Nuclear-matter constraints from chiral effective field theory

Ingo Tews

In collaboration with J. Carlson, S. Gandolfi, A. Gezerlis, E. Kolomeitsev, J. Lattimer, J. Lynn, A. Ohnishi, A. Schwenk, S. Reddy

Orsay-Workshop: "Bridging nuclear ab-initio and energy-density-functional theories", October 5, 2017, IPN Orsay, France

JINA–CEE

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Outline

- Motivation
- Chiral effective field theory e.g. Epelbaum *et al.*, PPNP (2006) and RMP (2009)
  - Systematic basis for nuclear forces, naturally includes many-body forces
  - Very successful in calculations of nuclei and nuclear matter
- Local chiral interactions
  - Can be constructed up to N^2LO Gezerlis, IT, et al., PRL & PRC (2013, 2014, 2016)
- QMC results for neutron matter and nuclei
- Symmetry energy constraints from lower bound of neutron matter
- Extensions to higher density using speed of sound

Motivation

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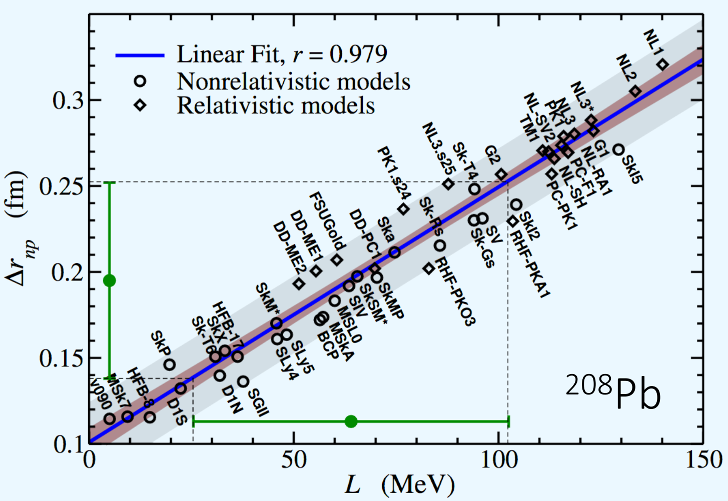
What are the fundamental forces that govern strongly-interacting matter?

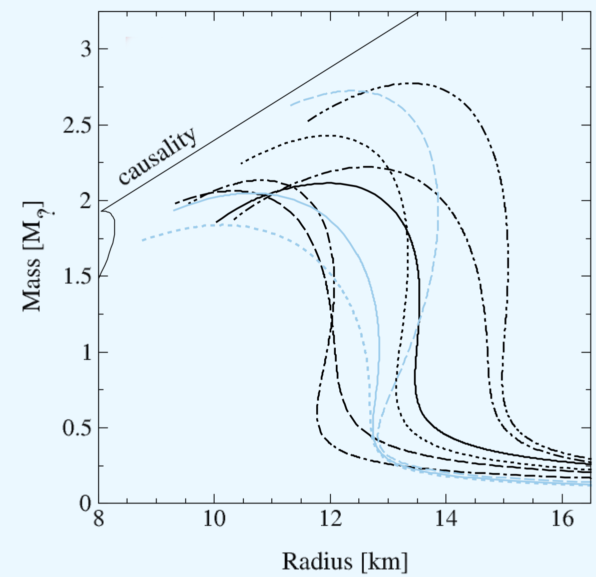
EOS of Infinite Matter at T=0: Ideal testbed, good matching point

How does the nuclear structure emerge?

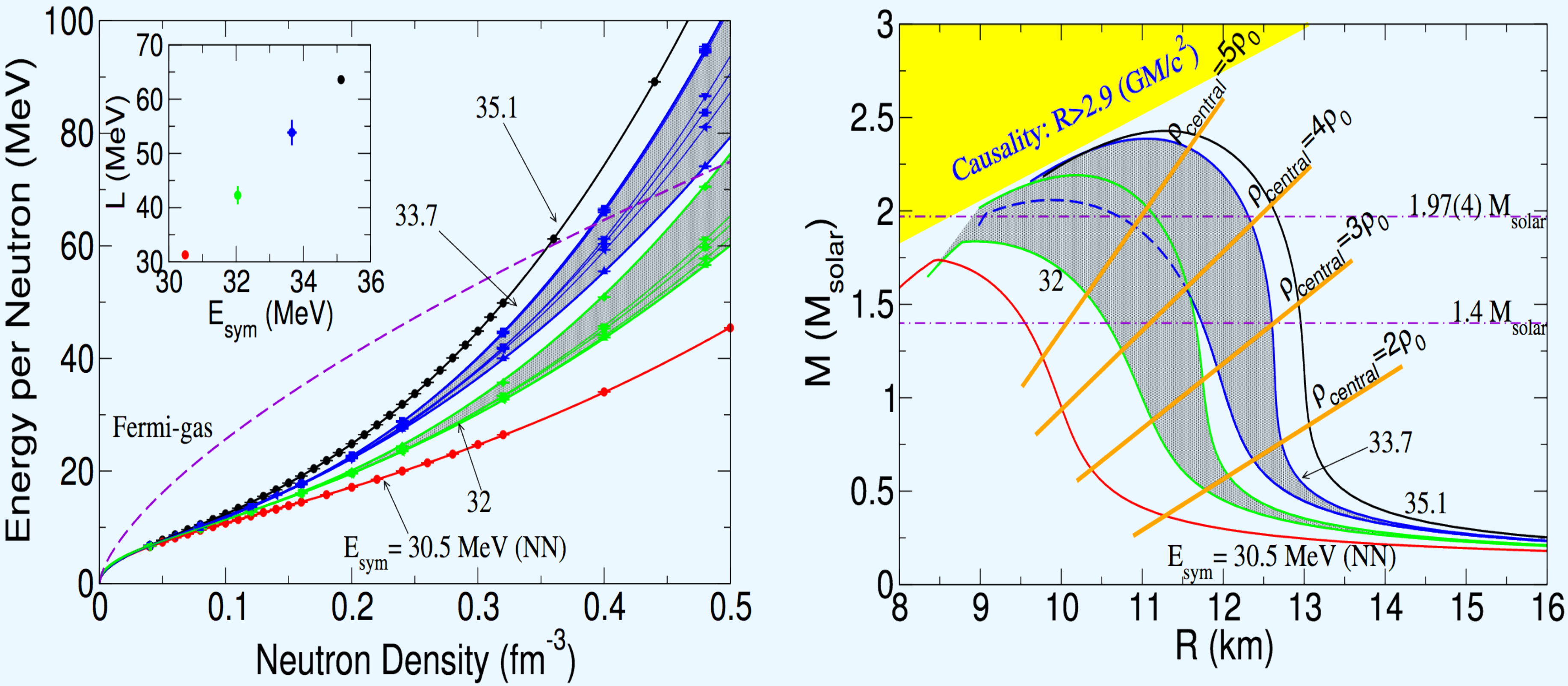
How can we understand astrophysical phenomena?



Roca-Maza et al. PRL (2011)



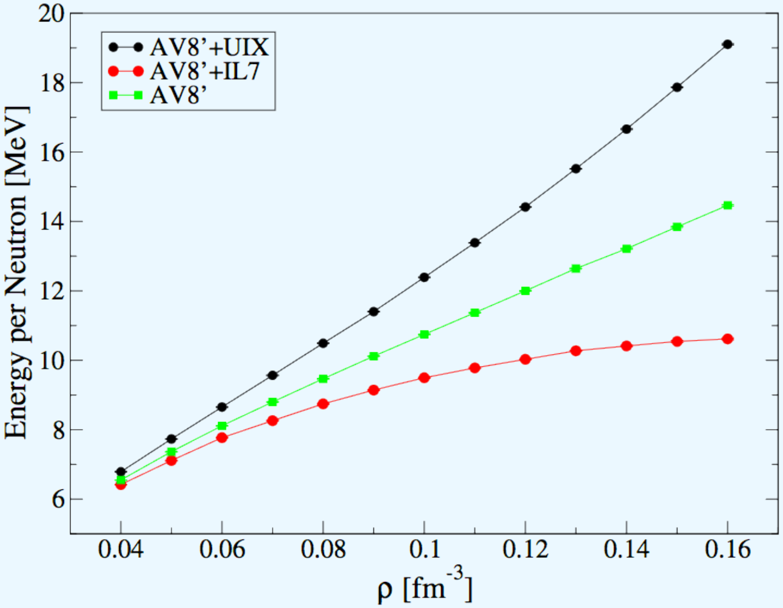
Phenomenological interactions



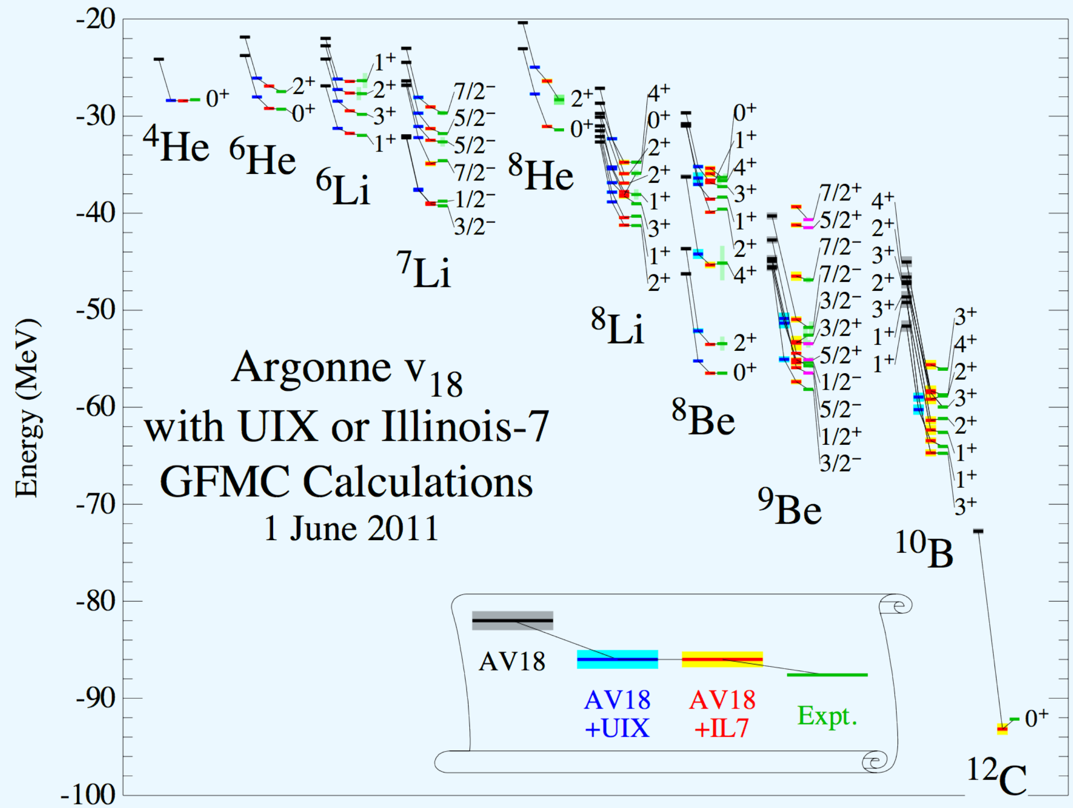
Gandolfi, Carlson, Reddy, PRC (2012)

- Phenomenological interactions give a very good description of properties of light nuclei with uncertainties of 1-2% in Quantum Monte Carlo calculations.
- Not clear how to systematically improve these interactions, especially in the three-body sector, and no systematic uncertainties.

Phenomenological interactions: 3N



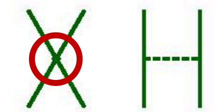
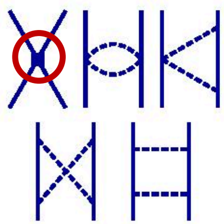
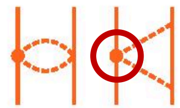
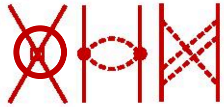
Maris et al., PRC (2013)



Different phenomenological 3N interactions :

- Urbana IX stiff enough to support heavy neutron stars but fails in nuclei
- Illinois 7 too soft to support heavy neutron stars but very good for nuclei

