# Nuclear Energy Density Functionals for Astrophysical Applications

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# Prelude

Nuclear models required for astrophysical applications (e.g. nucleosynthesis, supernovae, neutron stars) should be:

- **versatile**: applicable to compute various properties (equation of state, transport properties, reaction rates, etc.) of various systems (nuclei, nuclear matter) under various conditions/phases
- **thermodynamically consistent**: avoid spurious instabilities
- **as microscopic as possible**: make reliable extrapolations
- **numerically tractable**: systematic calculations over a wide range of temperatures, pressures, compositions, magnetic fields.

The (single-reference) nuclear energy density functional (EDF) theory appears to be currently the most suitable approach for astrophysics.

*Duguet, Lect. Notes Phys. 879 (Springer-Verlag, 2014), p. 293*

*Dobaczewski & Nazarewicz, in ”50 years of Nuclear BCS” (World Scientific Publishing, 2013), pp.40-60*

# Route to phenomenological EDFs

For simplicity, we consider **semilocal EDFs** $E = \int \mathcal{E}(\boldsymbol{r})\, \mathrm{d}^3\boldsymbol{r}$ obtained from **generalized Skyrme effective interactions**

$$v_{ij} = t_0(1 + x_0 P_\sigma)\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}t_1(1 + x_1 P_\sigma)\frac{1}{\hbar^2}\left[p_{ij}^2\,\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\,p_{ij}^2\right]$$
$$+t_2(1 + x_2 P_\sigma)\frac{1}{\hbar^2}\boldsymbol{p}_{ij}.\delta(\boldsymbol{r}_{ij})\,\boldsymbol{p}_{ij} + \frac{1}{6}t_3(1 + x_3 P_\sigma)n(\boldsymbol{r})^\alpha\,\delta(\boldsymbol{r}_{ij})$$
$$+\frac{1}{2}\,t_4(1 + x_4 P_\sigma)\frac{1}{\hbar^2}\left\{p_{ij}^2\,n(\boldsymbol{r})^\beta\,\delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\,n(\boldsymbol{r})^\beta\,p_{ij}^2\right\}$$
$$+t_5(1 + x_5 P_\sigma)\frac{1}{\hbar^2}\boldsymbol{p}_{ij}\cdot n(\boldsymbol{r})^\gamma\,\delta(\boldsymbol{r}_{ij})\,\boldsymbol{p}_{ij}$$
$$+\frac{\mathrm{i}}{\hbar^2}W_0(\boldsymbol{\sigma_i} + \boldsymbol{\sigma_j})\cdot\boldsymbol{p}_{ij}\times\delta(\boldsymbol{r}_{ij})\,\boldsymbol{p}_{ij} + \frac{\mathrm{i}}{\hbar^2}W_1(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\cdot\boldsymbol{p}_{ij}\times(n_{qi} + n_{qj})^\nu\,\delta(\boldsymbol{r}_{ij})\,\boldsymbol{p}_{ij}$$

$$\textbf{pairing}\ v_{ij}^\pi = \frac{1}{2}(1 + P_\sigma)v^\pi[n_n(\boldsymbol{r}), n_p(\boldsymbol{r}), \nabla n_n(\boldsymbol{r}), \nabla n_p(\boldsymbol{r})]\delta(\boldsymbol{r}_{ij})$$

$\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$, $\boldsymbol{r} = (\boldsymbol{r}_i + \boldsymbol{r}_j)/2$, $\boldsymbol{p}_{ij} = -\mathrm{i}\hbar(\boldsymbol{\nabla}_i - \boldsymbol{\nabla}_j)/2$ is the relative momentum, and $P_\sigma$ is the two-body spin-exchange operator.

The parameters $t_i$, $x_i$, $\alpha$, $\beta$, $\gamma$, $\nu$, $W_i$ are fitted to experimental and/or microscopic nuclear data.

# Why not fitting directly the EDF?

Freely adjusted EDFs will generally contain **self-interactions errors** even in the single-reference framework.

Let us consider the simple functional $\mathcal{E} = C\,(n_n + n_p)^2$.

- A single nucleon $q$ interacts with itself:

$$\mathcal{E}(\boldsymbol{r}) = C|\varphi^{(q)}(\boldsymbol{r},\sigma)|^4 \neq 0$$

- The EDF must be supplemented with **time-odd densities**:

$$\mathcal{E} = C\,(n_n + n_p)^2 - C\,(s_n + s_p)^2 \text{ with } s_{n/p} = n^{\uparrow}_{n/p} - n^{\downarrow}_{n/p}$$

$$\Rightarrow \mathcal{E} = 0 \text{ for a single nucleon since } n_q(\boldsymbol{r})^2 = s_q(\boldsymbol{r})^2 = |\varphi^{(q)}(\boldsymbol{r},\sigma)|^4$$

The cancellation of self-interaction errors implies that the coupling coefficients in the functional cannot be completely freely adjusted.
*Chamel, Phys. Rev. C 82, 061307(R) (2010).*

