIPM. The CIPM MRA refers to the Mutual Recognition Arrangement of the BIPM drawn up by the CIPM.

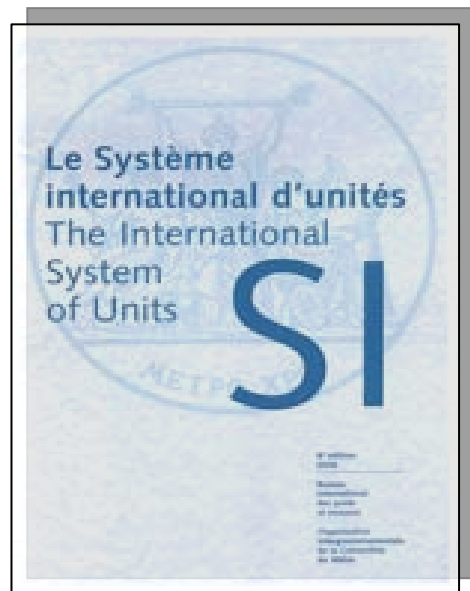
Among the Consultative Committees

CCM : Consultative Committee for Mass and Related Quantities

CCEM: Consultative Committee for Electricity and Magnetism

CCQM: Consultative Committee for Metrology in Chemistry

CCU : Consultative Committee for Units

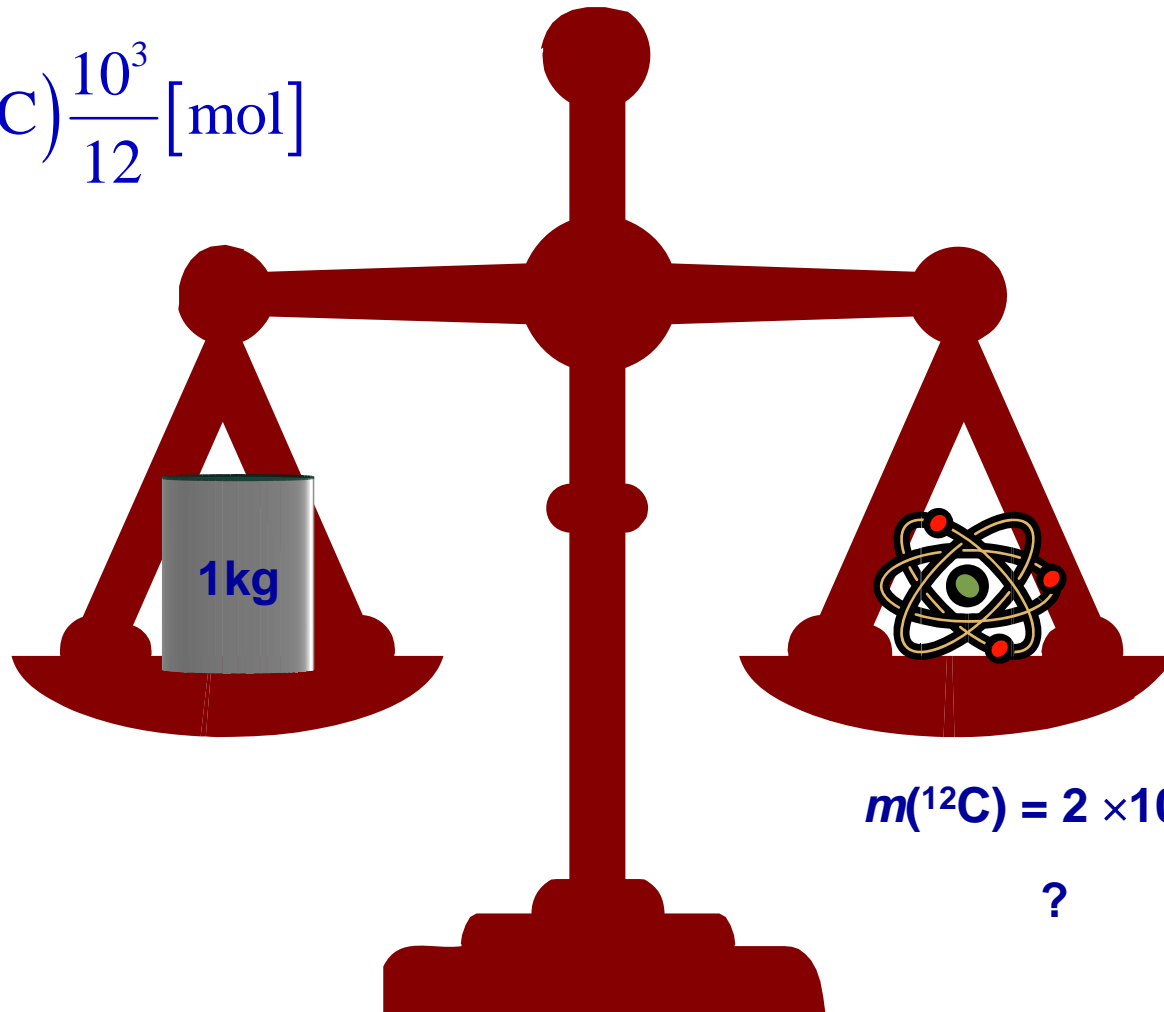


Experimental