Chiral effective field theory for nuclear forces

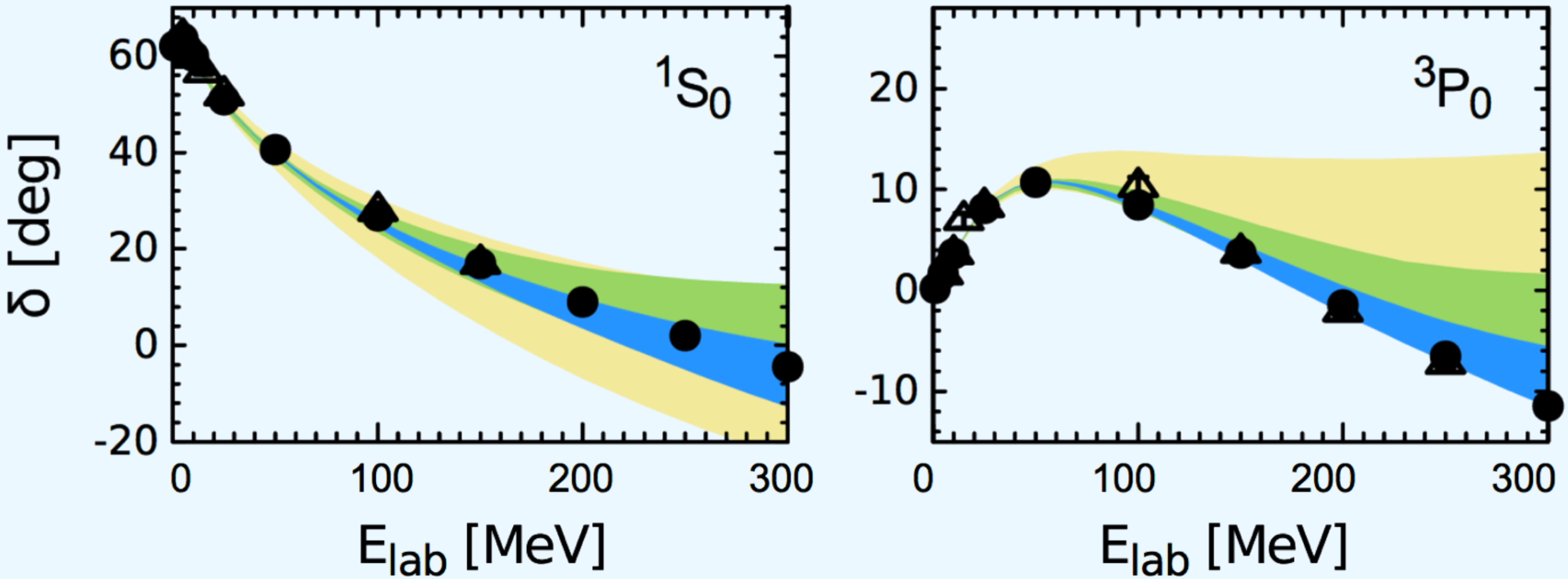
	NN	
LO $O\left(\frac{Q^0}{\Lambda^0}\right)$		2 LECs
NLO $O\left(\frac{Q^2}{\Lambda^2}\right)$		7 LECs
N²LO $O\left(\frac{Q^3}{\Lambda^3}\right)$		
N³LO $O\left(\frac{Q^4}{\Lambda^4}\right)$	 + ...	15 LECs

Systematic expansion of nuclear forces in low momenta Q over breakdown scale Λ_b:

- Pions and nucleons as explicit degrees of freedom
- Long-range physics explicit, short-range physics expanded in general operator basis, couplings (LECs) fit to experiment
- Separation of scales: Expand in powers of $\left(\frac{Q}{\Lambda_b}\right)^\nu \sim \left(\frac{1}{3}\right)^\nu$
- Power counting scheme
- Can work to desired accuracy with systematic error estimates

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Chiral effective field theory for nuclear forces



Epelbaum *et al.*, Eur. Phys. J (2015)

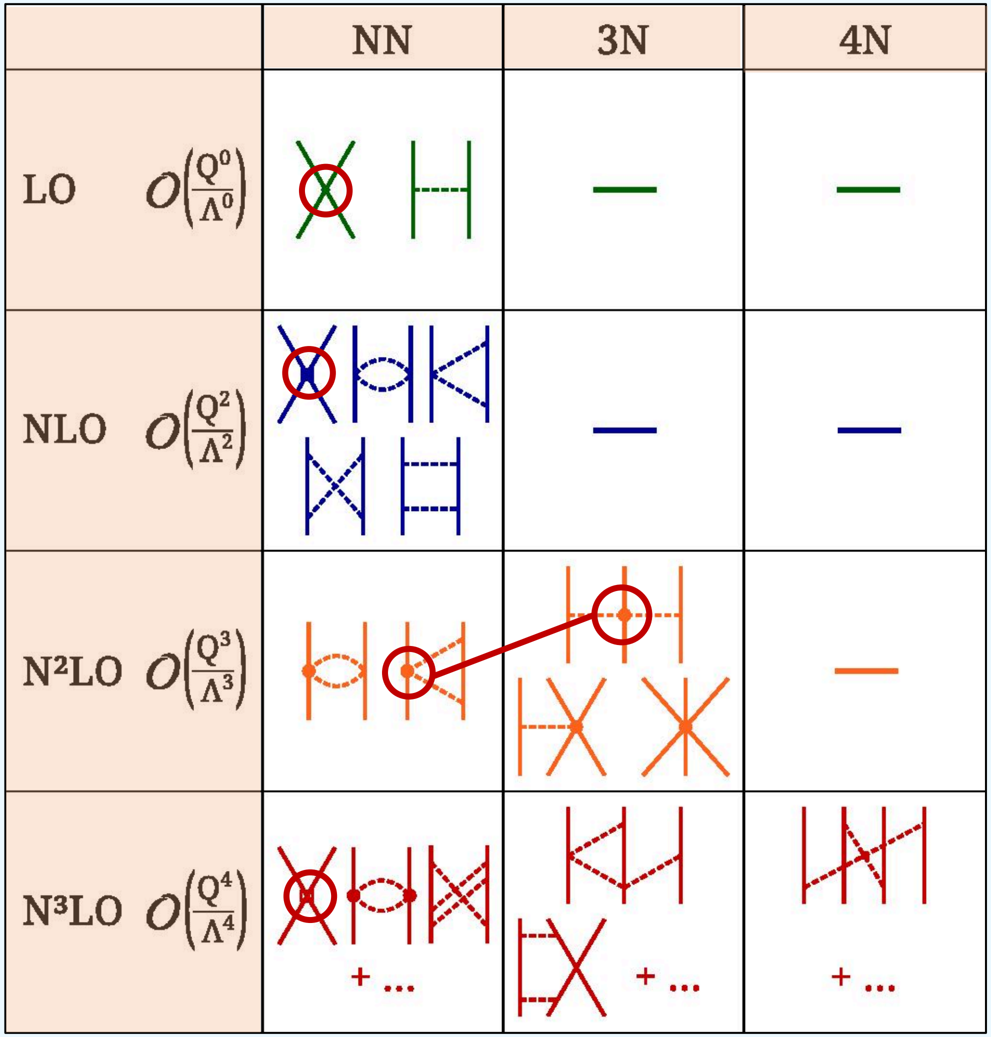
$$\Delta X^{\mathrm{N^2LO}} = \max\left(Q^4 \left|X^{\mathrm{LO}}\right|, Q^2 \left|X^{\mathrm{NLO}} - X^{\mathrm{LO}}\right|, Q \left|X^{\mathrm{N^2LO}} - X^{\mathrm{NLO}}\right|\right)$$

$$Q = \max(p/\Lambda_b, m_\pi/\Lambda_b)$$

Systematic expansion of the nuclear forces:

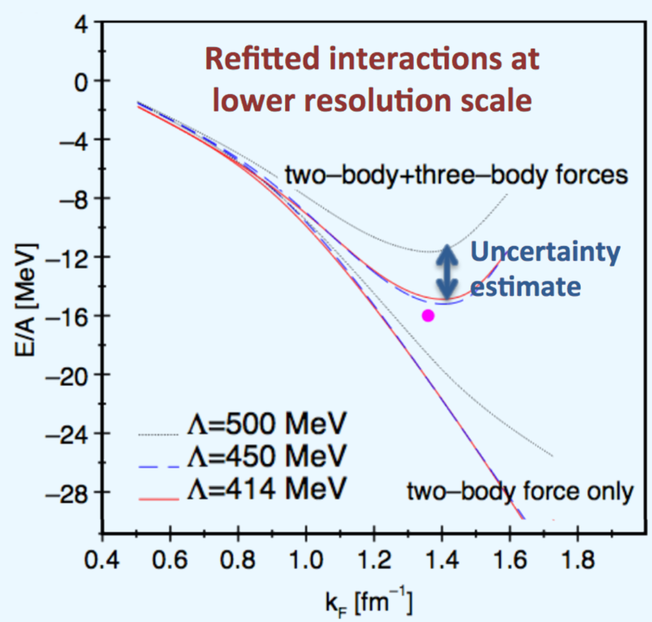
- Can work to desired accuracy
- Can obtain systematic error estimates

Chiral effective field theory for nuclear forces



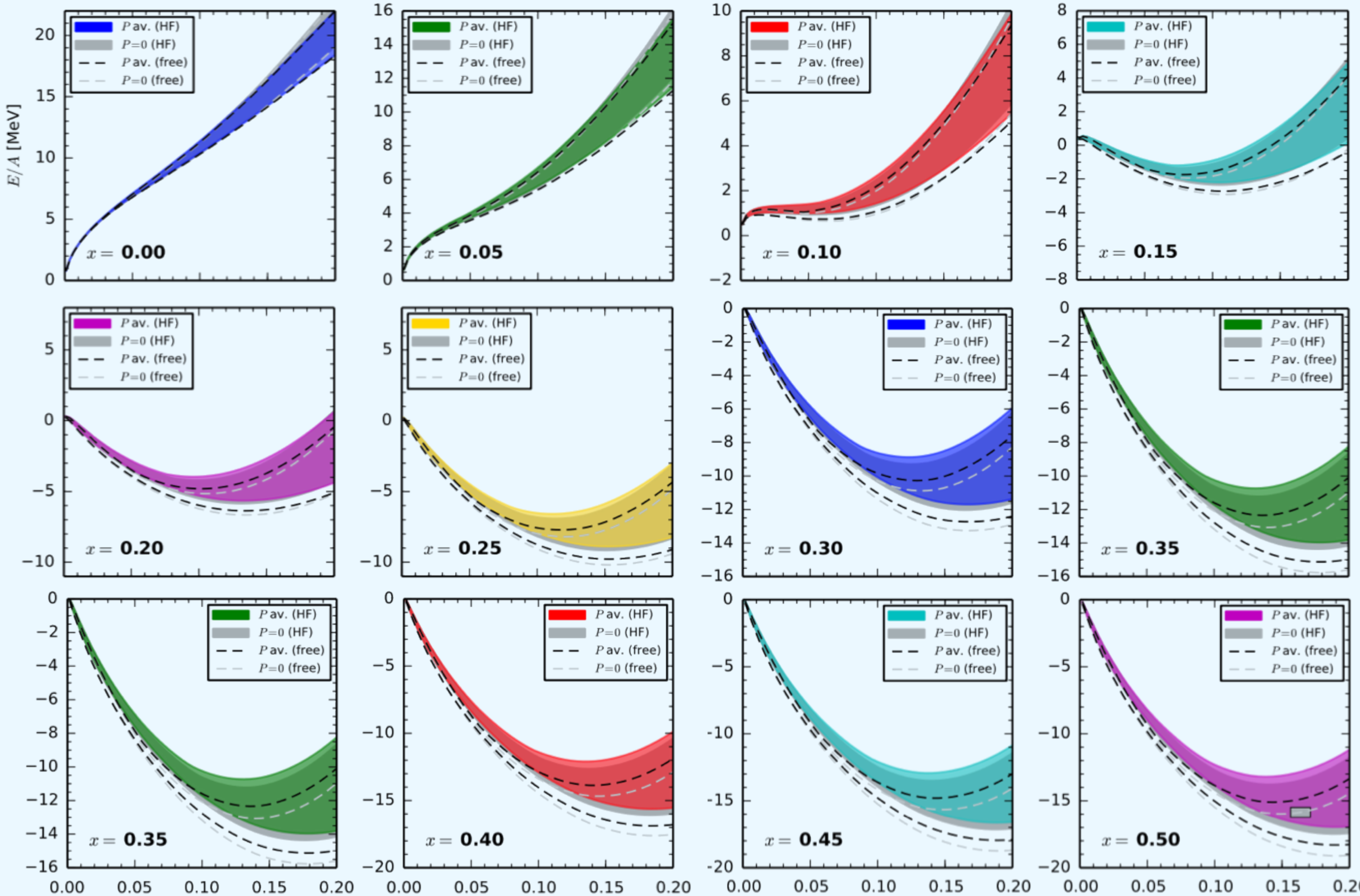
Many-body forces:

- Crucial for nuclear physics
- Natural hierarchy of nuclear forces
- Consistent interactions: Same couplings for two-nucleon and many-body sector
- Fitting: NN forces in NN system (NN phase shifts), 3N forces in 3N/4N system (Binding energies, radii)



Coraggio, Holt, Itaco, Machleidt, Marcucci, Sammarruca, PRC (2014)

EOS for asymmetric matter



MBPT

Drischler et al.,
PRC (2016)

