Quantum Monte Carlo calculations of neutron-rich matter

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"Bridging nuclear ab initio and EDF theories"
IPN Orsay
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Getting the TLAs out of the way

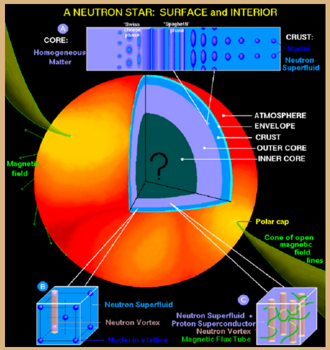
QCD = Quantum Chromodynamics

EFT = Effective Field Theory

QMC = Quantum Monte Carlo

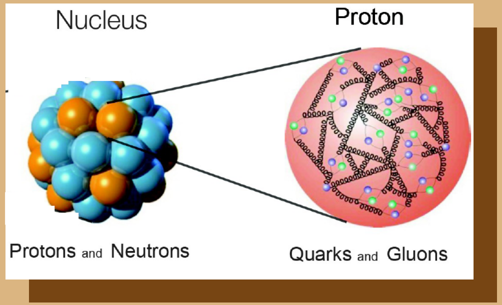
DFT = Density Functional Theory

Outline

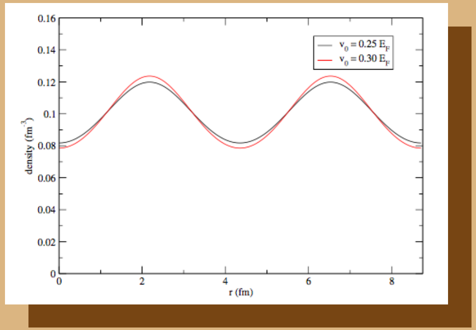


Credit: Dany Page

Motivation

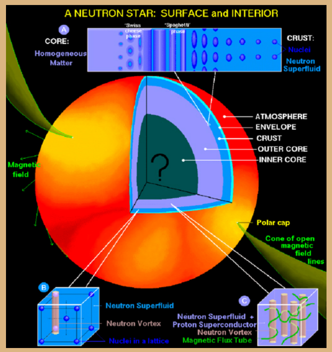


Nuclear methods



Recent results

Outline



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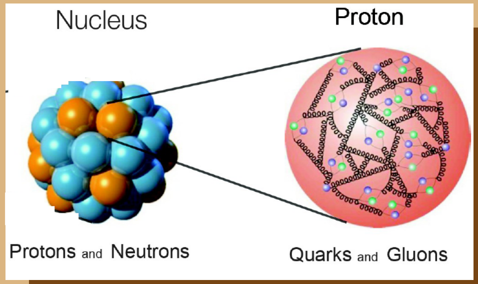

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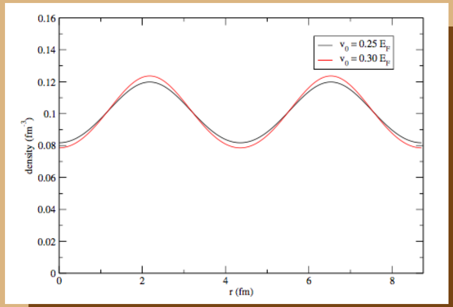
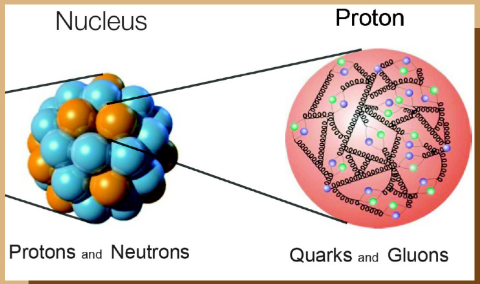


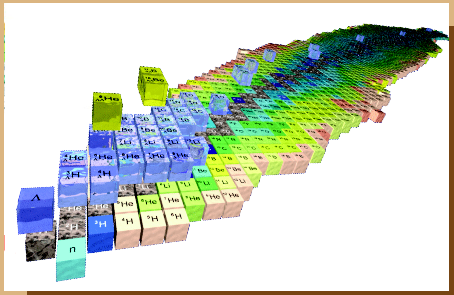
Recent results

Physical systems studied

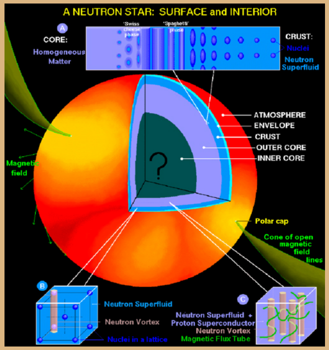
Nuclear forces



Nuclear structure



Nuclear astrophysics



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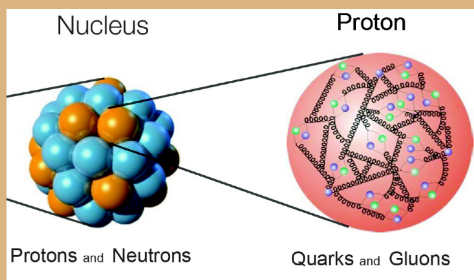

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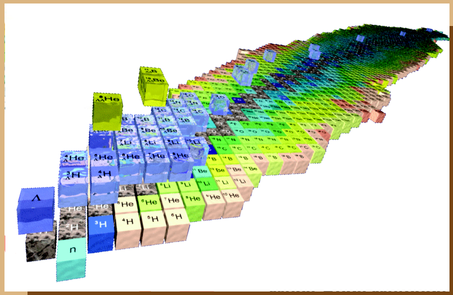

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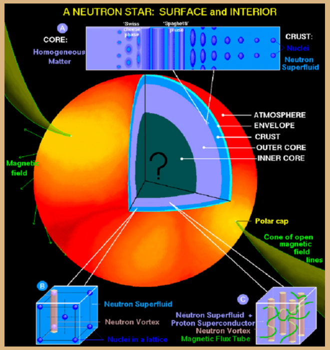
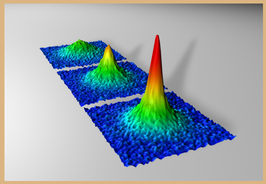
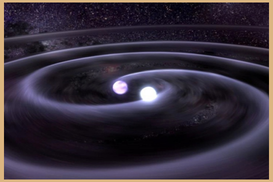
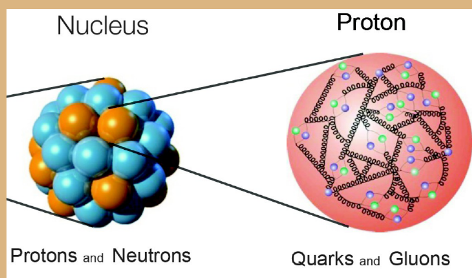




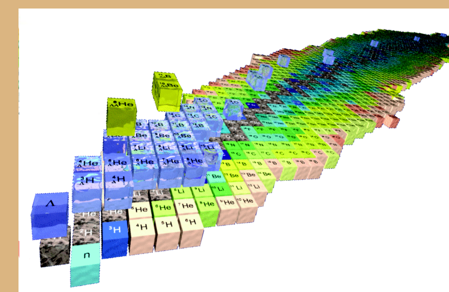


Physical systems studied

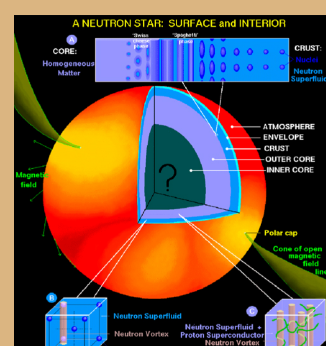
Few nucleons

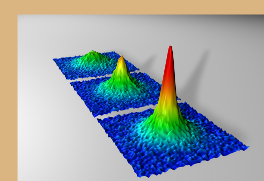


Many nucleons

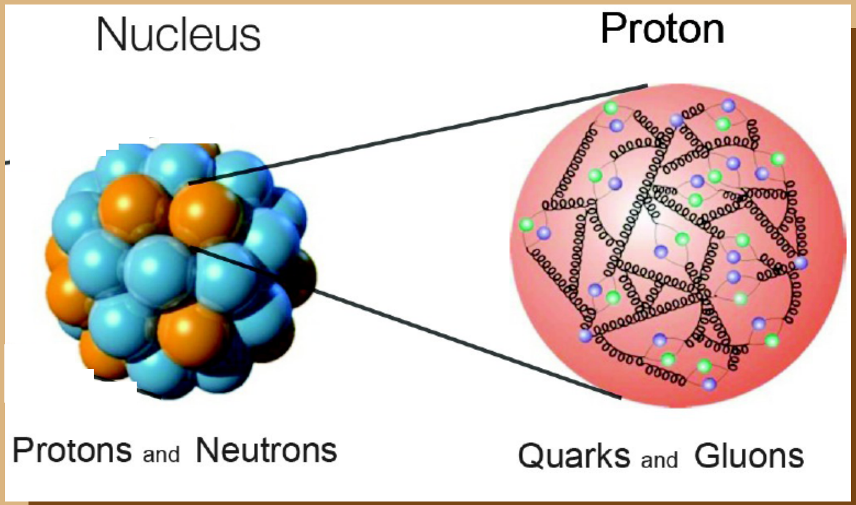


Very many nucleons



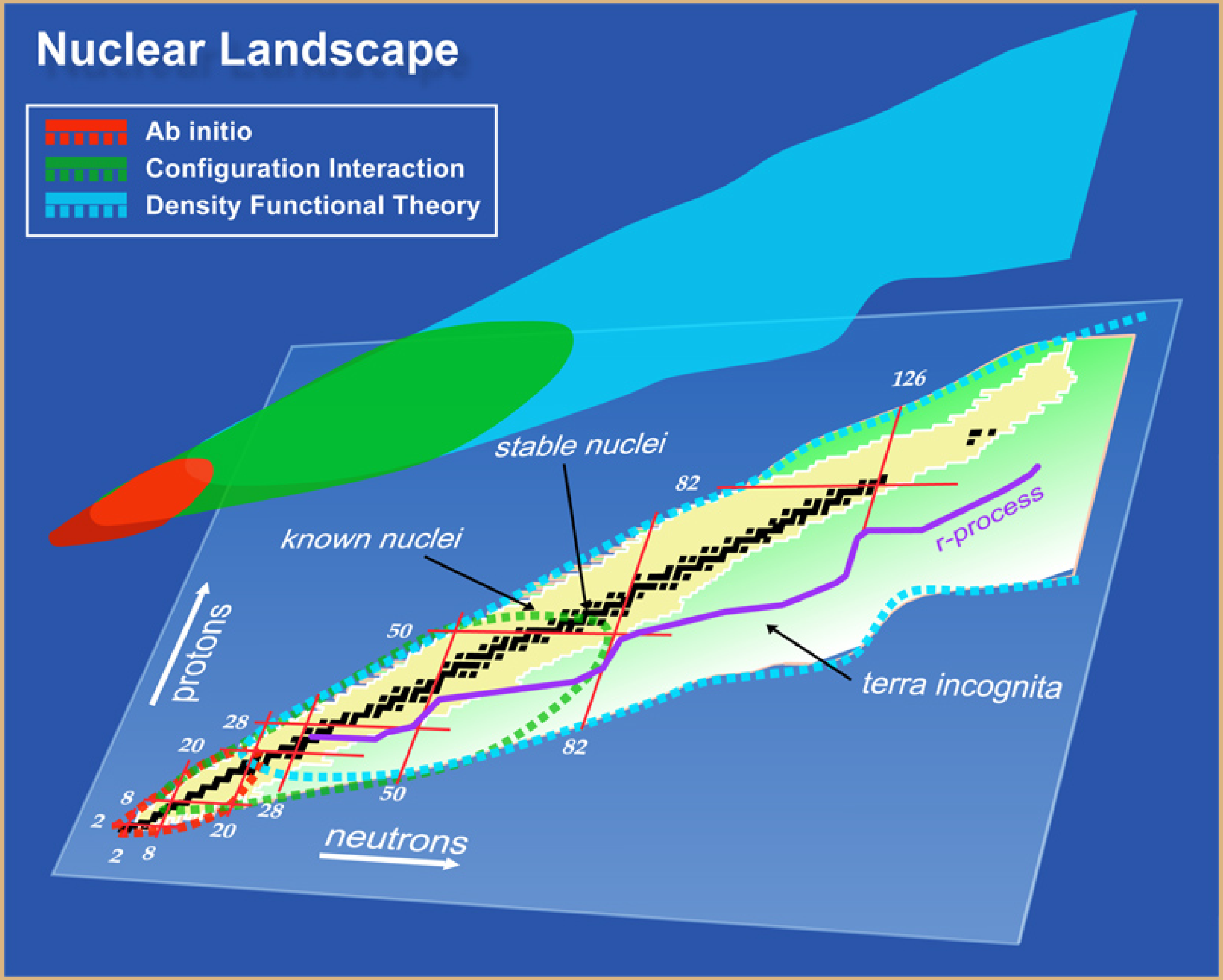


Key system: few nucleons



- No unique nuclear potential
- Preferable to use combination of phenomenological (high-quality) and more modern (conceptually clean) approach
- Desirable to make contact with underlying level
- New era, where practitioners design interactions themselves

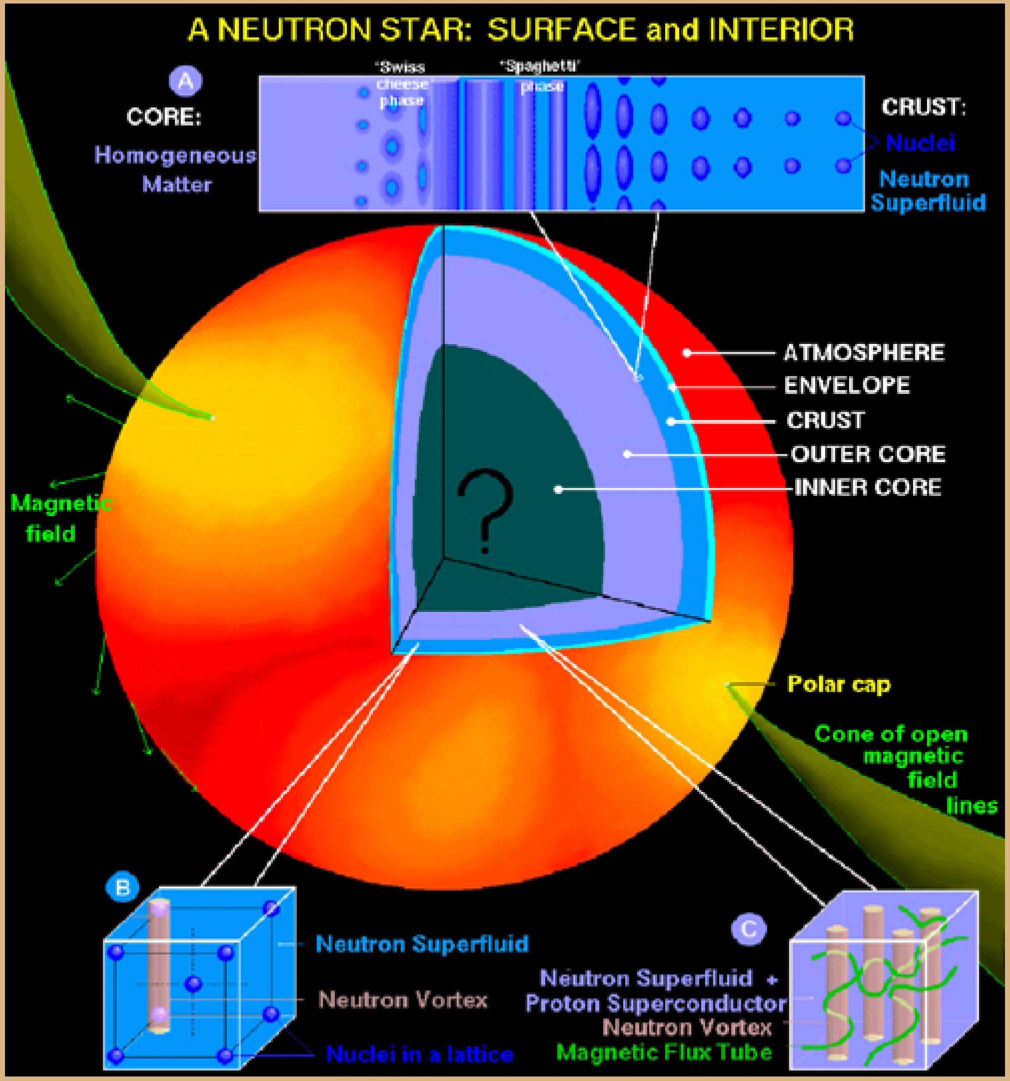
Key system: nuclei



- Experimental facilities continue to push the envelope
- Using complicated many-body methods we can try to "build nuclei from scratch"
- No universal theoretical method exists (yet?)
- Regions of overlap between different methods are crucial
- Goal is to study nuclei *from first principles* (when possible)

