Toward an EFT approach to nuclear system

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With M. Grasso, D. Lacroix, U. van Kolck, A. Boulet

Workshop: Bridging nuclear ab-initio and energy density functional theoris

05/10/2017

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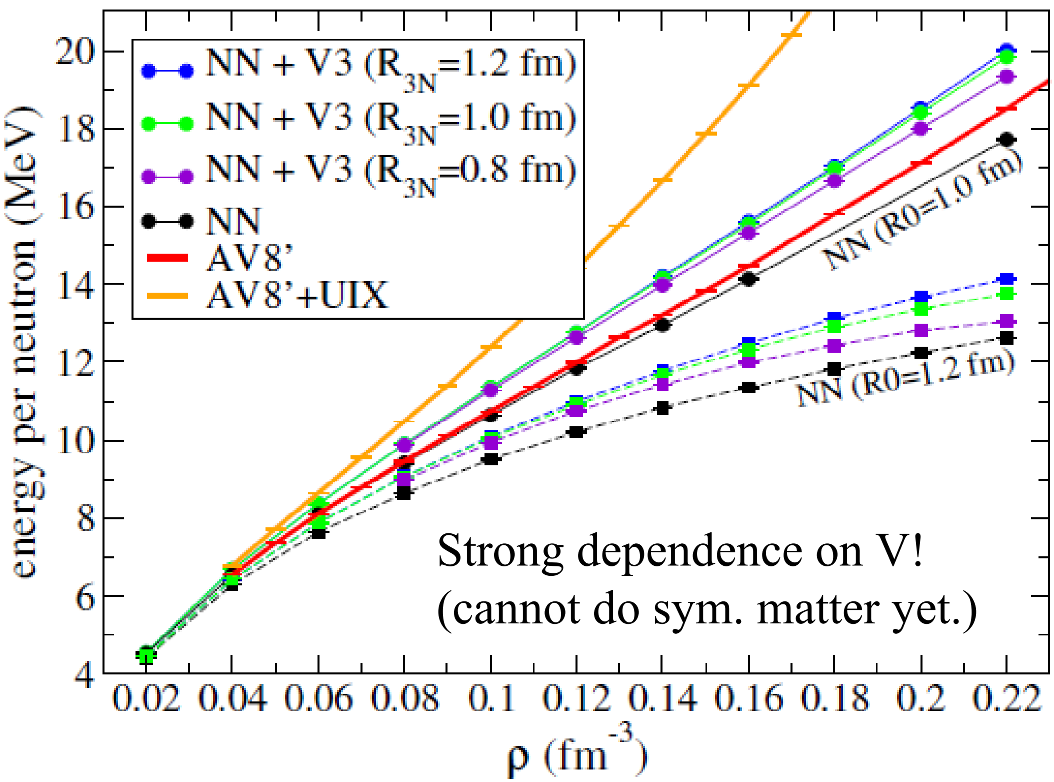
Motivation (to do EDF)

Nuclear matter: ab-inito

Equation of state of neutron matter at N^2LO.



S. Gandolfi, talk in ESNT workshop, 2017

- Assuming no problem in the ab-initio method, the same interaction (e.g., N^2LO under WPC[1]) with different fits/cutoffs give quite different EoS.
- A small uncertainty at 2-,3-body level seems to propagate to larger value in many-body system.

[1] Weinberg power counting (WPC) is not RG-invariant!

For details see: Nogga, Timmerman, van Kolck (2005), Yang, Elster, Phillips (2009)
, Ch. Zeoli R. Machleidt D. R. Entem (2012)

- Even with the correct power counting, it could be that one needs to go to very high order for the N^iLO interaction to have small enough theoretical error for many-body system.

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[1] Weinberg power counting (WPC) is *wrong !* **Or, to be confirmed**

For details see: Nogga, Timmerman, van Kolck (2005), Yang, Elster, Phillips (2009)
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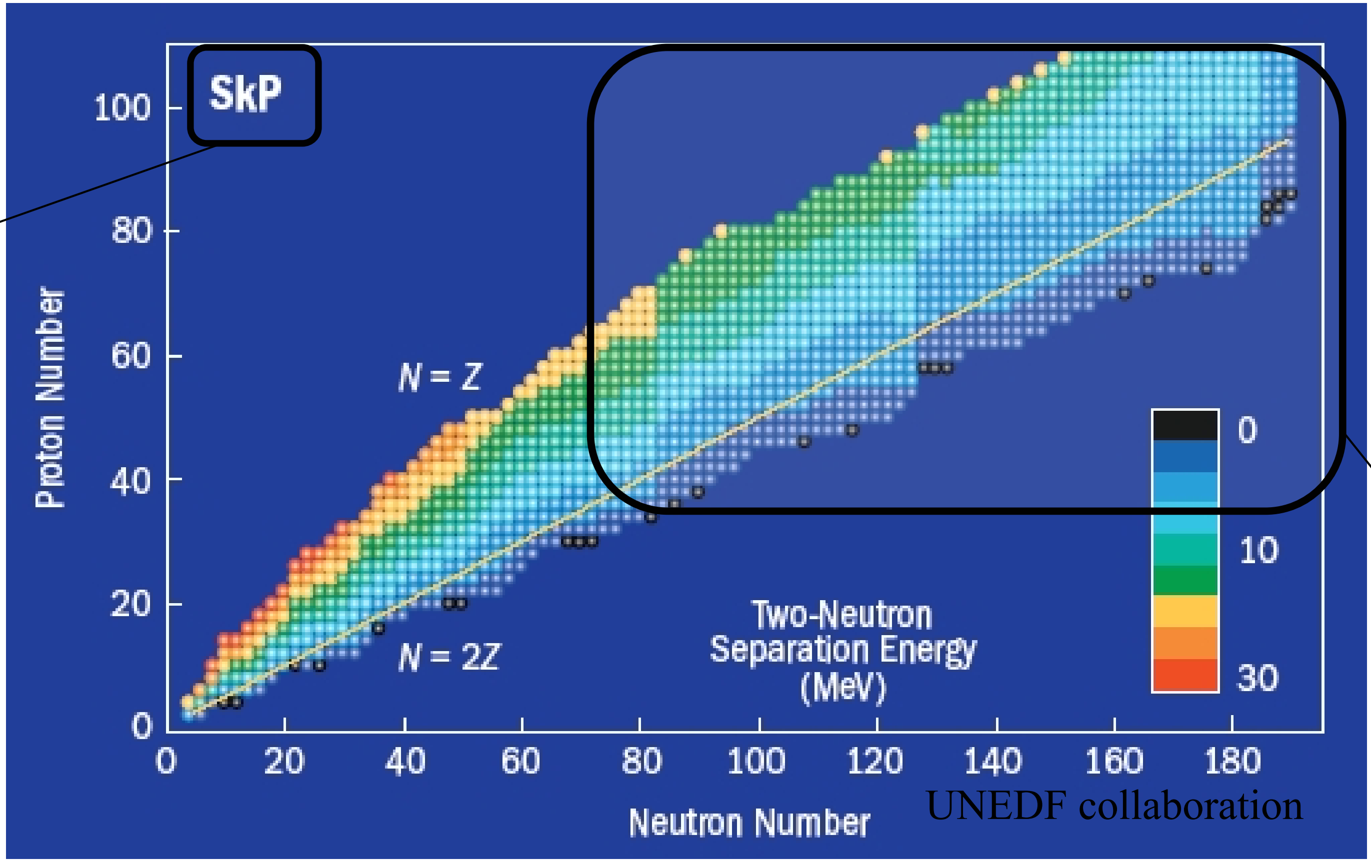
- Even with the correct power counting, it could be that one needs to go to very high order for the N^iLO interaction to have small enough theoretical error for many-body system.

On the other hand…

Mean field with Skyrme-type

Skyrme-type interaction works o.k. (able to do the fitting in EDF framework)



No way to get with ab-initio!

Need to think about other expansion (than on NN d.o.f.).

Disadvantages of current EDF approach

- The effective interaction is model-dep. (versions of Skyrme >20) =>lack of predictive power.
- Divergence occurs when goes beyond MF.