How can we link the kg to a fundamental constant?

A large mismatch between 1 kg and the mass of a carbon-12 atom

$$\frac{1}{N_A} = m(^{12}\text{C}) \frac{10^3}{12} [\text{mol}]$$



N_A by the X-ray Crystal Density (XRCD) method

n = number of ^{28}Si atoms in the sphere:

$$n = \left\{ \frac{m_{\text{sph}}}{m_{^{28}\text{Si}}} \right\} = \left\{ \frac{m_{\text{sph}}}{m_{\mathcal{K}}} \right\} \left\{ \frac{m_{\mathcal{K}}}{m_{^{12}\text{C}}} \right\} \left\{ \frac{m_{^{12}\text{C}}}{m_{^{28}\text{Si}}} \right\}$$

$$n = 8 \frac{V}{a^3} \quad \begin{array}{l} V = \text{volume of sphere} \\ a^3 = \text{volume of unit cell} \end{array}$$

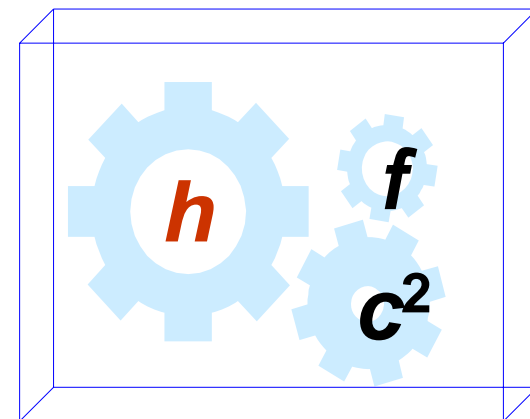
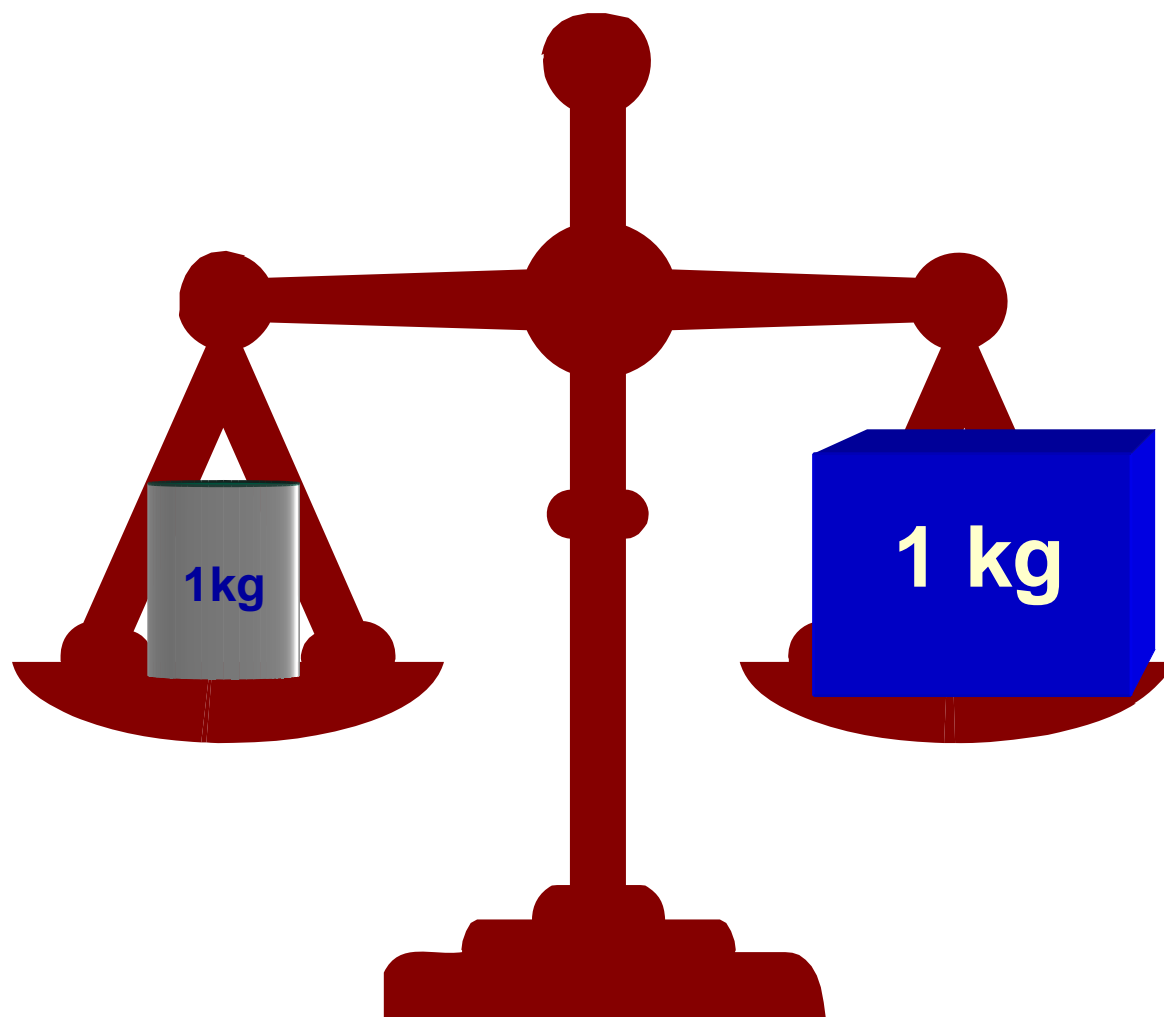
$$\left\{ \frac{m_{^{12}\text{C}}}{m_{\mathcal{K}}} \right\} = \left\{ \frac{m_{\text{sph}}}{m_{\mathcal{K}}} \right\} \left\{ \frac{m_{^{12}\text{C}}}{m_{^{28}\text{Si}}} \right\} \frac{a^3}{8V}$$