Key system: neutron stars

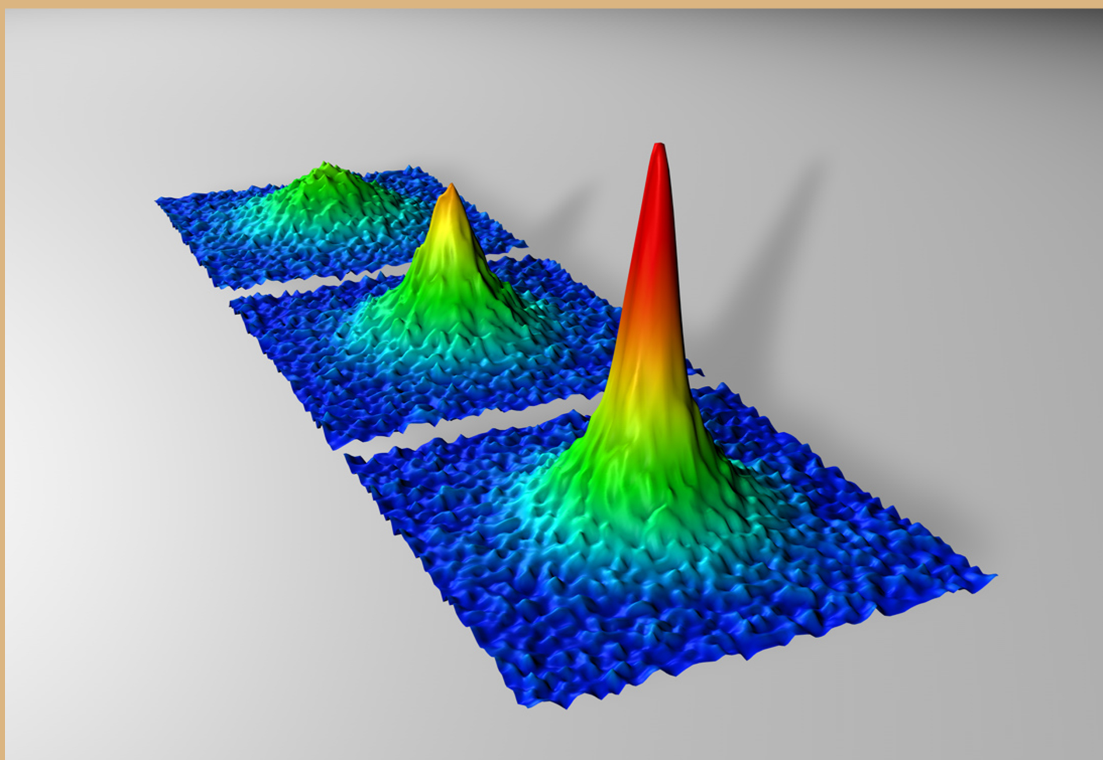
Neutron stars as ultra-dense matter laboratories



Credit: Dany Page

- Ultra-dense: 1.4 solar masses (or more) within a radius of 10 kilometres
- Terrestrial-like (outer layers) down to exotic (core) behaviour
- Observationally probed, i.e., not experimentally accessible
- Goal is to study neutron stars *from first principles* (when possible)

Key system: cold atoms

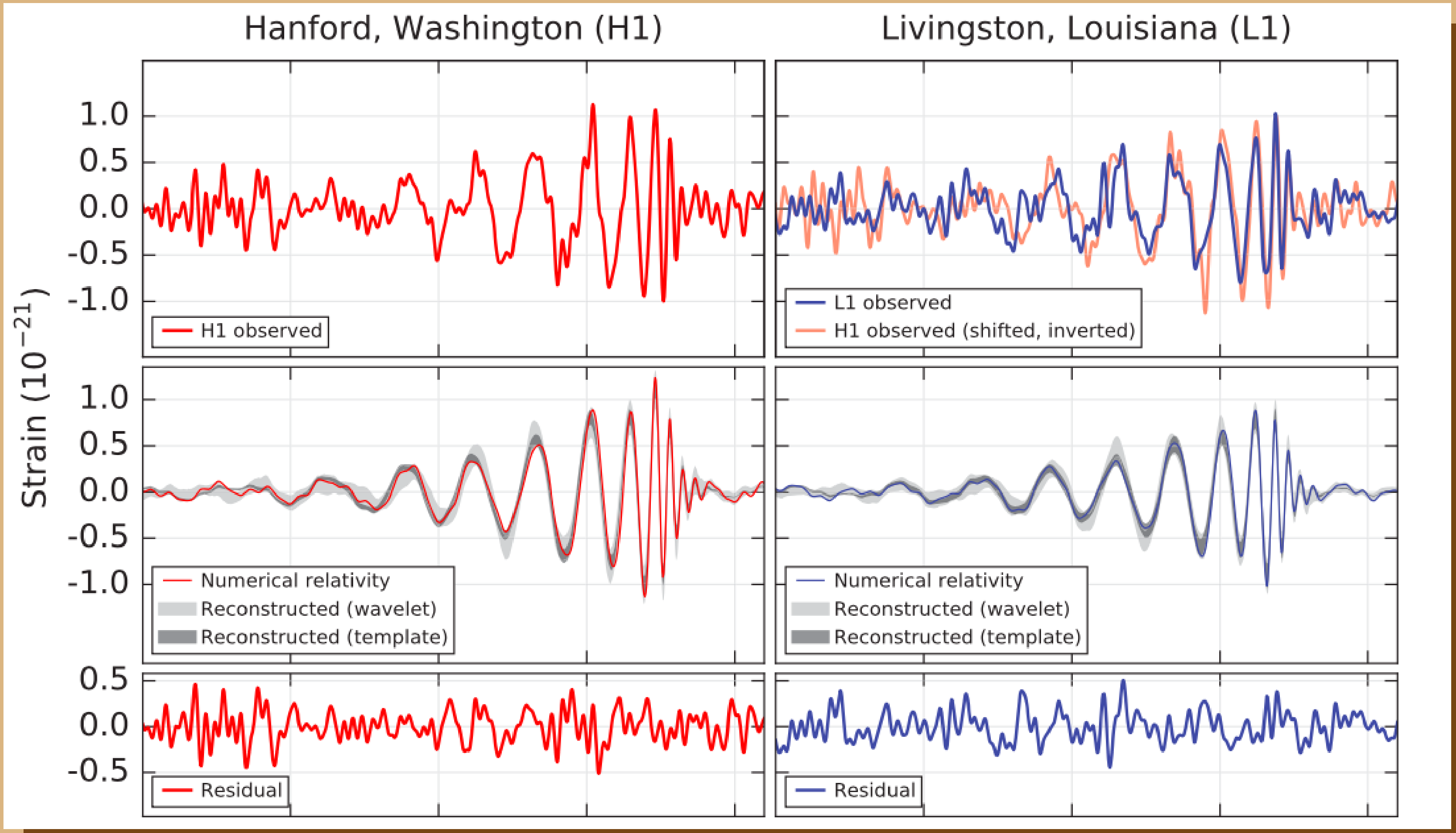


Credit: University of Colorado

- Starting in the 1990s, it became possible to experimentally probe degenerate bosonic atoms (beyond ^{4}He)
- Starting in the 2000s, the same happened for fermionic atoms (beyond ^{3}He)
- These are very cold and strongly interacting (as well as strongly correlated)
- Can be used to simulate other systems, investigating pairing, polarization, polaron physics, many species, reduced dimensionality

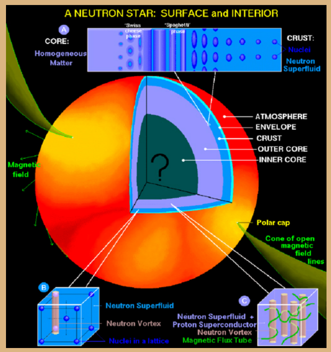
Key system: binaries

Credit: LIGO first detection PRL



- New era of gravitational wave astronomy (more like a microphone than a telescope)
- 3 (+2?) black-hole binary detections Neutron stars are lighter, but should be coming along shortly

Outline



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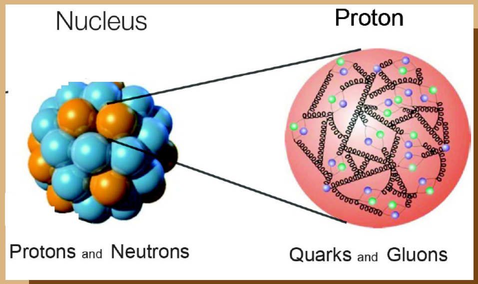
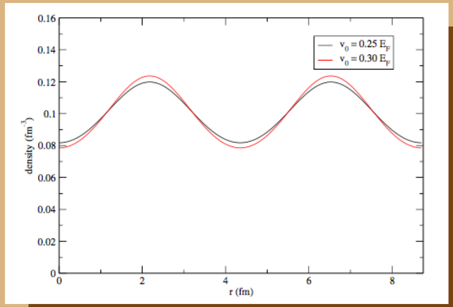
Motivation



Nuclear methods



Recent results

Nuclear interactions 1

Historically

"Effective Interactions" were employed in the context of mean-field theory.

Phenomenological

NN interaction fit to N-body experiment

Non-microscopic

NN interaction does not claim to (and will not) describe np scattering

Nuclear physics is difficult

Scattering phase shifts: different "channels" have different behavior.

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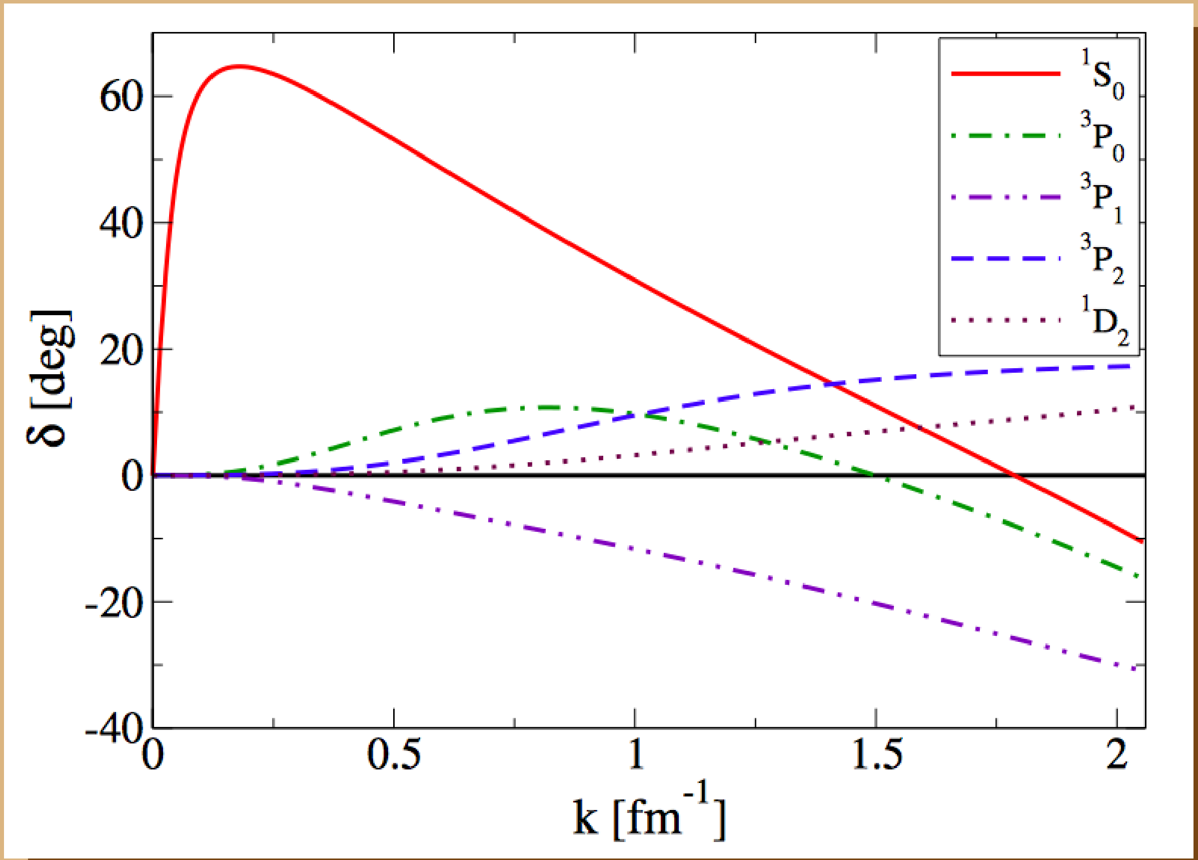

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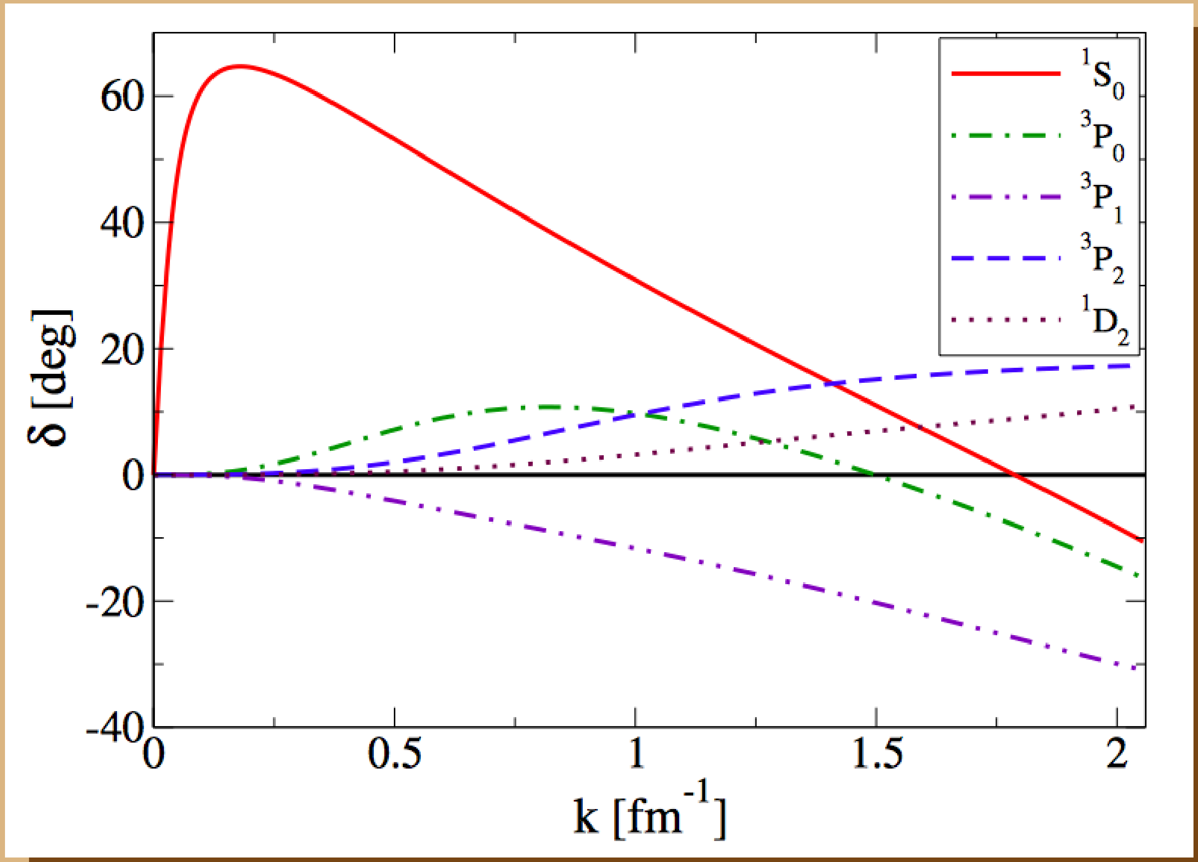
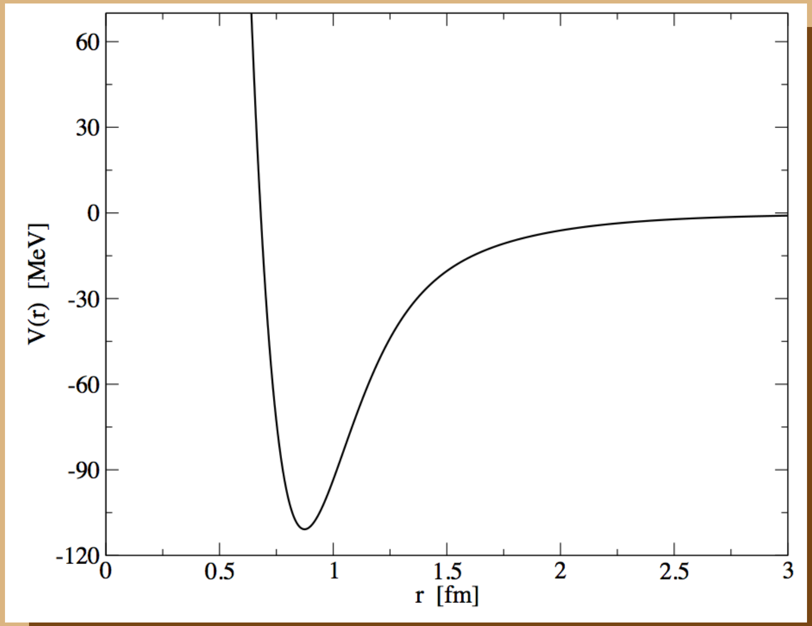