It would be good if one can find an EFT for it

DFT see J. Dobaczewski's talk Tuesday

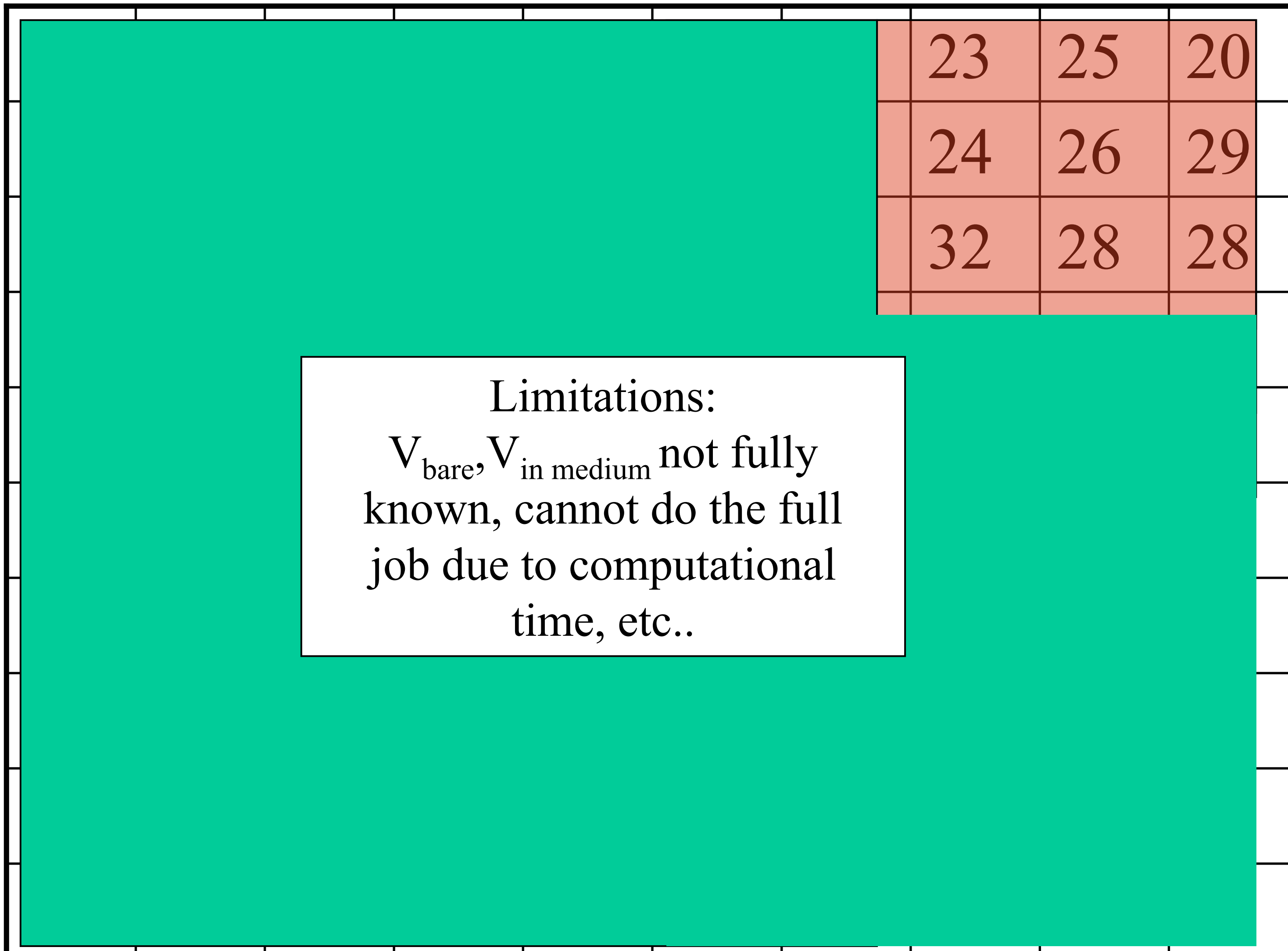
20	21	23	25	21	29	22	23	25	20
31	22	24	21	34	27	23	24	26	29
29	23	42	19	23	26	25	32	28	28
22	26	33	21	45	23	28	30	21	27
28	27	21	25	23	35	21	29	22	23
26	23	34	29	23	23	20	28	34	21
25	34	41	28	21	19	30	23	23	29
19	45	36	26	24	23	31	24	21	27
28	23	32	24	20	24	35	26	20	25
18	22	31	23	28	25	32	24	25	21

DFT see J. Dobaczewski's talk Tuesday

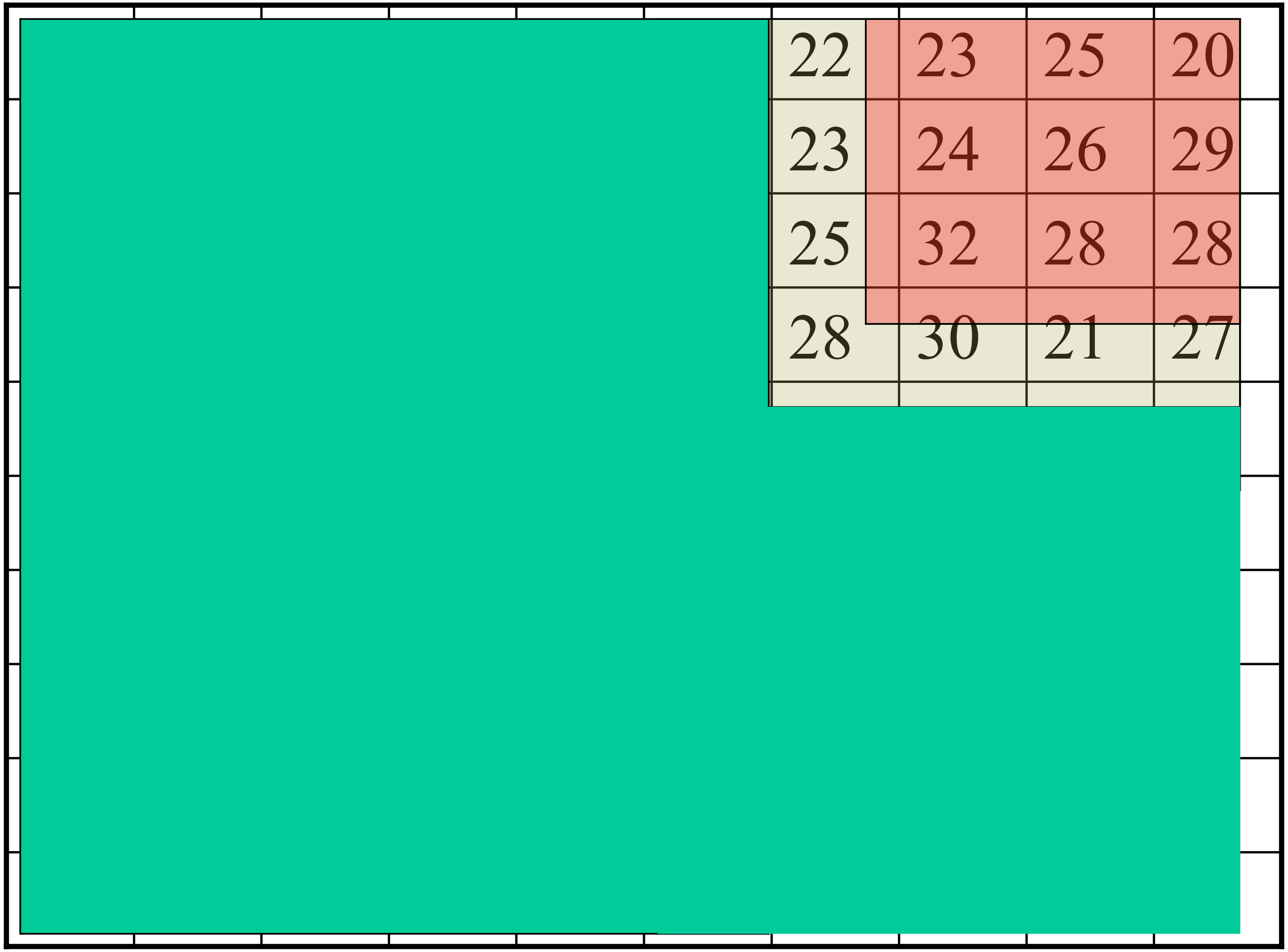
20	21	23	25	21	29	22	23	25	20	20
31	22	24	21	34	27	23	24	26	29	21
29	23	42	19	23	26	25	32	28	28	19
22	26	33	21	45	23	28	30	21	27	21
28	27	21	25	23	35	21	29	22	19	21
26	23	34	29	23	23	20	28	34	21	20
25	34	41	28	21	19	30	23	23	29	19
19	45	36	26	24	23	31	24	21	27	19
28	23	32	24	20	24	35	26	20	25	20
18	22	31	23	28	25	32	24	25	21	18