mass comparator ; mass spectrometer
X-ray interferometer; optical interferometer
purity: chemical, atomic, crystallographic

$$\frac{1}{N_A} = \left\{ \frac{m_{^{12}\text{C}}}{m_{\mathcal{K}}} \right\} \frac{10^3}{12} [\text{mol}] = \left\{ \frac{m_{^{12}\text{C}}}{m_{\mathcal{K}}} \right\} [\text{kg}] \cdot \frac{1}{12} \left[\frac{\text{mol}}{\text{kg}} \right] \cdot 10^3$$



How the Planck constant can be linked to the kilogram



$$m = h \frac{f}{c^2}$$

de Broglie-Compton
equation;

watt balance
equation for 1 kg is
“similar”.

Watt Balance

Part 1

$$m_{\mathcal{K}} g = I \cdot f(\vec{B}, \vec{r})$$

g is local grav. accel.; I is a current

Part 2

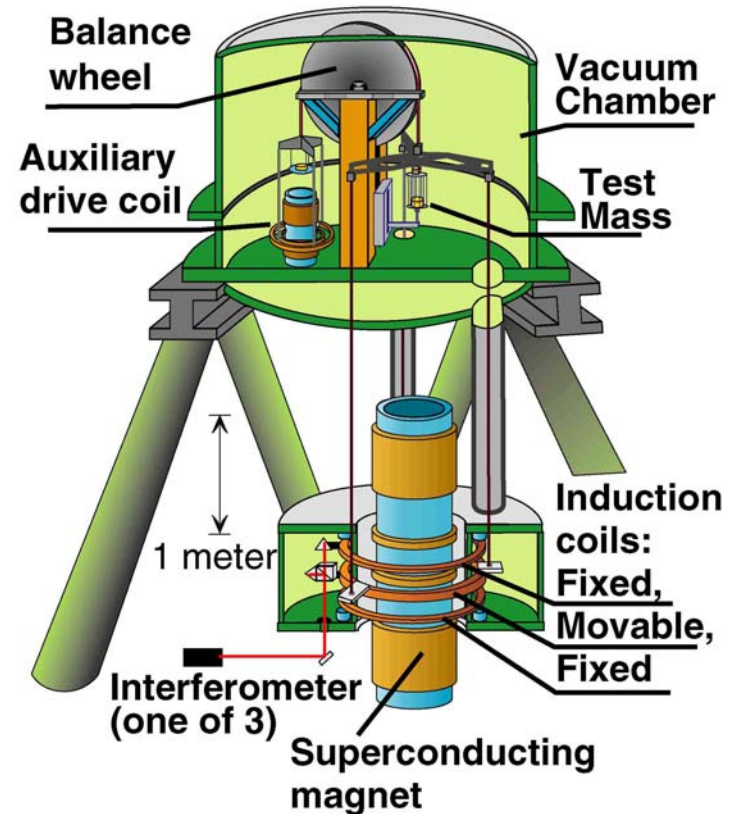
$$U = v \cdot f(\vec{B}, \vec{r})$$

U is a voltage; v is a velocity

$$m_{\mathcal{K}} g v = I U \quad [\text{Watt}] = \frac{U'}{R} [\text{Watt}]$$

$$m_{\mathcal{K}} = h \left(\frac{U'_{90} U_{90}}{R_{90} g v} \cdot (\text{cwnu}) \right)$$

(cwnu) means “constants with no uncertainty”



Schematic of NIST apparatus
(courtesy of NIST)

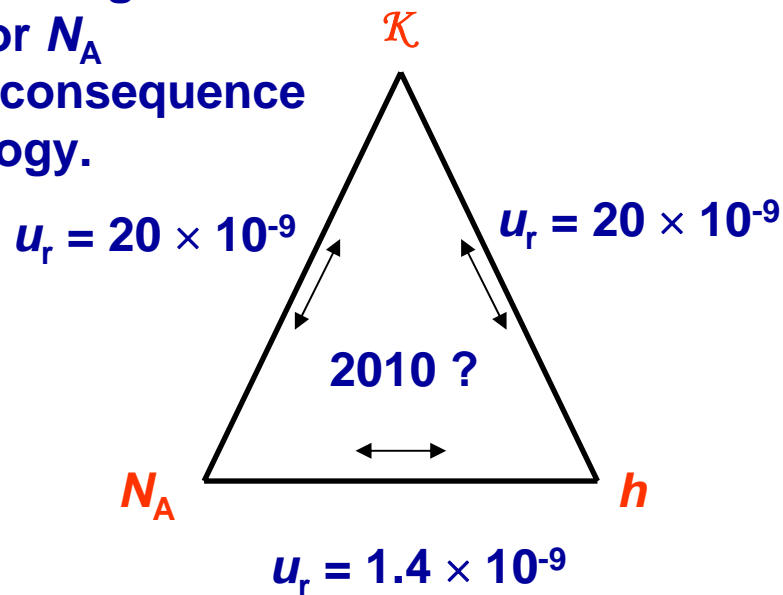
The international prototype, N_A , and h form a metrological triangle

$$N_A h = 0.012 \frac{m_e}{m(^{12}\text{C})} \frac{c \alpha^2}{2R_\infty}$$

$$u_r(N_A h) = 1.4 \times 10^{-9}$$

From CODATA 2006

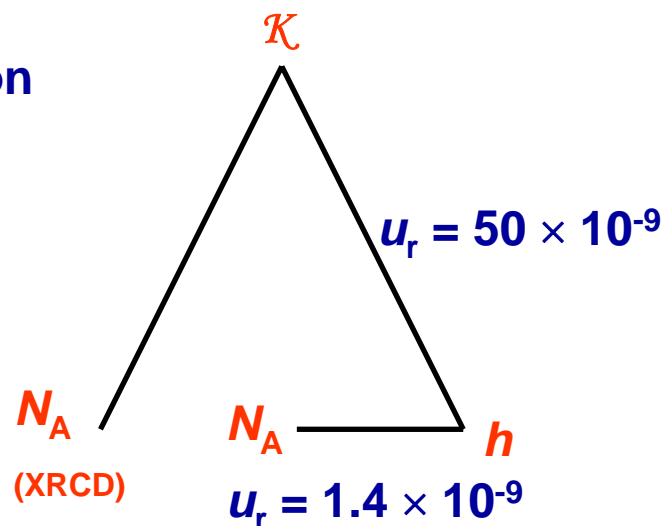
Whether the “new” kg is defined by h or N_A should have no consequence for mass metrology.