Any potential that reproduces them must be spin (and isospin) dependent

Nuclear interactions 2

Different approach: phenomenology treats NN scattering without connecting with the underlying level



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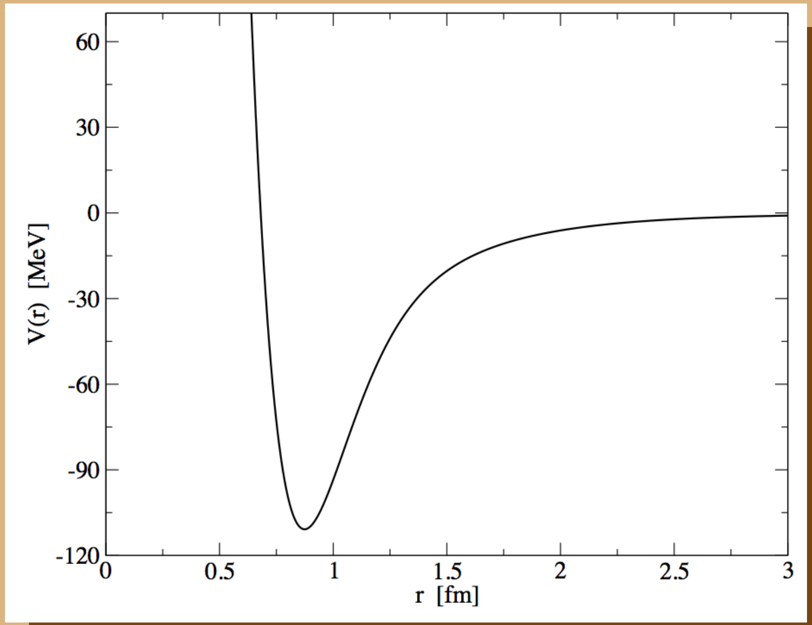
Different approach: phenomenology treats NN scattering without connecting with the underlying level

$$V_2 = \sum_{j<k} v_{jk} = \sum_{j<k} \sum_{p=1}^{8} v_p(r_{jk}) O^{(p)}(j,k)$$

$$O^{p=1,8}(j,k) = (1, \sigma_j \cdot \sigma_k, S_{jk}, \mathbf{L}_{jk} \cdot \mathbf{S}_{jk}) \otimes (1, \tau_j \cdot \tau_k)$$



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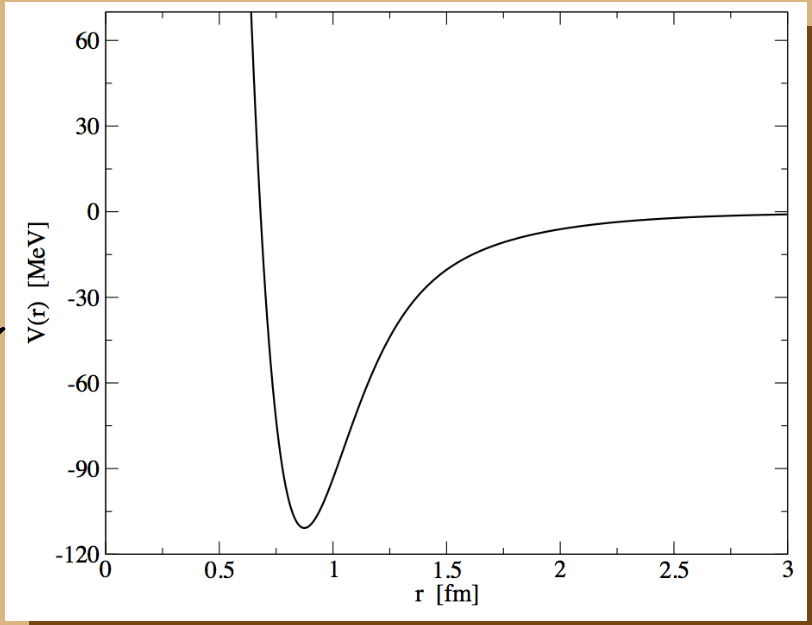

Such potentials are hard, making them non-perturbative at the many-body level (which is a problem for most methods on the market).

Softer, momentum-space formulations like CD-Bonn very popular

How to go beyond?

Historically, fit NN interaction to N-body experiment

Parallel approach, fit NN interaction to 2-body experiment, ignoring underlying level of quarks and gluons

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Historically, fit NN interaction to N-body experiment

Parallel approach, fit NN interaction to 2-body experiment, ignoring underlying level of quarks and gluons

Natural goal: fit NN interaction to 2-body experiment,
without ignoring underlying level

Chiral effective field theory

Nuclear Hamiltonian: chiral EFT

How to build on QCD in a systematic manner?

Exploit separation of scales: $a_{^1S_0} = (11\ \mathrm{MeV})^{-1}$

$$m_\pi = 140\ \mathrm{MeV}$$

$$\Lambda_\chi \approx m_\rho \approx\ 800\ \mathrm{MeV}$$

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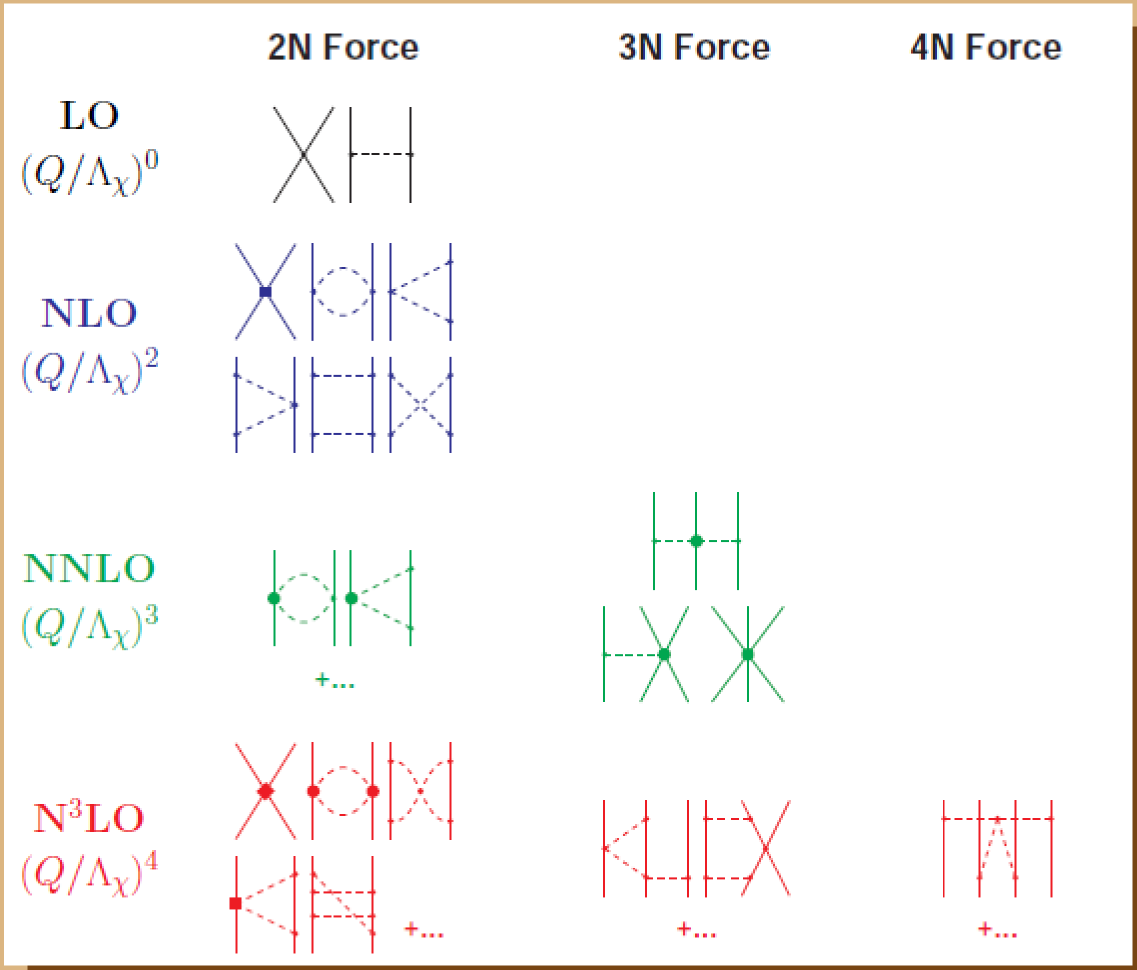
Chiral Effective Field Theory approach:

Use nucleons and pions as degrees of freedom

Systematically expand in $\dfrac{Q}{\Lambda_\chi}$

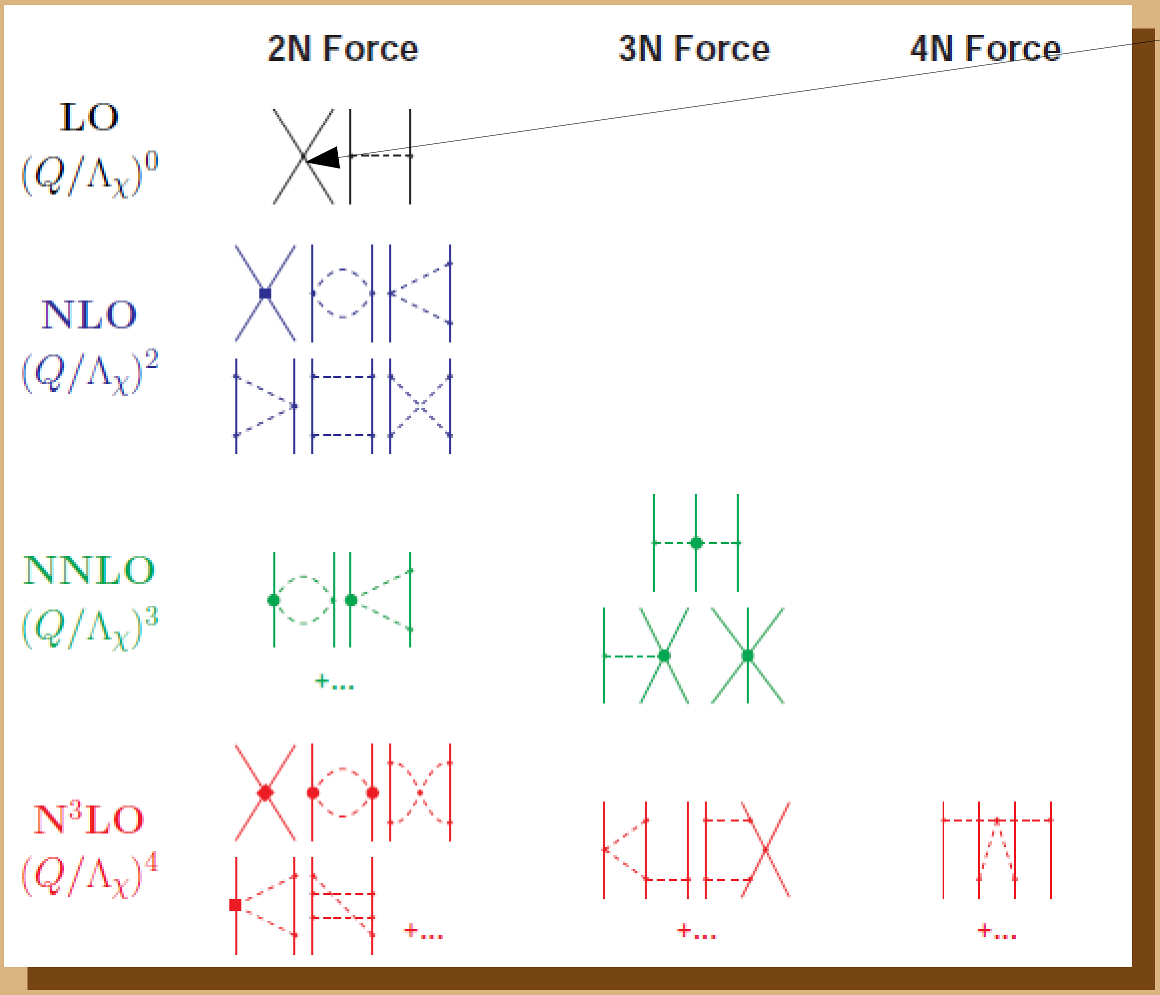
Program introduced by S. Weinberg, now taken over by the nuclear community

Nuclear interactions 3



- Attempts to connect with underlying theory (QCD)
- Systematic low-momentum expansion
- Consistent many-body forces
- Low-energy constants from experiment or lattice QCD
- Until recently non-local in coordinate space, so unused in continuum QMC
- Power counting's relation to renormalization still an open question

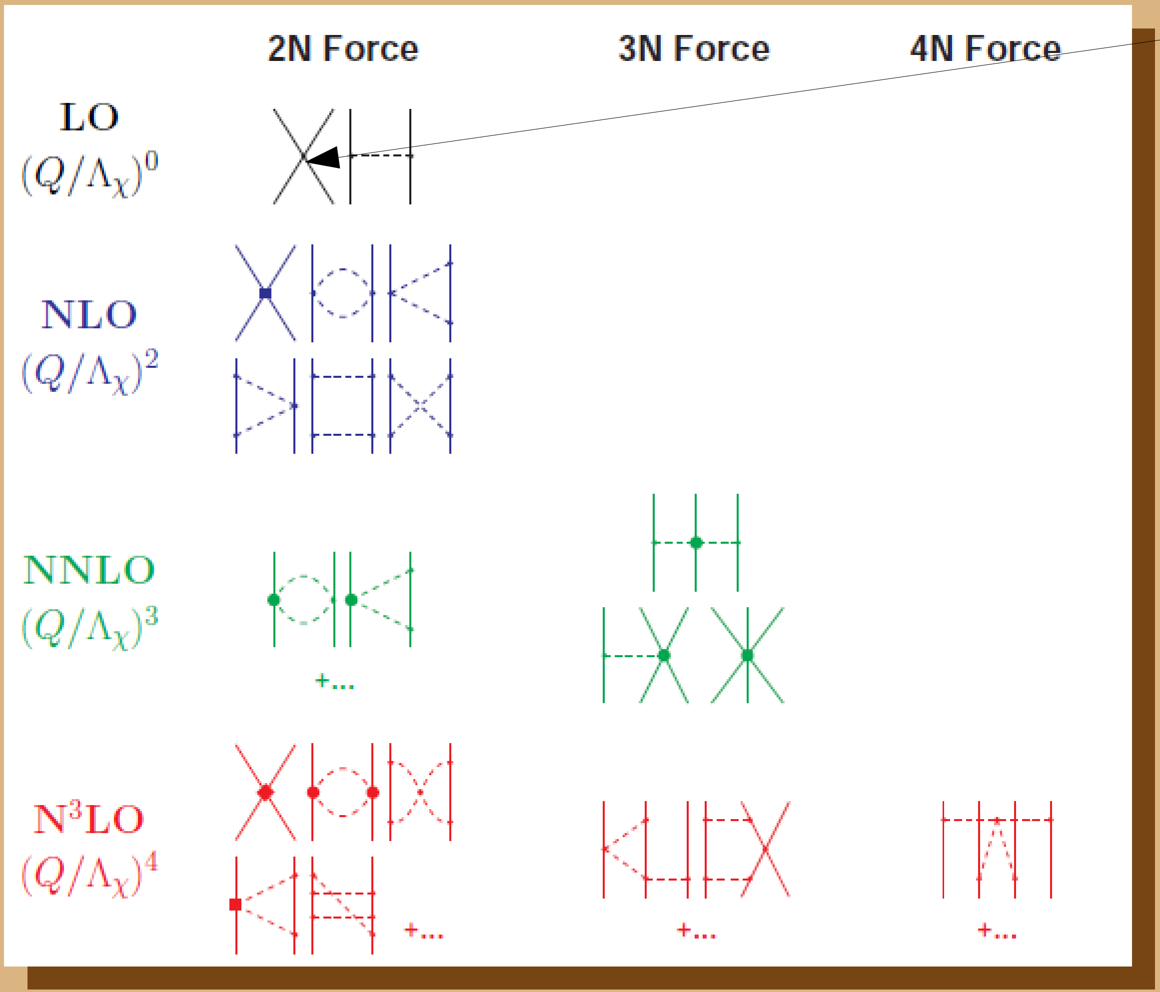
Nuclear Hamiltonian: chiral EFT



$$V_{\rm ct}^{(0)} = C_S + C_T \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

Merely the standard choice.

Nuclear Hamiltonian: chiral EFT



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Merely the standard choice.