=>18

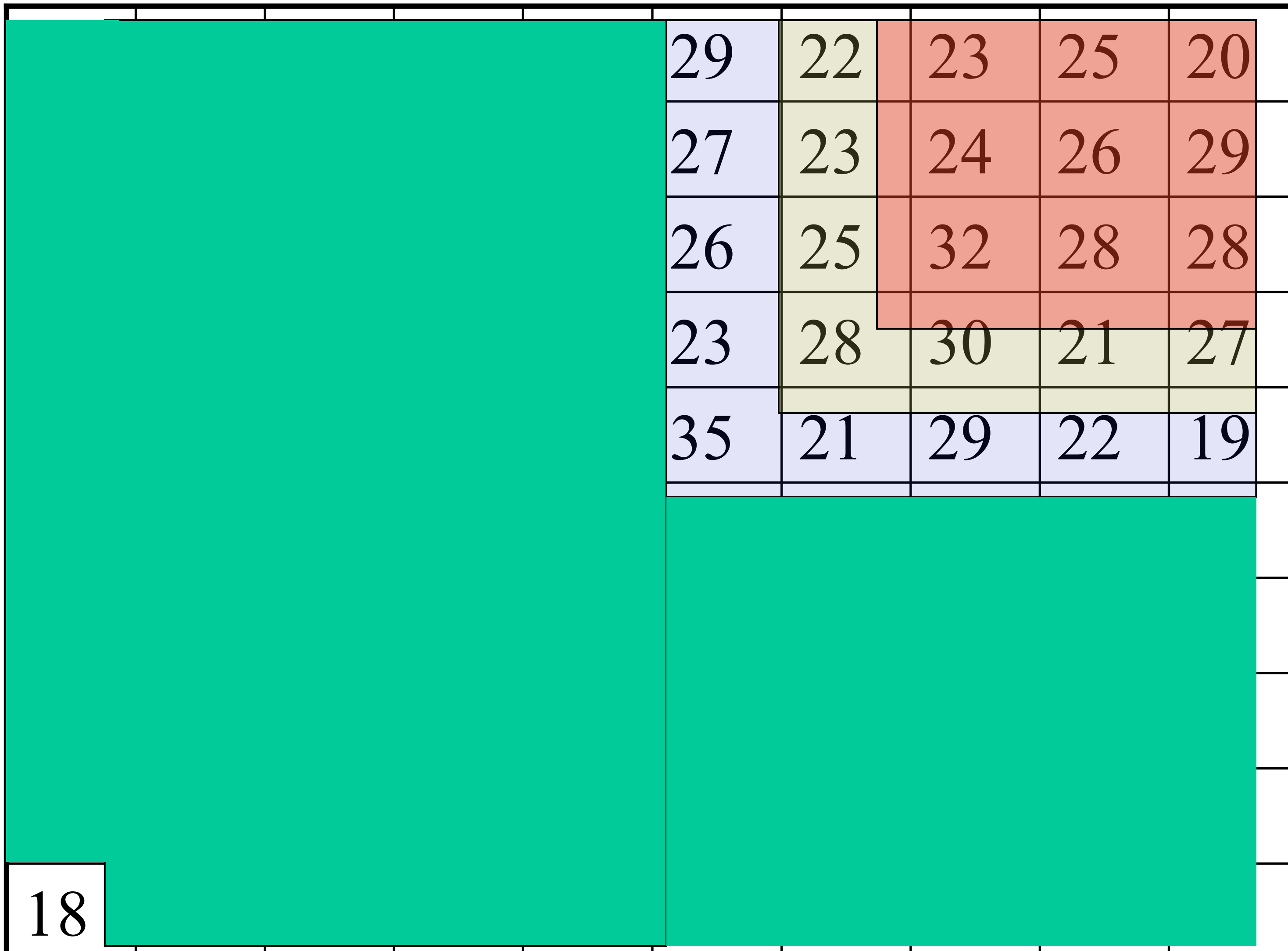
But life is difficult…



Need Strategy

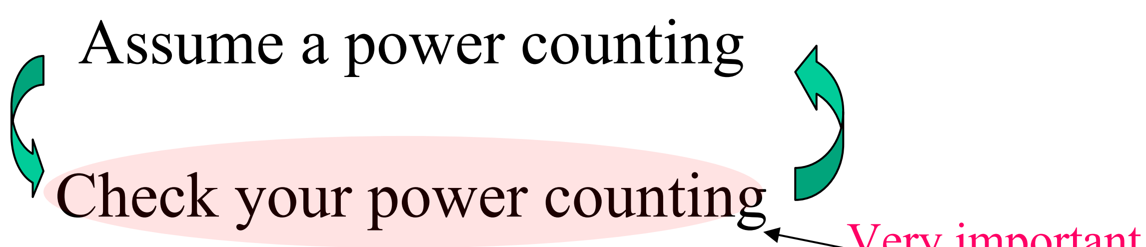


Need Strategy



Effective Field Theory

- Guidance: Underlying symmetries (if any)
- You always live with uncertainty/errors=>to control/reduce it, establish power counting.



- EFT breaks down at some point (because we ignored something, d.o.f., etc...).

What we already knew (expansion on $k_N a$)

Could do 'strict' EFT:

Pure neutron matter at very low density ($k_N a<1$, $\rho<10^{-6}$ fm^{-3}).

Lee & Yang formula (1957) describes the dilute system.

=> Can be re-derived by EFT with matching to ERE

E.g., L. Platter, H. Hammer, Ulf. Meissner, Nucl.Phys. A714 (2003), 250-264,
H. Hammer and R.J. Furnstahl, Nucl.Phys. A678 (2000) 277-294.

Skyrme completely wrong here!

Only 'EFT-inspired'

Tricks to extend to higher ρ **(up to 0.3 fm^{-3})**

Steele (2000), Schafer (2005), Kaiser (2011) => resum LO

To include symmetric matter too:

YGLO (PRC 94 , 031301(R) (2016)), M. Grasso et al (PRC 95, 054327 (2017))

See Marcella's talk Tuesday

See also: P.Papakonstantinou et al, arXiv:1606.04219.

What we already knew (expansion on 1/(**k_Na**))

Unitarity limit

- For $a \rightarrow \infty$, scale invariance gives $\frac{E}{E_{FG}} = \xi(a_s k_F, r_e k_F)$
- Nuclear system not far from unitarity.
  $|a_s=-18.9\ fm| >> $ range of interaction

'EFT-inspired' treatment

Neutron matter only

Expansion in $(a_s k_F)^{-1}$ + resum+input from ab-initio (QMC) calculations.

D Lacroix, Phys. Rev. A 94, 043614 (2016).
D Lacroix, A. Boulet, M. Grasso, C. J. Yang, PRC 95, 054306 (2017) .
A. Boulet and D Lacroix, arXiv:1709.05160
See Denis Lacroix's talk Monday.

Strict EFT maybe possible (within certain range of ρ)

C.J. Yang and U. van Kolck, in preparation.

Unitarity limit: Formula

D Lacroix, A. Boulet, M. Grasso, C. J. Yang, PRC 95, 054306 (2017) .

The proposed functional for Neutron matter

$$\frac{E}{E_{\rm FG}} = 1 - \frac{U_0}{1-(a_s k_F)^{-1}U_1} + \frac{R_0(r_e k_F)}{[1-R_1(ak_F)^{-1}]\,[1-R_1(a_s k_F)^{-1}+R_2(r_e k_F)]}$$

No free parameters: U_i, R_i from QMC data (with $V_{unitarity}$)

Validity: $\frac{1}{|a_s|} < k_F < \frac{1}{R} \Rightarrow 4*10^{-6} < \rho < 0.002[\mathrm{fm}^{-3}]$, or higher if there's an extra suppression in the coefficient in front of the range.

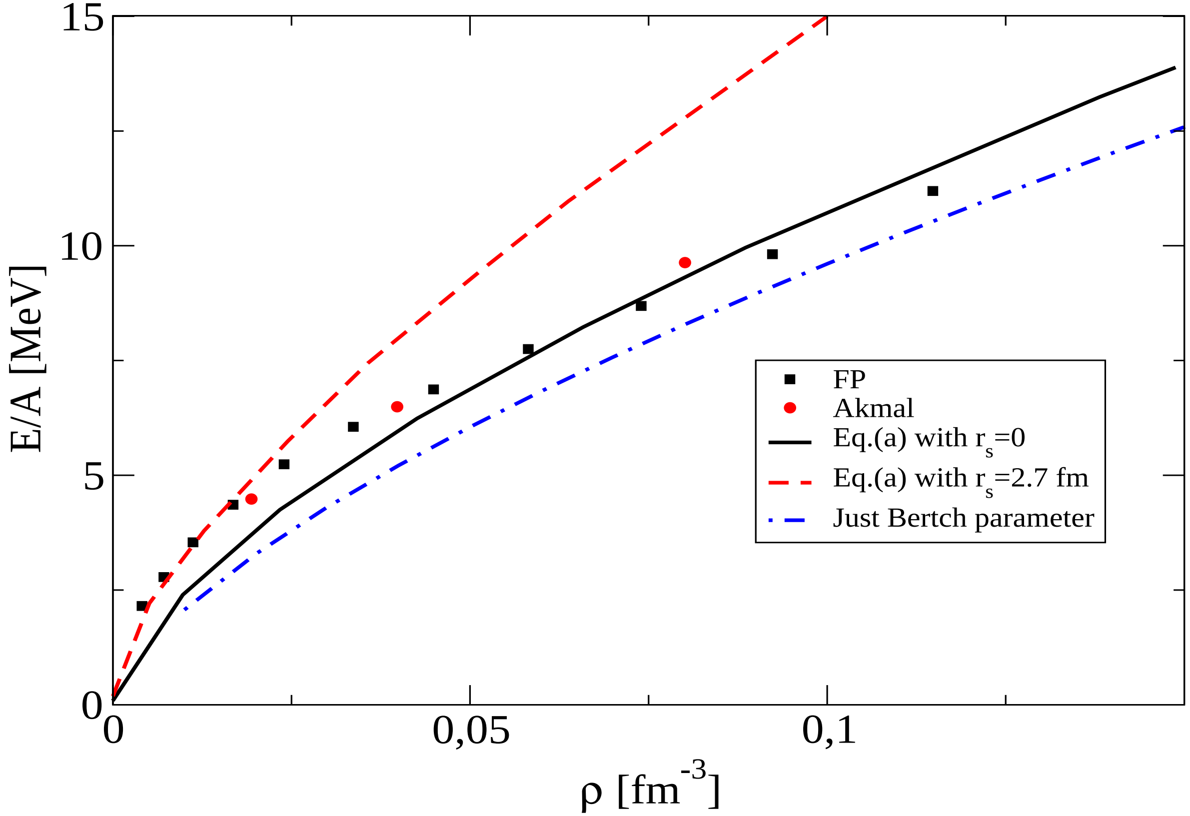
↓

The lower limit ($4*10^{-6}$) is exactly where Skyrme breakdown.

Hint: Skyrme is an UT-like expansion.

Unitarity limit: Results

D Lacroix, A. Boulet, M. Grasso, C. J. Yang, PRC 95, 054306 (2017) .



- Nuclear systems are not too far from the unitarity limit.
- Just a few more parameters might be sufficient to describe data up to ρ =0.3 fm^{-3}, this explains why Skyrme works!

