# Dark Energy and Modified Gravity without parametrizations

**Santiago Casas**

**CEA Paris-Saclay**
**Cosmostat / Dap / Irfu**

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# Motivation

➔ Euclid and future stage IV galaxy surveys:

• Huge amounts of data, challenging task to process

➔ Despite GW170817, many models on the market (e.g. talk by Valeria yesterday), many of them have free functions → need to be parametrized.

• To reach Euclid's full potential: We need nonlinear scales.

➔ We can only provide reliable N-body simulations for a very limited (and biased) subset of these models

➔ Covariance matrix calculation for different cosmologies is another enormous problem.

➔ Maybe we should focus on simple well-motivated models or model-independent observables that test gravity?

# Outline

- **Higgs-Dilaton cosmology:** A link between inflation and dark energy.
- Model with effectively less parameters than LCDM.

- **Model independent** determination of the gravitational slip $\eta$
- Obtained with present data
- Revisiting forecasts for Euclid

# Higgs-Dilaton cosmology: A link between Dark Energy and Inflation

$$\frac{\mathcal{L}_{SI+UG}}{\sqrt{-g}} = \frac{f(h,\chi)}{2}R - \frac{1}{2}(\partial h)^2 - \frac{1}{2}(\partial \chi)^2 - V(h,\chi)$$

- Inflation and Dark Energy share many essential properties.
- Scale invariant extension of the Standard Model.
- Non-minimally coupled to gravity, scale invariant potential.
- Standard Higgs causes inflation at early times, massless Dilaton produces a thawing quintessence field.

$$f(h,\chi) = \xi_h h^2 + \xi_\chi \chi^2 \,,$$

$$V(h,\chi) = \frac{\lambda}{4}\left(h^2 - \alpha\chi^2\right)^2 + \beta\chi^4$$

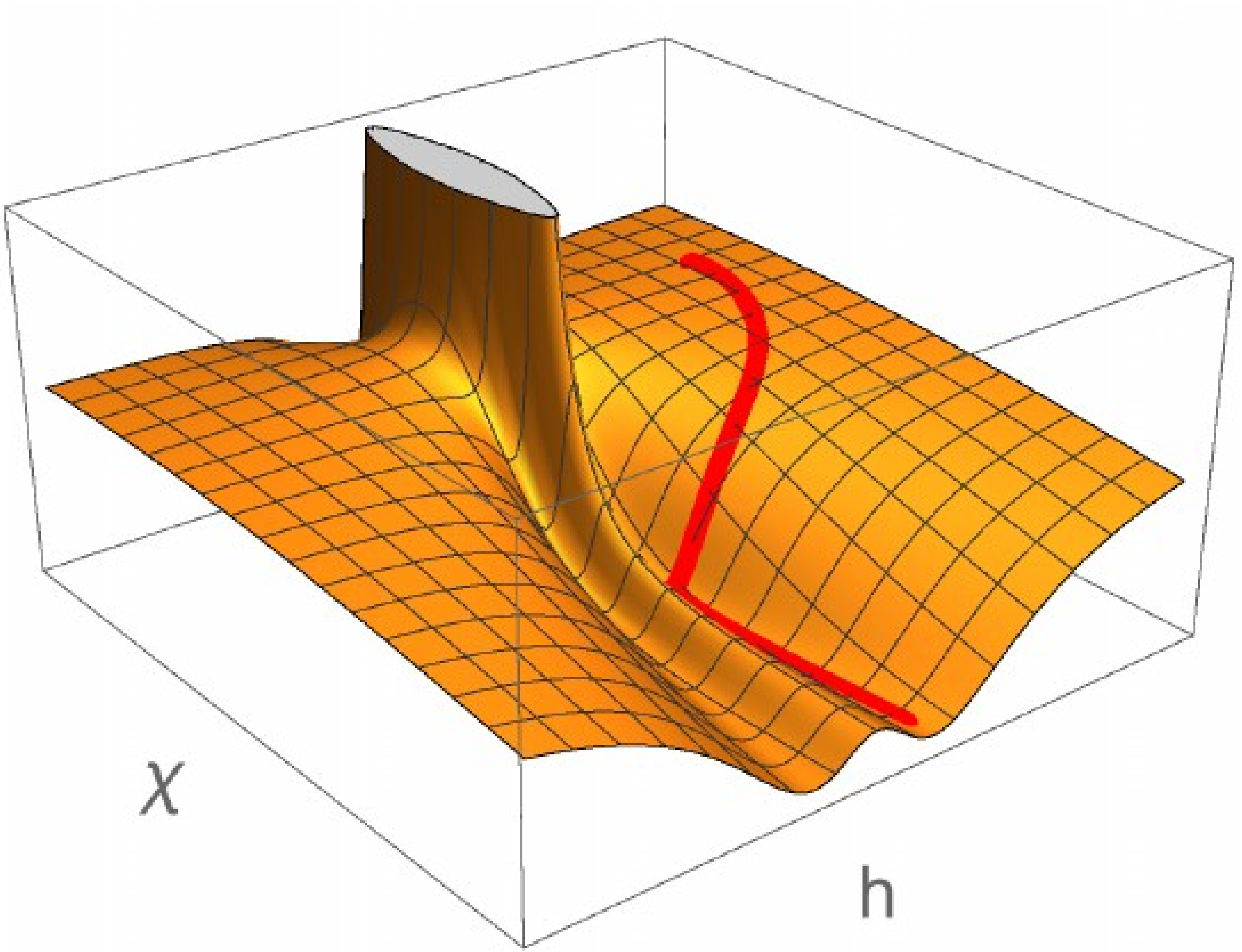
# Higgs-Dilaton cosmology: A link between Dark Energy and Inflation

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_P^2}{2} R - \frac{K(\Theta)}{2} (\partial\Theta)^2 - \frac{\Theta}{2} (\partial\Phi)^2 - U - U_{\Lambda_0}$$

• Conformal transformation and field redefinition: Einstein-frame Lagrangian.
• During inflation \Theta is close to zero, \Phi field is frozen, effectively single field inflation. → can compute standard slow-roll parameters.
• At late times, after reheating, \Theta field goes to minimum, we end up with quintessence Lagrangian with exponential potential:

$$\frac{\mathcal{L}}{\sqrt{-g}} \simeq \frac{M_P^2}{2} R - \frac{1}{2} (\partial\Phi)^2 - \frac{\Lambda_0}{c^2} e^{-\frac{4\gamma\Phi}{M_P}}$$

- \Phi field is the radial coordinate (\Theta angular one) from \Xi , h
- During inflation the system is bound to ellipsoidal trajectories (constant \Phi ) in the \Xi-h space.



# Higgs-Dilaton cosmology: A link between Dark Energy and Inflation

- The primordial power spectrum (scalar and tensor) after inflation:

$$P_s = A_s \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln\left(\frac{k}{k_*}\right)}, \quad P_t = A_t \left(\frac{k}{k_*}\right)^{n_t}$$

- Since the same fields are responsible for inflation and dark energy:

$$\frac{w'}{1-w} = -3(1+w) + 4\gamma\sqrt{3(1+w)\Omega_{\rm DE}}\,,$$

$$\Omega'_{\rm DE} = -3\Omega_{\rm DE}(1-\Omega_{\rm DE})w\,,$$

$$F(\Omega_{\rm DE}) = \left[\frac{1}{\sqrt{\Omega_{\rm DE}}} - \Delta \tanh^{-1}\sqrt{\Omega_{\rm DE}}\right]^2$$

$$n_s = 1 - \frac{2}{N_*} X \coth X$$

$$X \equiv 4cN_* = \frac{3N_*(1+w)}{4F(\Omega_{\rm DE})}$$

$$r = \frac{2}{|\kappa_c| N_*^2} X^2 \sinh^{-2} X\,.$$

- Primordial parameters are directly connected to late-time parameters.