Actually 4 terms in full set
consistent with the symmetries of QCD

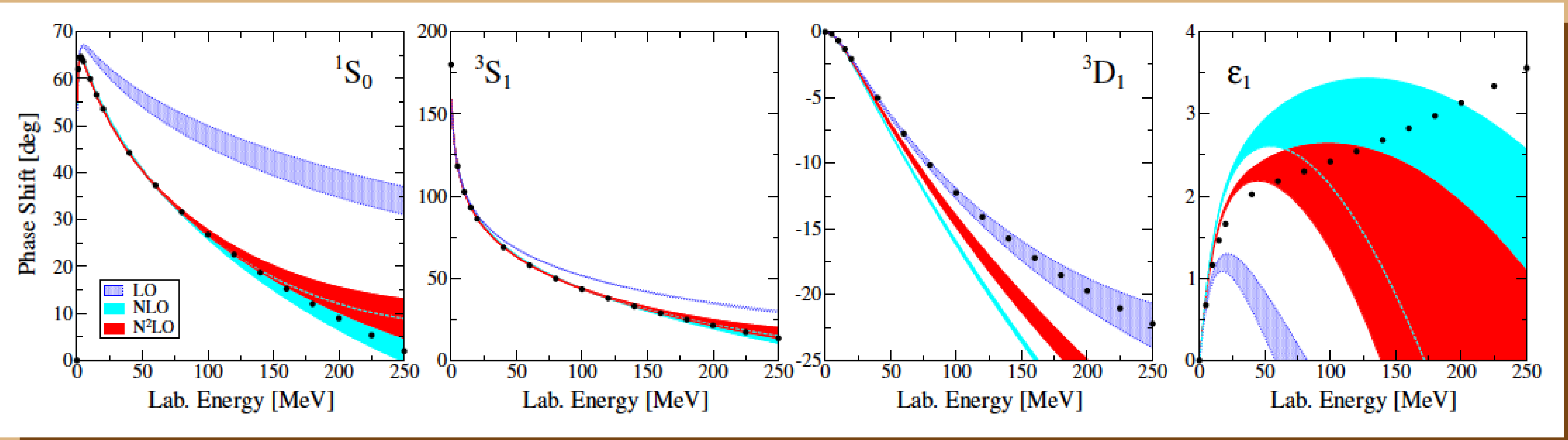
$$V_{\rm ct}^{(0)} = C_1 + C_2 \; \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + C_3 \; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + C_4 \; \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Pick 2 and antisymmetrize

A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. **111**, 032501 (2013).

A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. C **90**, 054323 (2014).

Local chiral EFT



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J. E. Lynn, J. Carlson, E. Epelbaum, S. Gandolfi, **A. Gezerlis**, K. E. Schmidt, A. Schwenk, I. Tews, Phys. Rev. Lett. **113**, 192501 (2014)

I. Tews, S. Gandolfi, **A. Gezerlis**, A. Schwenk, Phys. Rev. C **93**, 024305 (2016)

J. E. Lynn, I. Tews, J. Carlson, S. Gandolfi, **A. Gezerlis**, K. E. Schmidt, A. Schwenk, I. Tews, Phys. Rev. Lett. **116**, 062501 (2016)

P. Klos, J. E. Lynn, I. Tews, S. Gandolfi, **A. Gezerlis**, H.-W. Hammer, and A. Schwenk, Phys. Rev. C, **94**, 054005 (2017)

**But even with the interaction in place,
how do you solve the many-body problem?**

Nuclear many-body problem

$$H\Psi = E\Psi$$

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where $$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \cdots$$

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so

$$H\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A; s_1, \cdots, s_A; t_1, \cdots, t_A) = E\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A; s_1, \cdots, s_A; t_1, \cdots, t_A)$$

i.e. $2^A \binom{A}{Z}$ complex coupled second-order differential equations

Main many-body methods employed (by me)

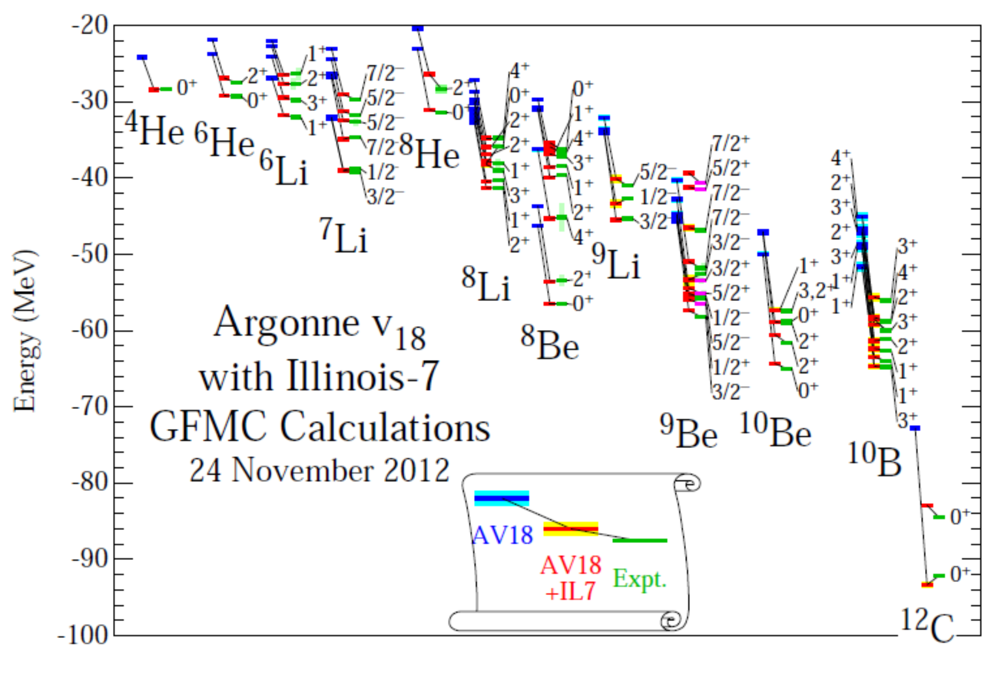
Two complementary methods

Quantum Monte Carlo

- Microscopic
- Computationally demanding (3N particle coordinates + spins)
- Limited to smallish N

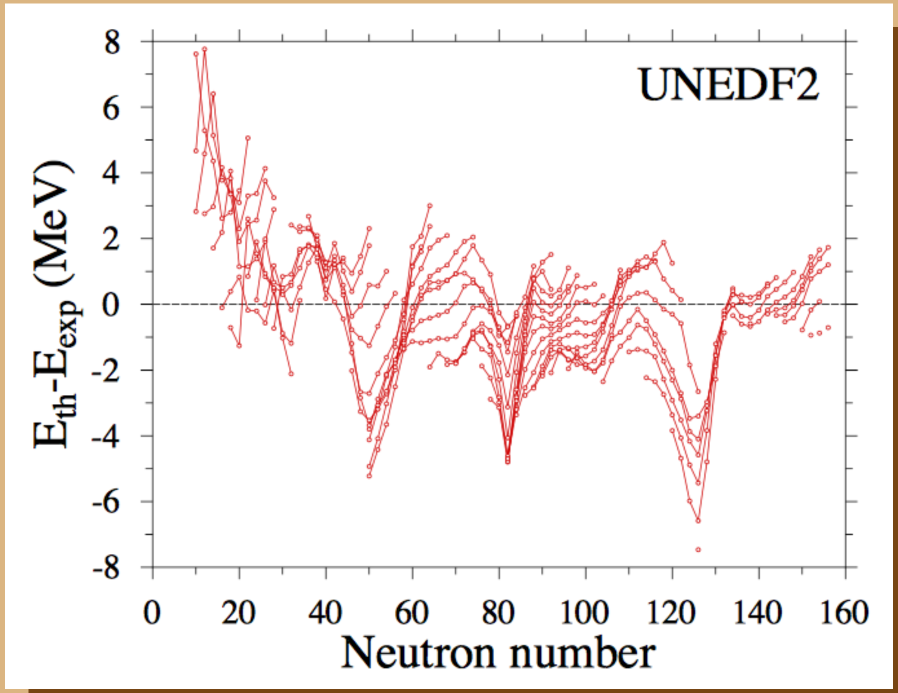
$$\Psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H}-E_T)\tau}\Psi_V$$

$$\to \alpha_0 e^{-(E_0-E_T)\tau}\Psi_0$$



Credit: Steve Pieper

Two complementary methods



Credit: W. Nazarewicz

Density Functional Theory

- More phenomenological (to date, but see major developments)
- Easier in crude form (orbitals → density → energy density)
- Can do any large N

$$E = \int d^3r \, \{\mathcal{E}[\rho(\mathbf{r})] + \rho(\mathbf{r}) V_{\text{ext}}(\mathbf{r})\}$$

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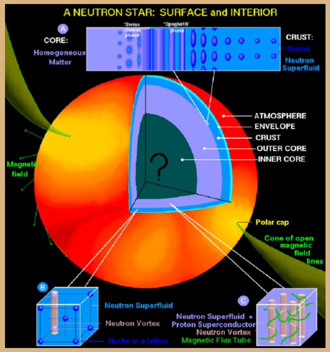
Research Strategies

i) Use QMC as a benchmark with which to compare DFT results
ii) Constrain DFT with QMC, then use DFT to make predictions

Outline



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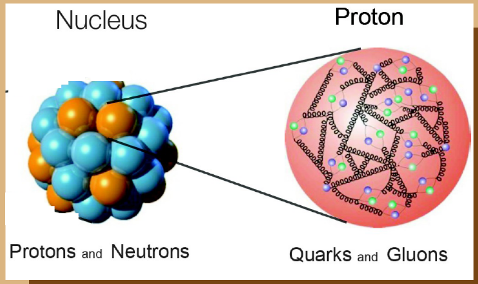
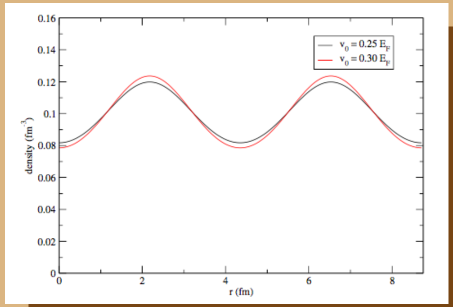
Motivation



Nuclear background



Recent results

Neutron matter: a selection

Connection with cold-atom experiment

QMC with chiral EFT

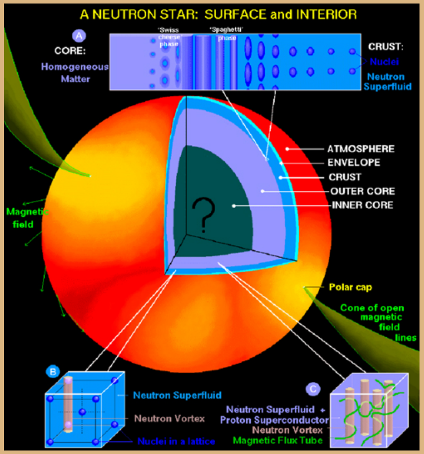
Inhomogeneous matter

1. Connection with cold-atom experiment

Connection between the two

Neutron matter

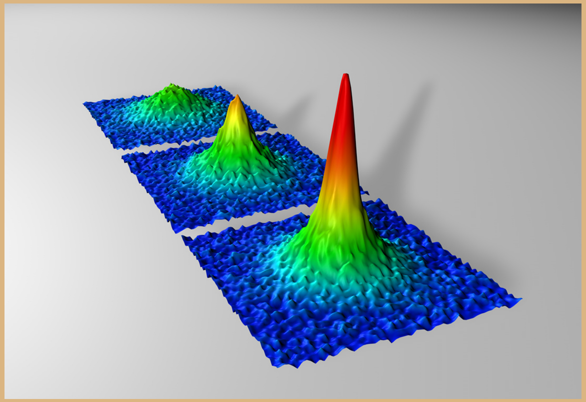
- MeV scale
- $O(10^{57})$ neutrons



Credit: Dany Page

Cold atoms

- peV scale
- O(10) or $O(10^5)$ atoms



Credit: University of Colorado

- Very similar E/E_{FG}
- Weak to intermediate to strong coupling

Connection between the two

Neutron matter

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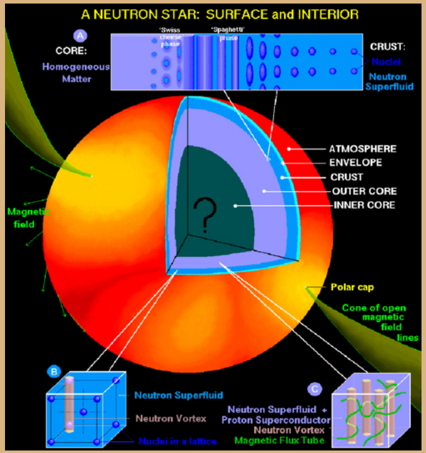

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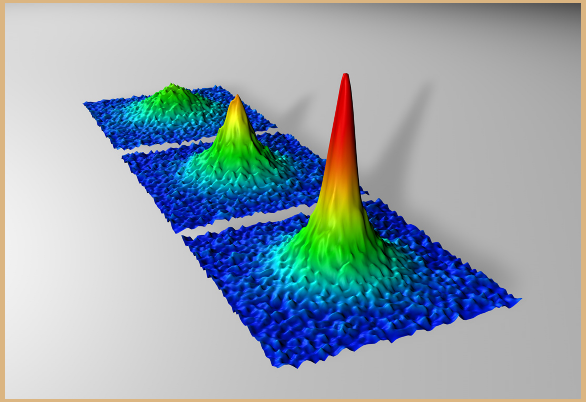
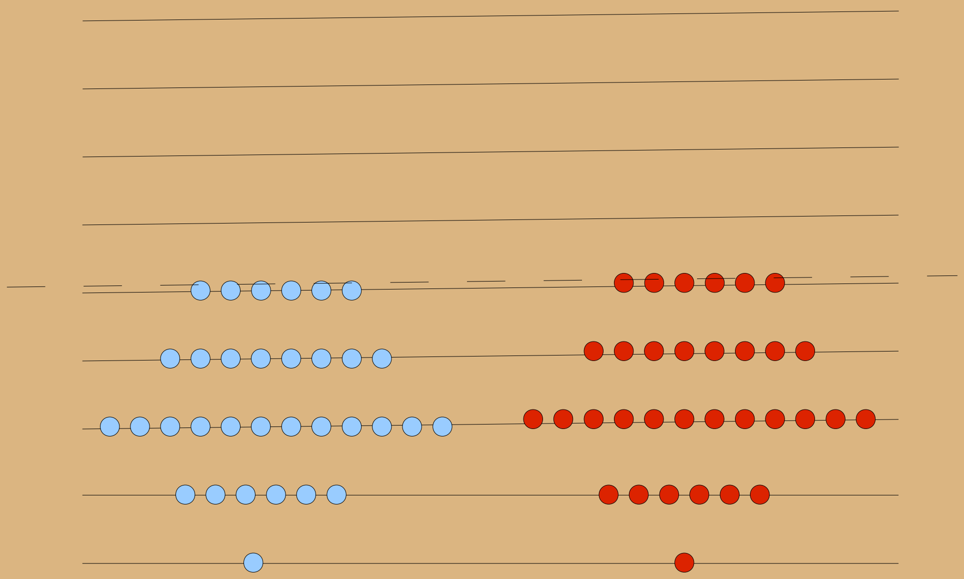


Credit: University of Colorado

A. Gezerlis, C. J. Pethick, and A. Schwenk
Pairing and superfluidity of nucleons in neutron stars
chapter in “Novel Superfluids: Volume 2”
(Oxford University Press, 2014)

Fermionic dictionary



Energy of a free Fermi gas: $E_{FG} = 3/5 N E_F$

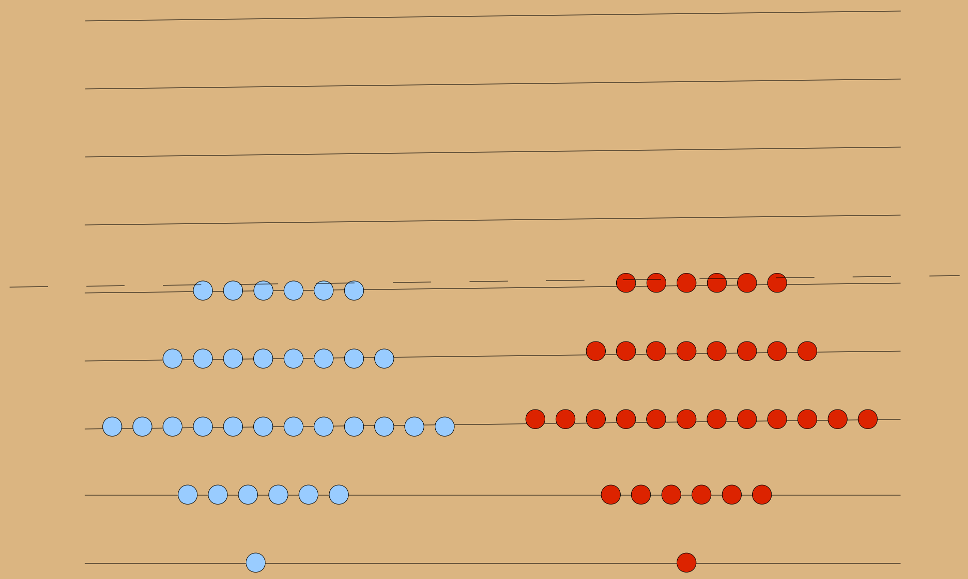
Fermi energy: $E_F = \hbar^2 k_F^2 / 2m$