# Fundamental constraints on semilocal EDFs

Most general semi-local EDF with all possible bilinear terms up to 2nd order in the derivatives:

$$\mathcal{E}_{\text{Sky}} = \sum_{t=0,1} C_t^n n_t^2 + C_t^{\Delta n} n_t \Delta n_t + C_t^\tau (n_t \tau_t - j_t^2)$$

$$+\frac{1}{2} C_t^{J^2} \left[ \left( \sum_\mu J_{t,\mu\mu} \right)^2 + \sum_{\mu,\nu} J_{t,\mu\nu} J_{t,\nu\mu} - \boldsymbol{s_t} \cdot \boldsymbol{F_t} \right] + C_t^s s_t^2 + C_t^{\Delta s} \boldsymbol{s_t} \cdot \Delta \boldsymbol{s_t}$$

$$+C_t^J \left[ \sum_{\mu,\nu} J_{t,\mu\nu} J_{t,\mu\nu} - \boldsymbol{s_t} \cdot \boldsymbol{T_t} \right] + C_t^{\nabla j} (\boldsymbol{s_t} \cdot \nabla \times \boldsymbol{j_t} + \rho_t \nabla \cdot \boldsymbol{J_t}) + C_t^{\nabla s} (\boldsymbol{\nabla} \cdot \boldsymbol{s_t})^2$$

Requiring the cancellation of ***one-particle*** **self-interaction errors**:

$$C_0^n + C_1^n + C_0^s + C_1^s = 0$$

$$C_0^\tau + C_1^\tau - C_0^J - C_1^J = 4(C_0^{\Delta n} + C_1^{\Delta n} + C_0^{\Delta s} + C_1^{\Delta s})$$

$$4(C_0^{\nabla s} + C_1^{\nabla s}) = C_0^{J^2} + C_1^{J^2}$$

$$C_0^\tau + C_1^\tau + 2(C_0^J + C_1^J) + C_0^{J^2} + C_1^{J^2} - 4(C_0^{\Delta s} + C_1^{\Delta s}) = 0$$

# Propagation of self-interaction errors

Self-interaction errors in the one-particle limit can contaminate systems consisting of many particles.

For instance, in polarized neutron matter the error in the energy density caused by self-interactions is given by

$$\delta\mathcal{E}^{\mathrm{pol}}_{\mathrm{NeuM}} = (C_0^n + C_1^n + C_0^s + C_1^s)n^2$$

If $C_0^n + C_1^n + C_0^s + C_1^s < 0$, self-interactions would thus drive a **ferromagnetic collapse of neutron stars**.

EDFs obtained from effective interactions are free from one-particle self-interaction errors but

- still contain *many-particle* self-interaction (and self-pairing) errors
  *Bender, Duguet, Lacroix, Phys. Rev. C 79, 044319 (2009).*
- induce additional relations between coupling coefficients.
  *Dobaczewski&Dudek, Phys.Rev. C52, 1827 (1995);55, 3177(E) (1997).*

# Nuclear uncertainties

Phenomenological EDFs cannot be uniquely defined due to experimental and theoretical uncertainties.

**How to quantify these uncertainties?**

The energy per nucleon of nuclear matter at $T = 0$ around saturation density $n_0$ and for asymmetry $\eta = (n_n - n_p)/n$, is usually written as

$e(n, \eta) = e_0(n) + S(n)\eta^2 + o\left(\eta^4\right)$ where

$e_0(n) = a_v + \frac{K_v}{18}\epsilon^2 - \frac{K'}{162}\epsilon^3 + o\left(\epsilon^4\right)$ with $\epsilon = (n - n_0)/n_0$

$S(n) = J + \frac{L}{3}\epsilon + \frac{K_{sym}}{18}\epsilon^2 + o\left(\epsilon^3\right)$ is the symmetry energy

The lack of knowledge is embedded in $a_v$, $K_v$, $K'$, etc.

To make meaningful comparisons, EDFs with different values of these parameters should be **fitted using the same protocole**.

# Brussels-Montreal Skyrme EDFs (BSk)

For application to extreme astrophysical environments, functionals should reproduce **global properties of both finite nuclei and infinite homogeneous nuclear matter**.