Quantum Monte Carlo method

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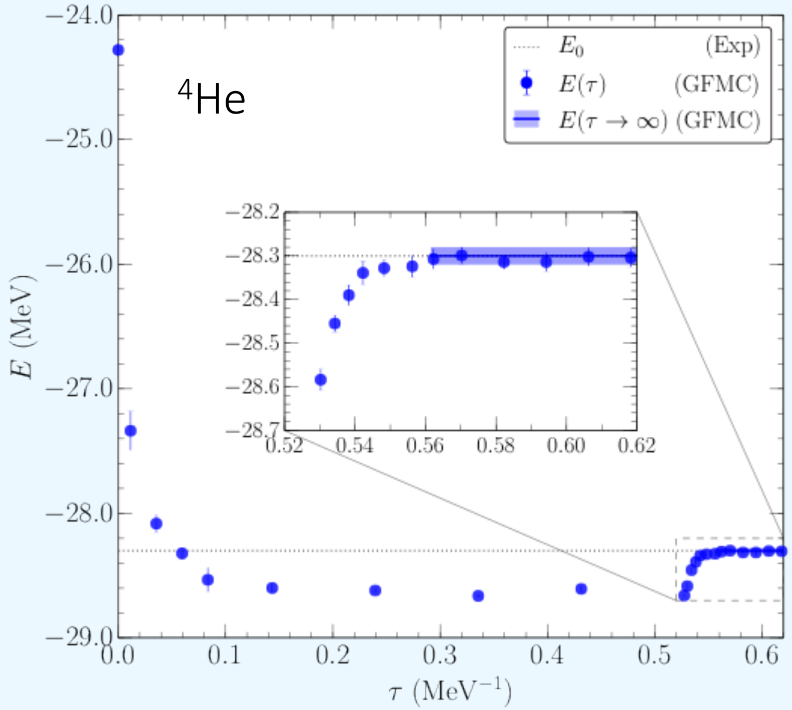
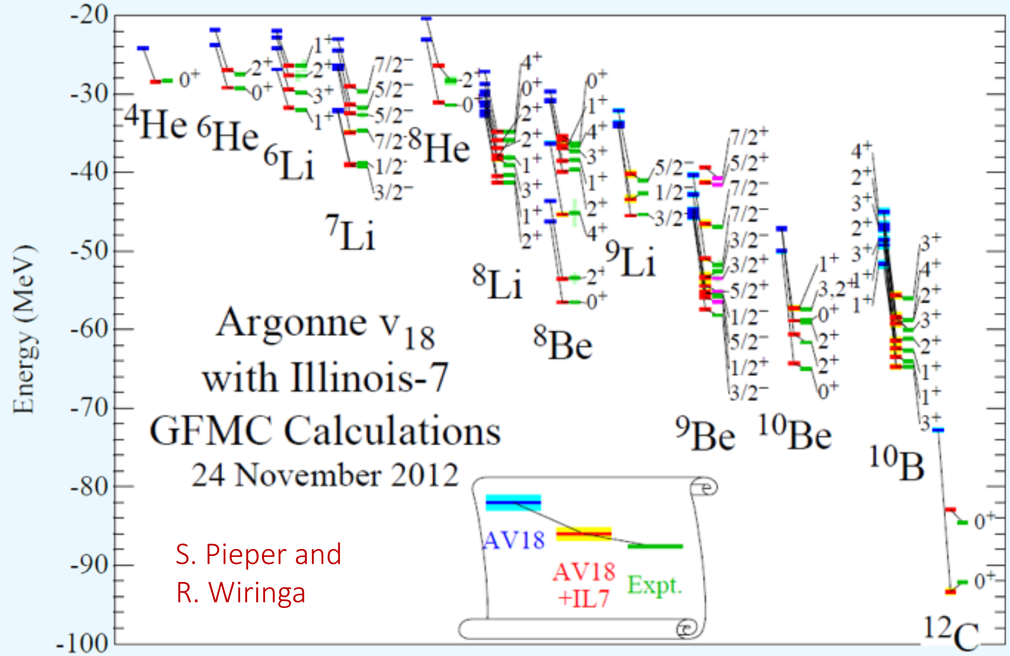


Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, arXiv:1706.07668



- ➢ Very precise method for strongly interacting systems.
- ➢ Needs as input local interactions but chiral EFT generally nonlocal!

Local chiral interactions

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To evaluate the propagator for small timesteps $\Delta\tau$ we need local potentials:

$$\langle r'|V|r\rangle = \begin{cases} V(r)\delta(r-r') & \text{if local} \\ V(r',r) & \text{if nonlocal} \end{cases}$$

Chiral Effective Field Theory interactions generally nonlocal:

- Momentum transfer $\mathbf{q} = \mathbf{p}' - \mathbf{p}$
- Momentum transfer in the exchange channel $\mathbf{k} = \frac{1}{2}(\mathbf{p} + \mathbf{p}')$
- Fourier transformation: $\mathbf{q} \to \mathbf{r}$, $\mathbf{k} \to \nabla_{\mathbf{r}}$

Sources of nonlocalities:

- Usual regulator in relative momenta

$$f(p) = e^{-(p/\Lambda)^{2n}}$$

- k-dependent contact operators

Solutions:

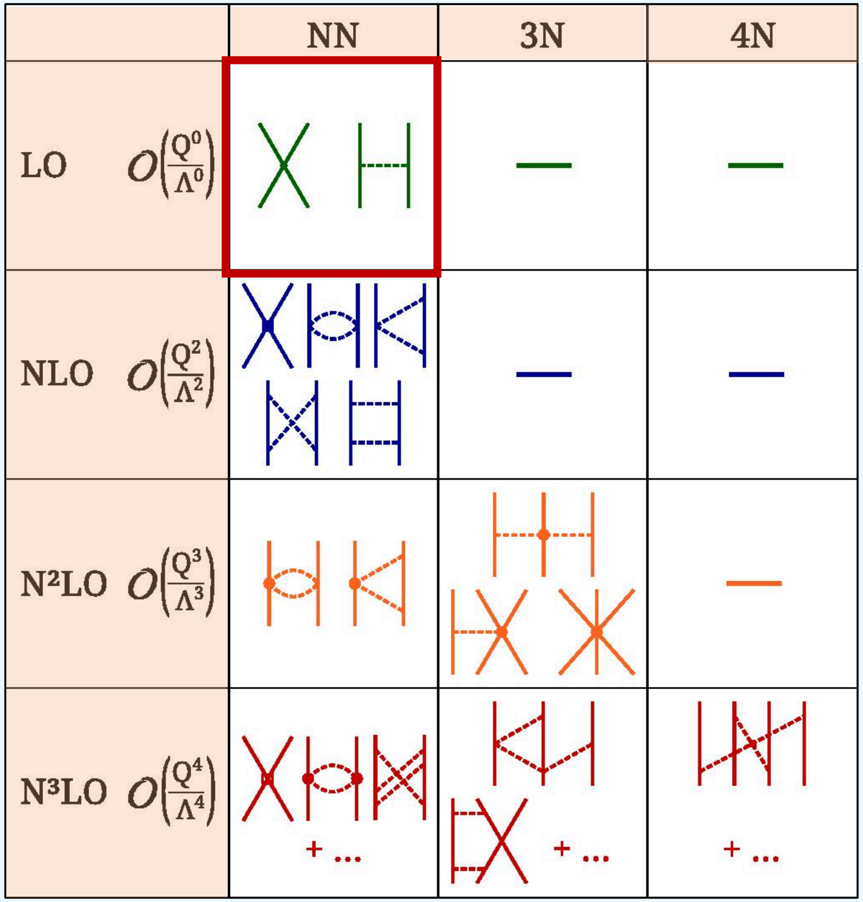
- Choose local regulators:

$$V_{\text{long}}(r) \to V_{\text{long}}(r)\left(1 - e^{-(r/R_0)^4}\right)$$

$$\delta(r) \to \delta_{R_0}(r) = \alpha e^{-(r/R_0)^4}$$

- Use Fierz freedom to choose local set of contact operators.

Local chiral interactions



Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

- Leading order $V^{(0)} = V^{(0)}_{\mathrm{cont}} + V^{\mathrm{OPE}}$
- Pion exchanges local

$$\begin{aligned} V_{\mathrm{long}}(r) &= V_C(r) + W_C(r)\,\tau_1\cdot\tau_2 \\ &+ (V_S(r) + W_S(r)\,\tau_1\cdot\tau_2)\,\sigma_1\cdot\sigma_2 \\ &+ (V_T(r) + W_T(r)\,\tau_1\cdot\tau_2)\,S_{12} \end{aligned}$$

→ local regulator

$$V_{\mathrm{long}}(r) \;\to\; V_{\mathrm{long}}(r)\left(1 - e^{-(r/R_0)^4}\right)$$

- Contact potential:

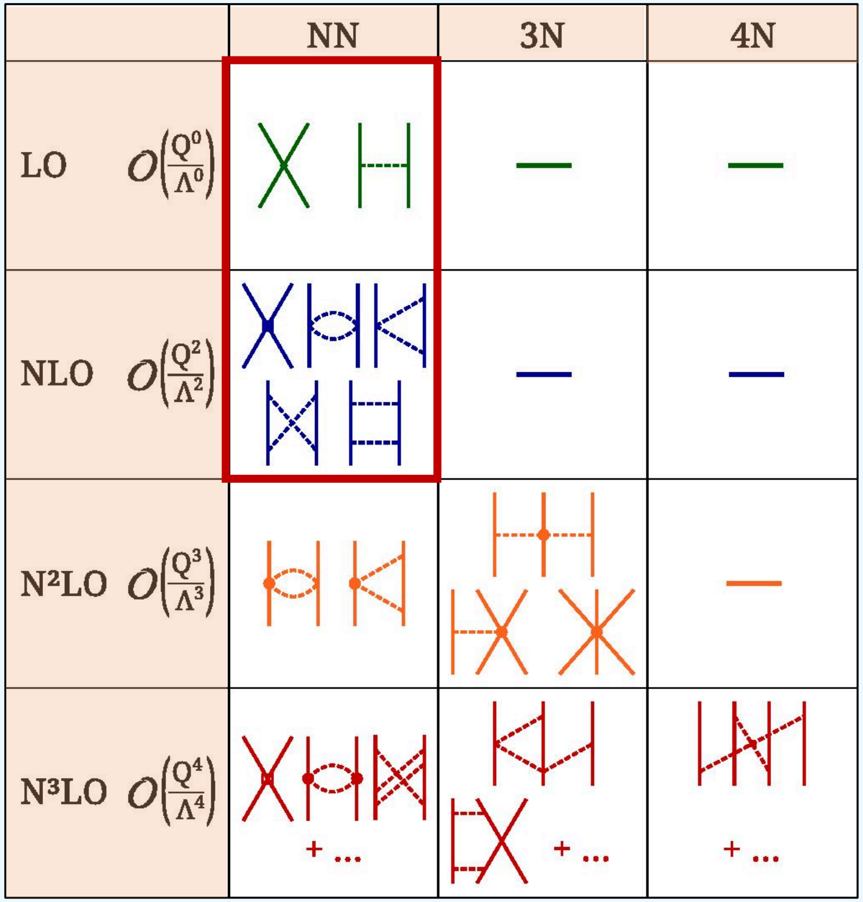
$$\begin{aligned} V^{(0)}_{\mathrm{cont}} &= \alpha_1 \mathbb{1} + \alpha_2\,\sigma_1\cdot\sigma_2 + \alpha_3\,\tau_1\cdot\tau_2 \\ &+ \alpha_4\,\sigma_1\cdot\sigma_2\,\tau_1\cdot\tau_2 \end{aligned}$$

→ Only two independent (Pauli principle)

$$V^{(0)}_{\mathrm{cont}} = C_S \mathbb{1} + C_T\,\sigma_1\cdot\sigma_2$$

$$\delta(\mathbf{r}) \;\to\; \delta_{R_0}(\mathbf{r}) = \alpha e^{-(r/R_0)^4}$$

Local chiral interactions

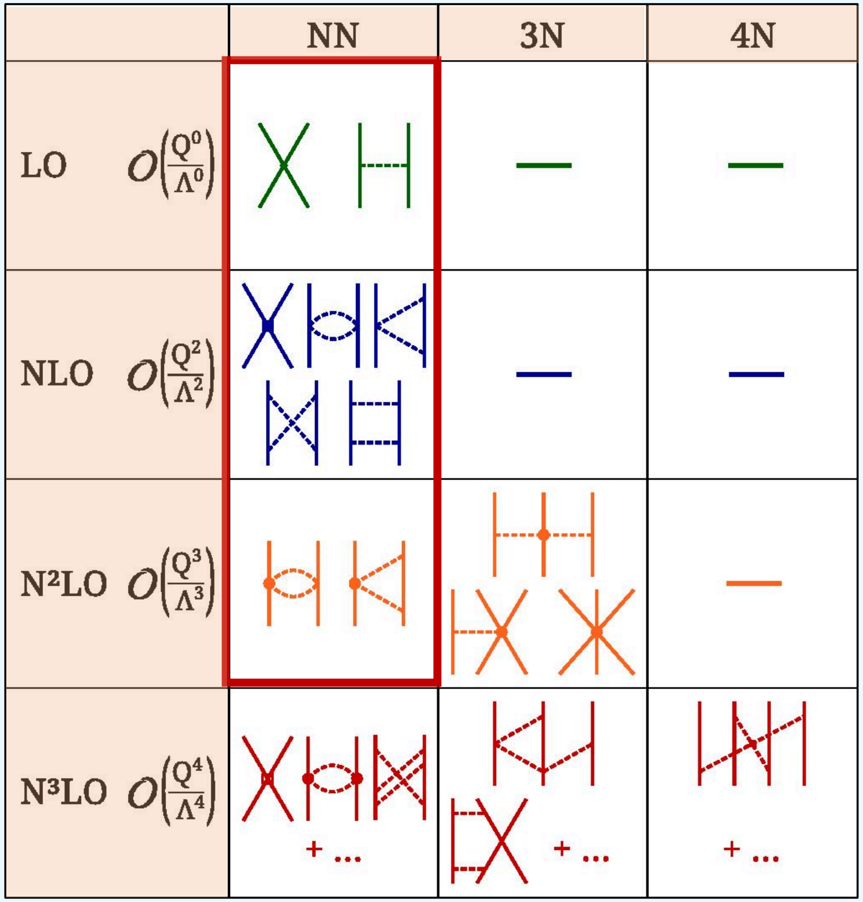


- Choose local set of short-range operators at NLO (7 out of 14)

$$
\begin{aligned}
V_{\text{cont}}^{(2)} = &\ \gamma_1\, q^2 + \gamma_2\, q^2\, \sigma_1 \cdot \sigma_2 + \gamma_3\, q^2\, \tau_1 \cdot \tau_2 \\
&+ \gamma_4\, q^2 \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \\
&+ \gamma_5\, k^2 + \gamma_6\, k^2\, \sigma_1 \cdot \sigma_2 + \gamma_7\, k^2\, \tau_1 \cdot \tau_2 \\
&+ \gamma_8\, k^2 \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \\
&+ \gamma_9\, (\sigma_1 + \sigma_2)(\mathbf{q} \times \mathbf{k}) \\
&+ \gamma_{10}\, (\sigma_1 + \sigma_2)(\mathbf{q} \times \mathbf{k}) \tau_1 \cdot \tau_2 \\
&+ \gamma_{11} (\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q}) \\
&+ \gamma_{12} (\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q}) \tau_1 \cdot \tau_2 \\
&+ \gamma_{13} (\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) \\
&+ \gamma_{14} (\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) \tau_1 \cdot \tau_2
\end{aligned}
$$

Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

Local chiral interactions



Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

- Choose local set of short-range operators at NLO (7 out of 14) and local regulators

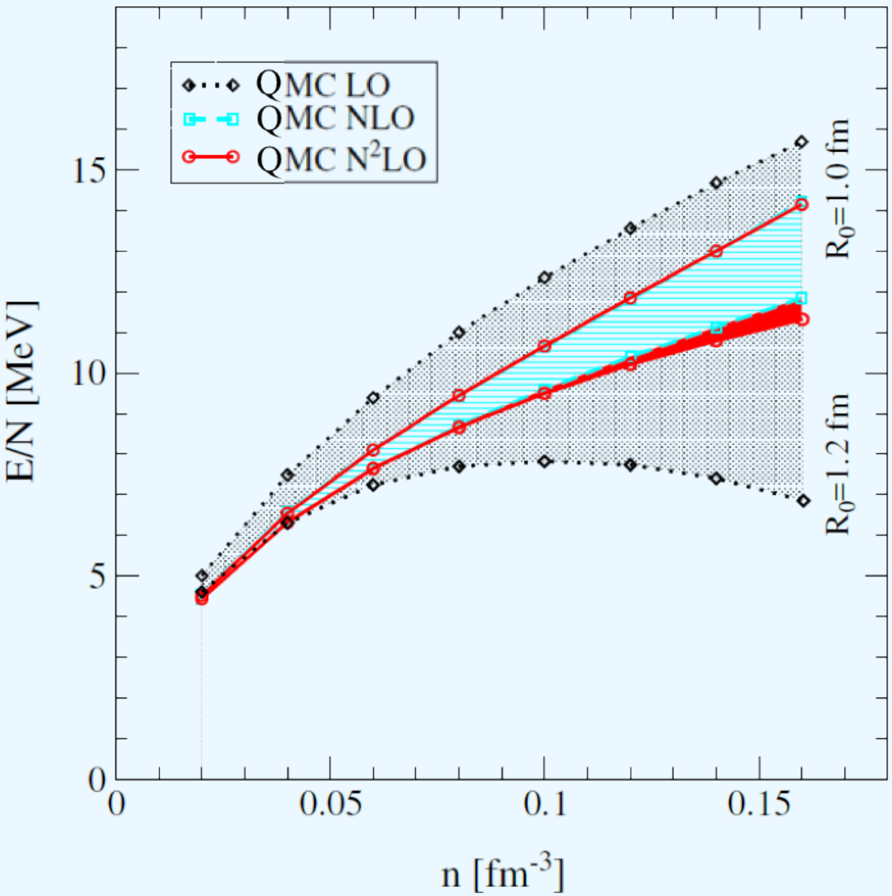
- Pion exchanges up to N^2LO are local

- This freedom can be used to remove all nonlocal operators up to N^2LO

  Gezerlis, IT, Epelbaum, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013)
  Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRC (2014)

- LECs fit to phase shifts

QMC results for NN forces



Gezerlis, IT, Epelbaum, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, PRL (2013) and PRC (2014)

NN-only calculation using AFDMC:

- QMC:
  Statistical uncertainty of points negligible
- Bands include NN cutoff variation

$$R_0 = 1.0 - 1.2\,\mathrm{fm}$$

- Order-by-order convergence up to saturation density

QMC with chiral 3N forces

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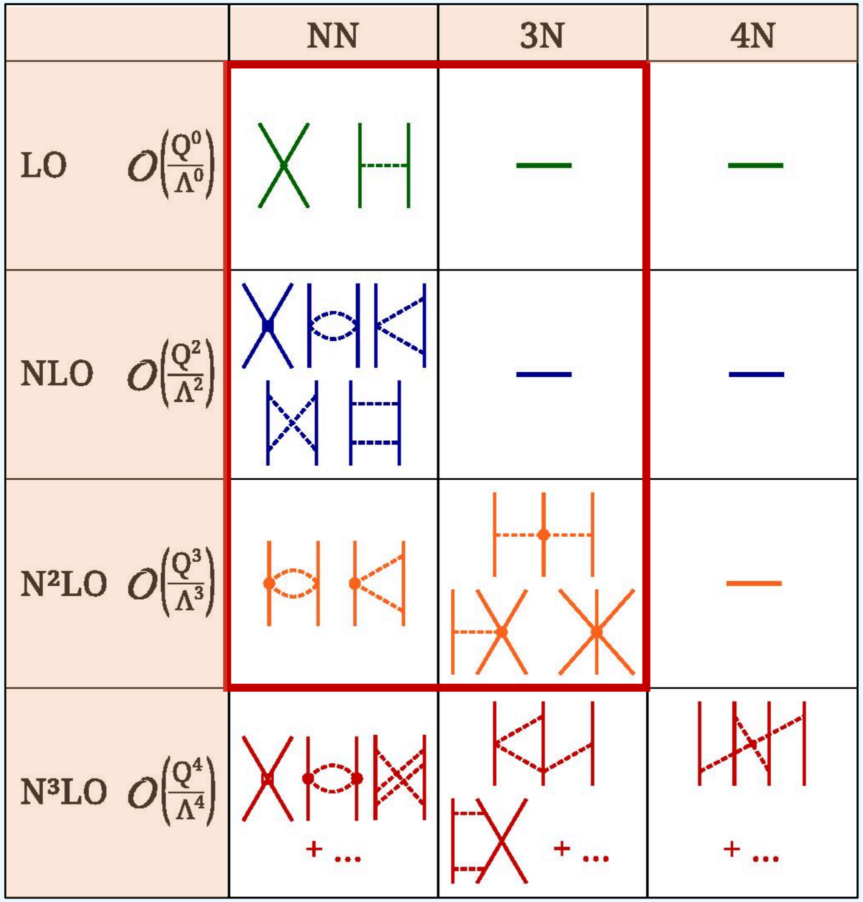
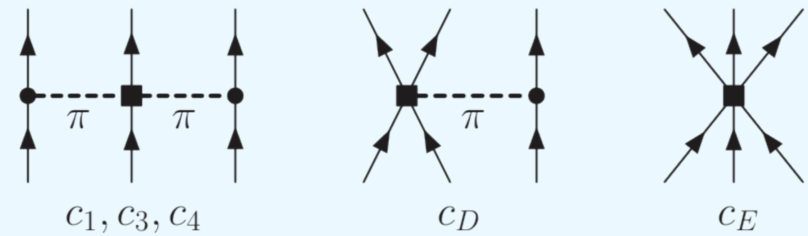


Inclusion of leading 3N forces:



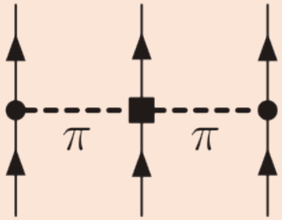
Three topologies:
- Two-pion exchange V_C
- One-pion-exchange contact V_D
- Three-nucleon contact V_E

Only two new couplings: c_D and c_E.

Fit to uncorrelated observables:
- Probe properties of light nuclei: ^{4}He E_B
- Probe T=3/2 physics: n-α scattering

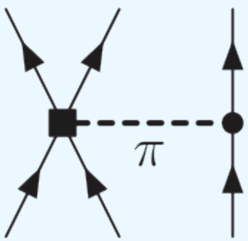
Weinberg, van Kolck, Kaplan, Savage, Wise,
Epelbaum, Kaiser, Machleidt, Meißner, Hammer ...

QMC with chiral 3N forces



c_1, c_3, c_4

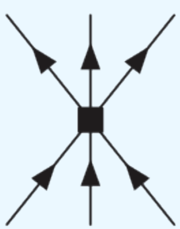
- Two-pion-exchange:
  - c_1 term: Tucson-Melbourn S-wave interaction
  - $c_{3,4}$ term: Fujita-Miyazawa interaction
- Usually most important contribution in PNM.



c_D

- Usually V_D and V_E vanish in neutron matter:
  - c_D due to spin-isospin structure
  - c_E due to Pauli principle

  see also Hebeler, Schwenk, PRC (2010)

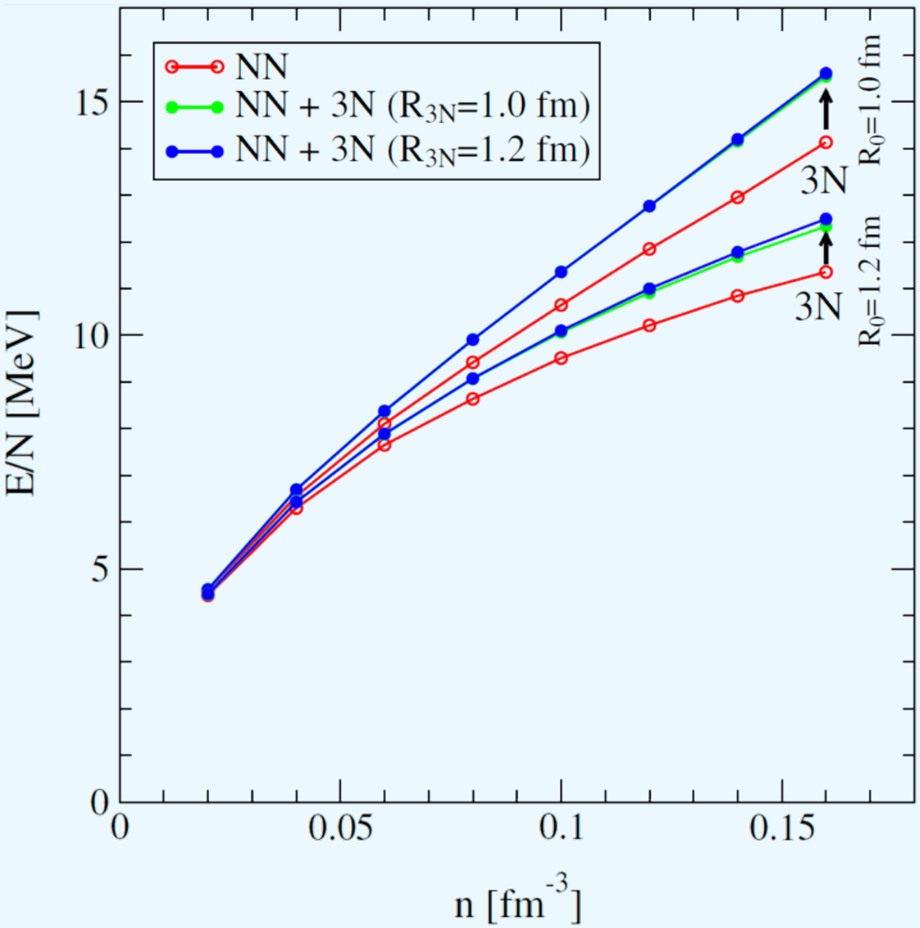


c_E

- Only true for regulator symmetric in particle labels like commonly used nonlocal regulators, not for local regulators

local 3N, see also Navratil, Few Body Syst. (2007)

QMC results with 3N TPE



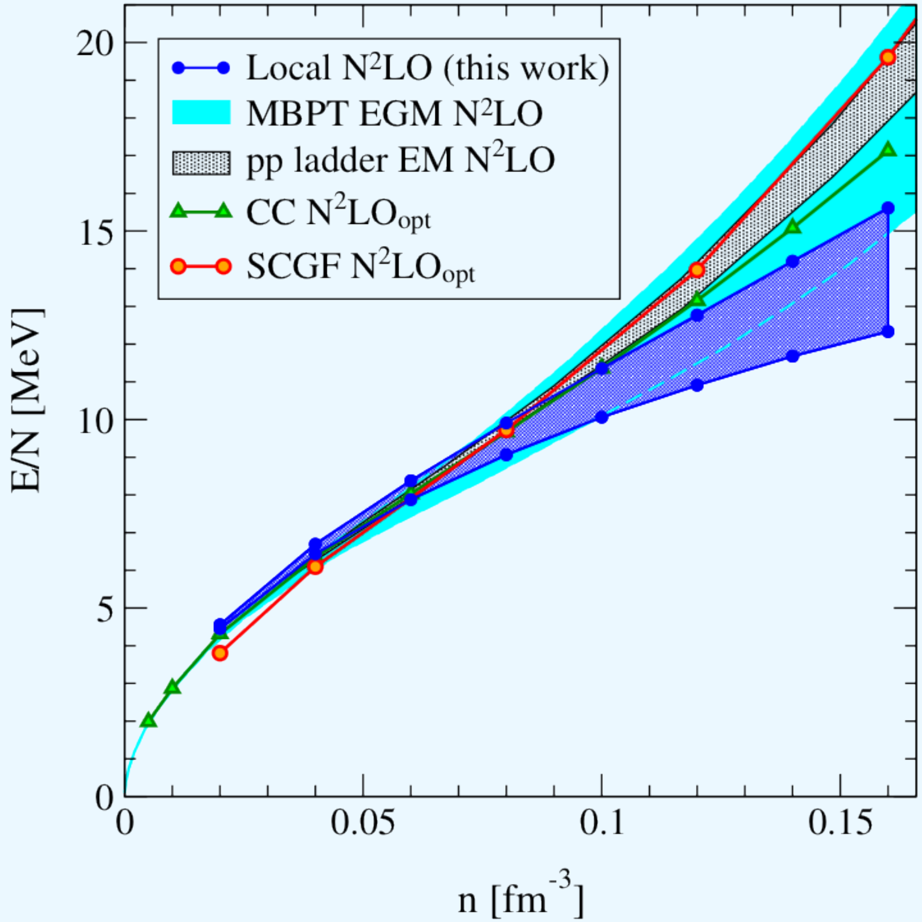
IT, Gandolfi, Gezerlis, Schwenk, PRC (2016)