# Fisher Forecast for Galaxy Clustering

- The observed power spectrum for galaxy clustering:

$$P_{\rm obs}(z,k,\mu;\theta) = P_{\rm s}(z) + \frac{D_A^2(z)_{ref}H(z)}{D_A^2(z)H(z)_{ref}} b^2(z)\left(1+\beta(z)\mu^2\right)^2 P(k,z) e^{-k^2\mu^2(\sigma_z^2/H(z)+\sigma_v^2(z))}$$

- Including Redshift Space Distortions (RSD), Baryon Acoustic Oscillations (BAO), Fingers of God and Alcock-Pazcynski effects.

$$F_{ij} = \frac{1}{8\pi^2}\int_{-1}^{+1} \mathrm{d}\mu \int_{k_{\rm min}}^{k_{\rm max}} \mathrm{d}k\, k^2 \frac{\partial \ln P_{\rm obs}(k,\mu,z)}{\partial\theta_i}\frac{\partial \ln P_{\rm obs}(k,\mu,z)}{\partial\theta_j} V_{\rm eff}(k,\mu,z)$$

- The Fisher matrix is very sensitive to the variations of the power spectrum w.r.t. cosmological parameters and to the geometry and galaxy content of the survey.

# General modifications of Gravity

- We focus here on scalar perturbations of the metric.

$$ds^2 = a^2(\tau)\left(-(1+2\Psi)d\tau^2 + (B_{,i}+S_i)d\tau dx^i - (-2\Phi\delta_{ij} + 2E_{,ij} + F_{i,j} + F_{j,i} + h_{ij})dx^i dx^j\right)$$

- Modifications to Einstein gravity can be parametrized by two functions of time and scale.

$$-k^2\Psi = 4\pi G a^2 \mu(a,k)\rho\delta$$

$$\eta(a,k) = \Phi/\Psi$$

- μ represents the change of gravitational constant $G_{\rm eff}$ , η the effective anisotropic stress.

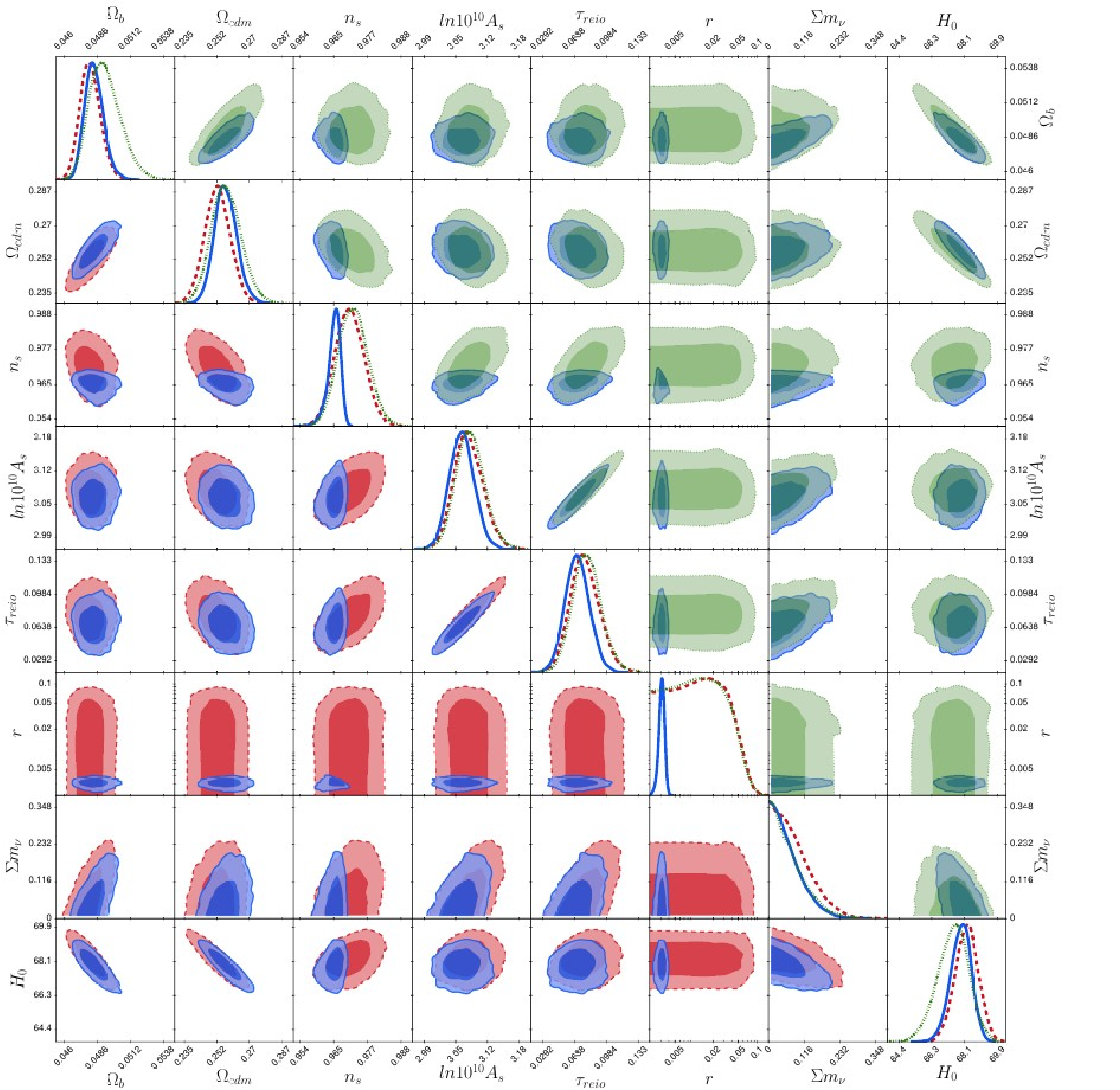
$$\mu(a,k) = 1 + f_1(a)\frac{1+c_1(\lambda H/k)^2}{1+(\lambda H/k)^2}$$

$$\eta(a,k) = 1 + f_2(a)\frac{1+c_2(\lambda H/k)^2}{1+(\lambda H/k)^2}$$

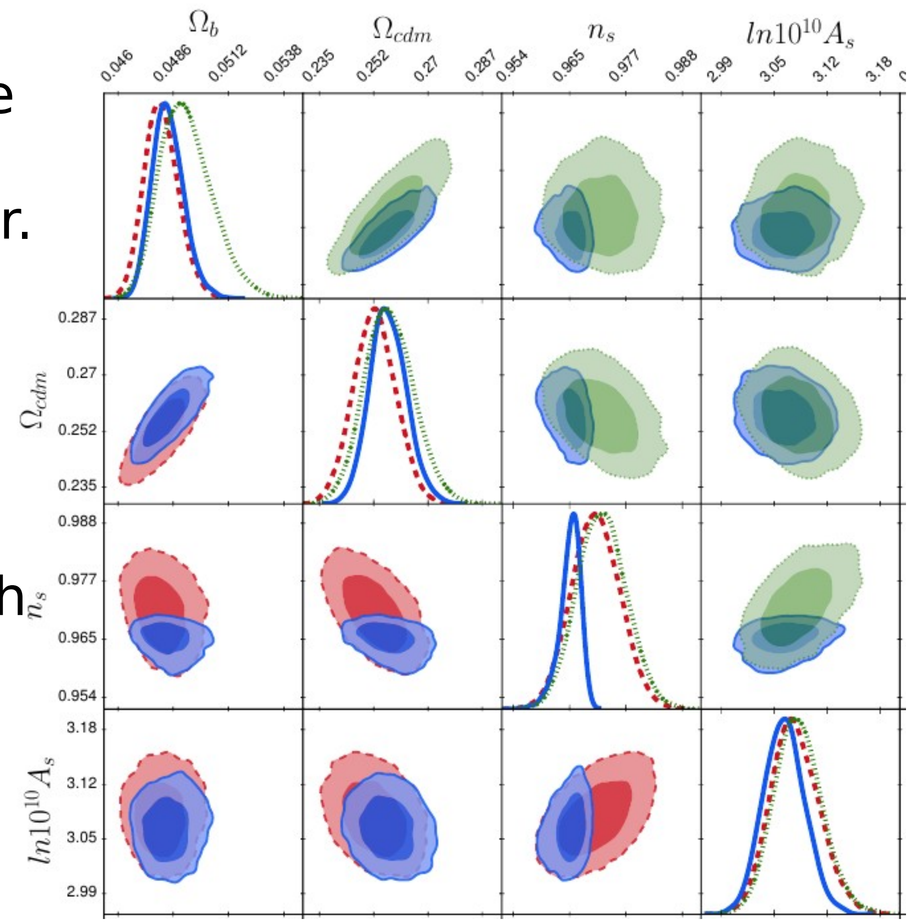
- GC measures μ, while WL measures the Weyl potential Σ.