**Experimental data**:

- all nuclear masses with $Z, N \geq 8$ from Atomic Mass Evaluation
- nuclear charge radii
- symmetry energy $29 \leq J \leq 32$ MeV
- incompressibility $K_v = 240 \pm 10$ MeV (ISGMR)
  *Colò et al., Phys.Rev.C70, 024307 (2004).*

**Many-body calculations**:

- equation of state of pure neutron matter
  *"RATP" and "T6" EDFs from Rayet et al., A&A116,183 (1982)*
- $^1S_0$ pairing gaps in nuclear matter
- effective masses in nuclear matter

# Phenomenological corrections for atomic nuclei

For atomic nuclei, we add the following corrections to the HFB energy:

- Wigner energy

$$E_W = V_W \exp\left\{ -\lambda\left(\frac{N-Z}{A}\right)^2 \right\} + V'_W|N-Z| \exp\left\{ -\left(\frac{A}{A_0}\right)^2 \right\}$$

  $V_W \sim -2$ MeV, $V'_W \sim 1$ MeV, $\lambda \sim 300$ MeV, $A_0 \sim 20$

- rotational and vibrational spurious collective energy

$$E_{\rm coll} = E_{\rm rot}^{\rm crank}\Big\{ b \ \tanh(c|\beta_2|) + d|\beta_2| \ \exp\{-l(|\beta_2| - \beta_2^0)^2\} \Big\}$$

This latter correction was shown to be in good agreement with calculations using 5D collective Hamiltonian.
*Goriely, Chamel, Pearson, Phys.Rev.C82,035804(2010).*

In this way, these collective effects do not contaminate the parameters ($\leq 20$) of the functional.

# Brussels-Montreal Skyrme EDFs

- **fit to realistic $^1S_0$ pairing gaps (no self-energy)** (BSk16-17)
  *Chamel, Goriely, Pearson, Nucl.Phys.A812,72 (2008)*
  *Goriely, Chamel, Pearson, PRL102,152503 (2009).*
  *Chamel, Phys. Rev. C 82, 014313 (2010).*
- **removal of spurious spin-isospin instabilities** (BSk18)
  *Chamel, Goriely, Pearson, Phys.Rev.C80,065804(2009)*
  *Chamel & Goriely, Phys. Rev. C 82, 045804 (2010).*
- **fit to realistic neutron-matter equations of state** (BSk19-21)
  *Goriely, Chamel, Pearson, Phys.Rev.C82,035804(2010)*
- **fit to different symmetry energies** (BSk22-26)
  *Goriely, Chamel, Pearson, Phys.Rev.C88,024308(2013)*
- **optimal fit of the 2012 AME with standard Skyrme** (BSk27*)
  *Goriely, Chamel, Pearson, Phys.Rev.C88,061302(R)(2013)*
- **generalized spin-orbit coupling** (BSk28-29)
  *Goriely, Nucl.Phys.A933,68(2015).*
- **fit to realistic $^1S_0$ pairing gaps with self-energy** (BSk30-32)
  *Goriely, Chamel, Pearson, Phys.Rev. C93,034337(2016).*

# Empirical pairing EDFs

The pairing EDF is generally assumed to be **local** and very often parametrized as

$$E_{\rm pair} = \int d^3r\, \mathcal{E}_{\rm pair}(\boldsymbol{r}) \,, \qquad \mathcal{E}_{\rm pair}(\boldsymbol{r}) = \frac{1}{4} \sum_{q=n,p} v^{\pi q}[n_n(\boldsymbol{r}), n_p(\boldsymbol{r})] \widetilde{n}_q(\boldsymbol{r})^2$$

$$v^{\pi q}[n_n, n_p] = V_\Lambda^{\pi q} \left(1 - \eta_q \left(\frac{n}{n_0}\right)^{\alpha_q}\right)$$

with a suitable **cutoff** prescription (regularization).

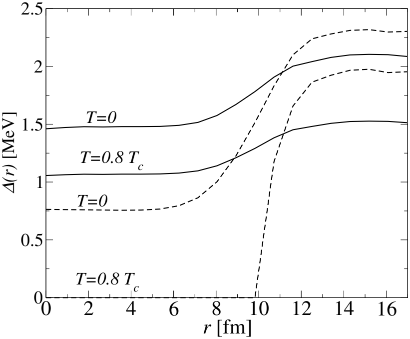
*Bertsch & Esbensen, Ann. Phys. 209, 327 (1991).*

## Drawbacks

- $V_\Lambda^{\pi q}$ must be refitted for any change in the cutoff.
- experimental nuclear data do not allow for an unambiguous determination of $\eta_q$ and $\alpha_q$.
- not enough flexibility to fit pairing gaps in nuclei and neutron matter, as required for reliable calculations of superfluidity in neutron-star crusts.

# Superfluidity in neutron star crusts

Pairing in neutron-star crusts is highly non-local: both bound and unbound neutrons contribute to superfluidity.