Fermi wave number: k_F

Number density: $\rho = g k_F^3 / 6\pi^2$

Scattering length: a

Fermionic dictionary



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Fermi energy: $E_F = \hbar^2 k_F^2 / 2m$

Fermi wave number: k_F

Number density: $\rho = g k_F^3 / 6\pi^2$

Scattering length: a

In what follows, the dimensionless quantity $k_F a$ is called the "coupling"

Motivation: Problems

Weak coupling

- $k_F a \to 0$
- Studied for decades
- Experimentally difficult
- Pairing exponentially small
- Analytically known

Strong Coupling

- $k_F a \to \infty$
- More recent (2000s)
- Experimentally probed
- Pairing significant
- Non-perturbative

Motivation: Problems

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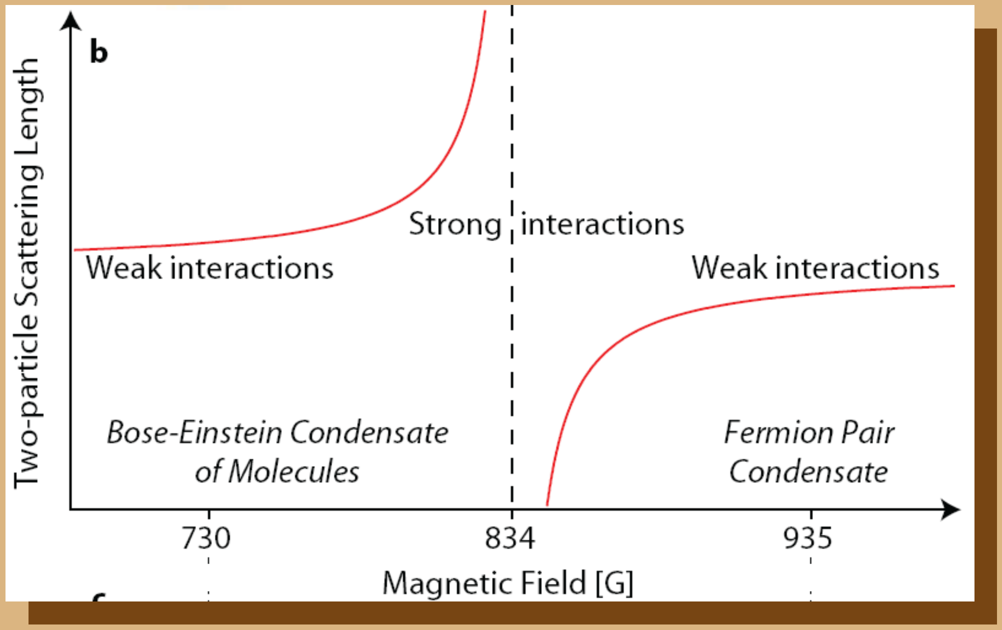
Strong Coupling

- $k_F a \to \infty$
- More recent (2000s)
- Experimentally probed
- Pairing significant
- Non-perturbative

Connection:

Using “Feshbach” resonances one can tune the coupling

Credit: Thesis of Martin Zwierlein



Cold atoms to the rescue

Theoretical **many-body problem formulated by George Bertsch more than 15 years ago:**

"What is the ground-state energy of a gas of spin-1/2 particles with infinite scattering length, zero range interaction?"

$$E = \xi E_{FG} \quad E_{FG} = \frac{3}{5} N \frac{\hbar^2 k_F^2}{2m}$$

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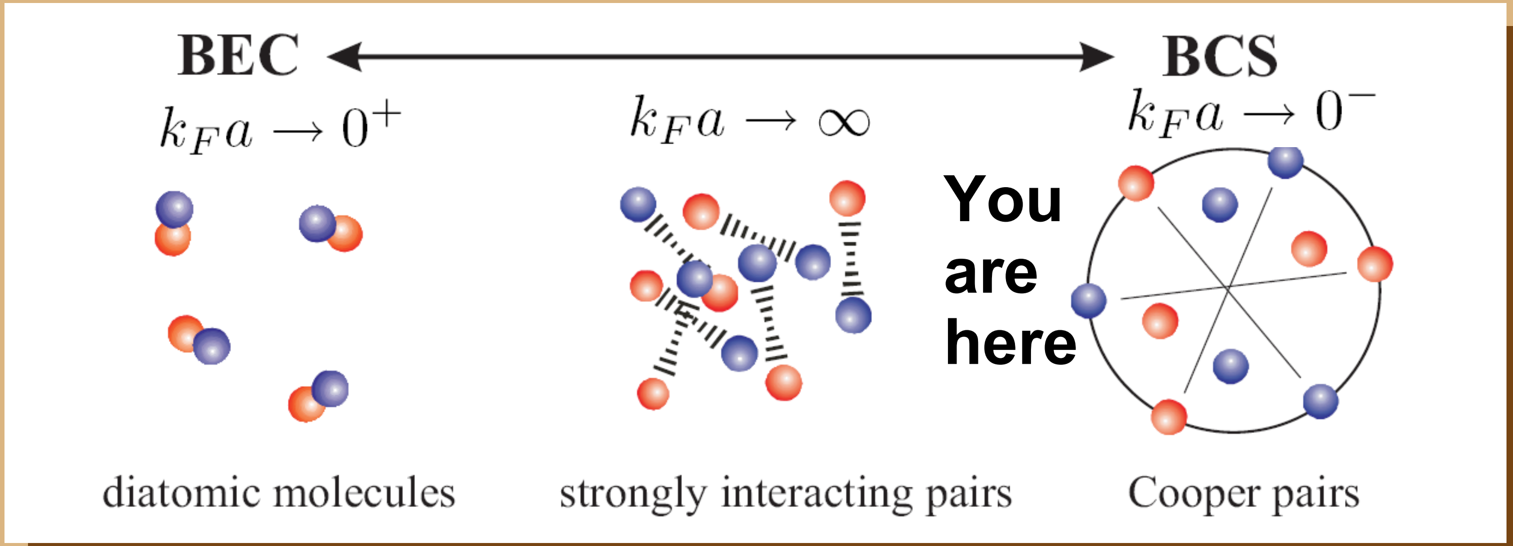
$$E = \xi E_{FG} \quad E_{FG} = \frac{3}{5} N \frac{\hbar^2 k_F^2}{2m}$$

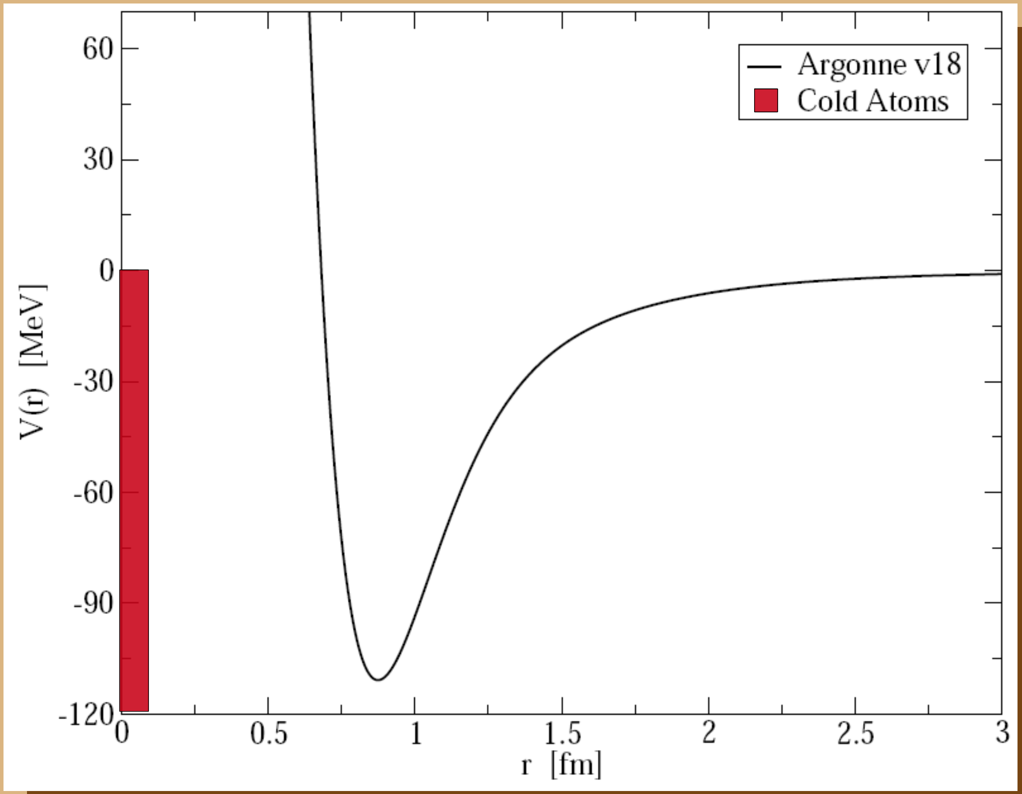
Now within *direct* experimental reach!



Credit: Ph.D. Thesis of Cindy Regal

Hamiltonian: unity in diversity

$$\mathcal{H} = -\frac{\hbar^2}{2m}\sum_{k=1}^{N}\nabla_k^2 + \sum_{i<j'} v(r_{ij'})$$



Neutron matter

$^1S_0$ channel of AV18 – later AV4

a = -18.5 fm, r_e = 2.7 fm

Cold atoms

basically any well-behaved potential

a = tunable, r_e =tunable/infinitesimal

What do we know for sure?

Weak Coupling

Equation of state: $$\frac{E}{E_{FG}} = 1 + \frac{10}{9\pi} k_F a + \frac{4}{21\pi^2}(11 - 2\ln 2)(k_F a)^2$$

Pairing gap: $$\frac{\Delta}{E_F} = \frac{1}{(4e)^{1/3}} \Delta_{\mathrm{BCS}}$$

What do we know for sure?

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Pairing gap: $$\frac{\Delta}{E_F} = \frac{1}{(4e)^{1/3}}\Delta_{\mathrm{BCS}}$$

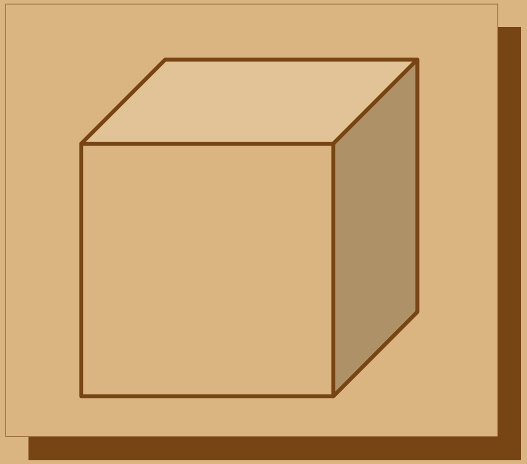
Strong Coupling

Mean-field BCS is easy but unreliable:

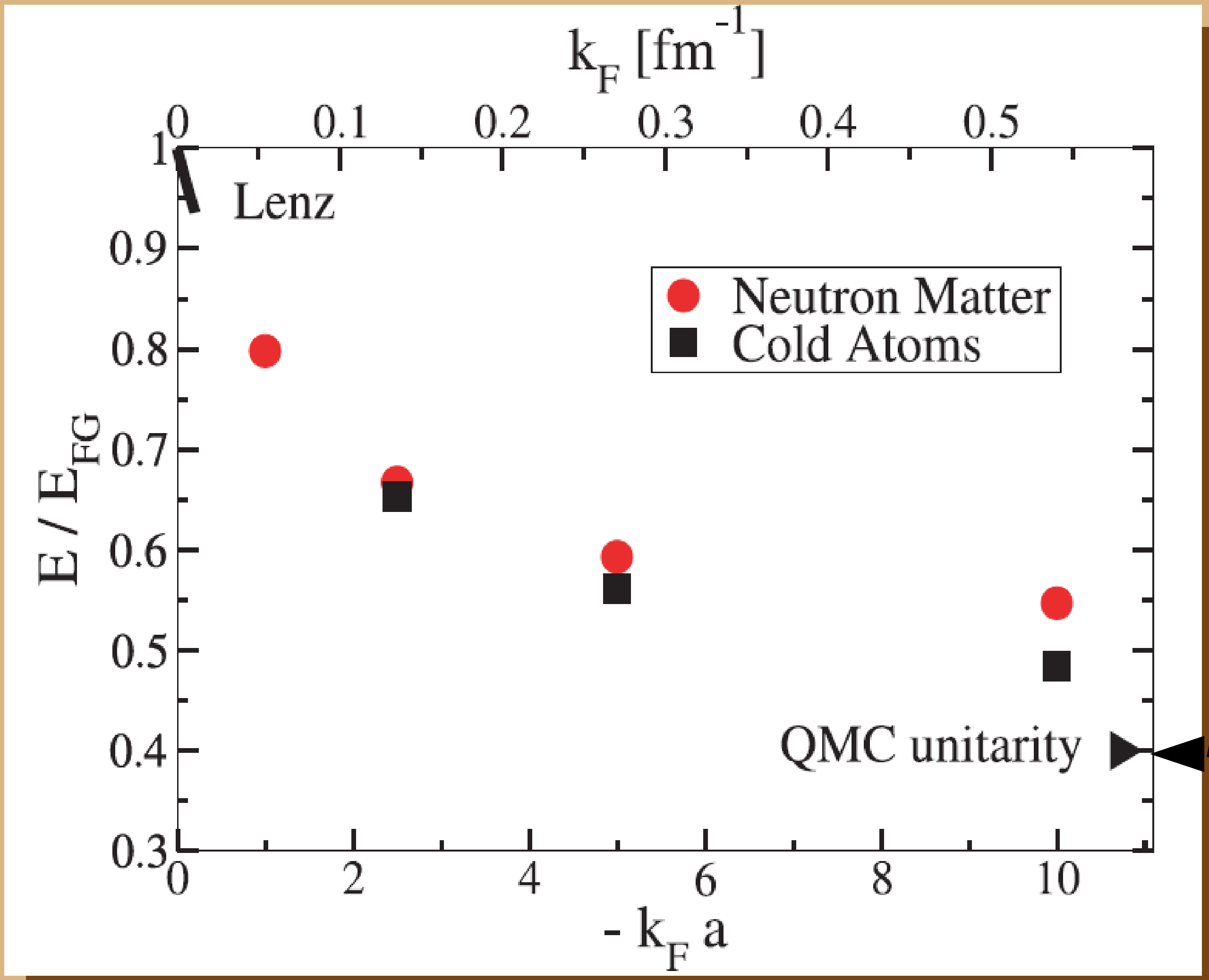
$$\Delta(\mathbf{k}) = -\sum_{\mathbf{k}'}\langle\mathbf{k}|V|\mathbf{k}'\rangle\frac{\Delta(\mathbf{k}')}{2\sqrt{\xi(\mathbf{k})^2 + \Delta(\mathbf{k})^2}}$$

