# Phenomenological analysis of Charged Lepton Flavor Violation processes 

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## Outline

Introduction

- The Standard Model
- Charged Lepton Flavor Violation (CLFV) processes

Effective field theories (EFT)

- What is it?
- Why do we need EFT?
- Renormalization Group Equations
$\mu$-e conversion in nuclei
- Motivation
- BSM model
- Prospects and conclusions


## Introduction

Standard Model

- Particles of matter: quarks and leptons (3 generations)
- Force carriers : gauge bosons (and Higgs boson)

- In the Standard model, the flavor is conserved.
- In CLFV processes, the initial and final number of flavors are different.
- CFLV forbidden in SM $\rightarrow$ need BSM physics

Examples:
$\mu \rightarrow e$ conversion in nuclei, $\mu \rightarrow e \gamma, \mu \rightarrow e e \bar{e}$ and mesons decays such as $K \rightarrow \bar{\mu} e$
Flavor physics is sensitive to new physics at scales $\Lambda_{N P}$
Eexperiment

## Effective fields theories (EFT)

## What is EFT?

- Theoretical framework used to parametrize observables at a given energy scale $\boldsymbol{\Lambda}$.
- Choose the relevant degrees of freedom to describe the dynamics
- Describe indirect effects of heavy New Physics (NP) on interactions between SM particles.
- Contact interactions


Why do we need EFT?
Assumption : new particles are too heavy to be produced

- Remove particles with $m>\Lambda$ from the theory $\rightarrow$ obtain an effective Lagrangian
- Operators and coefficients $\rightarrow$ parametrize the low energy EFT
- Compute observables at a specific scale $\Lambda<\Lambda_{N P}$

$$
\mathcal{L}_{\text {eff }}=\mathcal{L}_{S M}+\mathcal{L}_{5}+\mathcal{L}_{6}+\ldots
$$

where

$$
\mathcal{L}_{d}=\sum_{i} \frac{C_{i}^{d}}{\Lambda_{N P}^{d-4}} \mathcal{O}_{i}^{d}
$$

Example : $O^{6}=\left(\bar{\mu} \gamma^{\mu}\left(1-\gamma^{5}\right) \nu\right)\left(\bar{e} \gamma_{\mu}\left(1-\gamma^{5}\right) \nu\right)$

## Renormalization Group Equations (RGEs)

In an EFT, operator coefficients run with the energy scale and can mix to other operators!!

EFT $\Rightarrow$ what is the scale of the physics we are interested in? $C_{i}^{d}\left(\Lambda_{N P}\right)$ run down to the scale of interest $\left(\boldsymbol{\Lambda}_{\text {exp }}\right)$
$\Rightarrow$ need a tool to relate the different scales

## Solutions: RGEs

Every parameters of $\mathcal{L}$ evolve with energy scale because of loops


(2)

(3)

- Counterterms $\delta$ obtained from loops
- Anomalous dimension $\gamma$ encodes the running
- $\gamma=-2 \alpha \frac{\partial \delta}{\partial \alpha}$

$$
\Lambda \frac{\partial}{\partial \Lambda}\left(C_{l}, \ldots C_{J}, \ldots\right)=\frac{\alpha_{e}}{4 \pi} \vec{C} \Gamma^{e}+\frac{\alpha_{s}}{4 \pi} \vec{C} \Gamma^{s}
$$

$\Rightarrow$ all coefficients organized in the row vector $\vec{C}, \Gamma$ is an anomalous dimension matrix

## $\mu-e$ conversion in nuclei

## Why CLFV and muons?

- No SM contribution in CLFV $\Longleftrightarrow$ Signal of BSM physics !!
- $\mu^{-}$are easy to produce : at least $\sim 10^{8} \mu /$ sec at J-PARC
- Future experiments : $\sim 10^{12} \mu / \mathrm{sec}$
- $\tau$ : $400 \tau /$ sec at super-KEKB

This is what we start with.


This is the process we are looking for.


BSM scenario: Leptoquarks

- Bosons that allow leptons and quarks to interact
- Triplet of SU(3) and carry B and L numbers
- Could explain many similarities between quarks and leptons
- $m_{L Q} \sim T e V$

Extend the SM with two Leptoquarks $S$ and $\tilde{S}$
$\mathcal{L}_{B S M}=\mathcal{L}_{S M}+D^{\mu} S^{\dagger} D_{\mu} S+m^{2} S^{\dagger} S+D^{\mu} \tilde{S}^{\dagger} D_{\mu} \tilde{S}+\tilde{m}^{2} \tilde{S} \dagger \tilde{S}+$
$\left[\lambda_{L}^{*}\right]_{\mu d} \bar{l}_{\mu} i \tau_{2} q_{L, d}^{c} S+\left[\lambda_{R}^{*}\right]_{e u} \bar{e} u^{c} S+\left[\tilde{\lambda}^{*}\right]_{e d} \bar{e} d^{c} \tilde{S}+\left[\tilde{\lambda}^{*}\right]_{\mu d} \bar{\mu} d^{c} \tilde{S}+$ h.c