- Only three-nucleon two-pion exchange

  $\sim c_1$ and c_3

- Auxiliary-field diffusion Monte Carlo:
  - NN + 3N TPE forces
  - $R_0 = 1.0 - 1.2\,\mathrm{fm}$
  - $R_{3N} = 1.0 - 1.2\,\mathrm{fm}$
- 3N cutoff dependence small
- TPE 3N contributions ≈ 1 – 2 MeV at n_0

QMC results with 3N TPE



IT, Gandolfi, Gezerlis, Schwenk, PRC (2016)

- Only three-nucleon two-pion exchange $\sim c_1$ and c_3
- Auxiliary-field diffusion Monte Carlo:
  - NN + 3N TPE forces
  - $R_0 = 1.0 - 1.2\,\mathrm{fm}$
  - $R_{3N} = 1.0 - 1.2\,\mathrm{fm}$
- 3N cutoff dependence small
- TPE 3N contributions ≈ 1 – 2 MeV at n_0
- Less than for nonlocal regulators

See also Dyhdalo et al., PRC (2016)

QMC with chiral 3N forces

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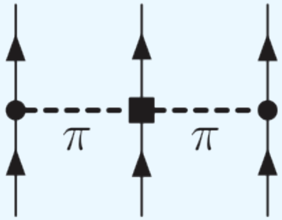
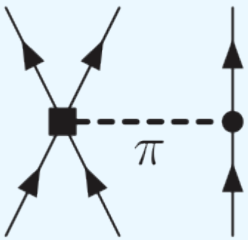
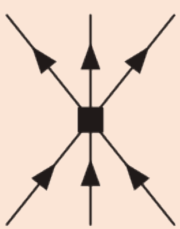


c_1, c_3, c_4



c_D



c_E

- For local regulator also V_E contributes to neutron matter:

$$V_E \sim c_E \sum_{i<j<k} \sum_{\text{cyc}} \mathcal{O}_{ijk}\, \delta_{R_{3N}}(r_{ij})\, \delta_{R_{3N}}(r_{kj})$$

- Fierz ambiguity:

$$\mathcal{O}_{ijk} = \{\mathbb{1},\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j,\, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j,\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j,\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k,\, [(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \cdot \boldsymbol{\sigma}_k][(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j) \cdot \boldsymbol{\tau}_k]\}.$$

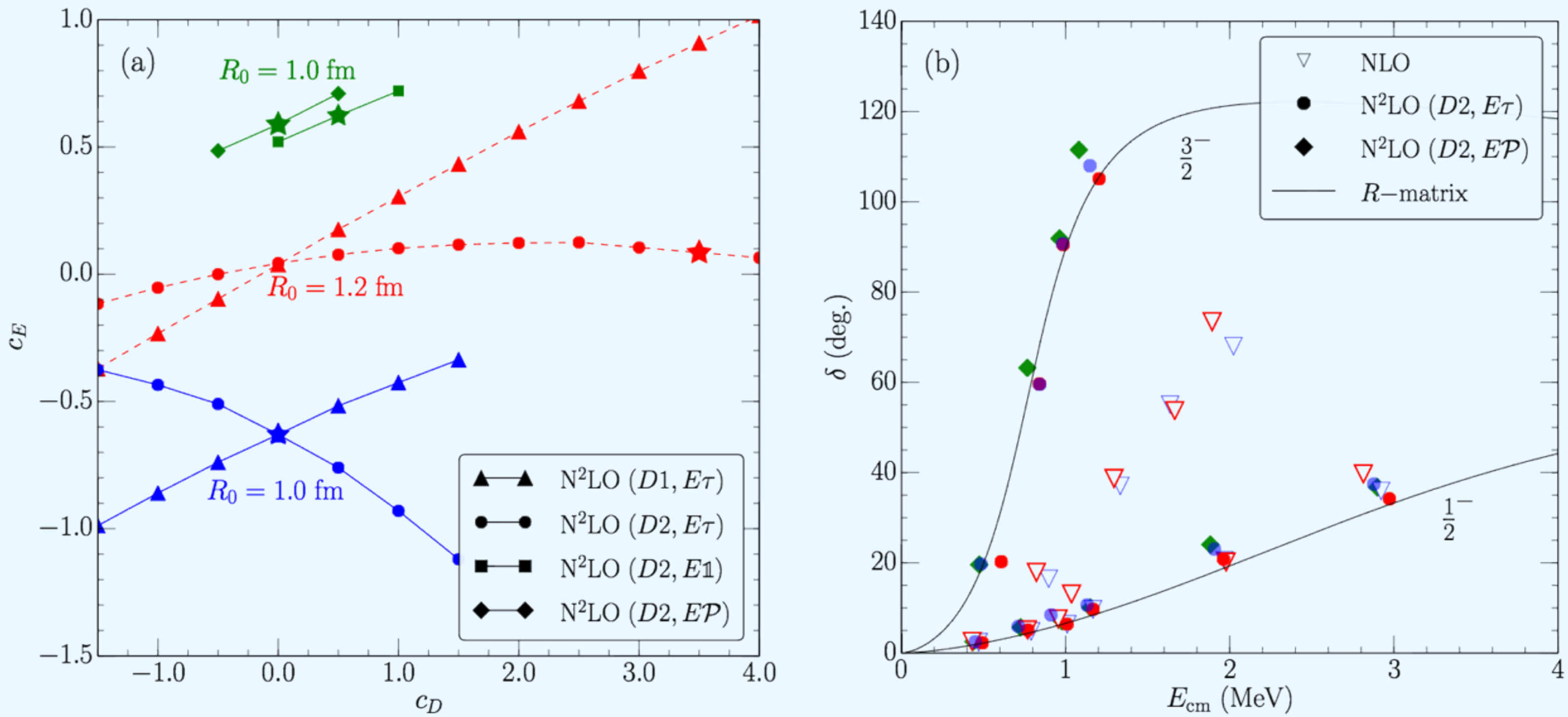
Epelbaum, Nogga, Gloeckle, Kamada, Meißner, Witala, PRC (2002)

- No Fierz rearrangement freedom for local regulators, choose different short-range structures to estimate the impact:

$$V_{E\tau} \sim \tau_i \cdot \tau_j$$
$$V_{E\mathbb{1}} \sim \mathbb{1}$$
$$V_{E\mathcal{P}} \sim \mathcal{P}_{S=1/2,\, T=1/2}$$

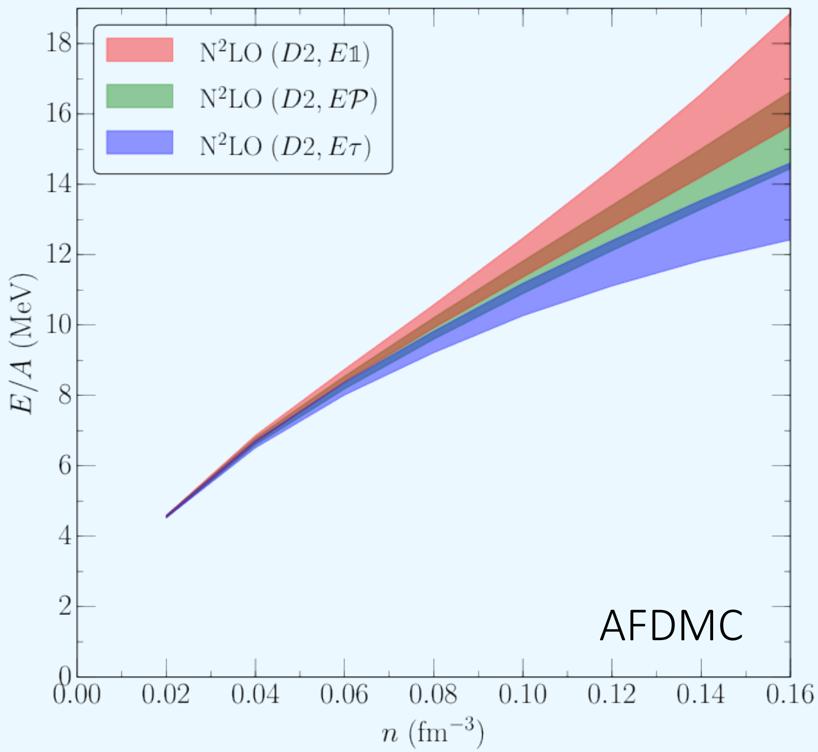
Fits of 3N LECs

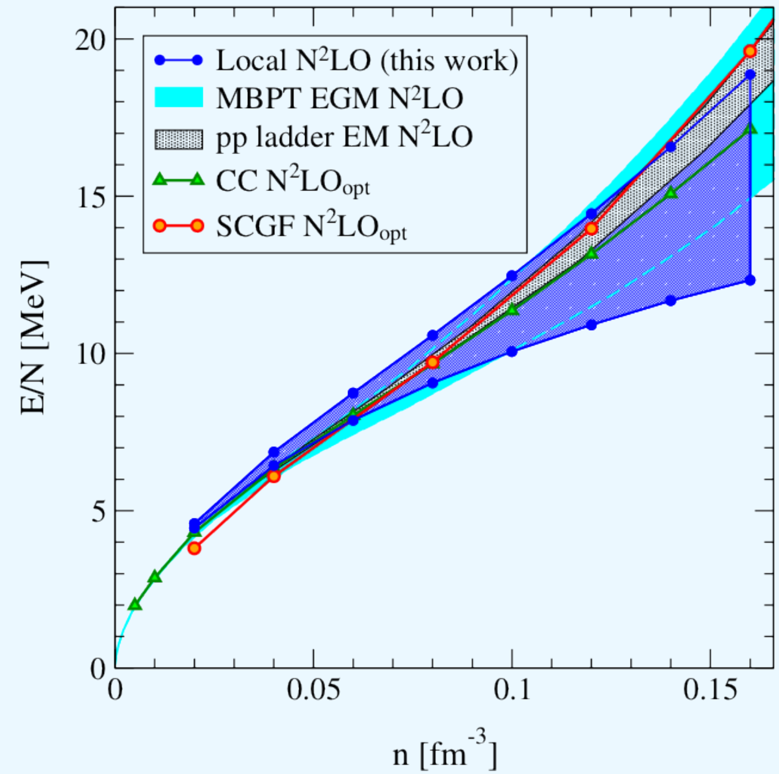
- Fit c_E and c_D to ^{4}He binding energy and n-α scattering (A≤5)



Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

Results



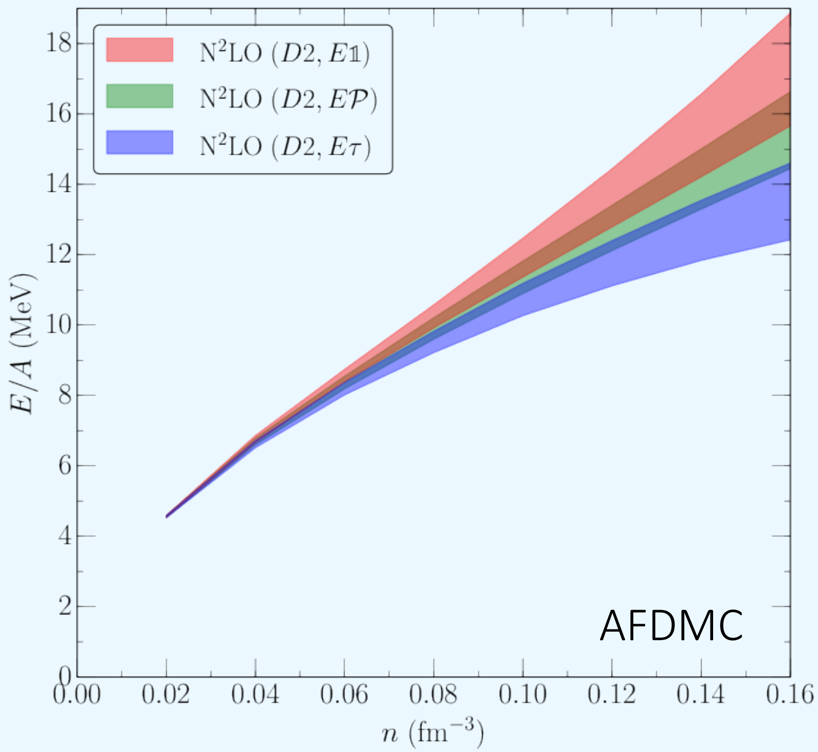


Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)

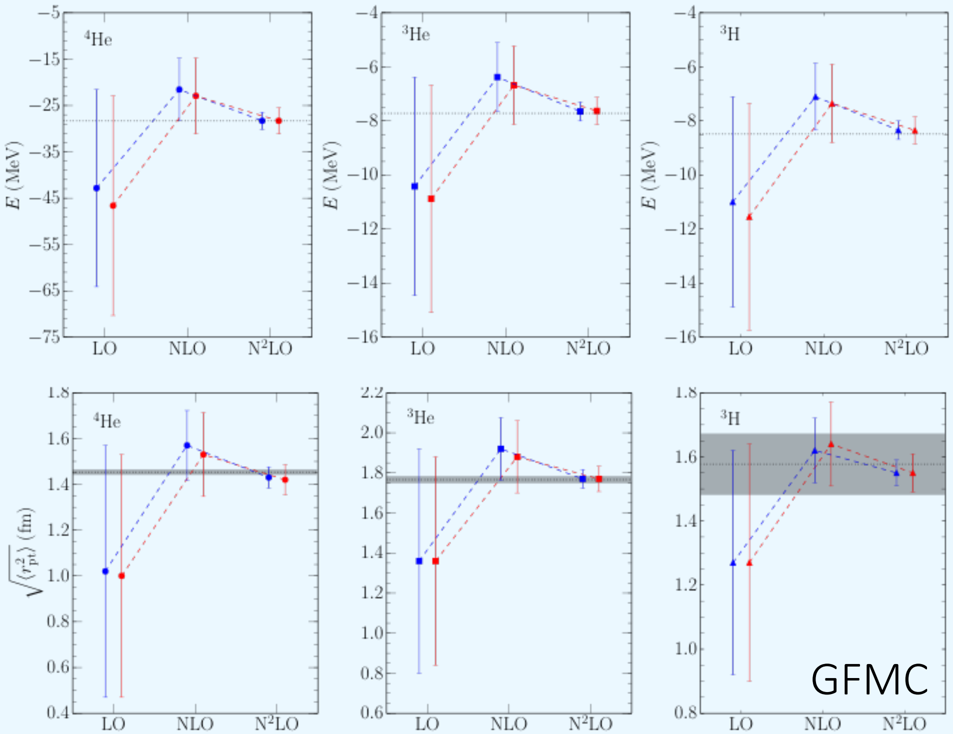
- Less repulsion from TPE, but additional contributions due to shorter-range 3N forces
- After inclusion of all contributions we find agreement of various approaches
  (different way of uncertainty estimate, see EKM, PRC 2015)

- Ambiguity in short-range structure leads to additional uncertainty.