what we are working toward

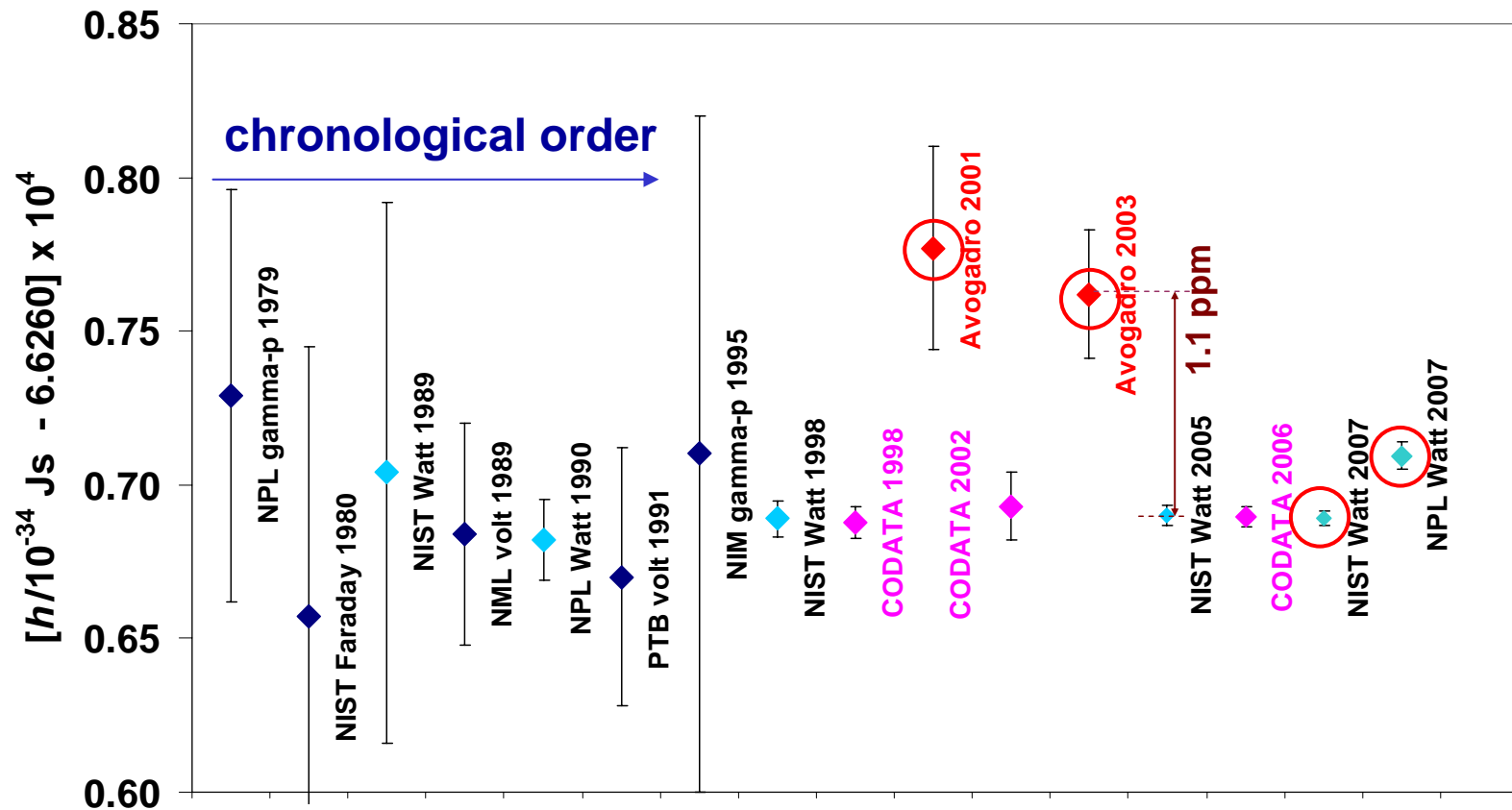
At present, the triangle does not close

**CODATA 2006
recommendation**



**CIPM, CCU, CCM, CCEM agree
that this situation must be clarified
before a redefinition of the kilogram**

History of measurements of the Planck constant



N_A and h are related by

$$N_A h = 0.012 \frac{m_e}{m(^{12}\text{C})} \frac{c\alpha^2}{2R_\infty} \quad u_r(N_A h) = 1.4 \times 10^{-9}$$

Steps to redefine the kilogram

CCU, CCM, etc. send proposals/counter proposals to the CIPM.