Ab initio GFMC is difficult but accurate:

$$\Psi_V = \prod_{i<j} f(r_{ij})\ \mathcal{A}[\prod \phi(r_{ij})]$$



Equations of state: results



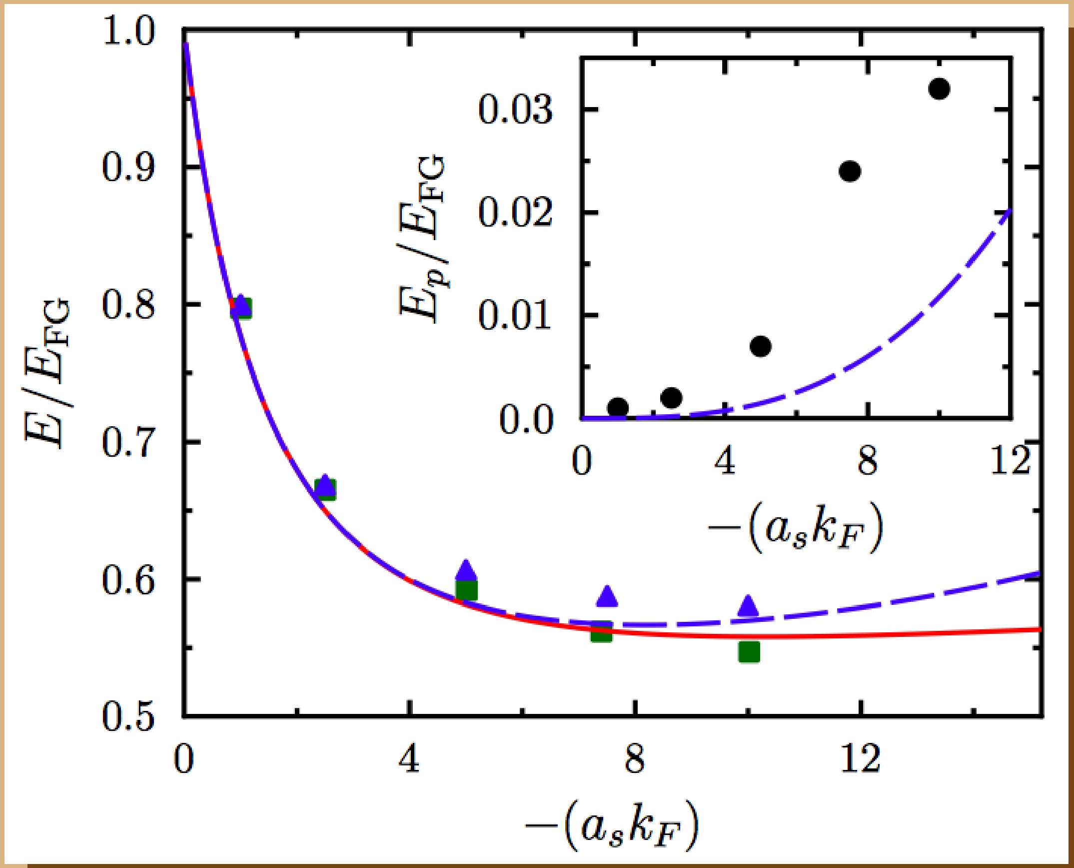
- Results identical at low density
- Range important at high density
- Duke and ENS experiments at unitarity (current QMC and MIT experiment are lower)

NEUTRONS

ATOMS

A. Gezerlis and J. Carlson, Phys. Rev. C **77**, 032801 (2008)
S. Gandolfi, A. Gezerlis, and J. Carlson, Ann. Rev. Nucl. Part. Sci. **65**, 303 (2015)

Equations of state: results

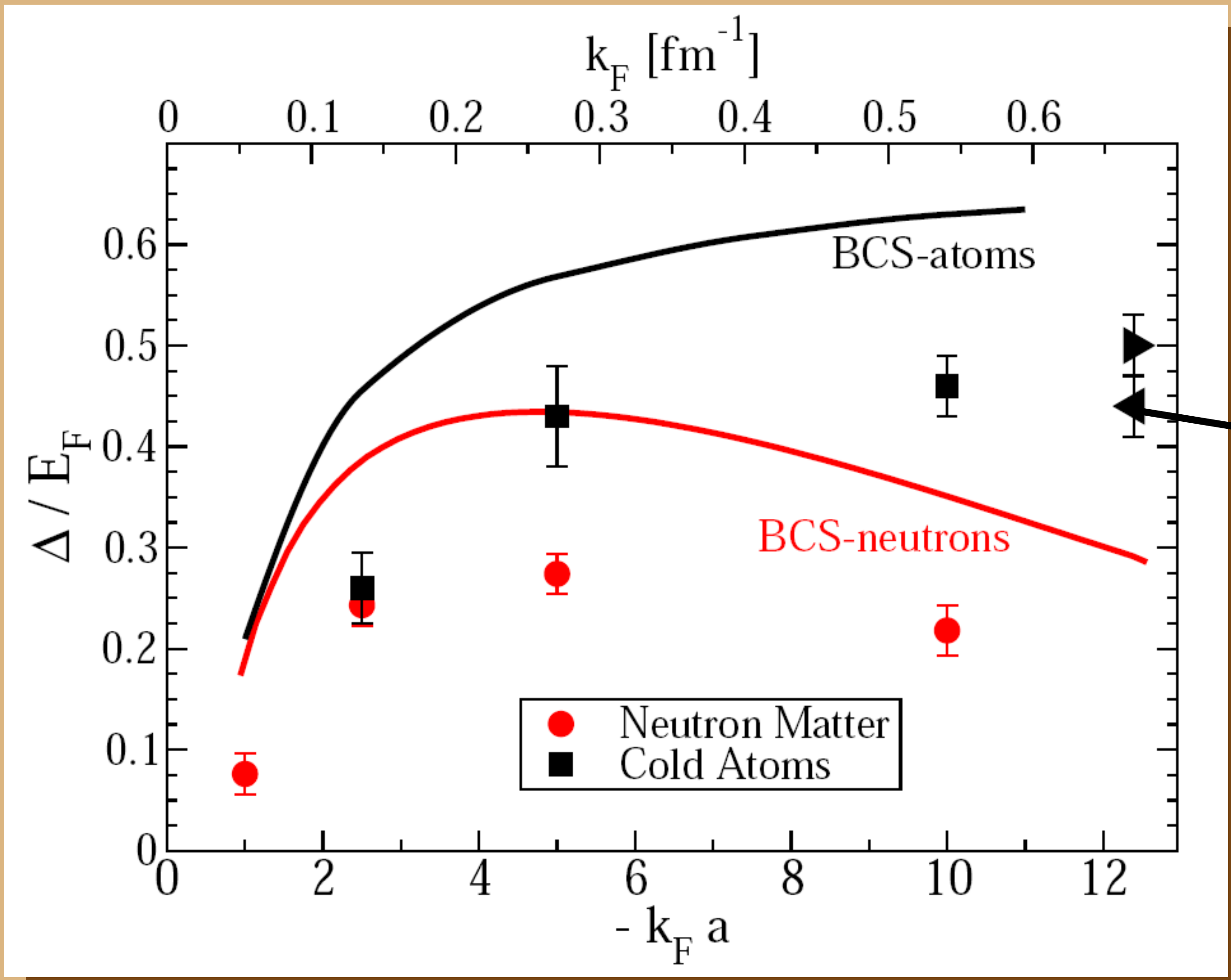


- DFT with no free parameters
- Probing effects of beyond s-wave interactions
- EOS is just the beginning

NEUTRONS

D. Lacroix, A. Boulet, M. Grasso, C.-J. Yang, Phys. Rev. C 95, 054306 (2017)

Pairing gaps: results



- Results identical at low density
- Range important at high density
- Two independent MIT experiments at unitarity

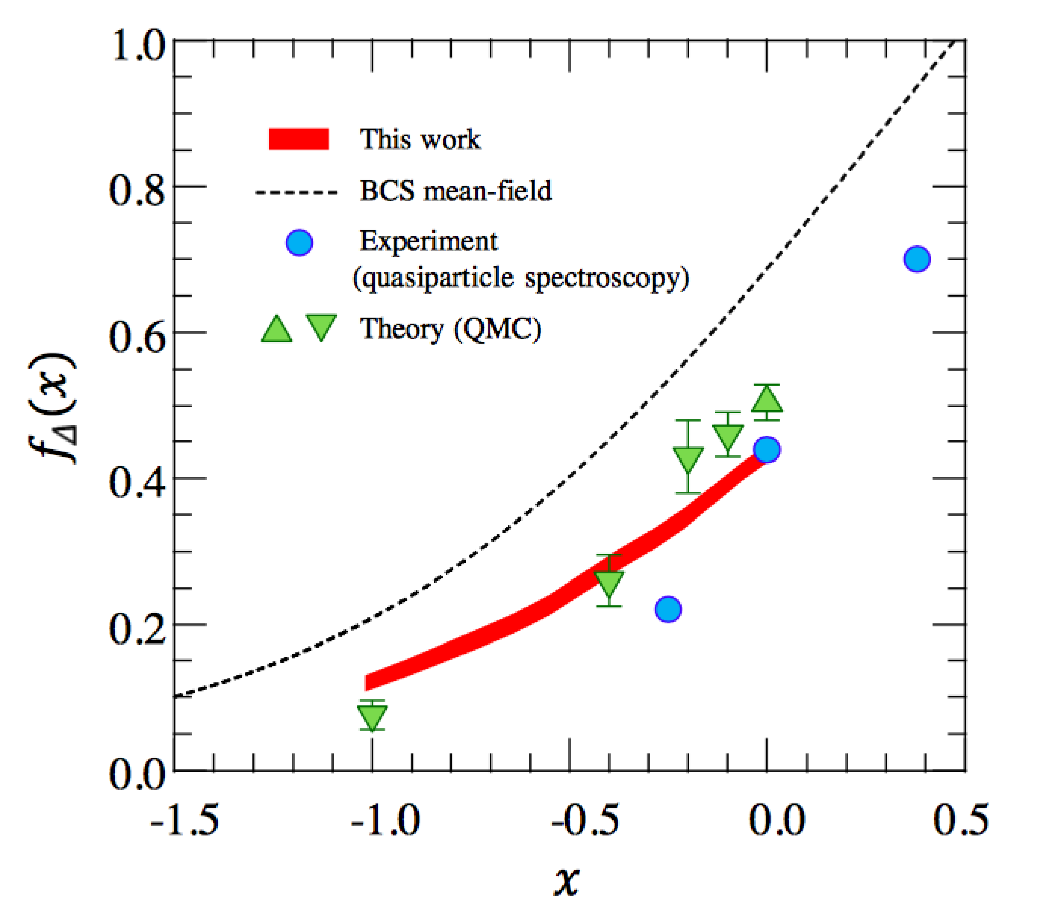
NEUTRONS

ATOMS

A. Gezerlis and J. Carlson, Phys. Rev. C **77**, 032801 (2008)

S. Gandolfi, A. Gezerlis, and J. Carlson, Ann. Rev. Nucl. Part. Sci. **65**, 303 (2015)

Experiment on cold-gas gaps away from unitarity



- New experiment at University of Tokyo
- ^{6}Li at $T/T_F < 0.06$
- Experimental extraction includes (some) beyond mean-field effects

ATOMS

M. Horikoshi et al, arXiv:1612.04026

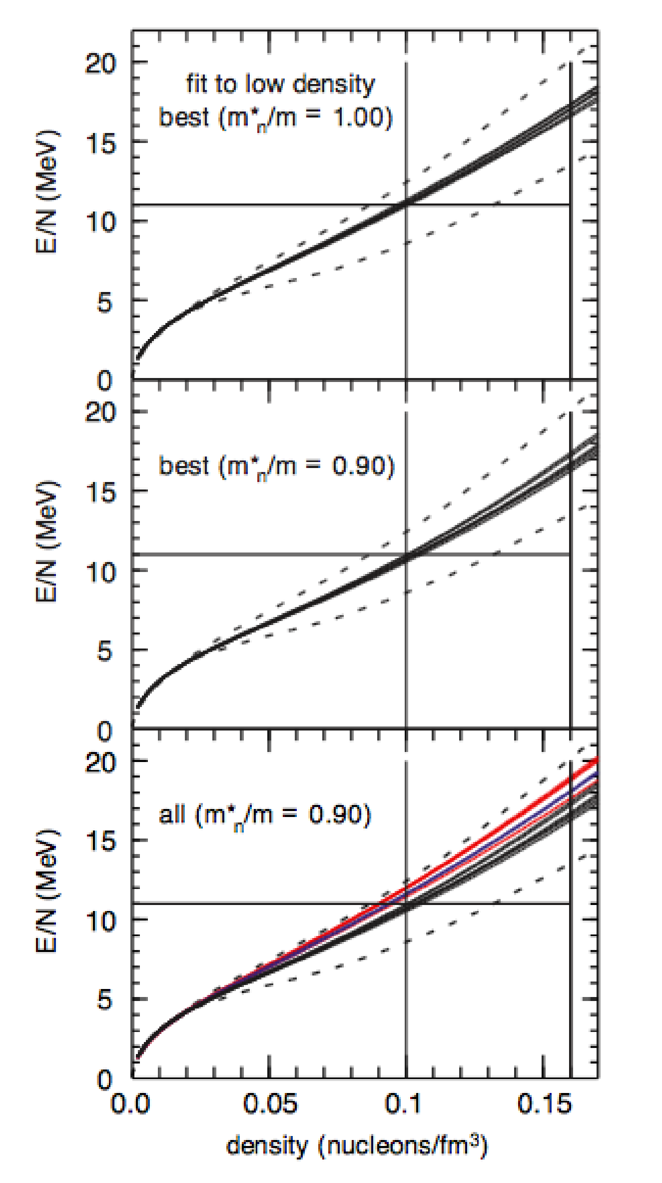
The meaning of it all

Neutron-star crust consequences

- Negligible contribution to specific heat consistent with cooling of transients
- Young neutron star cooling curves depend on the magnitude of the gap
- Superfluid-phonon heat conduction mechanism viable
- Constraints for Skyrme-HFB calculations of neutron-rich nuclei

2. QMC with chiral EFT

From low to high density

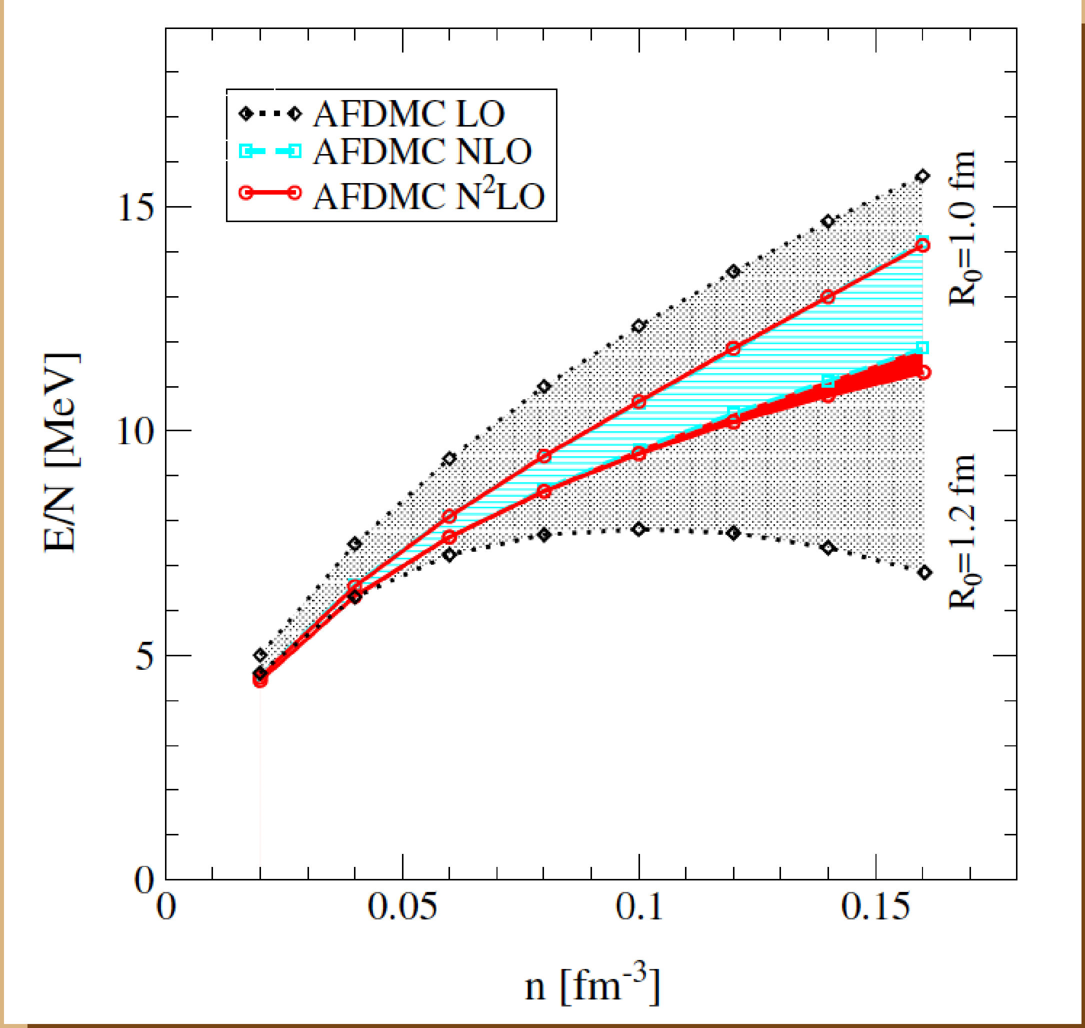


- Ab initio results for low-density matter under control
- Doubly-magic input better constrained at higher density