Results



Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, PRL (2016)



Lynn, IT, Carlson, Gandolfi, Gezerlis, Schmidt, Schwenk, arXiv:1706.07668

- Chiral interactions at N^2LO simultaneously reproduce the properties of $A \leq 5$ systems and of neutron matter (uncertainty estimate as in E. Epelbaum et al, EPJ (2015))
- Commonly used phenomenological 3N interactions fail for neutron matter
  Sarsa, Fantoni, Schmidt, Pederiva, PRC (2003)

Results for heavier systems

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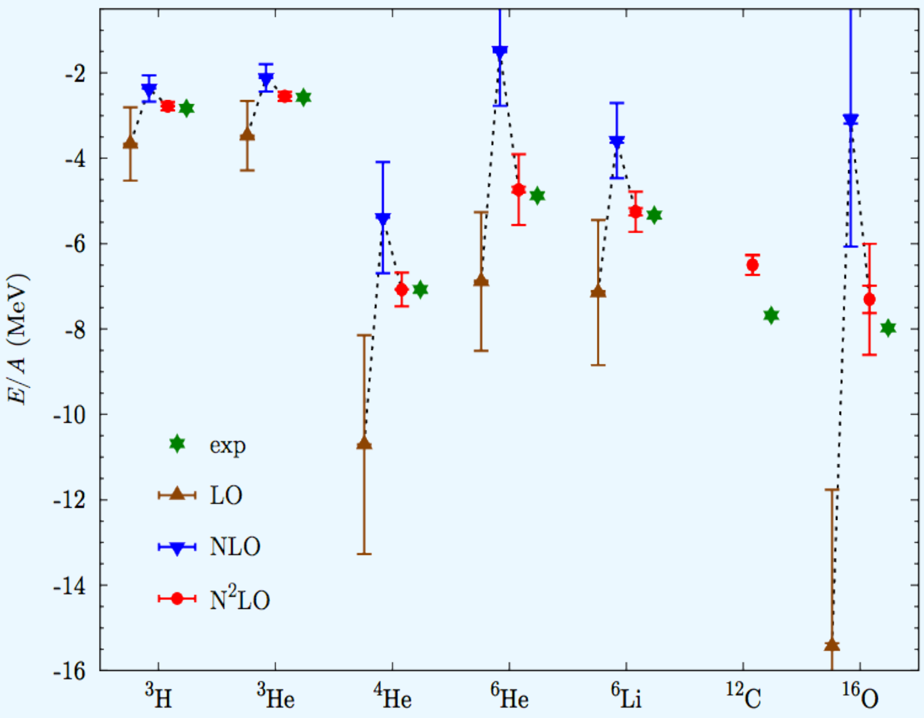
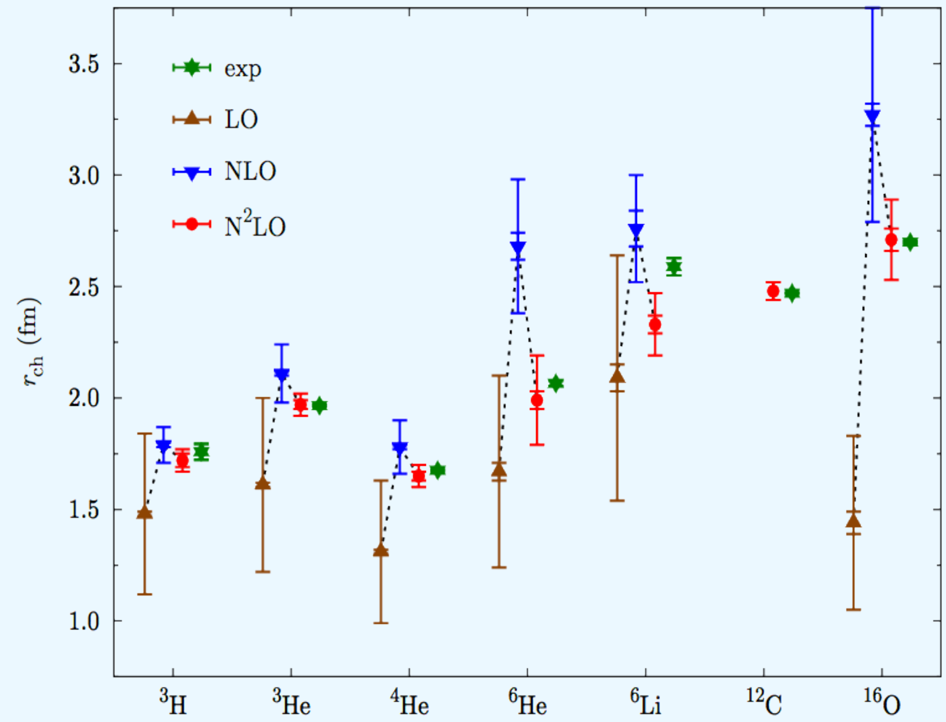
Results for AFDMC calculations of heavier systems:



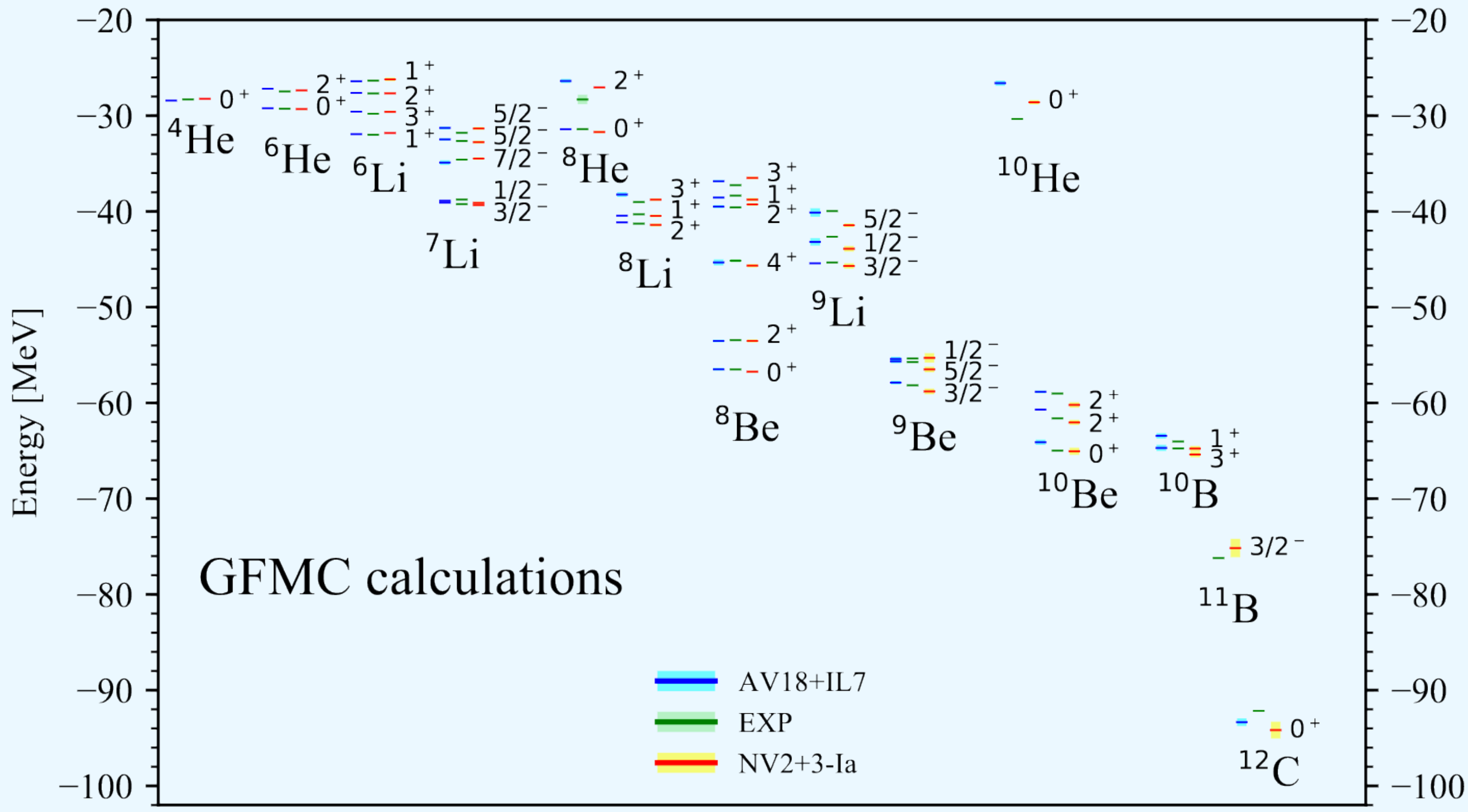


Excellent description of binding energies and charge radii for A ≤16.

Lonardoni et al., arXiv:1709.09143

Results for heavier systems

Results up to ^{12}C with explicit Deltas:



Piarulli et al., arXiv:1707.02883

Symmetry Energy

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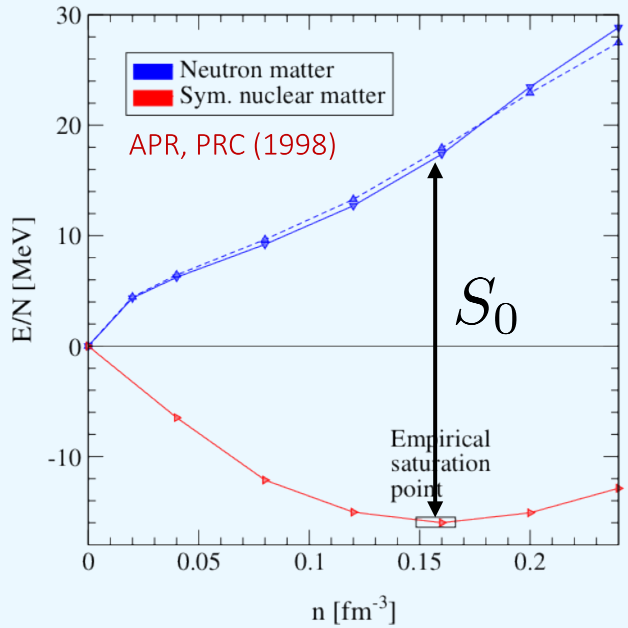
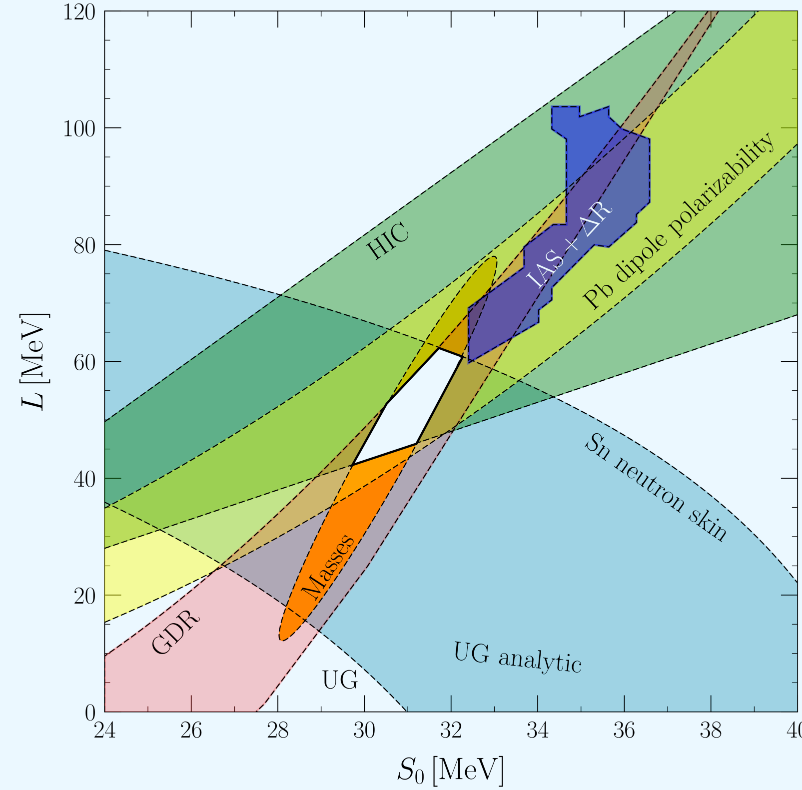




Lattimer, Lim, ApJ (2013)