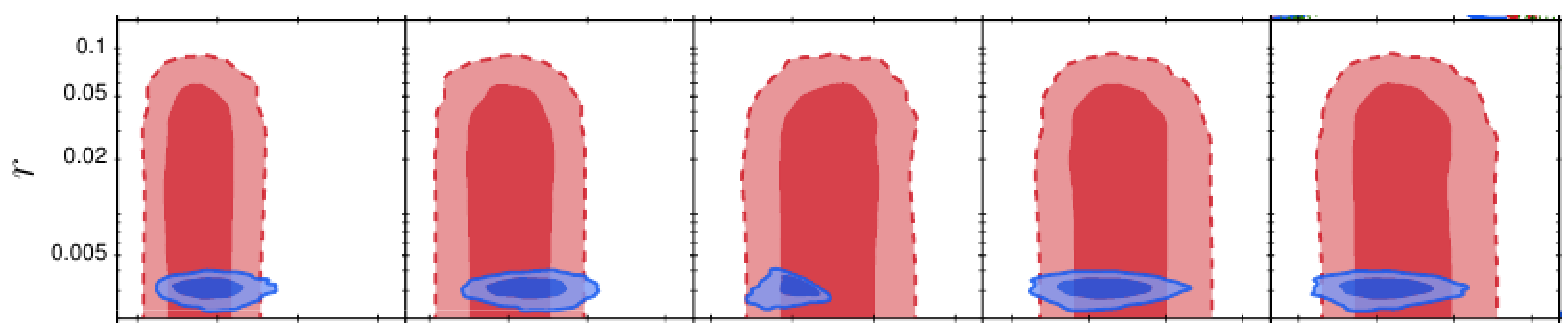
$$\Sigma(a,k) = (\mu(a,k)/2)(1+\eta(a,k))$$

- MCMC analysis, using latest data from Planck TT+pol, Keck/BICEP2, JLA, 6dF, SDSS, BOSS.
- Red: LCDM
- Blue: HD Model
- Green: A quintessence model with the same w(a) evolution as HDM.

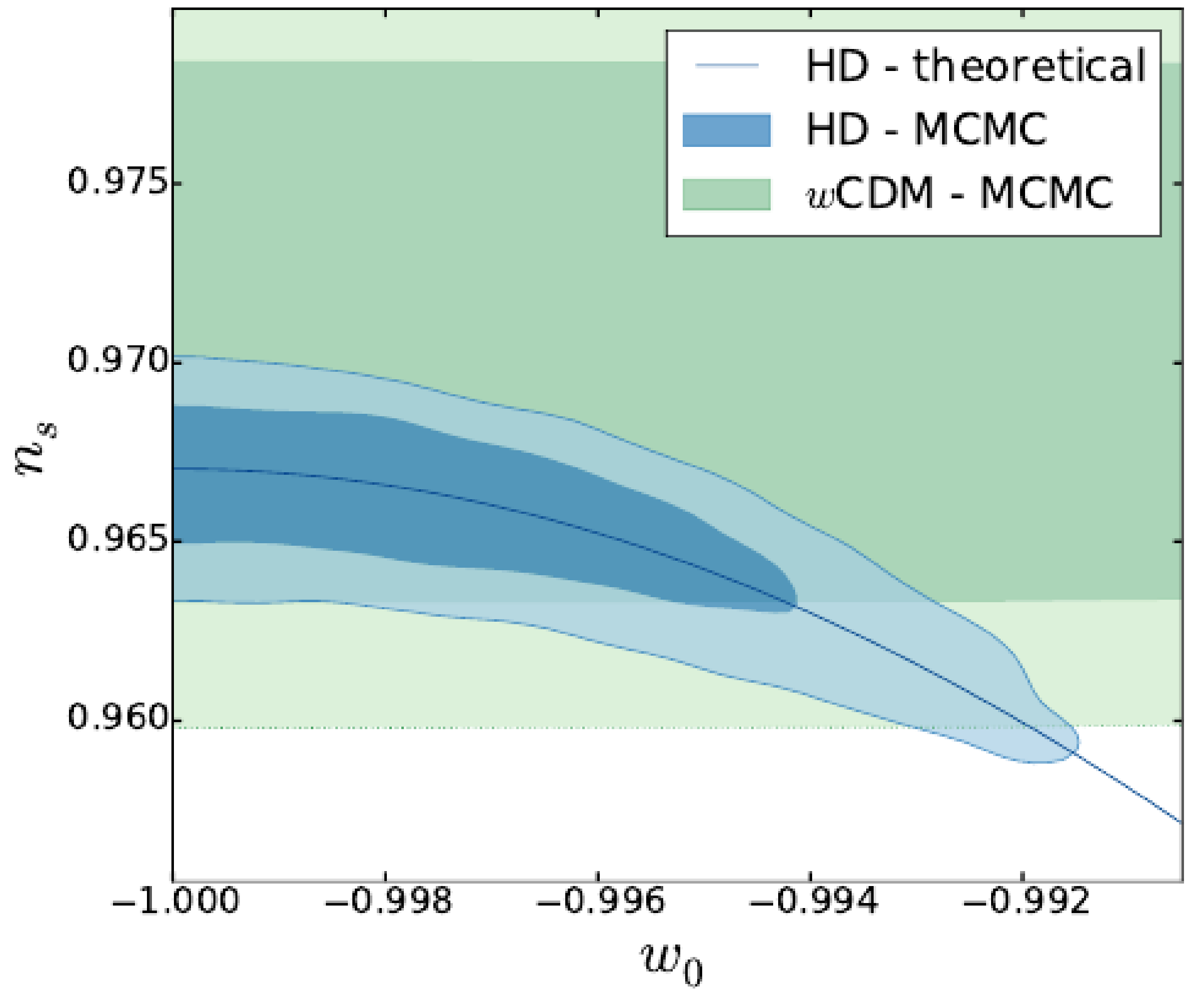


- For parameters outside the consistency relation: HDM and LCDM contours, similar.
- For **ns**, marginalized posterior is constrained at the consistency relation.
- For **r** , the consistency relations set strong upper bounds. Contours are much smaller.



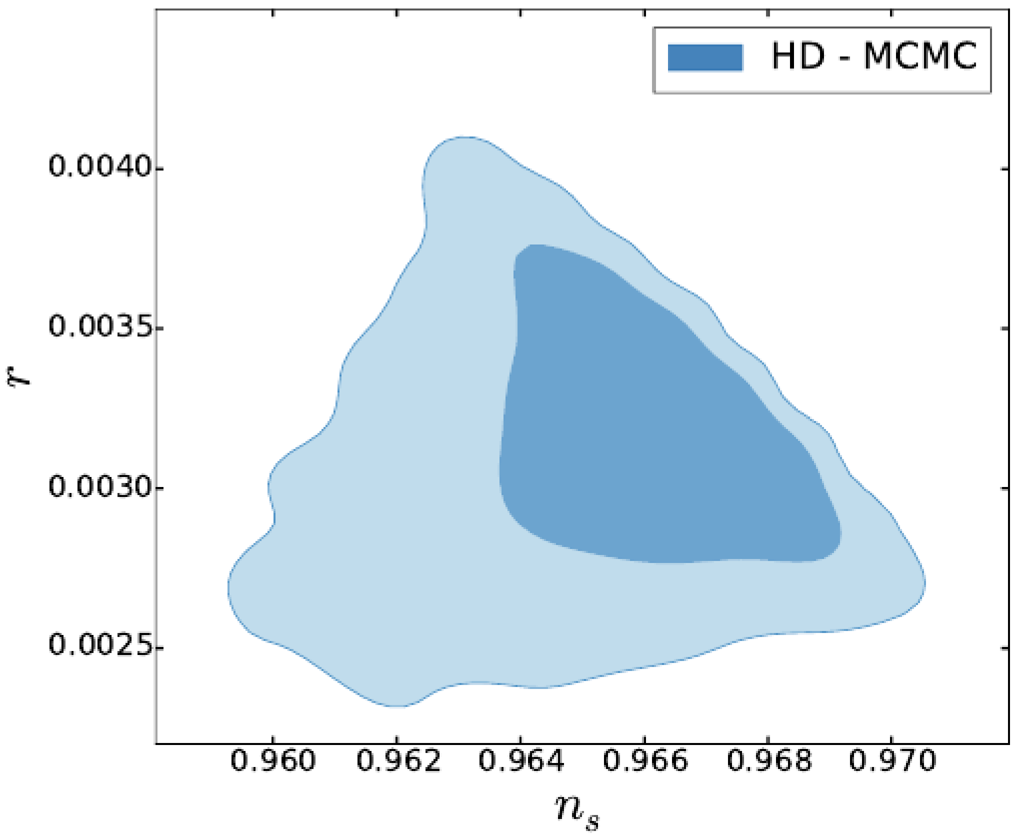



- With present data, w0-wa parameters of simple quintessential Dark Energy are still pretty much unconstrained.
- In HD cosmology, w can be very well constrained due to the cons. rel. (blue solid line).