Operators and coefficients at $\Lambda_{N P} \sim m_{L Q}$

$$
\begin{aligned}
& O_{T, X}^{u \mu}=\left(\bar{e} \sigma_{\alpha \beta} P_{X} \mu\right)\left(\bar{u} \sigma^{\alpha \beta} u\right), O_{S, X}^{u \mu}=\left(\bar{e} P_{X} \mu\right)(\bar{u} u) \\
& O_{V, X}^{d d}=\left(\bar{e} \gamma_{\alpha} P_{X} \mu\right)\left(\bar{d} \gamma^{\alpha} d\right), O_{A, X}^{d d}=\left(\bar{e} \gamma_{\alpha} P_{X} \mu\right)\left(\bar{d} \gamma^{\alpha} \gamma_{5} d\right) \\
& \sigma^{\mu \nu}=\frac{i}{2}\left[\gamma^{\mu}, \gamma^{\nu}\right], P_{X}=P_{R, L}=\frac{1 \pm \gamma^{5}}{2}
\end{aligned}
$$

For the Leptoquark S: $C_{S, X}^{u u^{u}}=\frac{\left[\lambda_{L}^{*}\right]_{\mu u}\left[\lambda_{R}\right]_{e u}}{4 m^{2}}, C_{T}^{u /}, X=\frac{\left[\lambda_{L}^{*}\right]_{\mu u}\left[\lambda_{R}\right]_{e u}}{8 m^{2}}$
For the Leptoquark $\tilde{S}: C_{V, X}^{d d}=C_{A, X}^{d d}=\frac{\left[\tilde{\lambda}^{*}\right]_{e d}[\tilde{\lambda}]_{\mu d}}{4 \tilde{m}^{2}}$

But we need these coefficients at $\Lambda_{\text {exp }} \sim 2 \mathrm{Gev}$ (nucleon level)

Running down to the experimental scale


$$
C_{l}\left(\Lambda_{e x p}\right)=C_{J}\left(\Lambda_{N P}\right) \lambda^{a \jmath}\left(\delta_{J I}-\frac{\alpha_{e} \widetilde{\Gamma}_{J l}^{e}}{4 \pi} \log \frac{\Lambda_{N P}}{\Lambda_{\text {exp }}}\right)
$$

- $\lambda=\frac{\alpha_{s}\left(\Lambda_{N P}\right)}{\alpha_{s}\left(\Lambda_{\text {exp }}\right)} \rightarrow$ QCD running
- $a_{J}=\frac{r^{s} \mu_{j}}{2 \beta_{0}} \rightarrow$ QCD running
- $\widetilde{\Gamma}_{J /}^{e} \rightarrow$ QED/QCD running and mixing


## Branching ratio

$$
B R=B R_{S I}+B R_{S D}(\mathrm{SI}=\text { Spin Independent, } \mathrm{SD}=\text { Spin Dependent })
$$

$$
\begin{aligned}
& B R_{S I}=2 B_{0}\left|Z\left[C_{V, R}^{p p}+C_{S, L}^{p p}+\frac{m_{\mu}}{m_{p}} C_{T, L}^{p p}\right] F_{p}\left(m_{\mu}\right)+N\left[C_{V, R}^{n n}+C_{S, L}^{n n}+\frac{m_{\mu}}{m_{n}} C_{T, L}^{n n}\right] F_{n}\left(m_{\mu}\right)\right|^{2} \\
& B R_{S D}=8 B_{0} \frac{J+1}{J}\left|\left(S_{p} C_{A, R}^{p p}+S_{n} C_{A, R}^{n n}\right)^{2}+\left(2 S_{p} C_{T, L}^{p p}++2 S_{n} C_{T, L}^{n n}\right)^{2}\right| \frac{S_{A}\left(m_{\mu}\right)}{S_{A}(0)}
\end{aligned}
$$

- $\mathrm{B}_{0}=G_{F}^{2} m_{\mu}^{5}(\alpha Z)^{3} /\left(\pi^{2} \Gamma_{c a p}\right)$
- $S_{p, n}, F_{p, n}$ and $\frac{S_{A}\left(m_{\mu}\right)}{S_{A}(0)}$ are nuclear factors
- Different coefficients ( $\tilde{S}$ and $S$ ) in $B R_{S I}$ and $B R_{S D}$
- $B R_{S D}$ highly suppressed