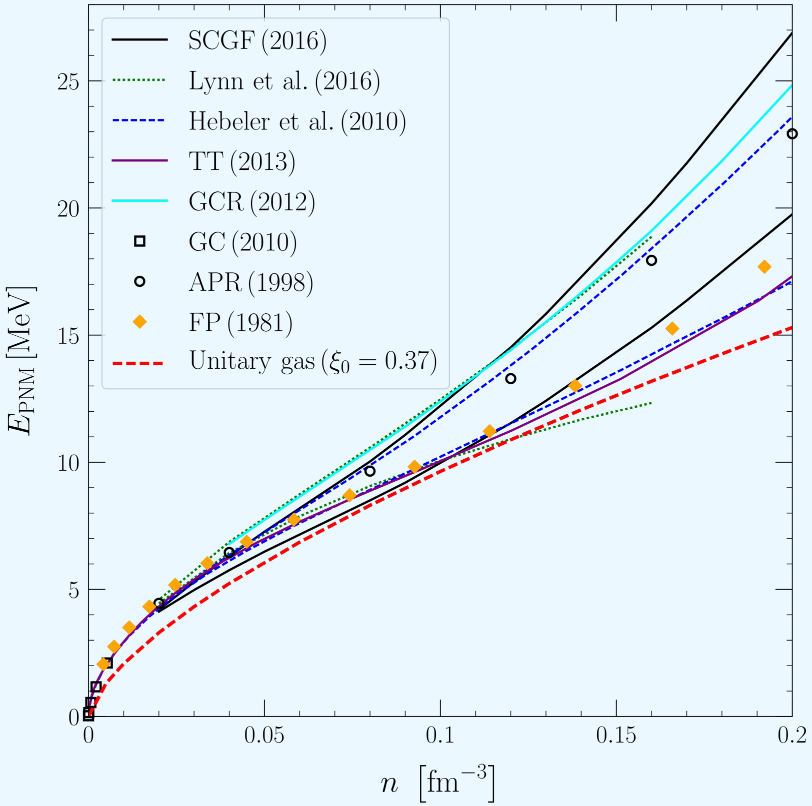
Typically, extrapolation to asym. nucl. matter from sym. nucl. matter ($u=n/n_0$):

$$E(u,x) \simeq E(u,1/2) + S(u)(1-2x)^2$$

Symmetry energy S(u) can be expanded as

$$S(u) = S_0 + \frac{L}{3}(u-1) + \frac{K_{\mathrm{sym}}}{18}(u-1)^2 + \mathcal{O}[(u-1)^3]$$

Symmetry Energy



IT, Lattimer, Ohnishi, Kolomeitsev, arXiv:1611.07133 (accepted for ApJ)

Unitary gas:

- Gas interacting via two-body interactions with infinite scattering length and vanishing effective range
- Then, system has no scale except density, and can be described by a dimensionless parameter, ξ (Bertsch parameter)
- Details of the interaction become irrelevant (universality)
- Experiment and theory: $\xi \approx 0.37$

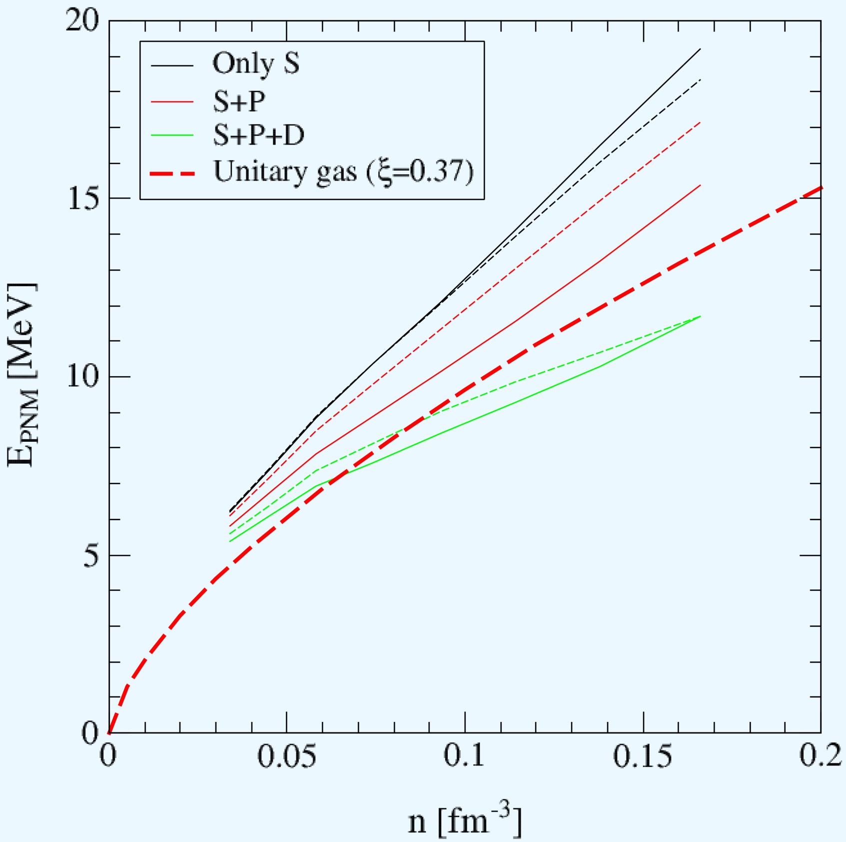
  Ku et al., Science (2012), Carlson et al. (2012)

Empirical observation:

Unitary gas energy seems to be lower bound to neutron-matter energy

- Constraints on S and L

Symmetry Energy



Justification for this conjecture:

- Finite negative scattering length and effective range effects increase the energy with respect to Unitary gas
  Carlson et al. (2012, Gandolfi et al. (2015), Schwenk, Pethick (2005)
- P- and D-wave contributions attractive but small
- 3N contributions strongly repulsive, compensate for P- and D-wave attraction

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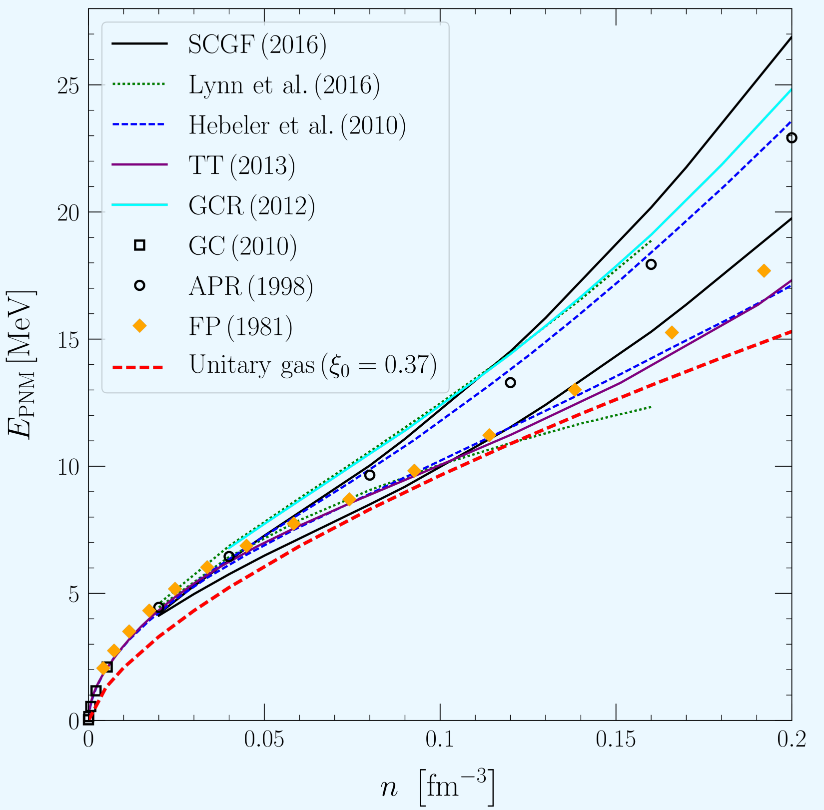
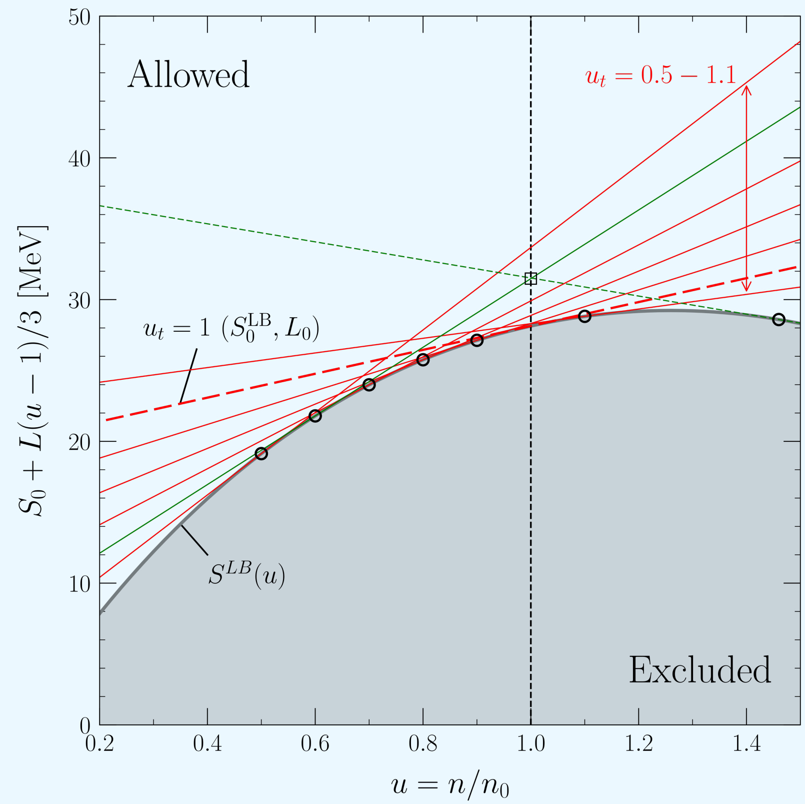
Symmetry Energy





IT, Lattimer, Ohnishi, Kolomeitsev, arXiv:1611.07133 (accepted for ApJ)

$$S_0 + \frac{L}{3}(u-1) \geq E^0_{\rm UG} u^{2/3} - \left[E_0 + \frac{K_n}{18}(u-1)^2 + \frac{Q_n}{162}(u-1)^3\right]$$

$$K_n = K_0 + K_{\rm sym}, \quad Q_n = Q_0 + Q_{\rm sym}$$

Symmetry Energy

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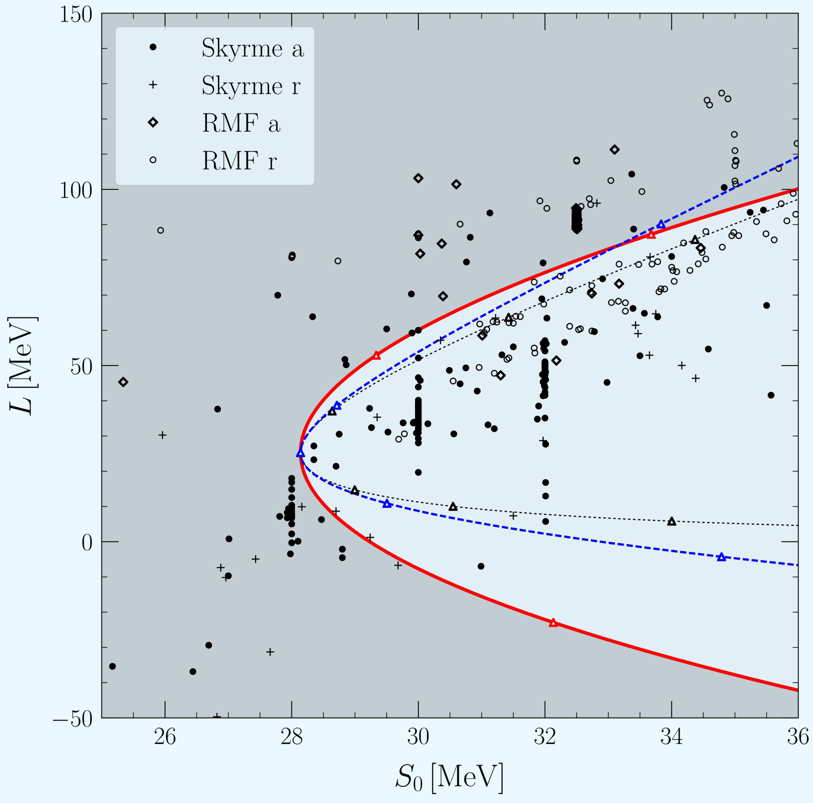
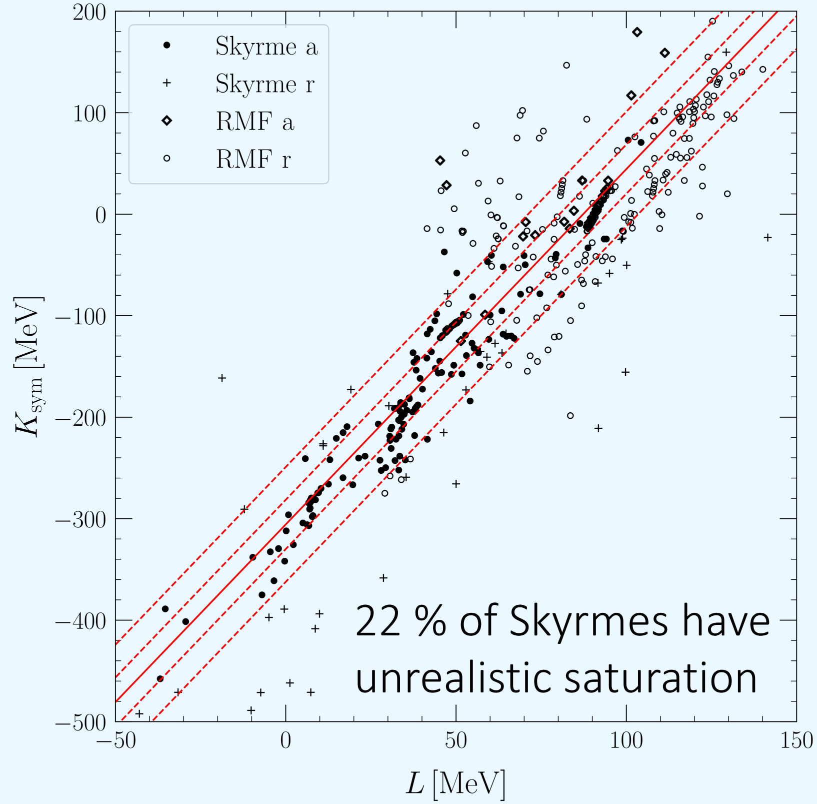
Conservative parameter choice leads to exclusion boundaries for S_0 and L :

$$E_0 = -15.5 \text{ MeV}, \; n_0 = 0.157 \text{ fm}^{-3}, \; K_n = K_0 = 270 \text{ MeV},$$
$$K_{\text{sym}} = 0, \; Q_n = 0 \text{ MeV or } -750 \text{ MeV}, \; \xi_0 = 0.365,$$