- Tensor to scalar ratio can be constrained very well.
- If future experiments suggest a point in the **r-ns** plane outside the consistency relation, model can be ruled out.
- For models very close in parameter space: Prior is very important!

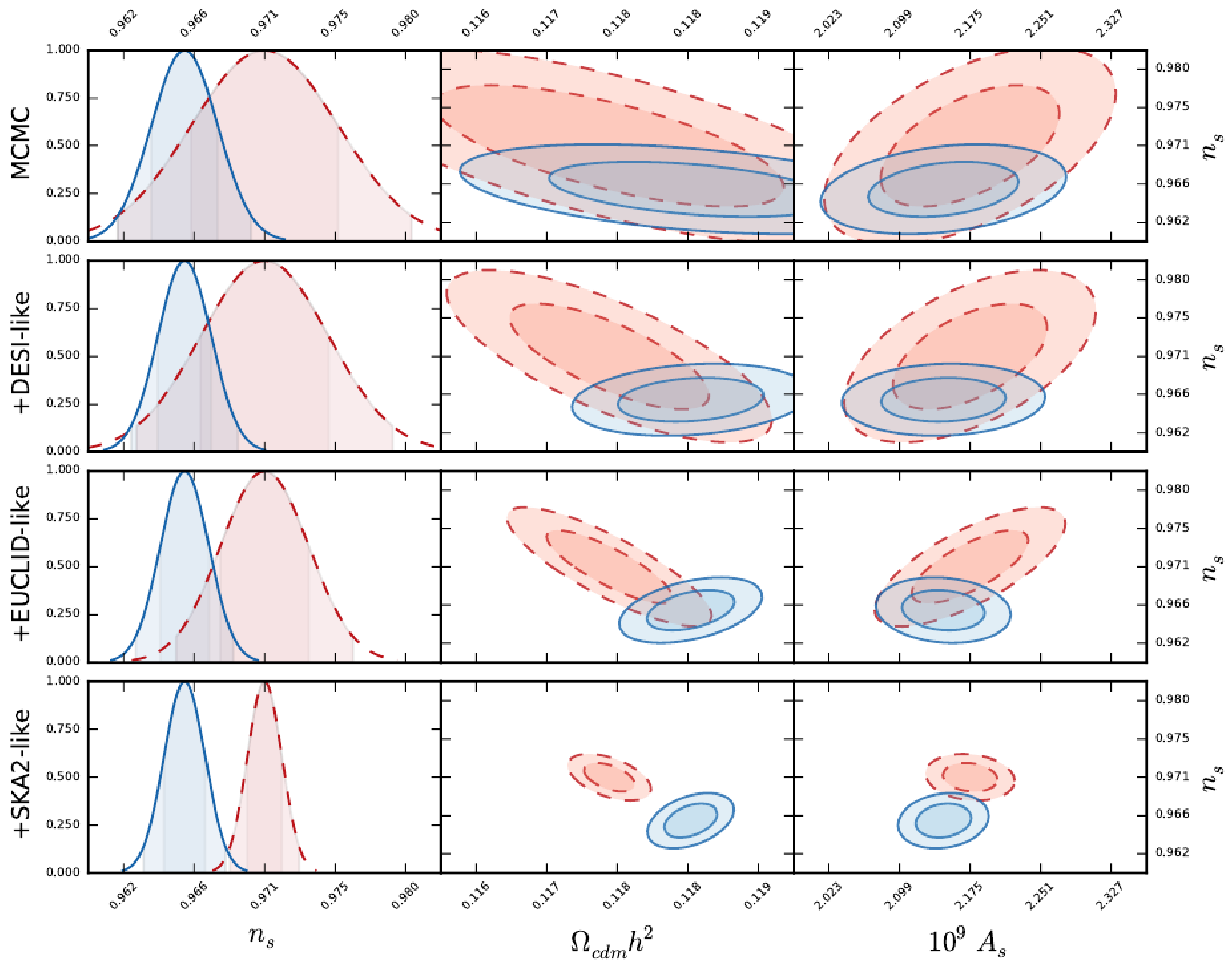
→ It should be part of the model.



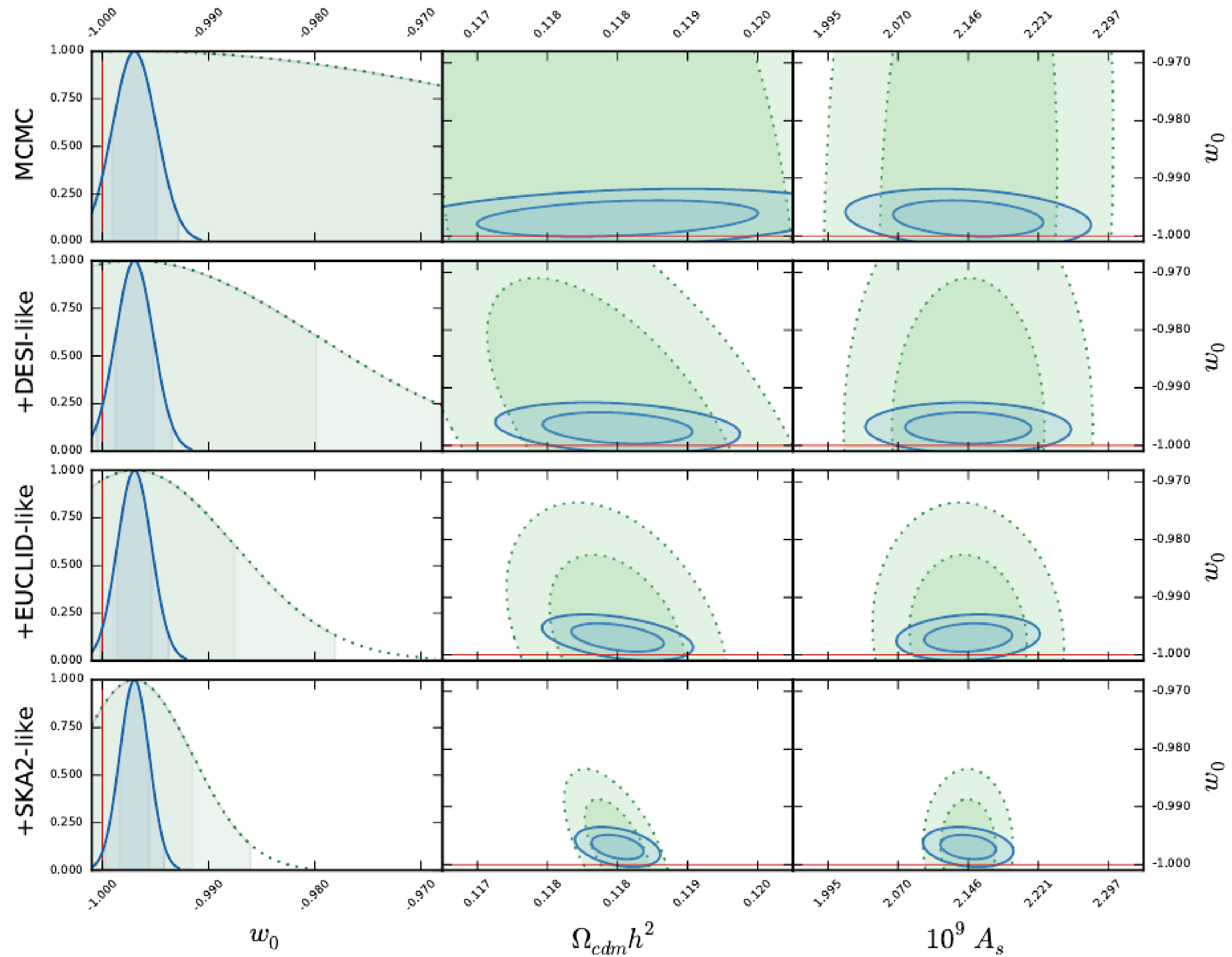

$$B(M) = \frac{p(\mathbf{x}|M)}{p(\mathbf{x}|M_{\Lambda\mathrm{CDM}})} = \frac{\pi(M_{\Lambda\mathrm{CDM}})}{\pi(M)} \frac{p(M|\mathbf{x})}{p(M_{\Lambda\mathrm{CDM}}|\mathbf{x})}$$