Further link between Skyrme and unitarity limit

Compare unitarity expansion:

$$\frac{E}{E_{\rm FG}} = 1 - \frac{U_0}{1-(a_s k_F)^{-1}U_1} + \frac{R_0(r_e k_F)}{[1-R_1(ak_F)^{-1}]\,[1-R_1(a_s k_F)^{-1}+R_2(r_e k_F)]}$$

to low ρ expansion:

$$\frac{E^{(1)}}{E_{\rm FG}} = \frac{10}{9\pi}(\nu-1)(k_F a_s) + (\nu-1)\frac{1}{6\pi}(k_F r_e)(k_F a_s)^2 \quad \text{(here } \nu=2\text{)}$$

can be rewritten in terms of t_i and x_i in Skyrme

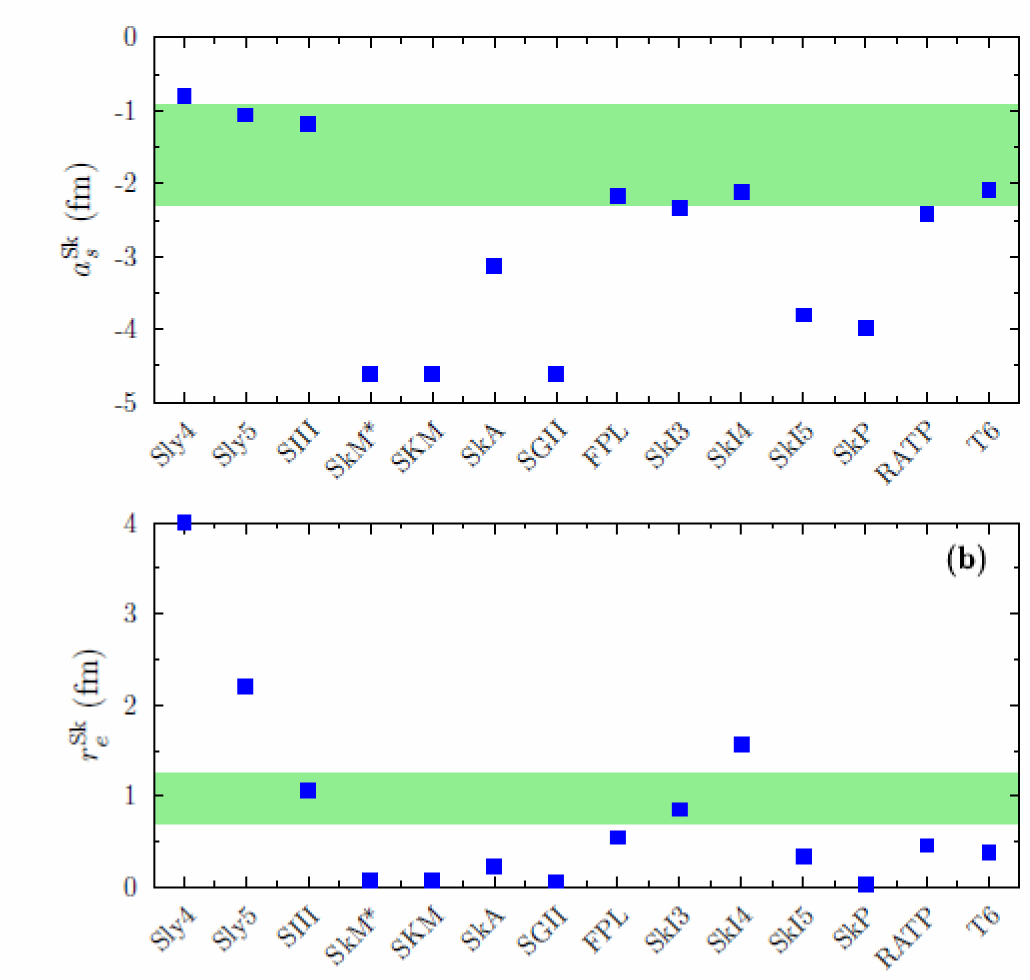
For the first few terms to match each other b/w the above Eqs., then the bare a_s, r_e in the positive power k_F-expansion become ρ-dep.:

$$\tilde{a}_s(k_F) = -\frac{1}{k_F}\frac{U_1}{[1-(a_s k_F)^{-1}U_1]}, \qquad \tilde{r}_e(k_F) = \frac{1}{k_F^3\tilde{a}_s^2(k_F)}\frac{R_1^2(r_e k_F)}{1-R_1(a_s k_F)^{-1}][1-R_1(a_s k_F)^{-1}+R_2(r_e k_F)]}.$$

Insert values of U_i, R_i from QMC, and vary k_F within typical density relevant to nuclear system ρ =0.01~0.2 [fm^{-3}], one finds:

$$\begin{cases} -2.3\ \text{fm} \leqslant \tilde{a}_s(\rho) \leqslant -0.92\ \text{fm}, \\ +0.69\ \text{fm} \leqslant \tilde{r}_e(\rho) \leqslant +1.26\ \text{fm}. \end{cases}$$

Compare $\widetilde{a}_s(k_F)$, $\widetilde{r}_e(k_F)$ generated by QMC and by Skyrme t_i, x_i:



Skyrme-like approaches are not far from the unitarity expansion!

Choose Skyrme-like interaction as the starting point for EFT approach

- Include more parameters won't necessarily help.

→ Limited predictive power.

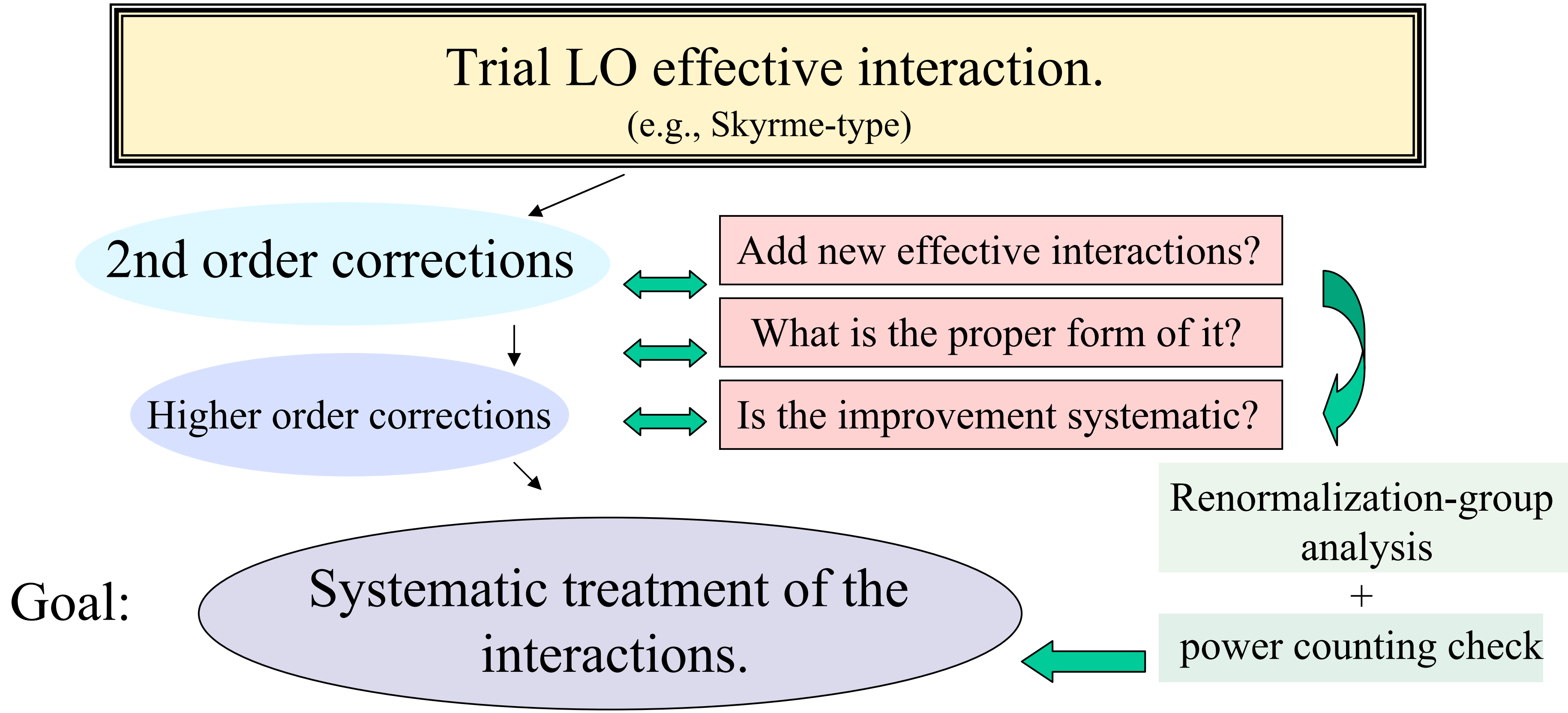
- Maybe the correct theory has a structure where different terms appears at different order.

→ Need to go beyond mean field to perform the test.

Scheme for EFT in EDF

or whatever the name it is

Try to bridge EFT ideas/techniques to mean field (and beyond) within EDF framework.



I know ***NOTHING*** about the exact form of LO, NLO, etc.

But, for any EFT the following must be true:

observables

order

Residual, ~O(1) if: 1. EFT works

2. $\Lambda \geq \overline{\Lambda}_{EFT}$

$$\mathcal{O}(k, p_{typ}; \Lambda; \overline{\Lambda}_{EFT}) = \sum_{i}^{n} \left(\frac{k, p_{typ}}{\overline{\Lambda}_{EFT}}\right)^{i} \mathcal{O}_i(k, p_{typ}; \overline{\Lambda}_{EFT}) + \mathscr{C}_n(\Lambda; k, p_{typ}, \overline{\Lambda}_{EFT}) \left(\frac{k, p_{typ}}{\overline{\Lambda}_{EFT}}\right)^{n+1}$$

cutoff

Breakdown scale

(given by 1st meson not included)

residual cutoff-dep.