Pairing field in the Wigner-Seitz cell at baryon density $\bar{n} = 0.06$ fm$^{-3}$ using BSk21

—— multiband ($\sim 10^2$) BCS theory

- - - local density approximation

*Chamel et al., in "50 years of Nuclear BCS" (World Scientific Publishing, 2013), pp.284-296.*

*Chamel et al., Phys.Rev.C81,045804 (2010).*

# Pairing EDFs from nuclear-matter calculations

Instead, we **fit *exactly* realistic $^1S_0$ pairing gaps** $\Delta_q(n_n, n_p)$ in infinite homogeneous nuclear matter for each densities $n_n$ and $n_p$.
*Chamel, Phys. Rev. C 82, 014313 (2010)*

$$v_\Lambda^{\pi q} = -\frac{8\pi^2}{\sqrt{\mu_q}}\left(\frac{\hbar^2}{2M_q^*}\right)^{3/2}\left[2\log\left(\frac{2\mu_q}{\Delta_q}\right) + \Lambda\left(\frac{\varepsilon_\Lambda}{\mu_q}\right)\right]^{-1}$$

$$\Lambda(x) = \log(16x) + 2\sqrt{1+x} - 2\log\left(1+\sqrt{1+x}\right) - 4$$

$$\mu_q = \frac{\hbar^2}{2M_q^*}(3\pi^2 n_q)^{2/3}$$

Regularization: s.p. energy cutoff $\varepsilon_\Lambda$ above the Fermi level.

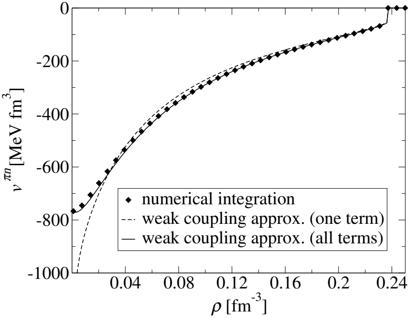
- **no free parameters** apart from the cutoff
- **automatic renormalization** of $v_\Lambda^{\pi q}$ with $\varepsilon_\Lambda$

# Accuracy of the "weak-coupling" approximation

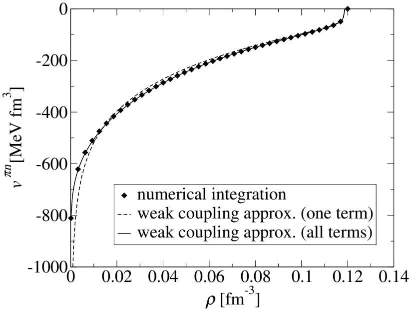
These formulas were obtained assuming $\Delta_q \ll \mu_q$ and $\Delta_q \ll \varepsilon_\Lambda$.

They are very accurate at any density because the density of states *is not* replaced by a constant as in the usual "weak coupling" approx.

symmetric nuclear matter




neutron matter




*Chamel, Phys. Rev. C 82, 014313 (2010)*

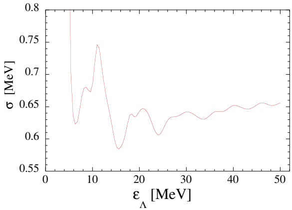
# Pairing cutoff and experimental phase shifts

In the limit of vanishing density, the pairing strength

$$v^{\pi q}[n_n, n_p \to 0] = -\frac{4\pi^2}{\sqrt{\varepsilon_\Lambda}} \left(\frac{\hbar^2}{2M_q}\right)^{3/2}$$

should coincide with the bare force in the $^1S_0$ channel.

A fit to the **experimental $^1S_0$ NN phase shifts** yields $\varepsilon_\Lambda \sim 7-8$ MeV.
*Esbensen et al., Phys. Rev. C 56, 3054 (1997).*

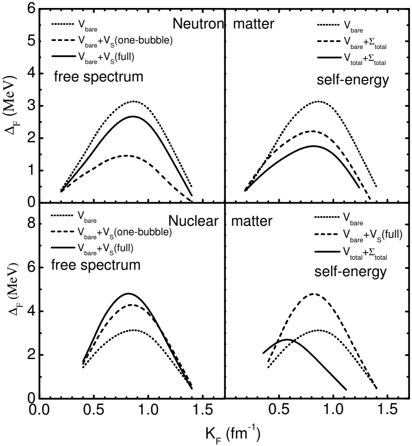



The fit to nuclear masses leads to a non monotonic dependence of the rms error on the cutoff.
*Chamel et al., in "50 Years of Nuclear BCS" (World Scientific Publishing Company, 2013), pp.284-296*

For the functionals BSk16-BS29, optimum mass fits were obtained with $\varepsilon_\Lambda \sim 16$ MeV, while we found $\varepsilon_\Lambda \sim 6.5$ MeV for BSk30-32.

# $^1S_0$ pairing gaps in neutron and symmetric matter

For consistency, we considered the gaps obtained from extended BHF calculations since effective masses as well as equations of state have been also calculated with this approach.




For comparison, we fitted functionals to different approximations for the gaps:

- **BCS**: BSk16
- **polarization+free spectrum**: BSk17-BSk29
- **polarization+self-energy**: BSk30-32.

*Cao et al., Phys.Rev.C74,064301(2006)*

# Other contributions to pairing in finite nuclei

Pairing in finite nuclei is not expected to be the same as in infinite nuclear matter because of

- **Coulomb and charge symmetry breaking effects**,
- **polarization effects in odd nuclei**,
- **coupling to surface vibrations**.