## Bounds on coefficients

Using experimental bounds on branching ratios :

- $B R_{S I}(\mu A I \rightarrow e A I) \leq 10^{-14}, B R_{S D}(\mu A I \rightarrow e A I) \leq 10^{-14}$
- $B R_{S I}(\mu T i \rightarrow e T i) \leq 10^{-14}, B R_{S D}\left(\mu^{47} T i \rightarrow e^{47} T i\right) \leq 10^{-14}$
- $B R_{S I}(\mu \mathrm{~Pb} \rightarrow e \mathrm{~Pb}) \leq 10^{-14}$
give the bounds on the coefficients: $C_{V, R}^{d d} \leq \sqrt{0.0103 \times B R}$, $C_{S, L}^{u \mu} \leq \sqrt{0.0011 \times B R} \ldots$

How to distinguish operators?

## $\Rightarrow$ Use different target nuclei

$$
a=\frac{\left.\frac{B R(A l \mu \rightarrow A l e)}{B R(X \mu \rightarrow X e)}\right|_{S}}{\left.\frac{B R(A l \mu \rightarrow A l e)}{B R(X \mu \rightarrow X e)}\right|_{\tilde{S}}} \quad b=\frac{\left.\frac{B R(N b \mu \rightarrow N b e)}{B R(X \mu \rightarrow X e)}\right|_{\text {scalar }}}{\left.\frac{B R(N b \mu \rightarrow N b e)}{B R(X \mu \rightarrow X e)}\right|_{\text {vector }}}
$$



## Conclusions and prospects

- Charged Lepton Flavor Violation $\Leftrightarrow$ signal of physics beyond the Standard Model
- Future experiments will improve their sensitivity
- Effective operators parametrization
- Coefficients run with the scale
- Different operators in SI and SD branching ratios
- Use various nuclei to distinguish operators and separate $S$ and $\tilde{S}$ contributions to the branching ratio

I'm a second year PhD student...


## BACKUP

Anomalous dimension and mixing
The operators coefficients below the scale $\Lambda_{N P}$ are organized in the vector $\vec{C}$ as following :

$$
\begin{aligned}
& \vec{C}=\left(\vec{C}_{V}^{u}, \vec{C}_{V}^{d}, \vec{C}_{A}^{u}, \vec{C}_{A}^{d}, \vec{C}_{S}^{u}, \vec{C}_{S}^{d}, \vec{C}_{T}^{u}, \vec{C}_{T}^{d}\right) \\
& \vec{C}_{V}^{f}=\left(C_{V L}^{f f}, C_{V R}^{f f}\right) \quad \vec{C}_{A}^{f}=\left(C_{A L}^{f f}, C_{A R}^{f f}\right) \\
& \vec{C}_{S}^{f}=\left(C_{S, L}^{f f}, C_{S, R}^{f f}\right) \quad \vec{C}_{T}^{f}=\left(C_{T, L}^{f f}, C_{T, R}^{f f}\right)
\end{aligned}
$$



In the basis of $\vec{C}$, the QED anomalous dimension matrix can be written $\Gamma^{e}=\left[\begin{array}{cc}\Gamma_{V A} & 0 \\ 0 & \Gamma_{S T}\end{array}\right]$ with

$$
\Gamma_{S T}=\left[\begin{array}{cccc}
\gamma_{S, S}^{u, u} & 0 & \gamma_{S, T}^{u, u} & 0 \\
0 & \gamma_{S, S}^{d, d} & 0 & \gamma_{S, T}^{d, d} \\
\gamma_{T, S}^{u, u} & 0 & \gamma_{T, L}^{u, u} & 0 \\
0 & \gamma_{T, S}^{d, d} & 0 & \gamma_{T, T}^{d, d}
\end{array}\right] \Gamma_{V A}=\left[\begin{array}{cccc}
0 & 0 & \gamma_{V, A}^{u, u} & 0 \\
0 & 0 & 0 & \gamma_{V, A}^{d, d} \\
\gamma_{A, V}^{u, u} & 0 & 0 & 0 \\
0 & \gamma_{A, V}^{d, d} & 0 & 0
\end{array}\right]
$$

$$
\begin{array}{c|cc} 
& C_{S, L}^{f f} & C_{S, R}^{f f} \\
\gamma_{S, S}^{f, f}= & C_{S, L}^{f f} & 6\left(1+Q_{f}^{2}\right) \\
C_{S, R}^{f f} & 0 & 6\left(1+Q_{f}^{2}\right)
\end{array} \quad \gamma_{T, S}^{f, f}=\begin{array}{cc|cc} 
& C_{S, L}^{f f} & C_{S, R}^{f f} \\
\hline & C_{T, L}^{f f} & -96 Q_{f} & 0 \\
0 & -96 Q_{f}
\end{array}
$$