No cutoff here! => ***physics cannot dep. on cutoff !***

H. W. Griesshammer, arXiv:1511.00490v3 [nucl-th].

Lepage plot: subtract at two Λ's to extract "n+1"

What will an EFT-based force look like?

- Leading order (LO): Need to make a guess.

=> Since Skyrme-type works so well, try it first!

Estimation of Breakdown scale

If require $O\left(\left(\frac{k_F}{M_{hi}}\right)^1\right) > O\left(\left(\frac{k_F}{M_{hi}}\right)^2\right)$

to be valid up to ρ=0.3 fm^{-3}.

Then M_{hi} need to be at least 400 MeV.

Also, the low bound cannot do better than the unitarity limit.

Then, only applicable for $\rho > 4*10^{-6}$[fm^{-3}].

Next-to-leading order (NLO) or higher:

1. Check renormalizability

C.J. Yang, M. Grasso, U. van Kolck, and K. Moghrabi, PRC 95, 054325 (2017)

2. Check power counting

Converging pattern

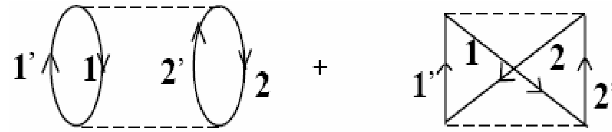
Lepage plot

3. Check reproduction of empirical result

C.J. Yang, M. Grasso, D. Lacroix, PRC 96, 034318 (2017)

Check renormalizability

2nd order results for nuclear matter



← No DR! use cutoff

$$\frac{\Delta E^{(2)}_{sym(l=0)}}{A} = -\frac{m k_F^4}{110880\hbar^2\pi^4}\left\{ \begin{bmatrix} -6534+1188 ln[2]+3564\lambda-19602\lambda^3-5940\lambda^5 \\ +(1782-20790\lambda^4) ln[\frac{\lambda-1}{\lambda+1}] \\ +(24948\lambda^5-5940\lambda^7) ln[\frac{\lambda^2-1}{\lambda^2}] \end{bmatrix}\widetilde{T}_{03}^2 + \begin{bmatrix} - \quad 14696+2112 ln[2]+5280\lambda-2860\lambda^3 \\ -48840\lambda^5-18480\lambda^7+(2640-55440\lambda^6) ln[\frac{\lambda-1}{\lambda+1}] \\ +(71280\lambda^7-18480\lambda^9) ln[\frac{\lambda^2-1}{\lambda^2}] \end{bmatrix} k_F^2\widetilde{T}_{03}\widetilde{T}_1 + \begin{bmatrix} -9886+1128 ln[2]+2520\lambda+147\lambda^3-3654\lambda^5 \\ -35280\lambda^7-15120\lambda^9+(1260-41580\lambda^8) ln[\frac{\lambda-1}{\lambda+1}] \\ +(55440\lambda^9-15120\lambda^{11}) ln[\frac{\lambda^2-1}{\lambda^2}] \end{bmatrix} k_F^4\widetilde{T}_1^2 \right\}$$

Diverge as Λ^5

$$\frac{\Delta E^{(2)}_{sym(l=1)}}{A} = -\frac{m k_F^8}{73920\hbar^2\pi^4}\left\{ \begin{bmatrix} -1033+156 ln[2]+420\lambda+140\lambda^3-840\lambda^5 \\ -5880\lambda^7-2520\lambda^9+(-210+6930\lambda^8) ln[\frac{\lambda-1}{\lambda+1}] \\ +(9240\lambda^9-2520\lambda^{11}) ln[\frac{\lambda^2-1}{\lambda^2}] \end{bmatrix}\widetilde{T}_2^2 \right\},$$

$$\frac{\Delta E^{(2)}_{neutr(l=0)}}{A} = -\frac{m k_{F_N}^4}{166320\hbar^2\pi^4}\left\{ \begin{bmatrix} -6534+1188 ln[2]+3564\lambda-19602\lambda^3-5940\lambda^5 \\ +(1782-20790\lambda^4) ln[\frac{\lambda-1}{\lambda+1}] \\ +(24948\lambda^5-5940\lambda^7) ln[\frac{\lambda^2-1}{\lambda^2}] \end{bmatrix} T_{03}^2 + \begin{bmatrix} - \quad 14696+2112 ln[2]+5280\lambda-2860\lambda^3 \\ -48840\lambda^5-18480\lambda^7+(2640-55440\lambda^6) ln[\frac{\lambda-1}{\lambda+1}] \\ +(71280\lambda^7-18480\lambda^9) ln[\frac{\lambda^2-1}{\lambda^2}] \end{bmatrix} k_{F_N}^2 T_{03}T_1 + \begin{bmatrix} -9886+1128 ln[2]+2520\lambda+147\lambda^3-3654\lambda^5 \\ -35280\lambda^7-15120\lambda^9+(1260-41580\lambda^8) ln[\frac{\lambda-1}{\lambda+1}] \\ +(55440\lambda^9-15120\lambda^{11}) ln[\frac{\lambda^2-1}{\lambda^2}] \end{bmatrix} k_{F_N}^4 T_1^2 \right\}$$

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- When $\Lambda \to \infty$, how the 2nd order terms behaves?

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2} k_F^4 \left[A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4\right],$$ finite terms

$$\frac{\Delta E_a^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2\right],$$ Diverge, k_F-dep appears in MF

$$\frac{\Delta E_d^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4\right],$$ Diverge, k_F-dep *not* in MF

Note that the above are regulator-dependent, except for the finite terms.

- Treatment I:

Absorb divergence into redefinition of parameters.

- Treatment II:

Add counter terms correspond to the divergences.

Treatment I:

No new term added, use special cases of α and t_i

C.J. Yang, M. Grasso, K. Moghrabi, U van Kolck, PRC 95, 054325 (2017)

- Idea: Absorb the Λ-divergence in 2nd order into mean field terms with the same k_F-dependence.

$$\frac{\Delta E_f^{(2)}(k_F)}{A} = \frac{3m}{2\pi^4\hbar^2} k_F^4 \left[A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4\right],$$ converge

$$\frac{\Delta E_a^{(2)}(k_F,\lambda)}{A} = -\frac{m}{8\pi^4\hbar^2} \lambda k_F^3 \left[B_0(\lambda) + B_1(\lambda) T_3 k_F^{3\alpha} + B_2(\lambda) k_F^2\right],$$ Diverge, k_F-dep appears in MF

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↓

Eliminate or re-absorb into first two lines by setting:

1. α=1/3 and t_1=t_2=0.
2. α=-1/6 and t_1=t_2=0, m=m^R.
3. α=2/3 and t_1=t_2=t_3=0.

Results: $\alpha = 2/3$

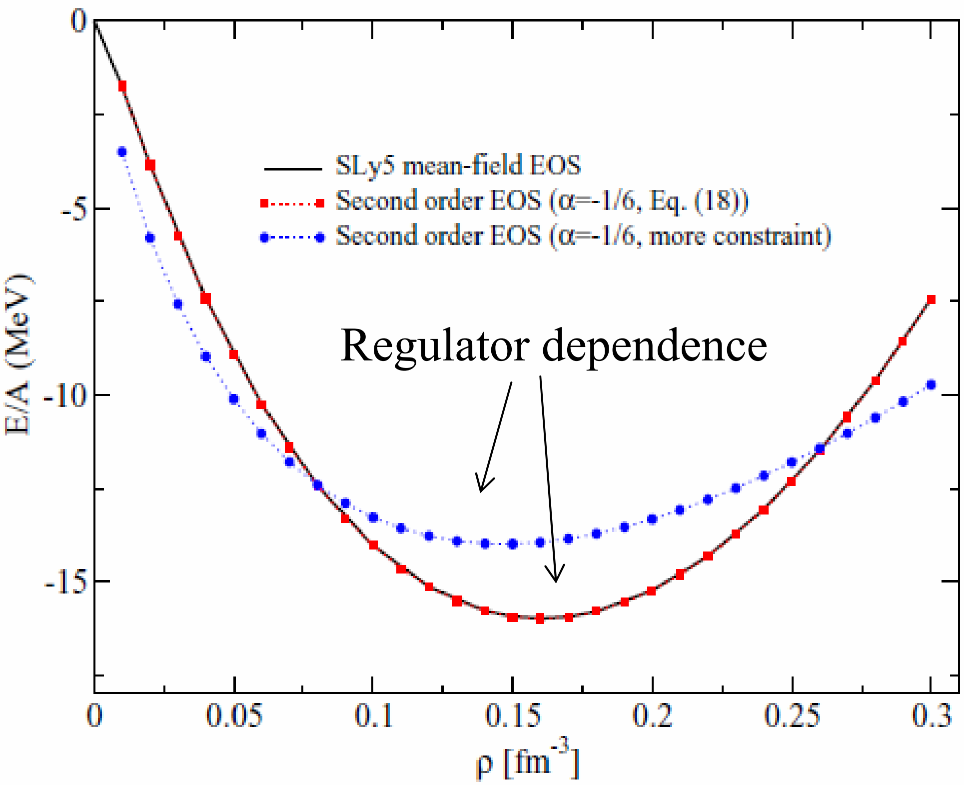
m (MeV)	t_0^R (MeV fm^3)	$\lvert x_0^R \rvert$	χ^2
939	−358.16	$< 10^{-4}$	346 850
−969.55	212.28	$< 10^{-4}$	15 989