The CIPM takes action, or it does not.

In 2005, the CIPM approved a Recommendation whose major points for mass metrologists are:

- approve in principle the preparation of new definitions and *mises en pratique* of the kilogram, the ampere and the kelvin so that if the results of experimental measurements over the next few years are indeed acceptable, all having been agreed with the various Consultative Committees and other relevant bodies, the CIPM can prepare proposals to be put to Member States of the Metre Convention in time for possible adoption by the 24th CGPM in 2011;

invites all Consultative Committees

- particularly the CCM, CCEM, CCQM and CCT, to consider the implications of changing the definitions of the above-mentioned base units of the SI, and to submit a report to the CIPM not later than June 2007;

Some consequences for mass metrologists

With m_X defined, the present uncertainty of $\{m_X/m_K\}$ will be attributed to the mass of the international prototype and will propagate to all other, macroscopic, masses derived from the international prototype.

Here X might be the mass of a fundamental constant such as $m(^{12}\text{C})$; or the product of a fundamental constant, such as h , multiplied by an appropriate factor; or...

Since the international prototype becomes a derived mass standard, the possibility is open to replace it by a better artefact or by the average of a group of artefacts.

Some consequences for everybody

- There will be no discontinuity in the kilogram, therefore no immediate consequences to measurements.
- The relative uncertainties of all mass standards, including the international prototype, will have an additional (but identical) component.
- Because the relative uncertainty component is exactly the same for all mass standards and all masses derived from mass standards, this component does not increase the uncertainty of comparisons between mass standards.
- Therefore, the consequence for end users will be negligible.
- Nevertheless, we must forge the strongest possible experimental links to a new definition to ensure that values of macroscopic masses remain traceable to the SI to sufficient accuracy.
- It seems unlikely that the public will understand the new definition of the kilogram. (This is a challenging problem in communications.)

Resolution of the CGPM, November 2007 - 1

...

The CGPM notes

- That any changes to the definitions of the SI units must be **self-consistent**.
- That it is desirable that definitions of the base units should be **easily understood**.

...

- **The importance of soliciting comments and contributions from the wider scientific and user communities.**

...

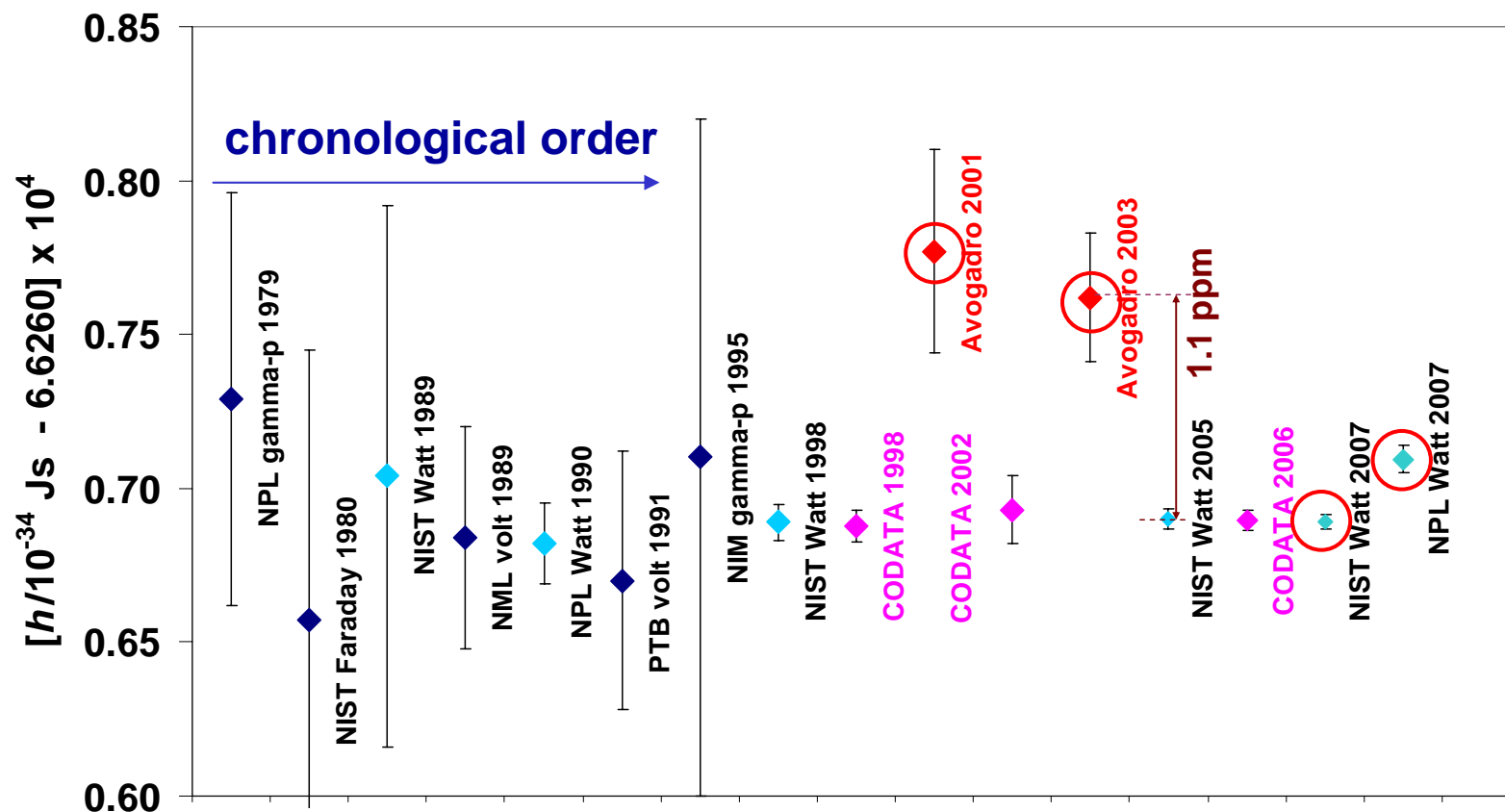
Resolution of the CGPM, November 2007 - 2