$$
C_{l}\left(\Lambda_{\text {exp }}\right)=C_{J}\left(\Lambda_{N P}\right) \lambda^{a \jmath}\left(\delta_{J I}-\frac{\alpha_{e} \widetilde{\Gamma}_{J l}^{e}}{4 \pi} \log \frac{\Lambda_{N P}}{\Lambda_{\exp }}\right)
$$

$$
C_{S, L}^{q q}\left(\Lambda_{e x p}\right)=24 \lambda^{a_{T} T} f_{T S} Q_{q} \frac{\alpha_{e}}{\pi} \log \frac{\Lambda_{N P}}{\Lambda_{\text {exp }}} C_{T, L}^{q q}\left(\Lambda_{N P}\right)+\lambda^{a_{S} 5}\left[1-\frac{3}{2} \frac{\alpha_{e}}{\pi} \log \frac{\Lambda_{N P}}{\Lambda_{e x p}}\left(1+Q_{q}^{2}\right)\right] C_{S, L}^{q q}\left(\Lambda_{N P}\right)
$$

Finally, the coefficients at the experimental scale $\Lambda_{\text {exp }}$ are obtained via the matching condition :

$$
C_{O, Y}^{N N}\left(\Lambda_{\exp }\right)=\sum_{q=u, d} G_{O}^{N, q} C_{O, Y}^{q q}\left(\Lambda_{\exp }\right)
$$

## Covariance matrix

In the basis $\left(C_{V, R}^{d d}, C_{S, L}^{u u}, C_{T, L}^{u u}, C_{A, R}^{d d}\right)$ the matrix is written:

$$
\begin{aligned}
& M^{-1}=\left[\begin{array}{cccc}
V^{2} & S V & T V & 0 \\
S V & S^{2} & S T & 0 \\
T V & S T & T^{2} & A T \\
0 & 0 & A T & A^{2}
\end{array}\right] \\
& T^{2}=2 B_{0}^{A l}\left|2 Z^{A l} \frac{m_{\mu}}{m_{p}} G_{T}^{p, u}+2 N^{A l} \frac{m_{\mu}}{m_{n}} G_{T}^{n, u}\right|^{2} F_{p, A l}^{2} \\
&+2 B_{0}^{T i}\left|2 Z^{T i} \frac{m_{\mu}}{m_{p}} G_{T}^{p, u}+2 N^{T i} \frac{m_{\mu}}{m_{n}} G_{T}^{n, u}\right|^{2} F_{p, T i}^{2} \\
&+32 B_{0}^{P b}\left|2 S_{p}^{P b} \frac{m_{\mu}}{m_{p}} G_{T}^{p, u}+2 S_{n}^{P b} \frac{m_{\mu}}{m_{n}} G_{T}^{n, u}\right|^{2} \\
&+8 B_{0}^{A l} \frac{J_{A l}+1}{J_{A l}} \frac{S_{A l}\left(m_{\mu}\right)}{S_{A l}(0)}\left|2 \tilde{S}_{p}^{A l} G_{T}^{p, u}+2 \tilde{S}_{n}^{A l} G_{T}^{n, u}\right|^{2} \\
&+8 B_{0}^{T i} \frac{J_{T i}+1}{J_{T i}} \frac{S_{T i}\left(m_{\mu}\right)}{S_{A l}(0)}\left|2 \tilde{S}_{p}^{T i} G_{T}^{p, u}+2 \tilde{S}_{n}^{T i} G_{T}^{n, u}\right|^{2} \\
& A T=8 B_{0}^{A l} \frac{J_{A l}+1}{J_{A l}} \frac{S_{A l}(m \mu)}{S_{A l}(0)}\left|\tilde{S}_{p}^{A l} G_{A}^{p, d}+\tilde{S}_{n}^{A l} G_{A}^{n, d}\right|\left|2 \tilde{S}_{p}^{A l} G_{T}^{p, u}+2 \tilde{S}_{n}^{A l} G_{T}^{n, u}\right| \\
&+8 B_{0}^{T i} \frac{J_{T i}+1}{J_{T i}} \frac{S_{T i}(m \mu)}{S_{T i}(0)}\left|\tilde{S}_{p}^{T i} G_{A}^{p, d}+\tilde{S}_{n}^{T i} G_{A}^{n, d}\right|\left|2 \tilde{S}_{p}^{T i} G_{T}^{p, u}+2 \tilde{S}_{n}^{T i} G_{T}^{n, u}\right|
\end{aligned}
$$