NEUTRONS

B. A. Brown and A. Schwenk, Phys. Rev. C 89, 011307 (2014)

Chiral EFT in QMC

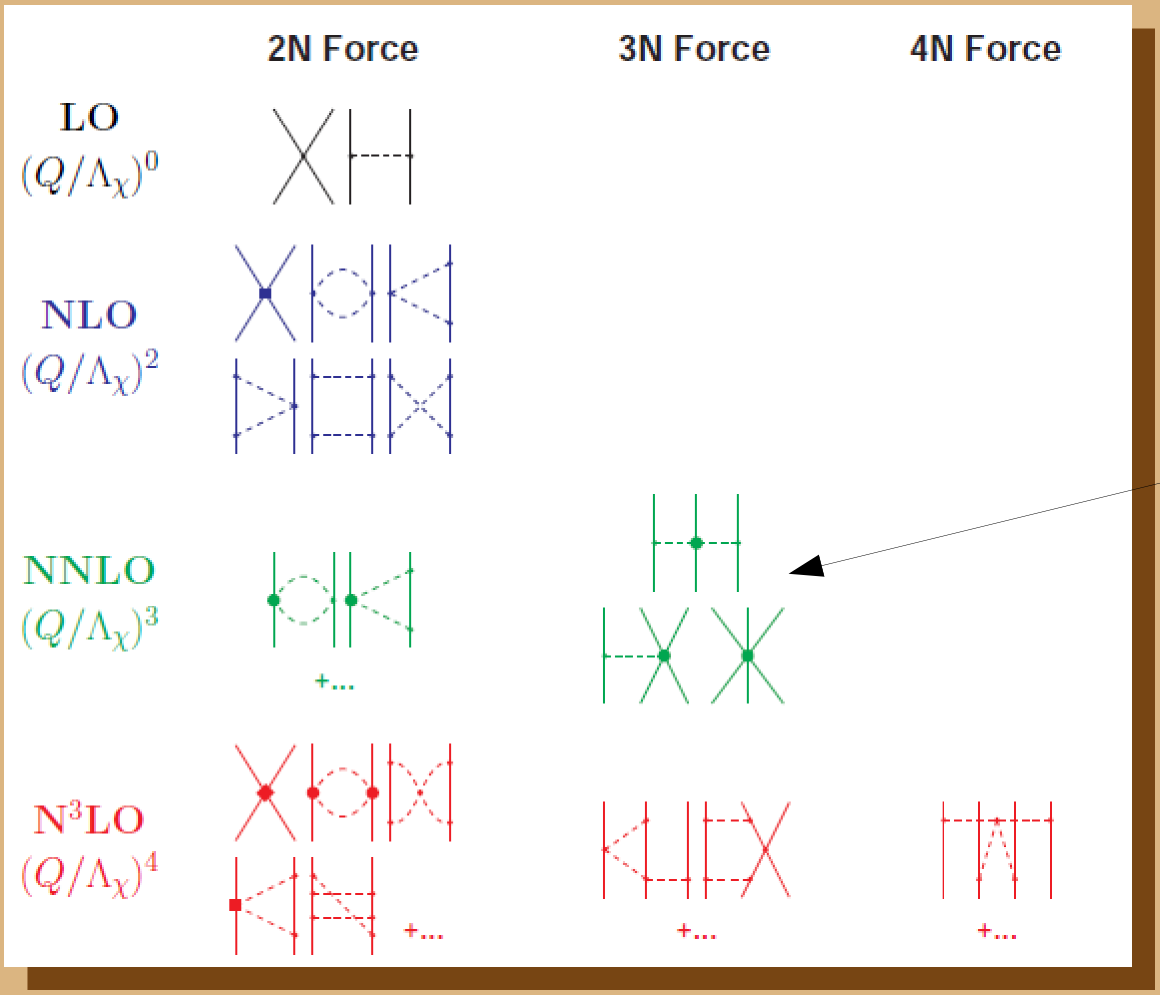


- Use Auxiliary-Field Diffusion Monte Carlo to handle the full interaction
- First ever non-perturbative systematic error bands
- Band sizes to be expected
- Many-body forces will emerge systematically

NEUTRONS

A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. **111**, 032501 (2013).
A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. C **90**, 054323 (2014).

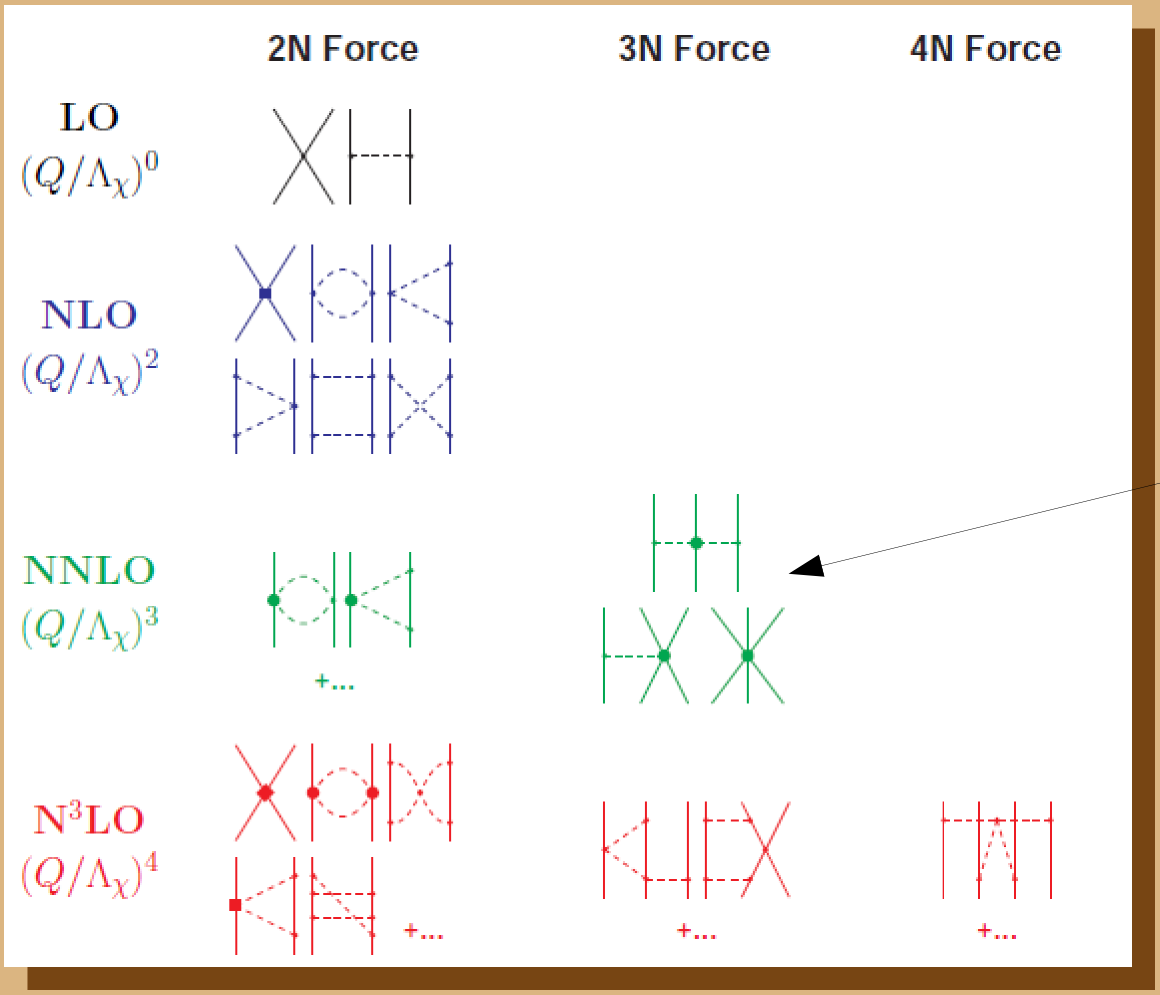
Nuclear Hamiltonian: chiral EFT



Leading three-nucleon force

- Two-pion exchange (parameter-free)
- One-pion exchange-contact (c_D)
- Three-nucleon contact (c_E)

Nuclear Hamiltonian: chiral EFT



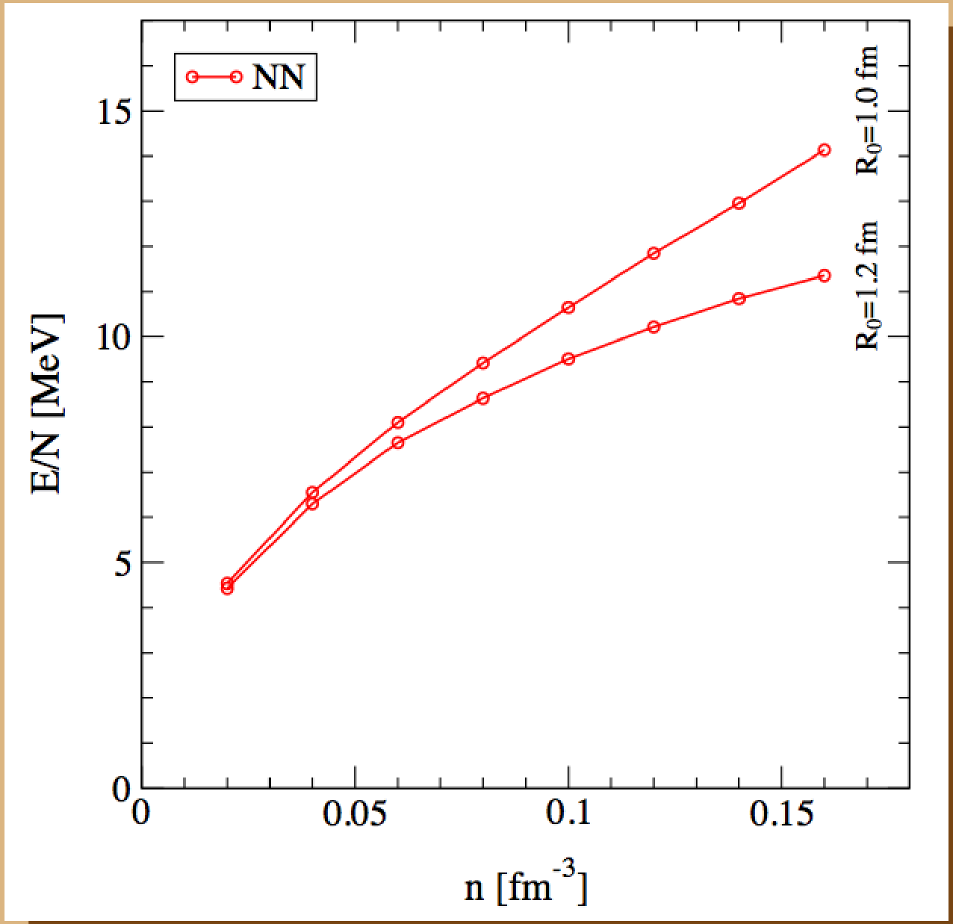
N2LO 3NF

- Two-pion exchange (parameter-free)
- One-pion exchange-contact (c_D)
- Three-nucleon contact (c_E)

V_D and V_E are merely regulator effects in PNM

3NF TPE in PNM

Overall error bands

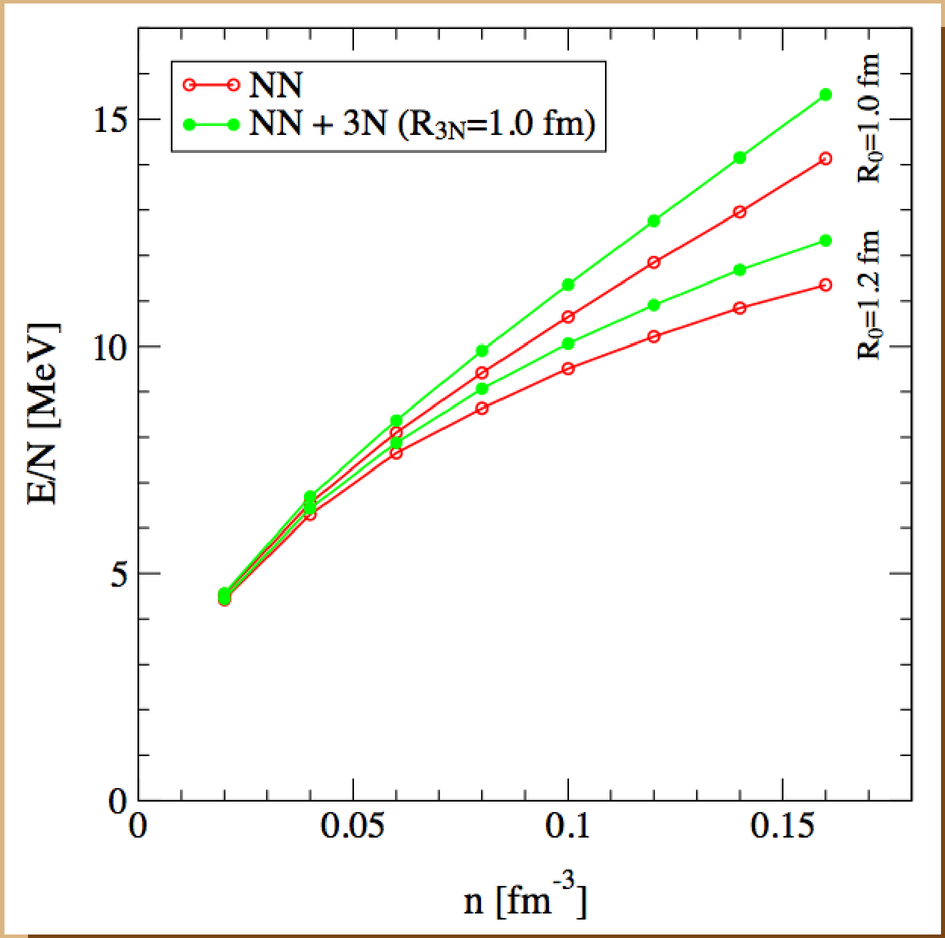


- NN error band already published

NEUTRONS

I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. C **93**, 024305 (2016)

Overall error bands

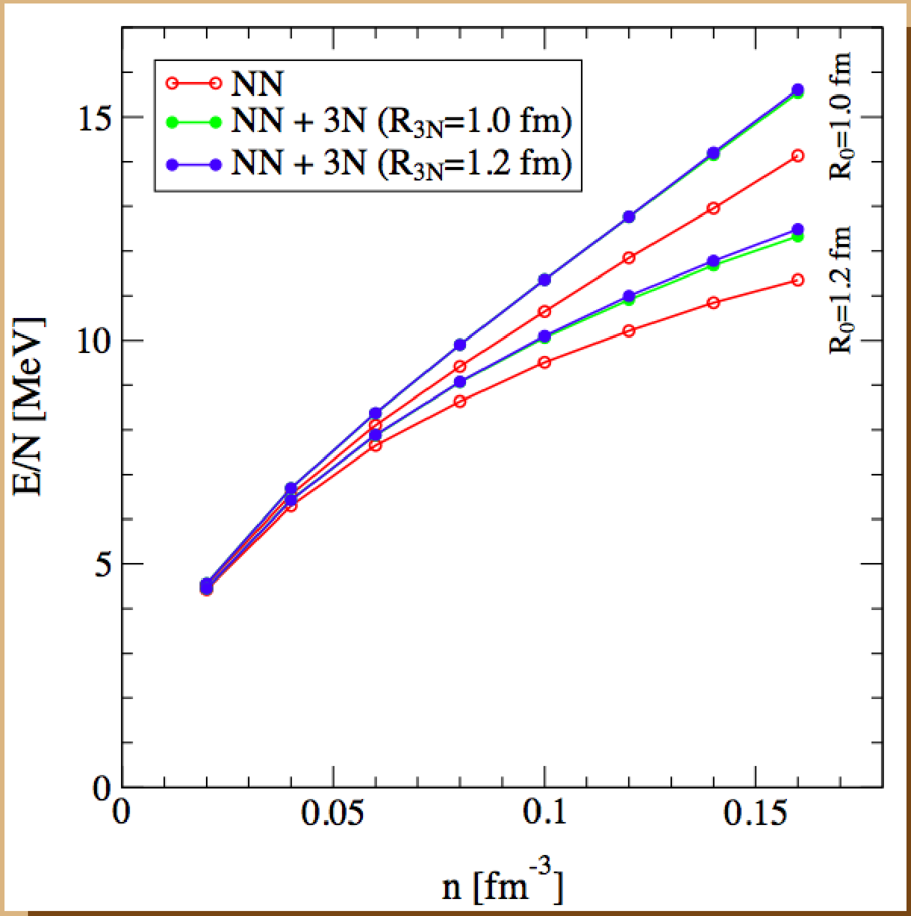


- NN error band already published
- Now vary 3NF cutoff within plateau

NEUTRONS

I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. C **93**, 024305 (2016)