The CGPM Recommends that NMIs and the BIPM

- pursue the relevant experiments so that the CIPM can decide whether it may be possible to redefine the kg, A, K and mol using fixed values of the fundamental constants at the time of the 24th CGPM (2011).
- together with CIPM and appropriate Working Groups, work on practical ways of realizing the new definitions (MeP) and consider the most appropriate way of explaining the new definitions to users.
- initiate awareness campaigns to alert user communities to the possibility of redefinitions (technical and legislative implications, MeP)...

Requests the CIPM to report to the 24th CGPM in 2011 and to undertake whatever preparations are considered necessary so that, if the results of experiments are found to be satisfactory and the needs of users met, formal proposals for changes to the definitions of the kg, A, K, and mol can be put to the 24th CGPM.

A reminder of the present situation

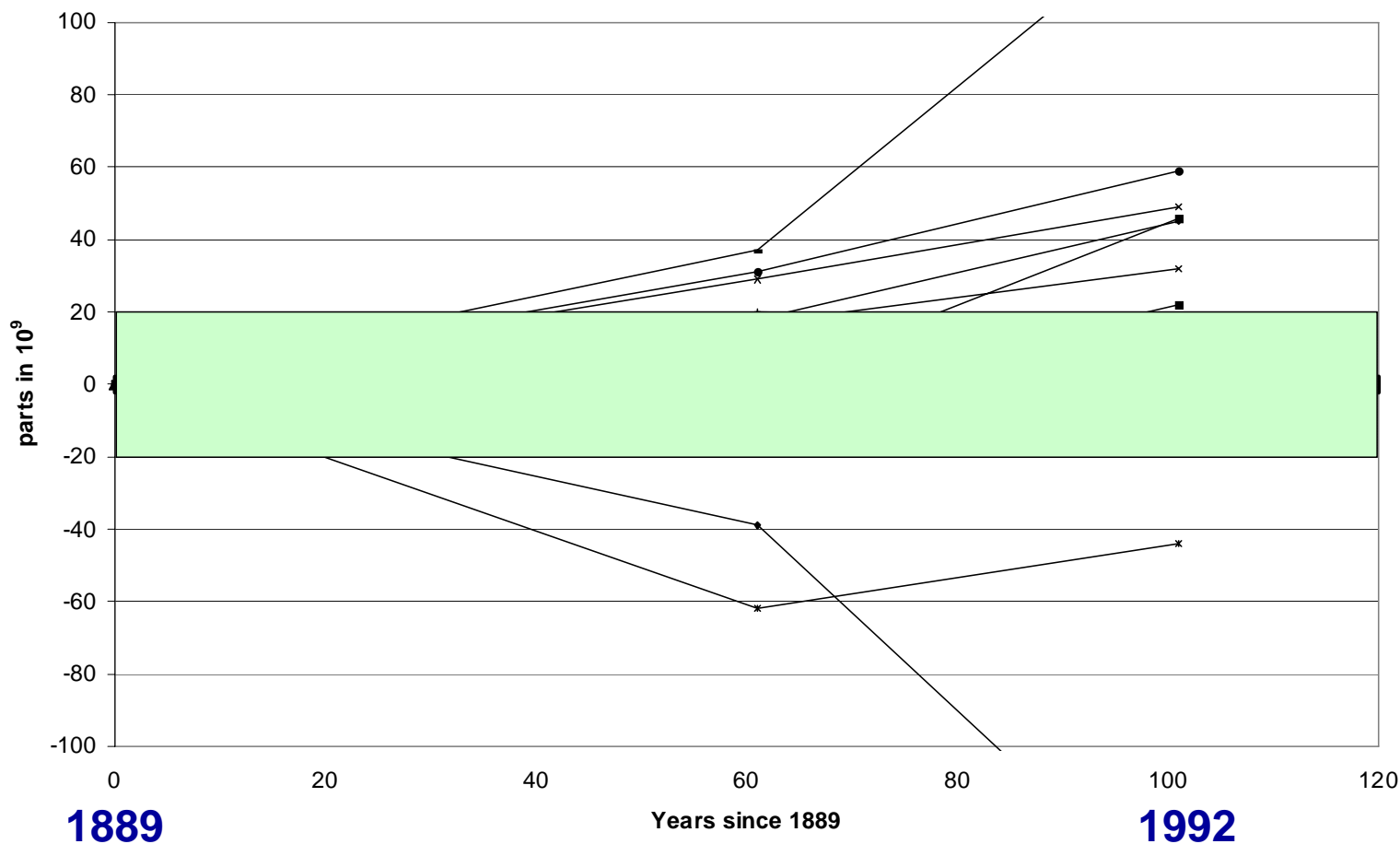


N_A and h are related by

$$N_A h = 0.012 \frac{m_e}{m(^{12}\text{C})} \frac{c\alpha^2}{2R_\infty} \quad u_r(N_A h) = 1.4 \times 10^{-9}$$

Changes in mass of national prototypes manufactured in 1889

CODATA/NPL_WB: 280×10^{-9} ; CODATA/XRCD: 1000×10^{-9}



Summary

- Artefacts are inherently unstable with respect to fundamental physical constants.