| **Model** | **$\Lambda$CDM** | **HD** | **$w$CDM** |
|---|---|---|---|
| $\ln B$ | 0.00 | 0.88 | −2.63 |

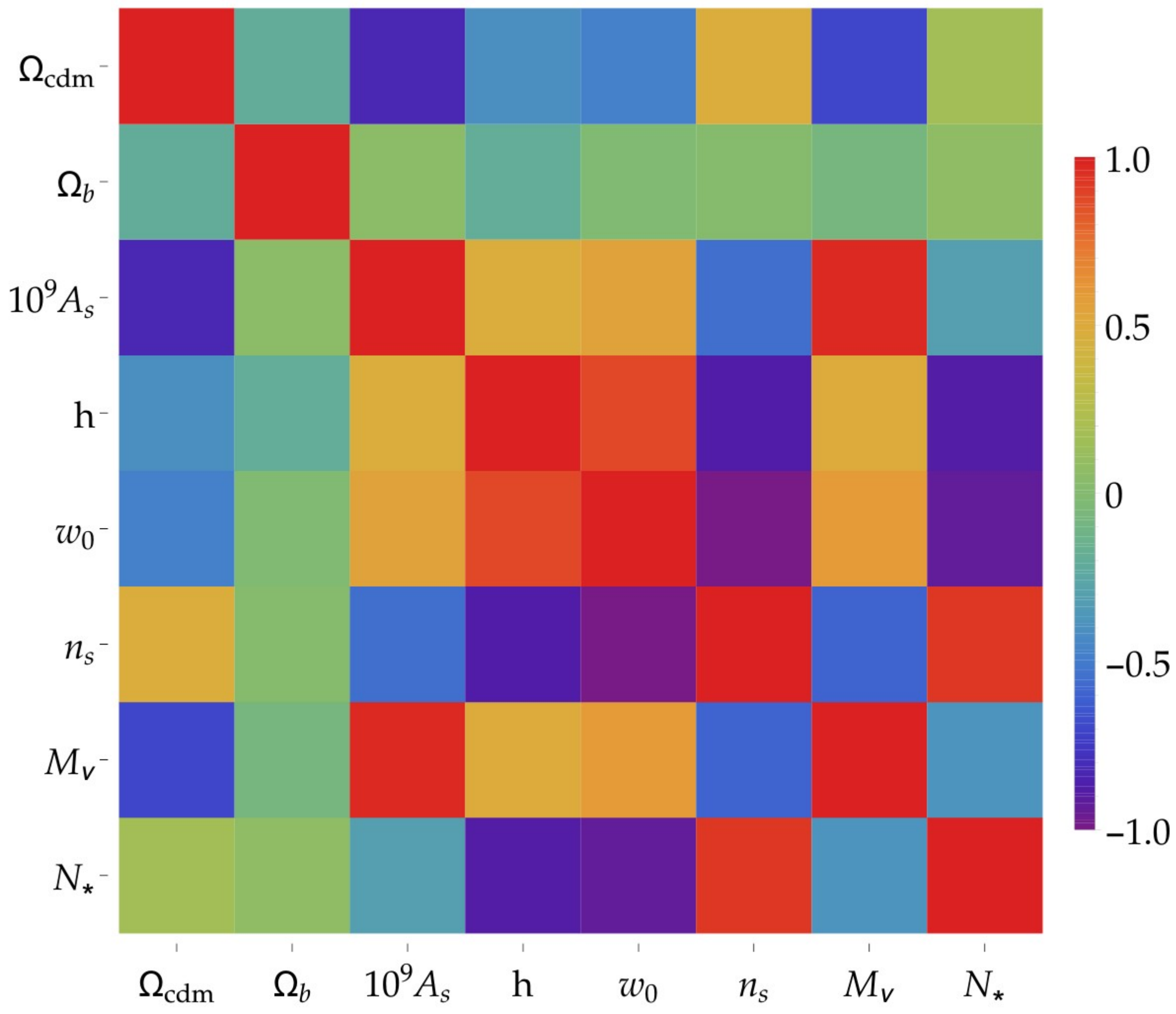
- Forecasts: Non-linear scales and combinations of observables are crucial for parameter estimation.



- Power of the consistency relation: very strong constraints on w.
- w=-1 could be ruled out at the 2-sigma level.



- Consistency relation induces strong correlations and flips standard correlations among parameters.



# Teaser:
# Model-independent determination of the gravitational slip

- Usually determination of cosmological parameters is model dependent: \Omega_m , growth rate and bias are not observable.
- We assume certain properties of Dark Matter or initial conditions (primordial power spectrum)
- One can define the following ARLE (amplitude, RSD, Lensing, Dimensionless Hubble) variables that cancel out these effects:

$$A = Gb\delta_{m0} \qquad R = Gf\delta_{m0},$$
$$L = \Omega_{m0}\Sigma\delta_m \qquad E = H/H_0 \ .$$

## Teaser:

# Model-independent determination of the gravitational slip

- At the linear level we can define only these model-independent observables :

$$P_1 \equiv \frac{R}{A} = \frac{f}{b},$$

$$P_2 \equiv \frac{L}{R} = \frac{\Omega_{\mathrm{m0}}\Sigma}{f},$$

$$P_3 \equiv \frac{R'}{R} = f + \frac{f'}{f} = \frac{(f\sigma_8(z))'}{f\sigma_8(z)}$$

- The Poisson and structure growth equations can be written in terms of ARLE parameters:

$$-k^2(\Psi - \Phi) = \frac{3(1+z)^3 L}{2E^2}$$

$$-k^2\Psi = R' + R\left(2 + \frac{E'}{E}\right)$$

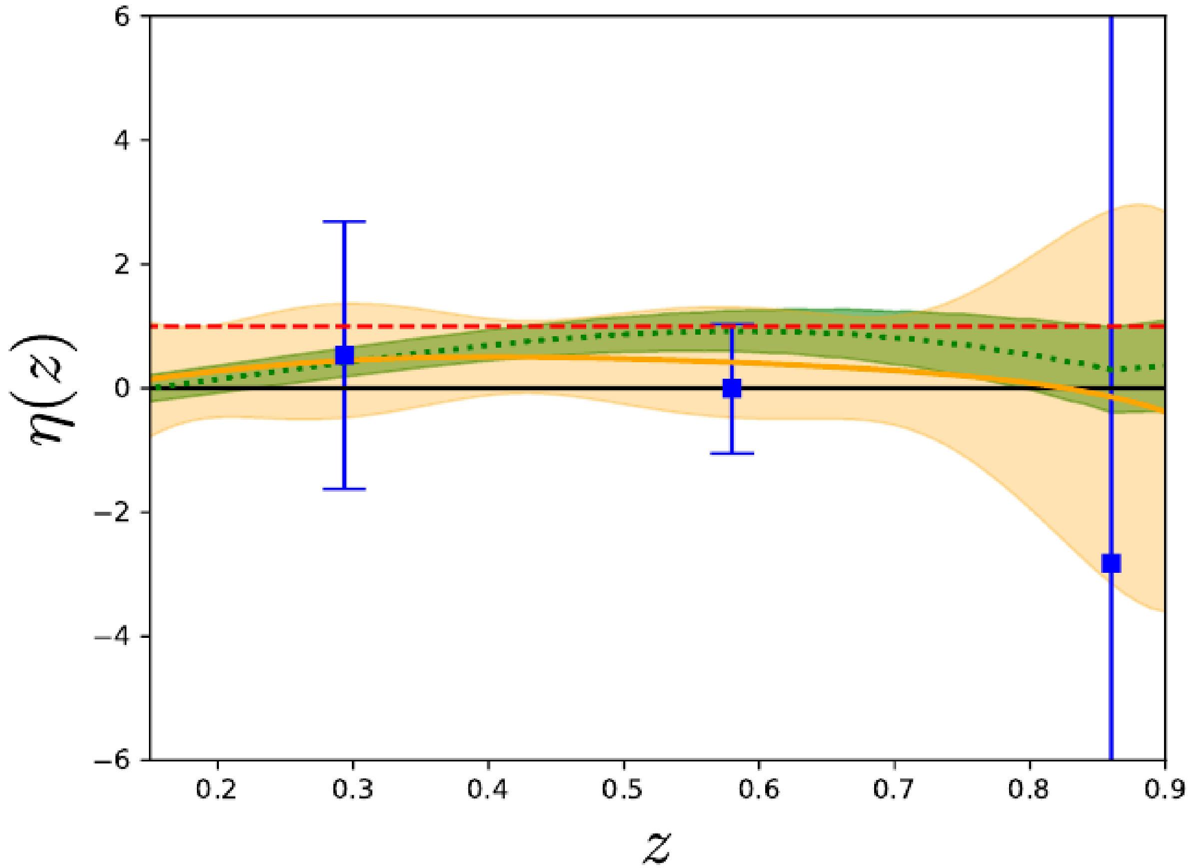
## Teaser:

# Model-independent determination of the gravitational slip

- From there we can obtain the gravitational slip in a model-independent way:

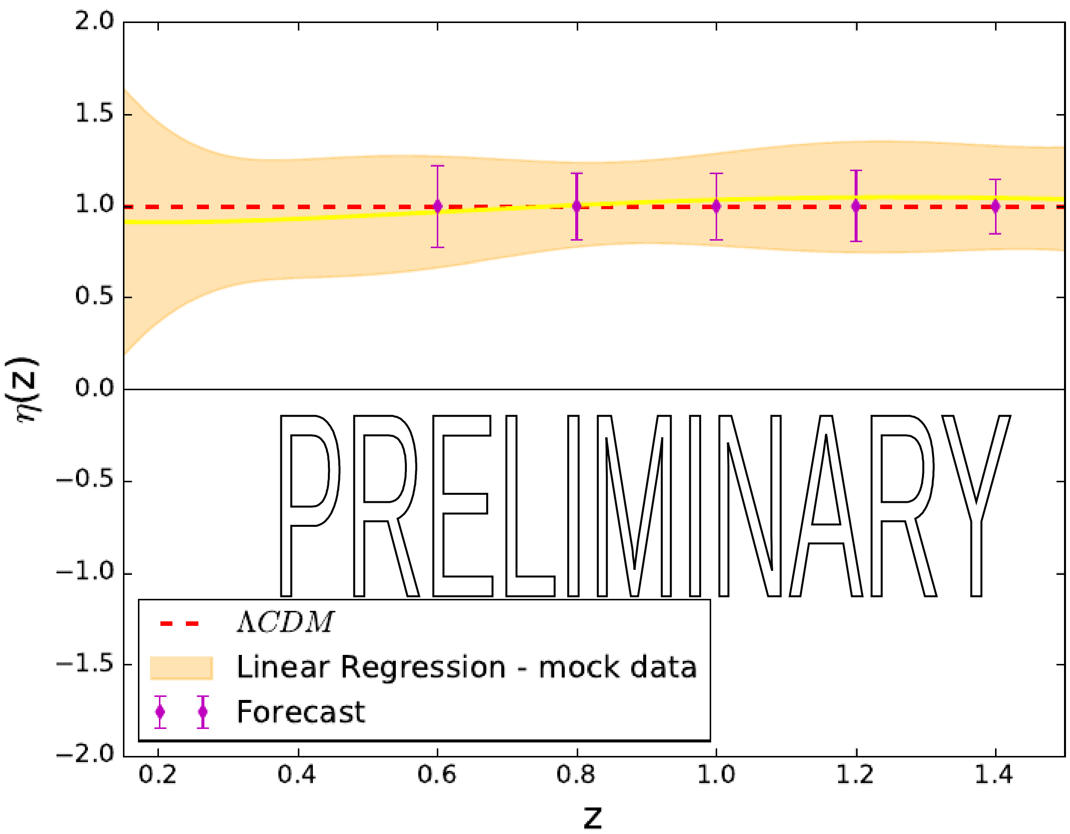
$$\eta_{obs} \equiv \frac{3P_2(1+z)^3}{2E^2\left(P_3 + 2 + \frac{E'}{E}\right)} - 1 = \eta$$

- To obtain this from data we use RSD data from BOSS, VIPERS, VIMOS, WiggleZ, SDSS.
- For P2 we use Eg statistics from the same collaborations.
- For Hubble data we use SN Ia, local measurements and cosmic chronometers.




- The hardest problem is to obtain continuous functions from sparse and noisy data.
- To do this we use: Gaussian Processes (green), Generalized Linear Regression (orange) and Binning (blue).
- For precise numbers: read the arXiv this week

# Model-independent determination of the gravitational slip




- One can rewrite the P(k) of galaxy and Cij of lensing and make forecasts for future surveys.
- For the forecasts done by Amendola, Guarnizo et al:
  \eta can be measured model-independently at all redshifts with errors of 10%.
- These forecasts should be revised with the new IST validated codes.

# Conclusions

- In order to fully understand data from Euclid and stage IV surveys, we need much more understanding about nonlinearities, the covariance matrix and baryonic effects.
- Since we still have too many models on the market, focusing on model-independent constraints is maybe the best way to test gravity without inserting too many assumptions.
- Models such as the Higgs-Dilaton cosmology, provide the full picture from inflation to reheating and late-time acceleration. And it even has less effective parameters than LCDM.
- Within this kind of unified models, Euclid can measure – together with next generation CMB experiments – the inflationary parameters with high precision.

# Backup slides

## The problem:

• Background very close to ΛCDM, important differences at the perturbation level (fifth force) and different structure formation between baryons and DE.

• Coupling parameter only constrained with Planck data[1]
→ Previous forecasts have used only linear quantities[2].
• No semi-analytical method available for the behavior in the non-linear regime.

## Proposed solution:

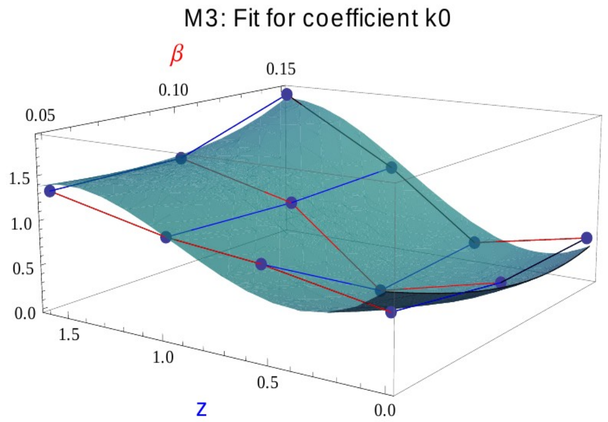
• Create fitting functions from N-body simulations and use them in forecasts.
• Have to be very careful with numerical and theoretical uncertainties.
• Even in conservative case, constraints improve by more than one order of magnitude.
• However, systematic bias from theoretical uncertainty is very important.

[1]Pettorino, Phys.Rev.D 88 (2013), [2]Amendola, Pettorino, Quercellini, Vollmer, Phys.Rev.D 85 (2012).