In an attempt to account for these effects, we include an additional phenomenological term in the pairing interaction (only for BSk30-32)

$$v^{\pi\, q} \to v^{\pi\, q} + \kappa_q |\nabla n|^2$$

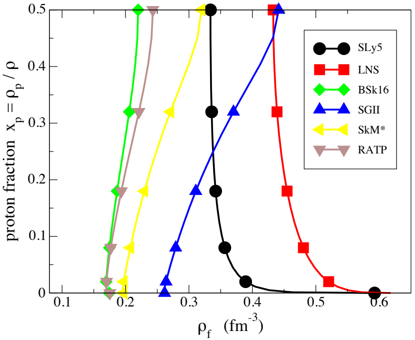
and we introduce renormalization factors $f_q^{\pm}$

$$v^{\pi\, q} \longrightarrow f_q^{\pm} v^{\pi\, q}$$

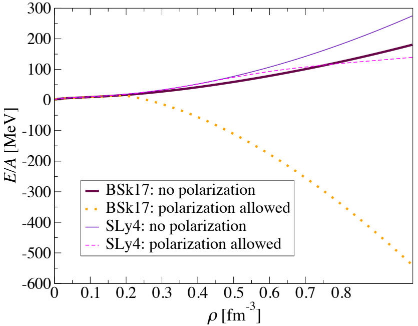
Parameters were determined by fitting nuclear masses. Typically $f_q^{\pm} \simeq 1 - 1.2$ and $f_q^{-} > f_q^{+}$, and $\kappa_q < 0$.

# Ferromagnetic instability

Unlike microscopic calculations, conventional Skyrme functionals predict a spurious ferromagnetic transition in nuclear matter




*Margueron et al., J.Phys.G36(2009),125102.*




*Chamel et al., Phys.Rev.C80(2009),065804.*

This instability can strongly affect the neutrino propagation in hot dense nuclear matter and leads to a collapse of neutron stars

# Stability of unpolarized matter restored

The ferromagnetic instability at $T = 0$ can be completely removed by **adding new terms** in the standard Skyrme interaction (BSk18)

$$\frac{1}{2}\,t_4(1+x_4P_\sigma)\frac{1}{\hbar^2}\left\{p_{ij}^2\,n(\boldsymbol{r})^\beta\,\delta(\boldsymbol{r}_{ij})+\delta(\boldsymbol{r}_{ij})\,n(\boldsymbol{r})^\beta\,p_{ij}^2\right\}$$

$$+t_5(1+x_5P_\sigma)\frac{1}{\hbar^2}\boldsymbol{p}_{ij}.n(\boldsymbol{r})^\gamma\,\delta(\boldsymbol{r}_{ij})\,\boldsymbol{p}_{ij}$$

*Chamel, Goriely, Pearson, Phys.Rev.C80,065804 (2009)*

**Dropping the $J^2$ terms** and their associated time-odd parts (>BSk19)

- removes spin and spin-isospin instabilities at *any* $T \geq 0$
- prevents an anomalous behavior of the entropy
  *Rios,Polls, Vidana, Phys. Rev. C 71, 055802 (2005)*
- considerably improves the values of Landau parameters (especially $G_0'$) and the sum rules

*Chamel & Goriely, Phys.Rev.C82, 045804 (2010)*

# Landau parameters and the $J^2$ terms

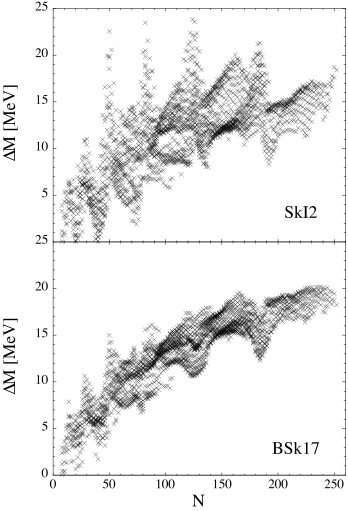
Landau parameters for **Skyrme forces fitted without the $J^2$ terms**. Values in parenthesis were obtained by dropping the time-odd counterparts of the form $\boldsymbol{s} \cdot \boldsymbol{T}$.

| | $G_0$ | $G_0'$ | $G_0^{\mathrm{NeuM}}$ |
|---|---|---|---|
| SGII | 0.01 (0.62) | 0.51 (0.93) | -0.07 (1.19) |
| SLy4 | 1.11 (1.39) | -0.13 (0.90) | 0.11 (1.27) |
| SkI1 | -8.74 (1.09) | 3.17 (0.90) | -5.57 (1.10) |
| SkI2 | -1.18 (1.35) | 0.77 (0.90) | -1.08 (1.24) |
| SkI3 | 0.57 (1.90) | 0.20 (0.85) | -0.19 (1.35) |
| SkI4 | -2.81 (1.77) | 1.38 (0.88) | -2.03 (1.40) |
| SkI5 | 0.28 (1.79) | 0.30 (0.85) | -0.31 (1.30) |
| SkO | -4.08 (0.48) | 1.61 (0.98) | -3.17 (0.97) |
| LNS | 0.83 (0.32) | 0.14 (0.92) | 0.59 (0.91) |

$\boldsymbol{s} \cdot \boldsymbol{T}$ terms can be cancelled by fine-tuning a tensor interaction, but this leads to other instabilities.