- The masses of “identical” 1 kg Pt-Ir artefacts are not stable with respect to each other.
- There is no experimental evidence (so far) that m_K is changing with respect to the fundamental physical constants.
- Present results between Watt Balance and XRCD values of h (or N_A) disagree at the 10^{-6} level.
- Two most recent Watt Balance results disagree by 3×10^{-7} .
- 1 kg calibrations are currently disseminated to $< 3 \times 10^{-8}$.
- An immediate redefinition of the kilogram would benefit electrical metrology and some important areas of physics.
- Many groups are active in this research and progress is being made.

Conclusion

There is much to do in the next two years if we are to redefine the kilogram in a useful way for everybody.

Watt Balance details

$$m_{\mathcal{K}} = \frac{U'U}{Rgv} \quad \text{In SI} \quad m_{\mathcal{K}} = \hbar \frac{f_1 f_2}{f_3 c^2} \left\{ \frac{n_1 n_2}{4i\beta\gamma} \right\}$$

$$U = \frac{nf}{K_J} = \frac{nf}{\left(\frac{2e}{h}\right)} \quad U_{90} = \frac{nf}{K_{J-90}} \quad \begin{aligned} g &= \beta c f_3 \\ v &= \gamma c \end{aligned}$$

$$m_{\mathcal{K}} = \frac{U'_{90} U_{90}}{R_{90} g v} \cdot \frac{K_{J-90}^2}{\left(\frac{2e}{h}\right)^2} \frac{R_{K-90}}{\frac{h}{e^2}} = \hbar \frac{U'_{90} U_{90}}{R_{90} g v} \left\{ \frac{K_{J-90}^2 R_{K-90}}{4} \right\}$$