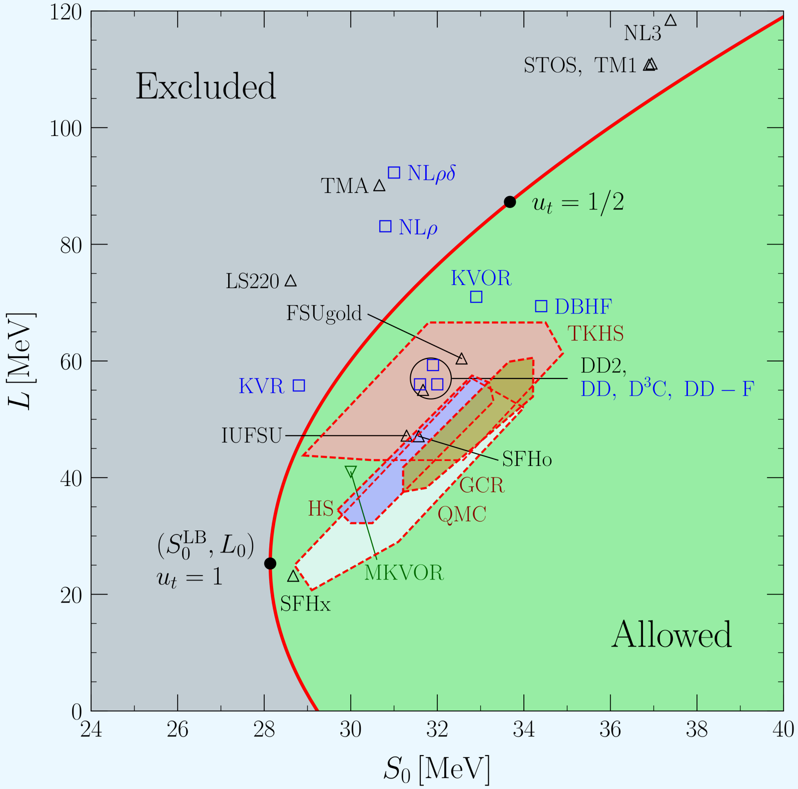
Comparsion with compilations of Dutra et al. (2012,2014):



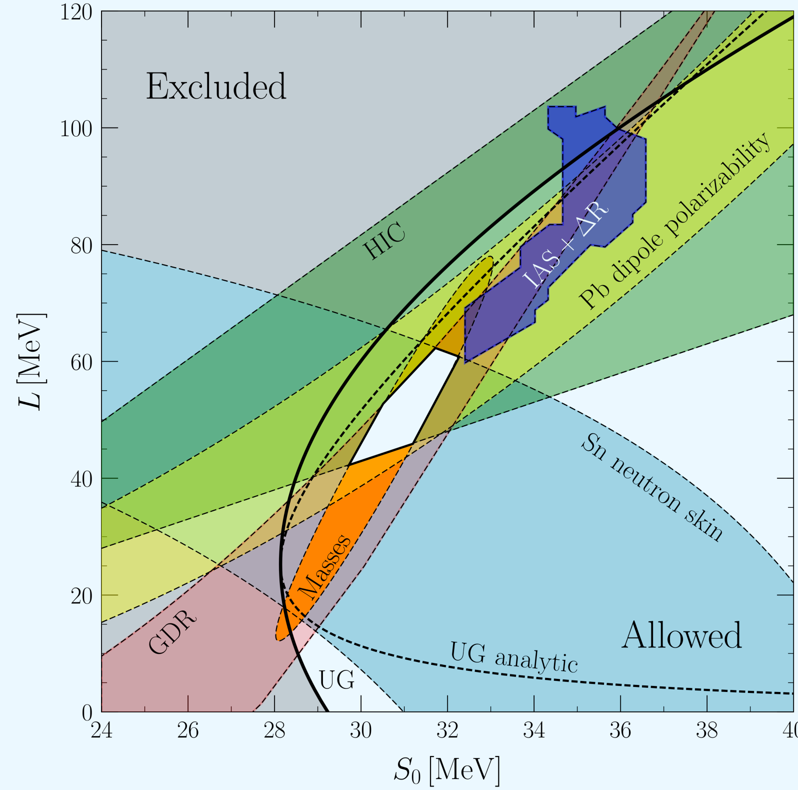


IT, Lattimer, Ohnishi, Kolomeitsev, arXiv:1611.07133 (accepted for ApJ)

Symmetry Energy



IT, Lattimer, Ohnishi, Kolomeitsev, accepted for ApJ.



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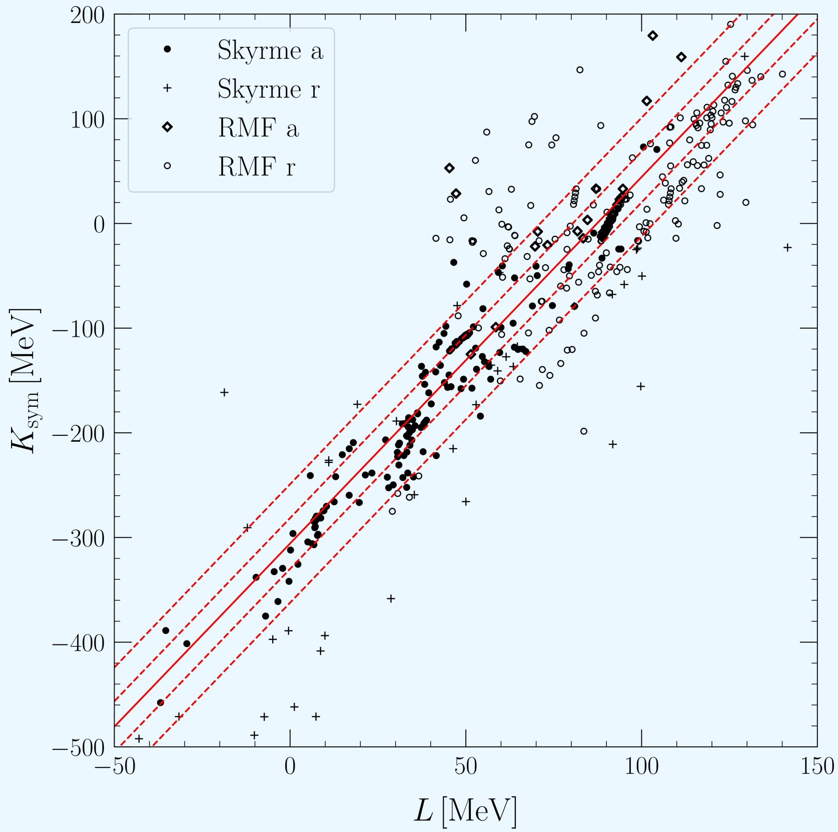
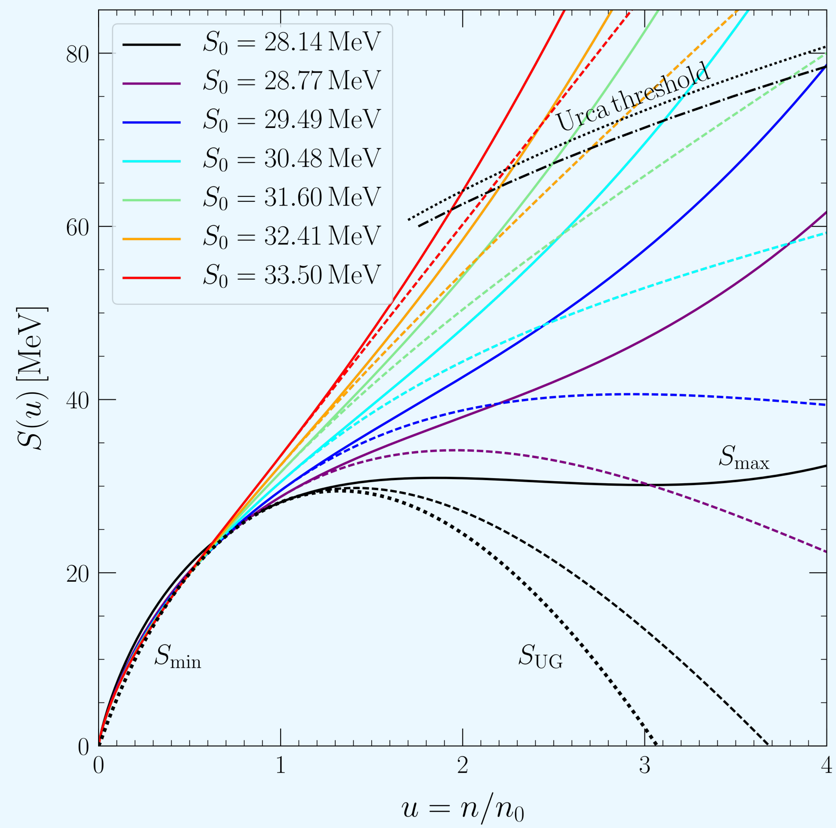
Put constraints on symmetry energy S_0
and its density dependence L.

$$S_0^{\rm LB} = 28.14 \text{ MeV} \quad \text{and} \quad L_0 = 25.28 \text{ MeV}$$

Symmetry Energy

Because for every S_0 there is an upper limit to L, when using correlations between K_{sym} and L and Q_{sym} and L, one can obtain upper limits for the symmetry energy:





IT, Lattimer, Ohnishi, Kolomeitsev, arXiv:1611.07133 (accepted for ApJ)

Summary

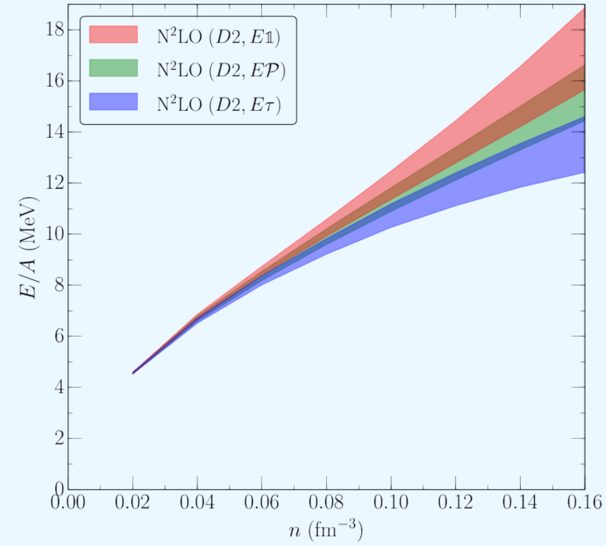
- QMC calculations of neutron matter, light nuclei, and n-alpha scattering with local chiral potentials up to N^2LO including NN and 3N forces can serve as nonperturbative benchmarks.

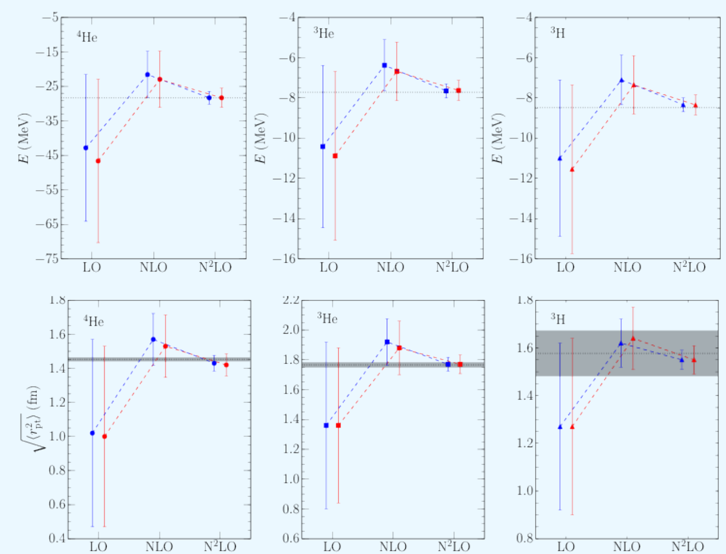
- Chiral interactions at N^2LO simultaneously reproduce the properties of $A \leq 16$ systems and of neutron matter, commonly used phenomenological 3N interactions fail.

- Further improvements necessary to calculate nuclei and neutron-matter EOS with improved uncertainties.

- Constraints on symmetry energy and its slope parameter can be obtained from lower bound of neutron-matter energy.

- High-density extension based on speed of sound.





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Thank you for your attention.