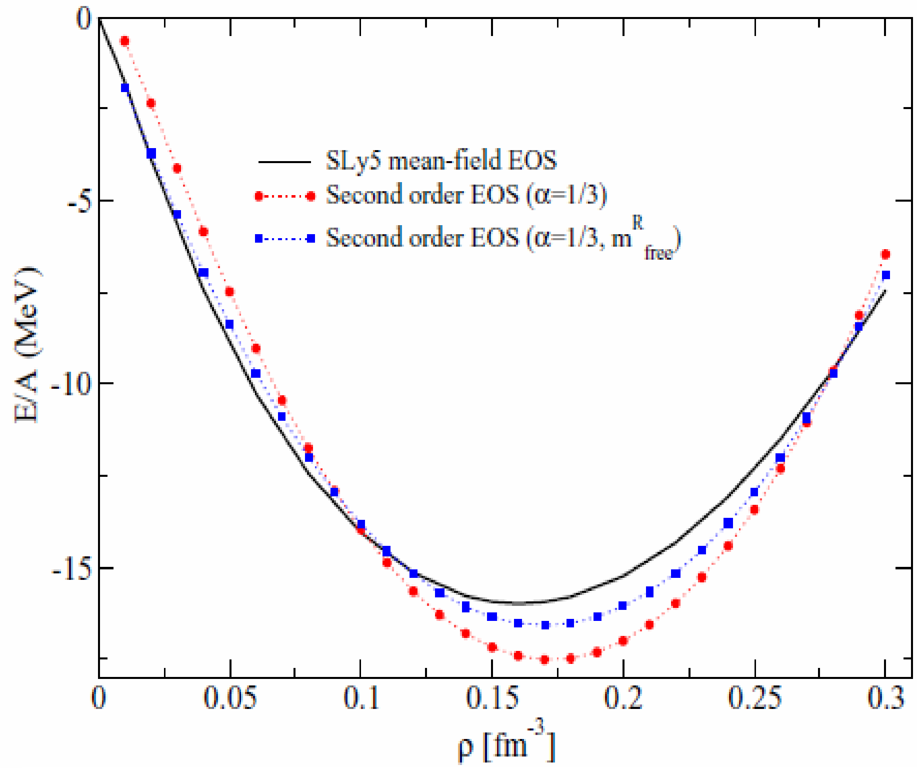
No saturation! Only t_0 !

Results: α=-1/6 Results: α=1/3

m^R (MeV)	t_0^R (MeV fm^3)	T_3^R (MeV fm$^{5/2}$)	x_0^R	x_3^R	χ^2
591.9	793.15	−1570.8	1.465	−0.1759	<0.1

m (MeV)	t_0^R (MeV fm^3)	T_3^R (MeV fm^4)	$\lvert x_3^R \rvert$	χ^2
939	−1244.1	247.11	$<10^{-4}$	1364
23845	−580.16	46.248	$<10^{-2}$	188





Lessons

1. The leading order quite possible just contains only t_0-t_3 terms.

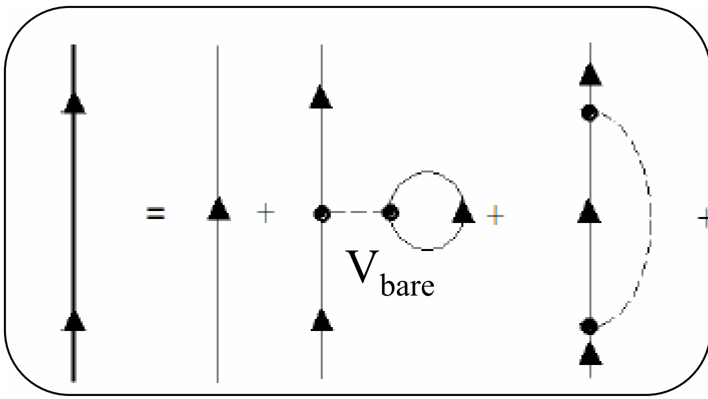
2. However, the regulator dependence tells us the power counting cannot be established in this way.

More general consideration (adding counter terms at NLO):

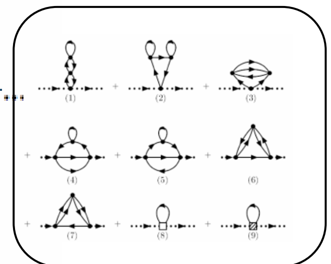
C.J. Yang, M. Grasso, D. Lacroix, PRC 96, 034318 (2017)

Diagrammatic explanation of the idea

Dressing of propagator → V_{eff}



Leading order (LO)



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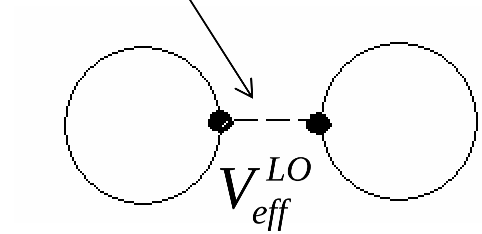
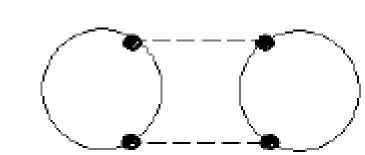


Then, NLO includes (at least):



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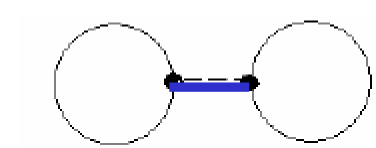
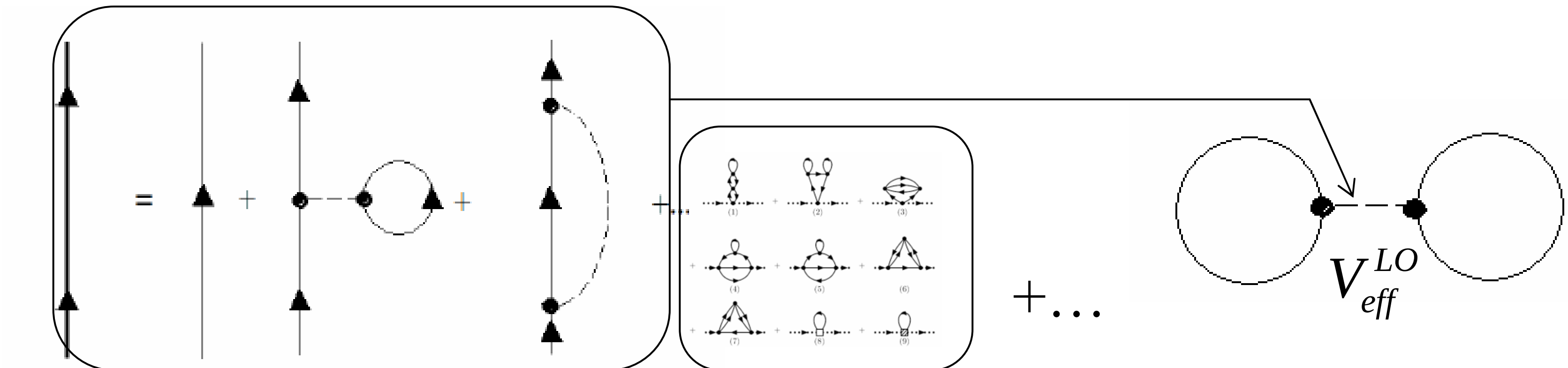


$V_{eff}^{LO} G V_{eff}^{LO}$ — *diverge* at least in Λk_F^3

V_{eff}^{NLO} — new counter term(s)

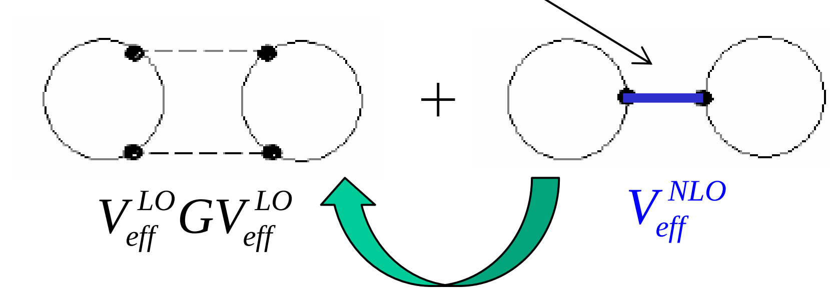
$V_{eff}^{Sly5} G V_{eff}^{Sly5}$ evaluated in: C.J. Yang, M. Grasso, X. Roca-Maza, G. Colo, and K. Moghrabi, PhysRevC.94.034311

Dressing of propagator→V_{eff}



Leading order (LO)

Then, NLO includes:



* V_{eff}^{NLO} contains (at least) contact terms to renormalize $V_{eff}^{LO} G V_{eff}^{LO}$.

Counter term part of the NLO potential

V_{eff}^{NLO} : For t_0-t_3 model, the divergence from $V_{eff}^{LO} G V_{eff}^{LO}$ is:

$\underbrace{O(k_F^3), O(k_F^{3+3\alpha})}_{k_F^n-dep.\text{ appears in }MF}, \underbrace{O(k_F^{3+6\alpha})}_{new\ k_F^n-dep.}.$ **3 different k_F-dep.**

If want to keep α free, =>Minimun contact term required: $Ck_F^{3+6\alpha}$.