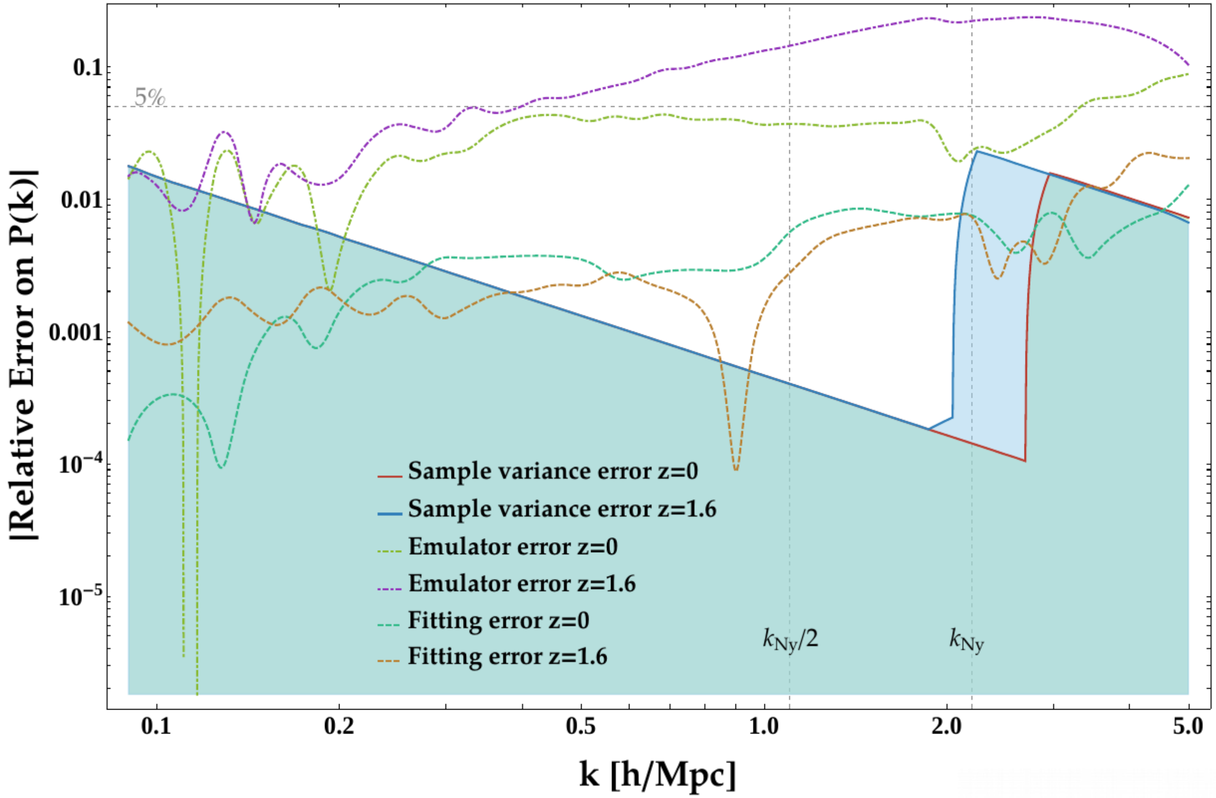
# Fitting functions

- Use CoDECS EXP simulations with three different couplings.
- We developed an automatic method that corrects numerical anomalies around the Nyquist frequency.
- Multidimensional nonlinear fit: Tested 8 “sigmoidal” models for goodness of fit, each with 5 coefficients depending polynomially (3rd order) on the parameters.

| Model Name | Analytical expression | $N_p$ |
|---|---|---|
| M1 | $f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \tanh((k - k_0) \cdot b)$ | 5 |
| M2 | $f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \arctan((k - k_0) \cdot b)$ | 5 |
| M3 | $f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \mathrm{Gudermannian}((k - k_0) \cdot b)$ | 5 |
| M4 | $f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \mathrm{erf}((k - k_0) \cdot b)$ | 5 |
| M5 | $f(k) = 1 + a_0 + a_1 \cdot k + \frac{c \cdot k}{1+e^{-(k-k_0)\cdot b}}$ | 5 |
| M6 | $f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot (\frac{1}{4} + e^{(-b+(k-k_0))^2})$ | 5 |
| M7 | $f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \frac{b\cdot(k-k_0)}{\sqrt{1+b^2\cdot(k-k_0)^2}}$ | 5 |
| M8 | $f(k) = h - a_1 \cdot k + c \cdot \left(A + \frac{(\mathcal{K}-A)\cdot k}{\left(1+Q\cdot e^{-B(k-M)}\right)^{1/\nu}}\right)$ | 9 |



# Statistical and Systematic Errors




$$n_{eff}(k,z) = n(z)/(1+n(z)\sigma_p(k,z))$$

$$\delta\theta_i = -\left[F^{\theta\theta}\right]^{-1}_{ik} F^{\theta\psi}_{kj}\delta\psi_j$$

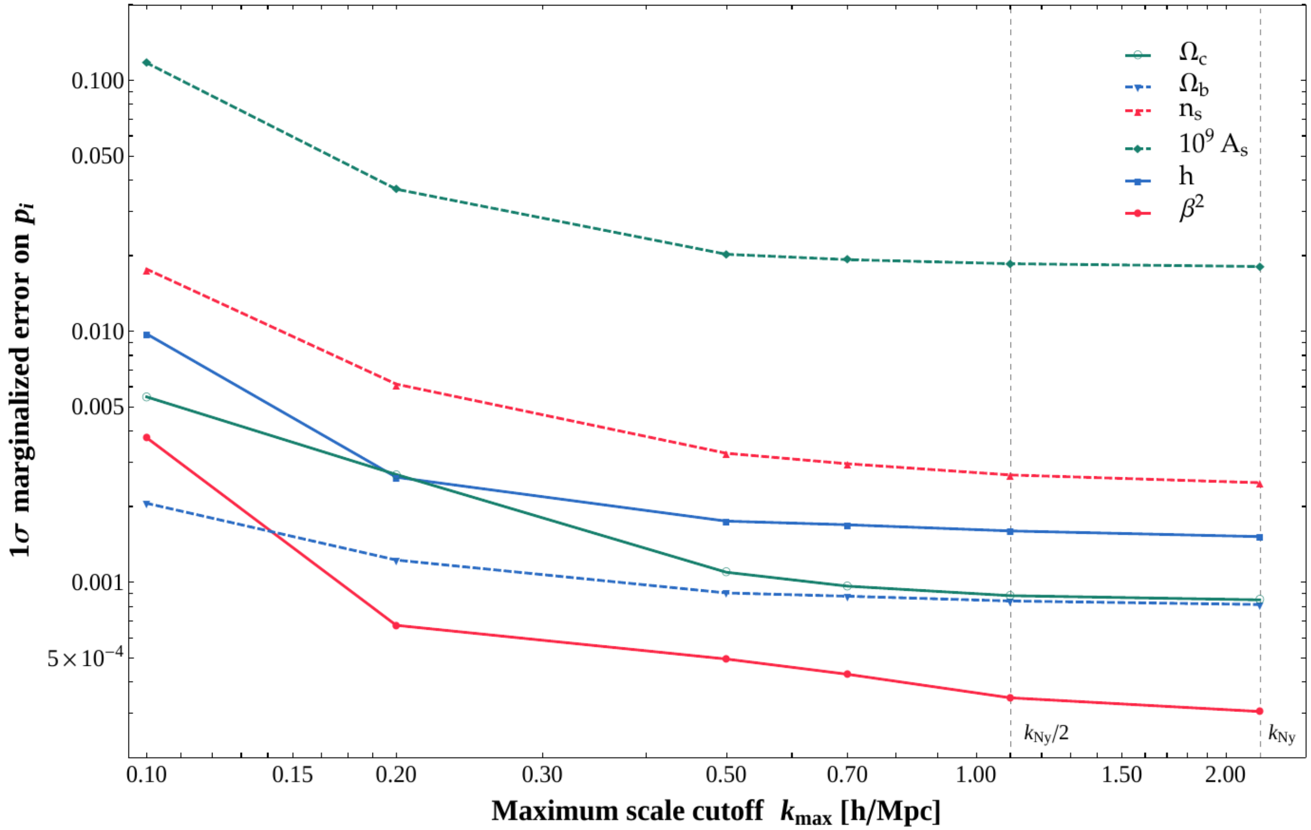
Implement noise as reduced statistics.

Imperfect knowledge of nl-PS gives rise to large systematic biases.