*Li-Gang Cao, Col‘o, Sagawa, Phys. Rev. C 81, 044302 (2010)*

# Impact of the $J^2$ terms




**Warning:**
Adding (SkI2) or removing (BSk17) a posteriori the $J^2$ terms without refitting the EDF can induce large errors on nuclear properties (e.g. masses)!

*Chamel & Goriely, Phys.Rev.C82, 045804 (2010)*

# Stability of unpolarized matter restored

The ferromagnetic instability at $T = 0$ can be completely removed by **adding new terms** in the standard Skyrme interaction (BSk18)

$$\frac{1}{2} t_4(1 + x_4 P_\sigma)\frac{1}{\hbar^2}\left\{p_{ij}^2\, n(\boldsymbol{r})^\beta\, \delta(\boldsymbol{r}_{ij}) + \delta(\boldsymbol{r}_{ij})\, n(\boldsymbol{r})^\beta\, p_{ij}^2\right\}$$

$$+t_5(1 + x_5 P_\sigma)\frac{1}{\hbar^2}\boldsymbol{p}_{ij}.n(\boldsymbol{r})^\gamma\, \delta(\boldsymbol{r}_{ij})\, \boldsymbol{p}_{ij}$$

*Chamel, Goriely, Pearson, Phys.Rev.C80,065804 (2009)*

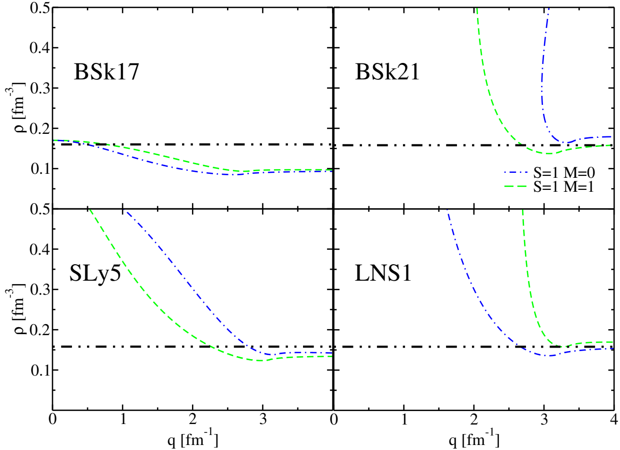
**Dropping the $J^2$ terms** and their associated time-odd parts (>BSk19)

- removes spin and spin-isospin instabilities at *any* $T \geq 0$
- prevents an anomalous behavior of the entropy
  *Rios,Polls, Vidana, Phys. Rev. C 71, 055802 (2005)*
- considerably improves the values of Landau parameters (especially $G_0'$) and the sum rules
- but also leads to unrealistic effective masses in polarized matter
- introduces self-interaction errors

*Chamel & Goriely, Phys.Rev.C82, 045804 (2010)*

# Spin and spin-isospin instabilities

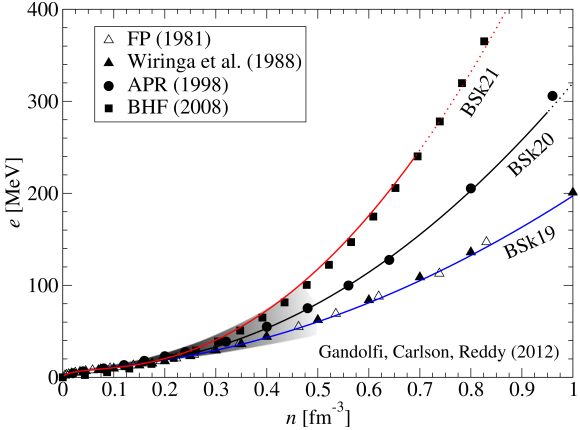
Although functionals >BSk18 are devoid of spurious long-wavelength instabilities, finite-size instabilities can still arise: e.g. neutron matter




*Pastore et al., Phys.Rev.C90, 025804 (2014)*

# Neutron-matter equation of state

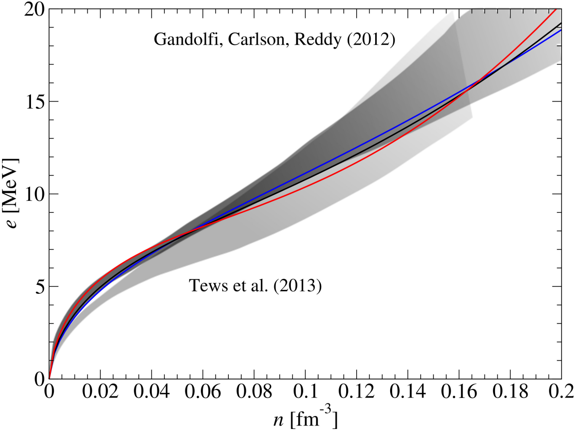
BSk19, BSk20 and BSk21 were fitted to realistic neutron-matter equations of state with different of degrees of stiffness:




*Goriely, Chamel, Pearson, Phys. Rev. C 82, 035804 (2010).*

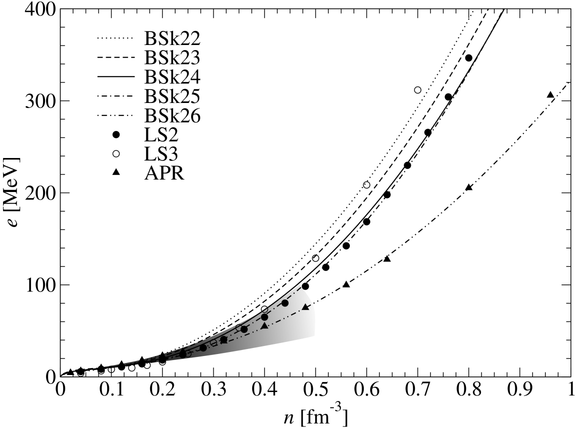
# Neutron-matter equation of state at low densities

All three EDFs yield similar equations of state at subsaturation densities consistent with ab initio calculations:



# Symmetry energy

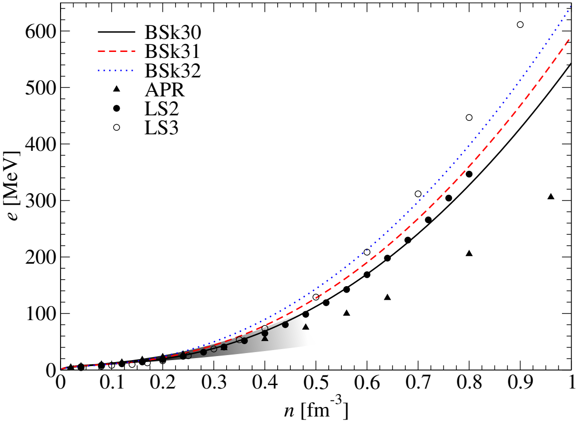
The EDFs BSk22-26 were fitted to realistic neutron-matter equations of state but with different values for $J = 29 - 32$ MeV:




*Goriely, Chamel, Pearson, Phys.Rev.C 88, 024308 (2013).*

# Symmetry energy, spin-orbit, pairing

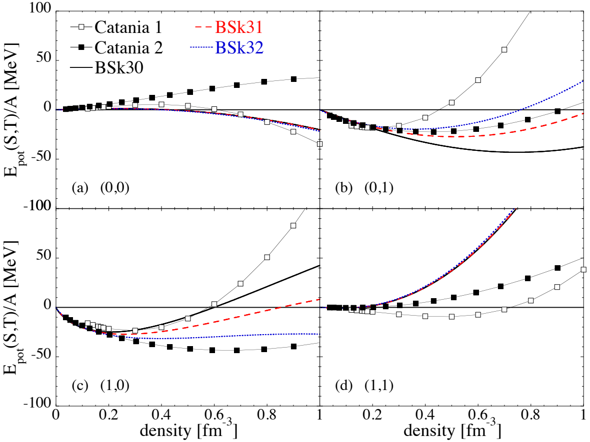
The EDFs BSk30-32 were fitted to realistic pairing gaps and include improved spin-orbit coupling but with different values for $J = 30 - 32$ MeV:




*Goriely, Chamel, Pearson, Phys.Rev.C 93,034337 (2016).*

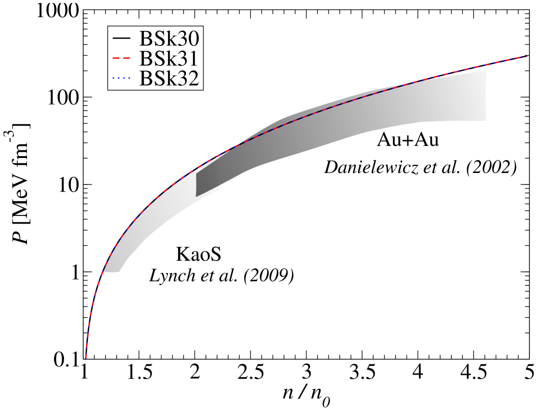
# Potential energy in spin-isospin channels

These EDFs are compatible with the potential energy in the different (S,T) channels, as predicted by BHF calculations:



# Empirical constraints from heavy-ion collisions

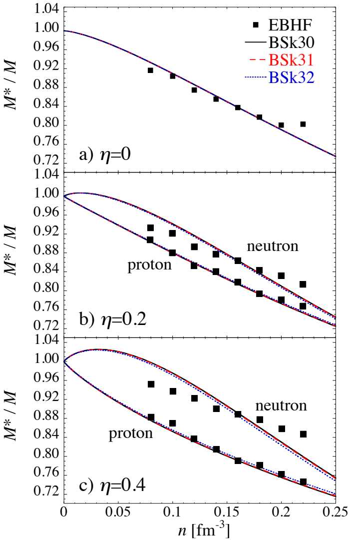
Our EDFs are also consistent with empirical constraints inferred from heavy-ion collisions:




*Danielewicz et al., Science 298, 1592 (2002)*

*Lynch et al., Prog. Part. Nuc. Phys.62, 427 (2009)*

# Nucleon effective masses in nuclear matter




Skyrme EDFs generally predict a wrong effective-mass splitting. This can be cured by introducing $t_4$ and $t_5$ terms.

Effective masses from BSk30-32 are consistent with

- isovector giant dipole resonances in finite nuclei,
- many-body calculations in asymmetric nuclear matter.

EBHF calculations from *Cao et al.,Phys.Rev.C73,014313(2006)*.

# Properties of finite nuclei

Fits to the 2353 measured masses with $Z$, $N > 8$ from the 2012 AME

| | HFB-30 | HFB-31 | HFB-32 |
|---|---|---|---|
| $\sigma(M)$ [MeV] | 0.573 | 0.571 | 0.586 |
| $\bar{\epsilon}(M)$ [MeV] | 0.003 | -0.004 | -0.007 |
| $\sigma(S_n)$ [MeV] | 0.474 | 0.464 | 0.489 |
| $\bar{\epsilon}(S_n)$ [MeV] | -0.008 | 0.000 | -0.007 |
| $\sigma(Q_\beta)$ [MeV] | 0.589 | 0.578 | 0.601 |
| $\bar{\epsilon}(Q_\beta)$ [MeV] | 0.009 | 0.006 | -0.004 |
| $\sigma(R_c)$ [fm] | 0.026 | 0.027 | 0.027 |
| $\bar{\epsilon}(R_c)$ [fm] | 0.001 | 0.002 | 0.000 |
| $\sigma_{\rm mod}(26\ \theta)$[fm] | 0.009 | 0.005 | 0.012 |
| $\theta(^{208}\mathrm{Pb})$[fm] | 0.133 | 0.151 | 0.170 |

*Goriely, Chamel, Pearson, Phys.Rev. C93,034337(2016).*

# Summary for SR-EDF afficionados

**Removal of one-particle self-interaction errors** can be a useful guide to guess the time-odd part of the EDF.

**Local pairing EDFs fitting $^1S_0$ pairing gaps in nuclear matter** can be constructed analytically.

**Introducing density-dependent $t_4$ and $t_5$ terms in the interaction**

- allows to fit stiff neutron-matter equations of state,
- leads to realistic effective masses and potential energy contributions in the $(S, T)$ channels,
- removes spurious spin-isopin instabilities (at least $T = 0$, $q = 0$).

**Removing $J^2$ and $\mathbf{s} \cdot \mathbf{T}$ terms in the EDF**

- considerably improves Landau parameters,
- removes spin-isopin instabilities at *any* $T \geq 0$ (but not any $q > 0$),
- prevents an anomalous behavior of the entropy,
- avoid instabilities in single-particle spectra of finite nuclei.
  *Lesinski et al., Phys. Rev. C 76, 014312 (2007)*

# General conclusions

- The EDF theory seems to offer the **best compromise** between versatility and computational cost for astrophysical applications.
- But **guidance from ab initio approaches** is crucial to make reliable extrapolations (especially for neutron stars!)
- The **Brussels-Montreal EDFs** were fitted using the **same protocole** to a wealth of data trying to span the current lack of knowledge of nuclear physics.
- In this way, the **impact of nuclear uncertainties on astrophysical observables** can be consistently assessed.

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### Assignements to ab initio gurus

We need more inputs to construct *consistent* EDFs: equation of state of (un)polarized asymmetric nuclear matter, effective masses, Landau parameters, pairing gaps, etc.

# Discussions

**Time-odd part of the EDF**

- How to get realistic effective masses in polarized matter?
- Finite-size spin-isospin instabilities at $n > 0.2$ fm$^{-3}$ would lead to inhomogeneous ferromagnetic phases in neutron star cores. But are they physical or spurious?
- $j^2$ terms in the EDF play a crucial role for the dynamics of neutron stars (entrainment between superfluid neutron and protons). Ab initio calculations of effective masses in asymmetric nuclear matter?

**Pairing part of the EDF**

- Challenge: neutron superfluidity in neutron-star crusts
- Isospin dependence of $^1S_0$ neutron and proton pairing gaps in asymmetric nuclear matter?
- Finite-size contributions to pairing?