Most general case: $Ak_F^3, Bk_F^{3+3\alpha}, Ck_F^{3+6\alpha}$.

In infinite matter, k_F^{3n} in-distinguishable with $3\pi^2\rho$

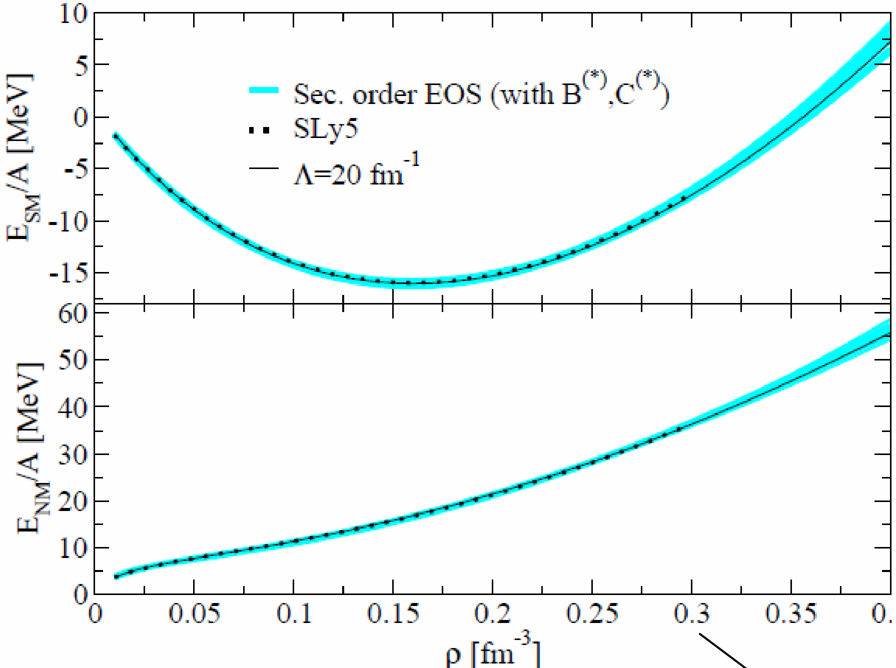
=> k_F^n-term in EOS *could* originated (at interaction level) from:

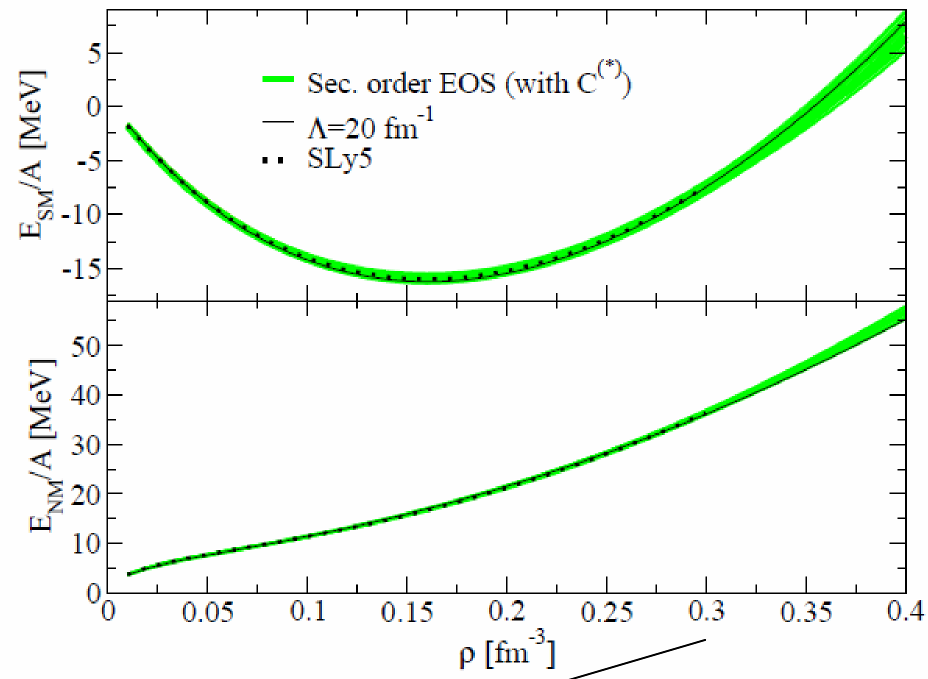
$(k-k')^{n-3v-3}\rho^v$,

where v is an extra parameter to be decided in the fitting to finite nuclei.

NLO results (based on t_0-t_3 as LO)
α <1/6 case

Color band: Λ=1.2~20 fm^{-1}

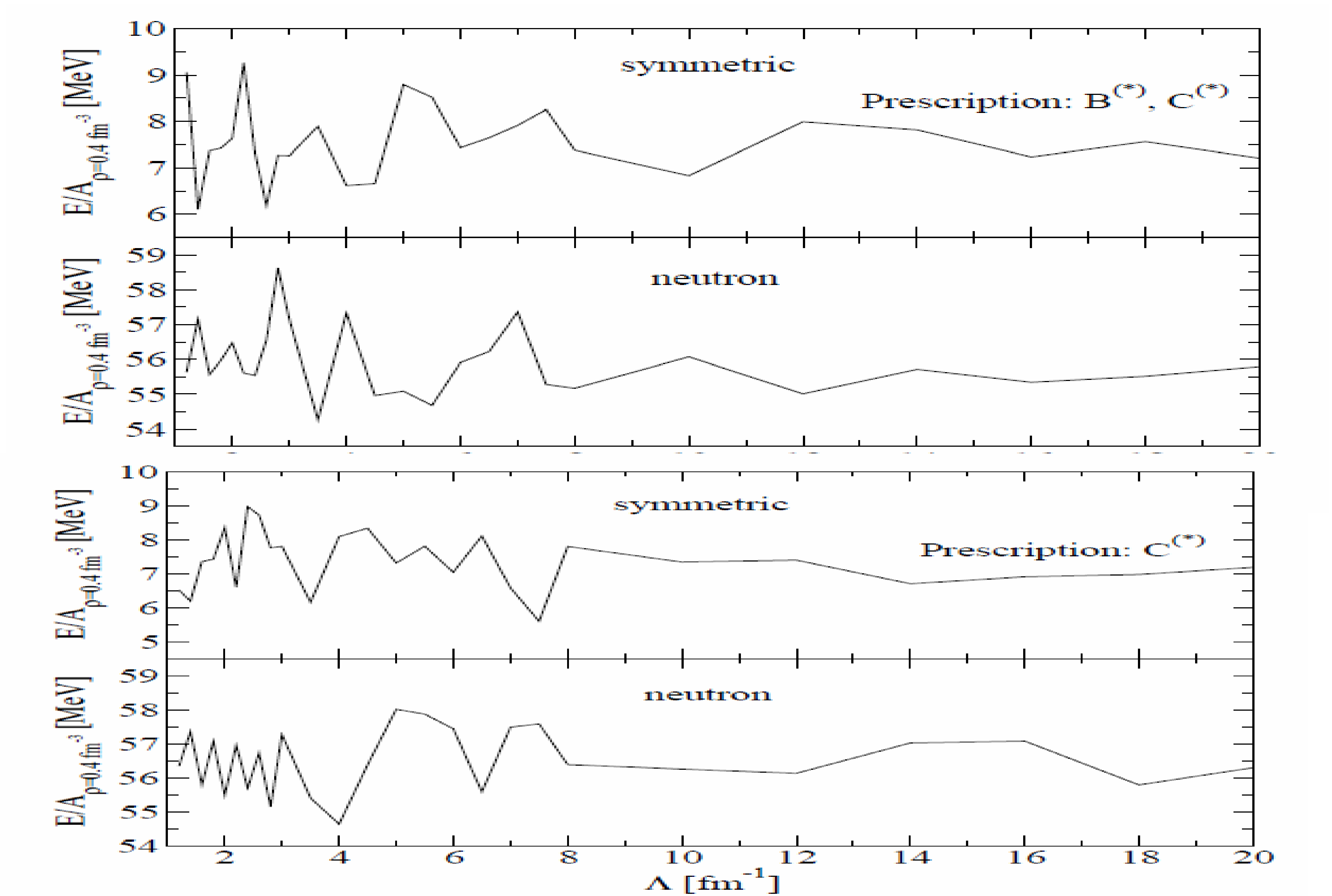




LECs fitted up to 0.3 fm^{-1}

Similar results (with different counter terms) tell us that the regulator-dependence is eliminated by adding counter terms!

Renormalization group (RG) check at ρ =0.4 fm^{-1}



Future work

- So far only perform calculations at EOS level, and has (too) many parameters and limited observables to fit. => Many sets of LECs fit equally well.
- Need to go to:

1. finite nuclei →
2. NNLO →

Power counting check
(e.g., Lepage plot)

Thank you!

Brainstorming I

- **Any alternative suggestion of LO interaction?**

❖ Could it be derived from more microscope/fundamental theories?

❖ Use multiple density-dep. term at LO?

❖ What's the upper and lower bound value for α (if any)?

❖ Should we keep α independent of cutoff ?

Brainstorming II

- How to do the same (2nd order) for finite nuclei?