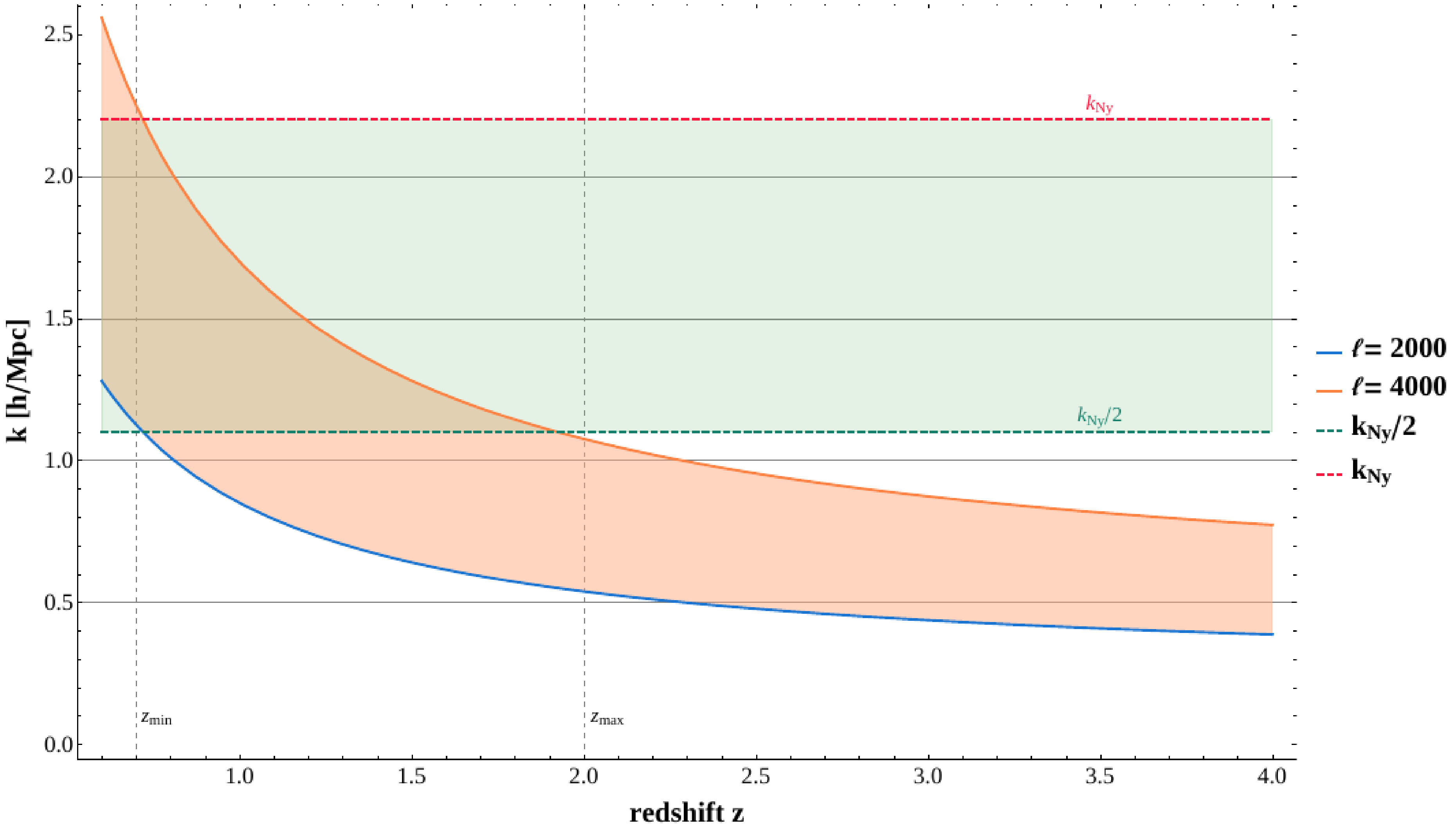
At very small scales, systematic biases dominate over statistical errors.

# Results

- The more one includes information from nonlinear scales, the better the constraints.
- At some point information gain saturates due to mode-mode coupling.

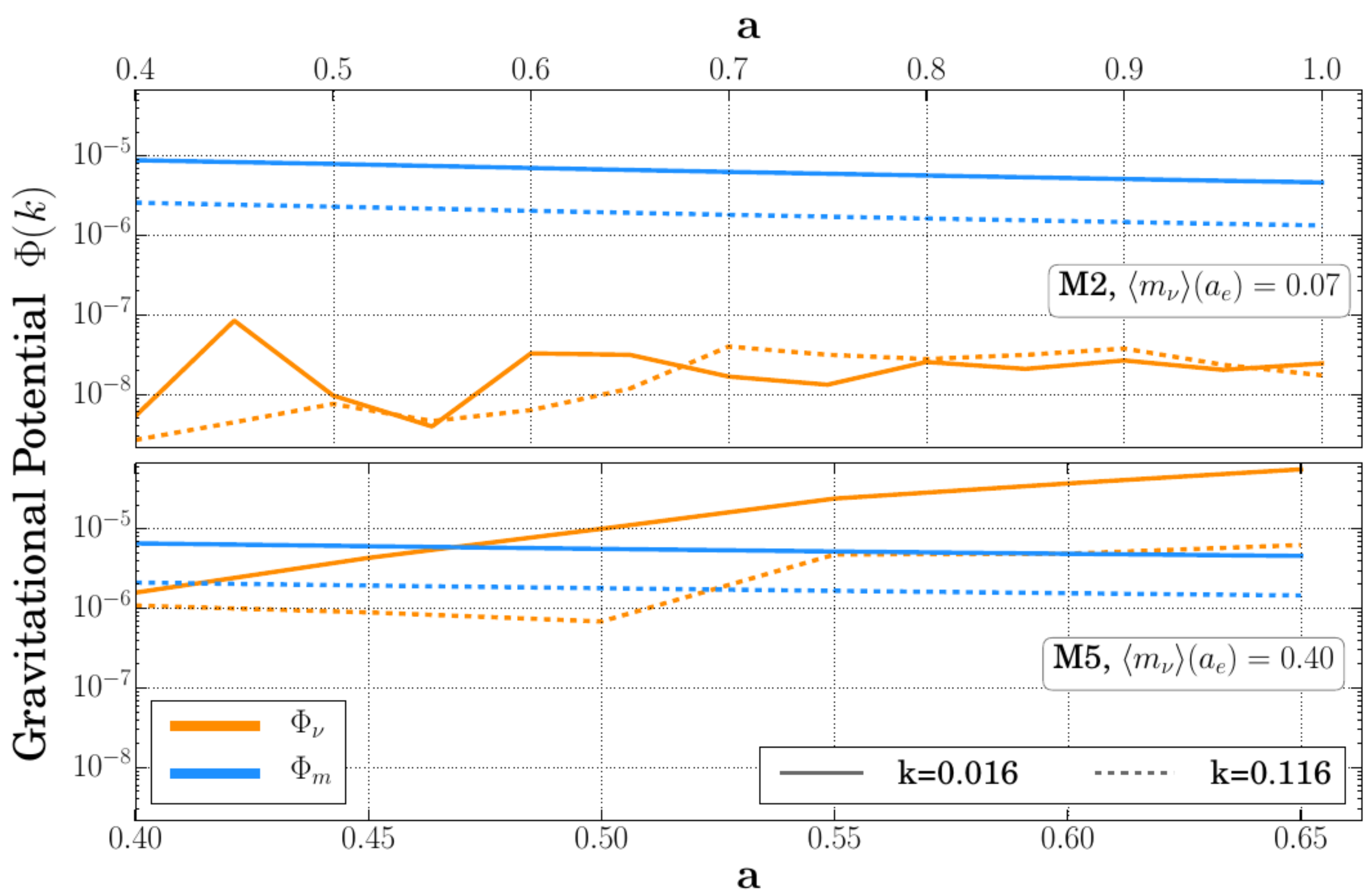


# Importance of non-linearities for WL




This is valid only under the Limber approximation

# Gravitational potentials




- For small masses, gravitational potentials from ν's, oscillate in time.
- For large masses they increase above the allowed limits by observations.

## Very difficult calculations!! Numerically and symbolically!!