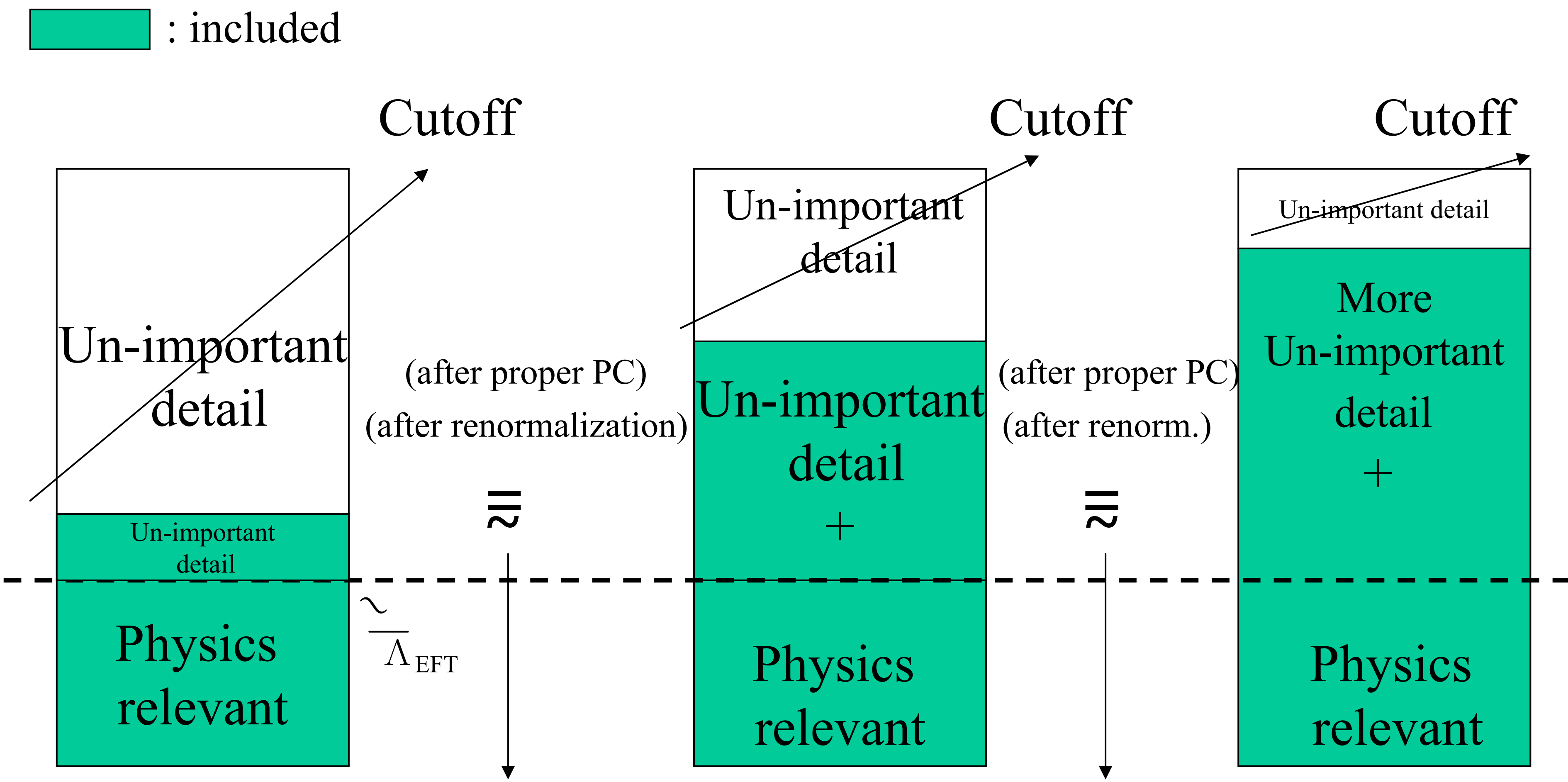
  M. Brenna, G. Colo, X. Roca-Maza PRC 90, 044316 (2014)

- Any idea to extend the EFT built from unitarity limit to symmetric matter.

  Should we have different power counting between pure neutron and symmetric matter?

Back up slides

Renormalization group (RG)



***Only source of error**: given by the high order terms.

If not so, → **the power counting isn't completely correct!**

(unimportant are not really unimportant)

Interaction & mean field EoS

Interaction: Skyrme without spin-orbit

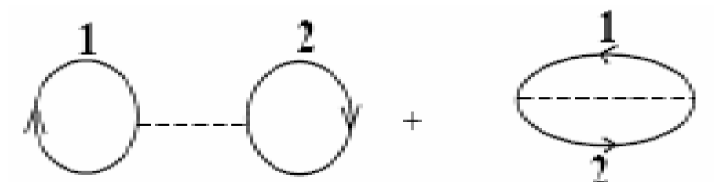
$$v = \underbrace{t_0(1+x_0P_\sigma)}_{S-wave\ \mathrm{O(0)}} + \frac{1}{2}\underbrace{t_1(1+x_1P_\sigma)(k'^2+k^2)}_{S-wave\ \mathrm{O(q^2)}} + \underbrace{t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\mathbf{k}}_{p-wave\ \mathrm{O(q^2)}}$$

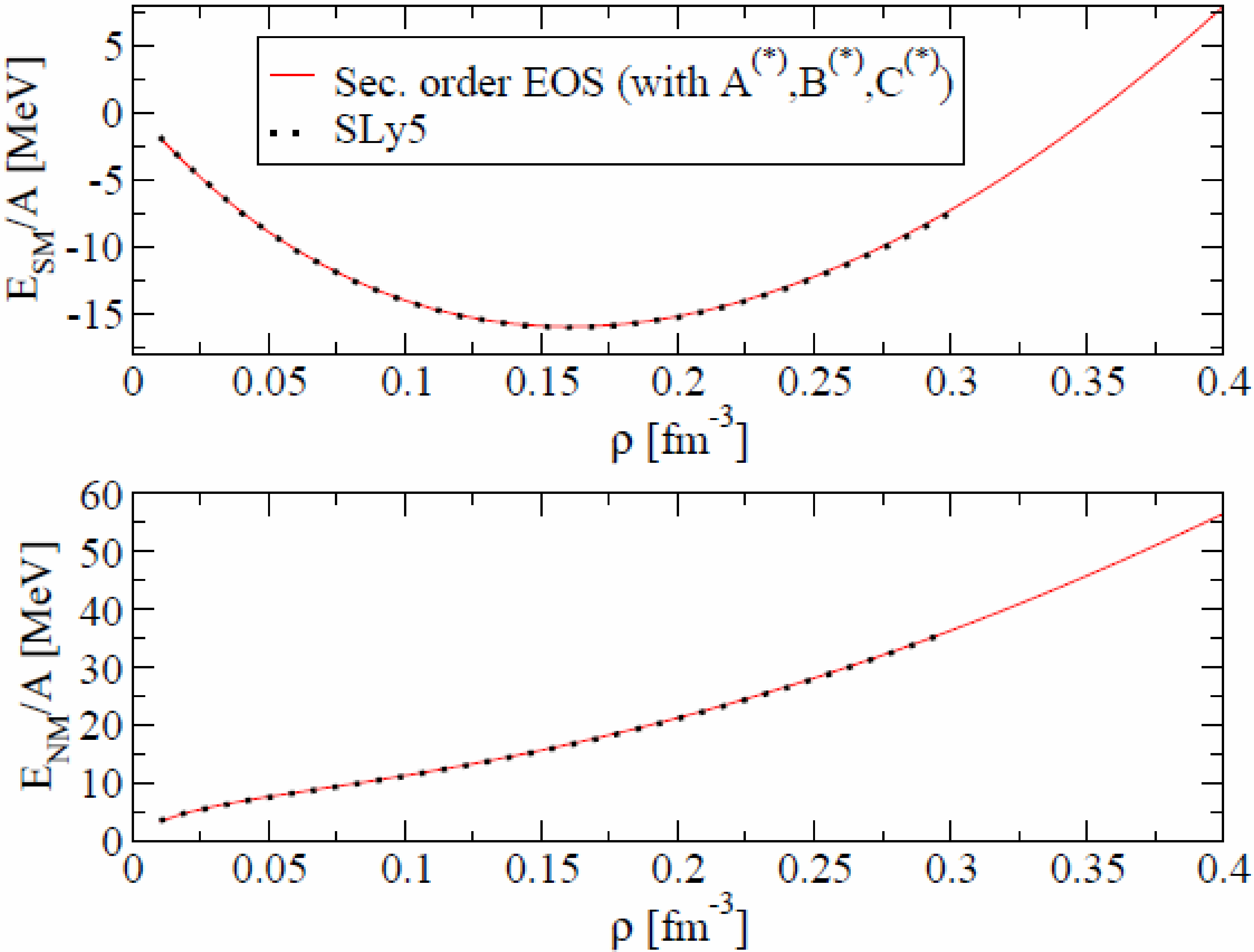
$$+\frac{1}{6}\underbrace{t_3(1+x_3P_\sigma)\rho^\alpha}_{s-wave,\ \text{higher body}}. \qquad \mathrm{P}_\sigma = \frac{1}{2}(1+\sigma_1\cdot\sigma_2)$$

No pion! Like pionless EFT, except for the density-dependent term.

$$EoS: \quad \frac{E}{A} \propto \frac{1}{\rho}\int_0^{k_{F1}} d^3\mathbf{k}_1 \int_0^{k_{F2}} d^3\mathbf{k}_2 v$$

$$\left(\mathbf{k} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}, \mathbf{k}' = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2} + \mathbf{q}\right)$$





Sec. order EOS (with A(*),B(*),C(*))
SLy5
E_SM/A [MeV]
ρ [fm^-3]
E_NM/A [MeV]
ρ [fm^-3]

Parameters v.s. cutoff