Overall error bands

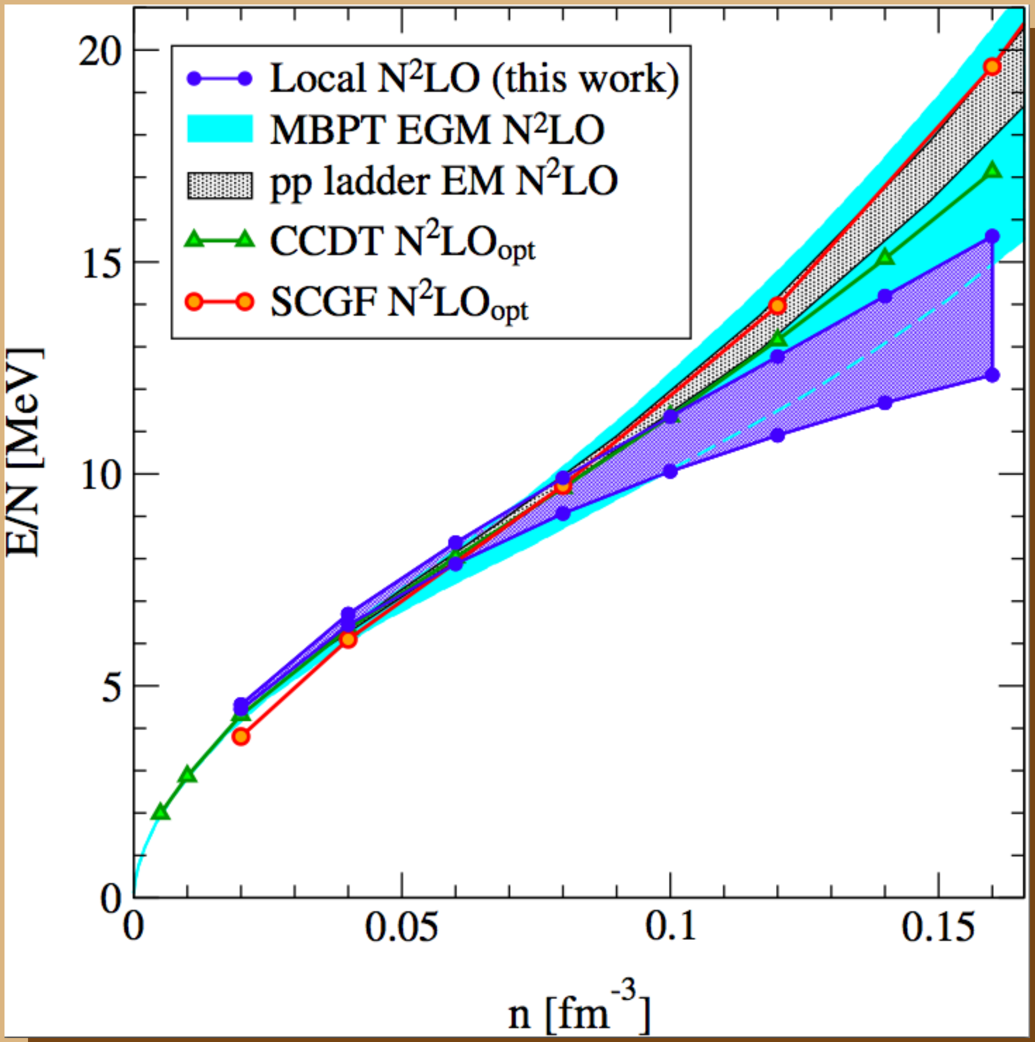


- NN error band already published
- Now vary 3NF cutoff within plateau
- 3NF cutoff dependence tiny in comparison with NN cutoff one
- 3NF contribution 1-1.5 MeV, cf. with MBPT 4 MeV with EGM

NEUTRONS

I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. C **93**, 024305 (2016)

Compare with other calculations at N2LO



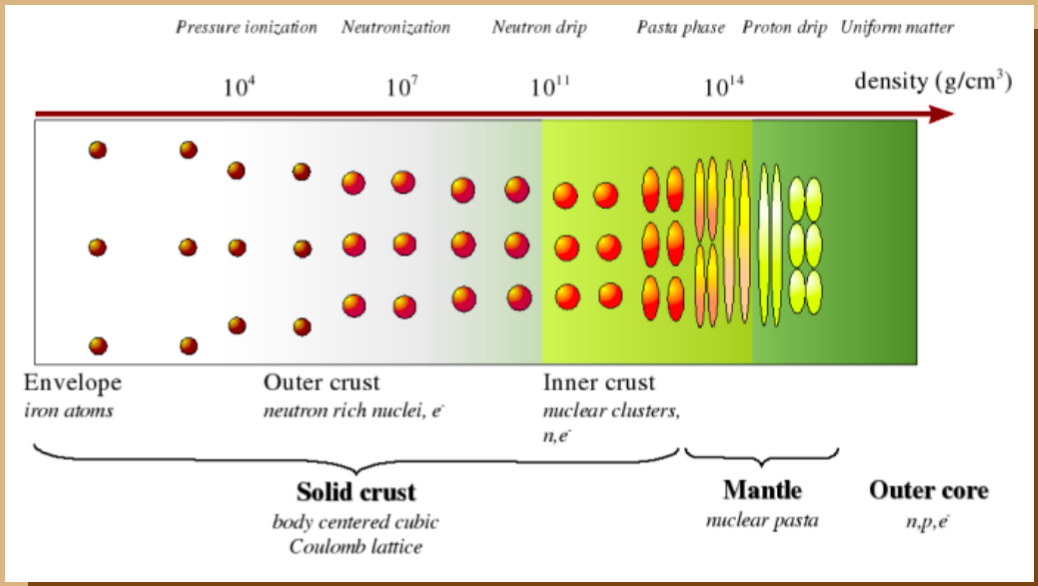
- Overall agreement across methods
- QMC band result of using more than one cutoff
- Band width essentially understood

NEUTRONS

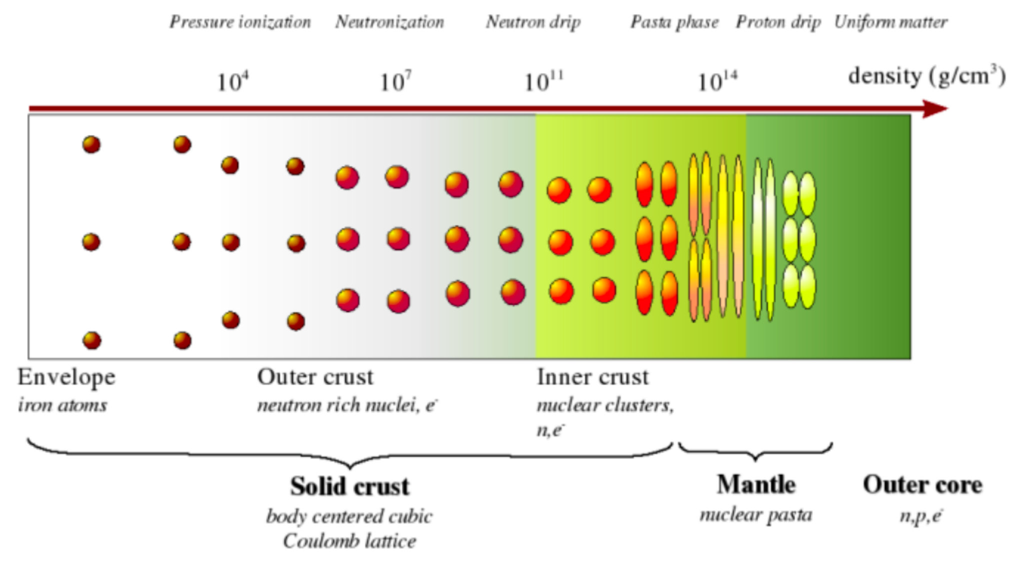
I. Tews, S. Gandolfi, A. Gezerlis, A. Schwenk, Phys. Rev. C **93**, 024305 (2016)

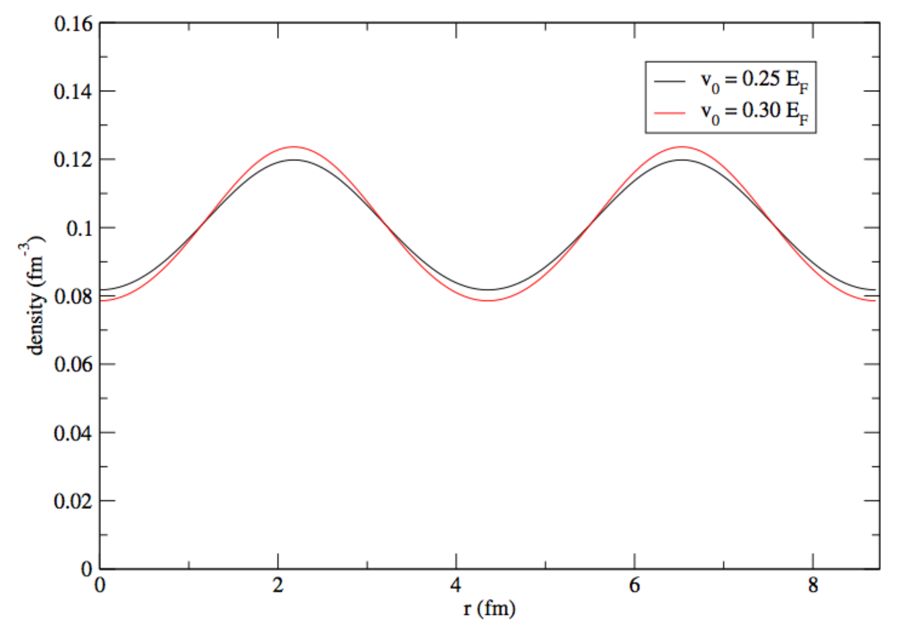
3. Inhomogeneous matter

Neutron star crusts inhomogeneous



Neutron star crusts inhomogeneous





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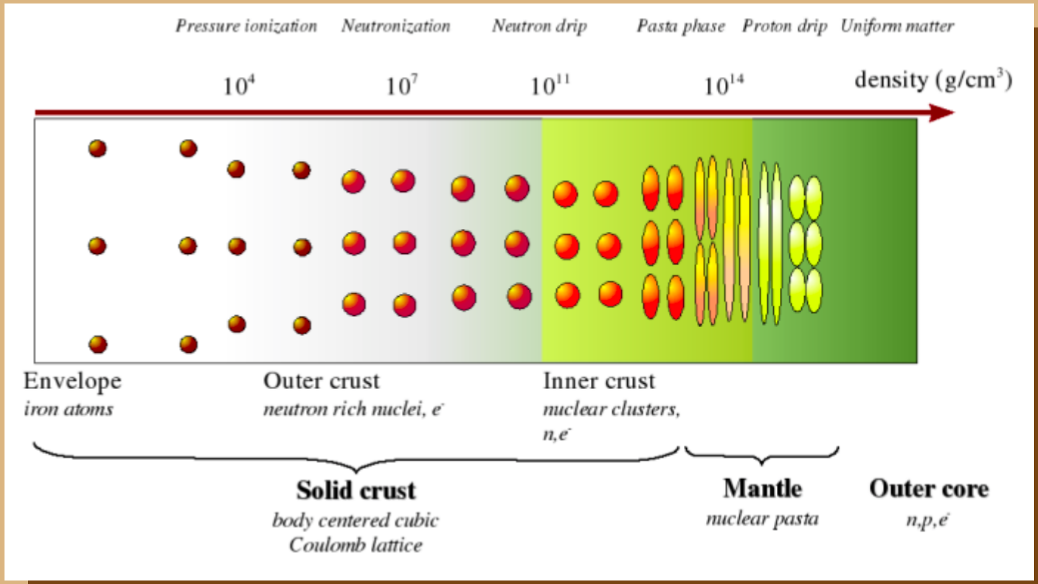
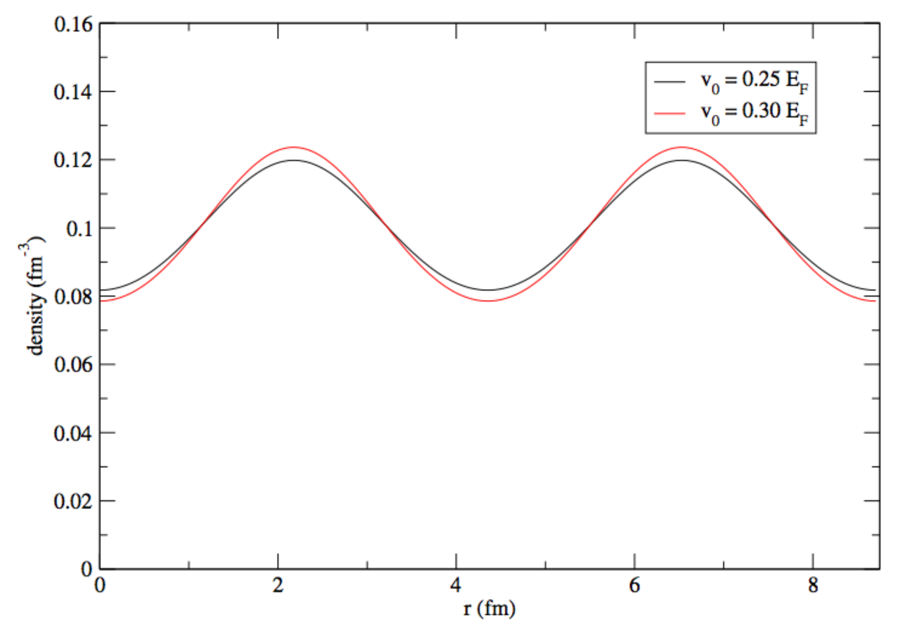


M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)



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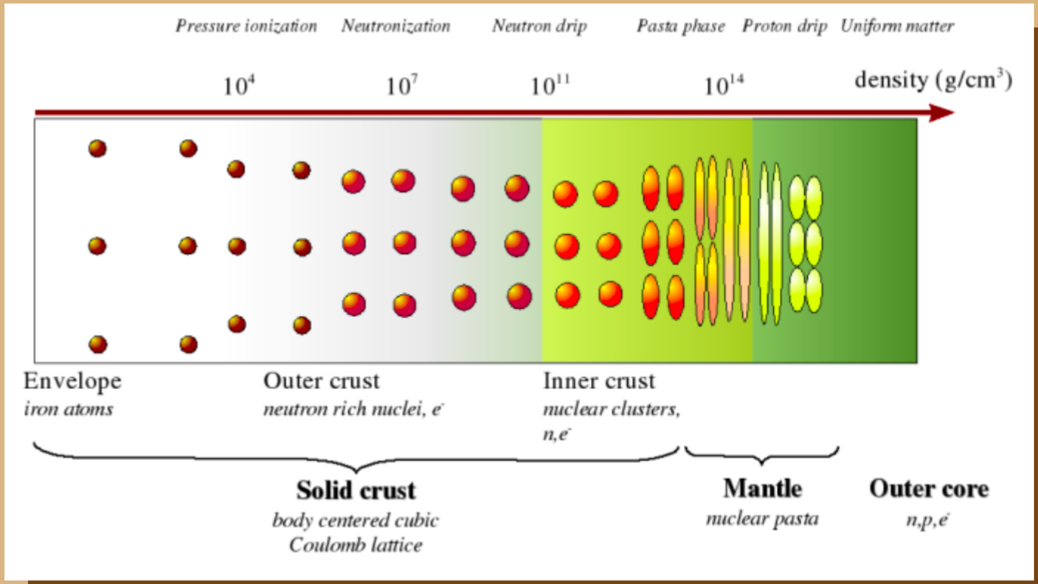
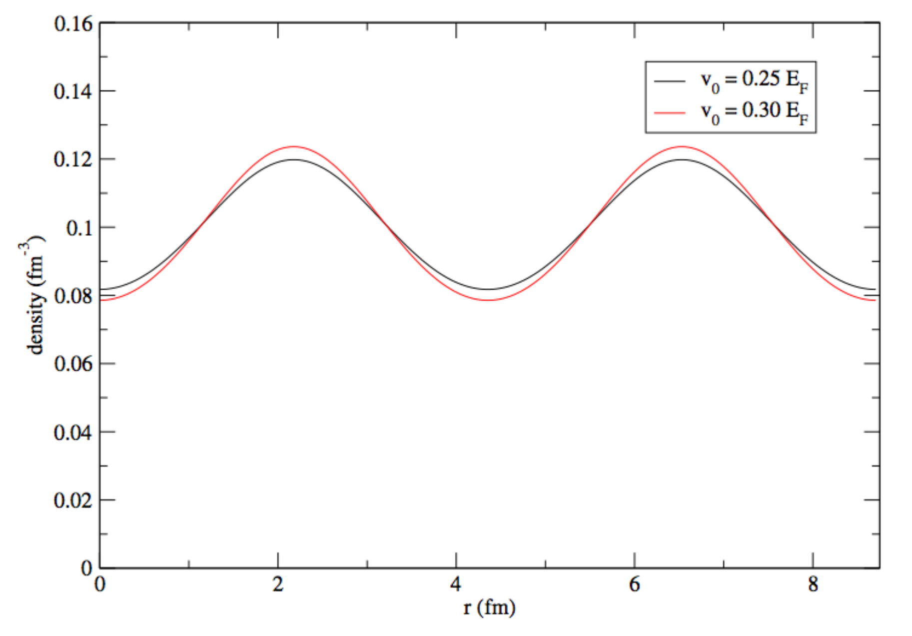


Situation identical to electrons in solids or atoms in optical lattices

M. Buraczynski and A. Gezerlis,
Phys. Rev. Lett. **116**, 152501 (2016)



Problem setup

Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \sum_i 2v_q \cos(\mathbf{q}\cdot\mathbf{r}_i)$$

Problem setup

Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \sum_i 2v_q \cos(\mathbf{q}\cdot\mathbf{r}_i)$$

non-relativistic kinetic energy

two-nucleon interaction

three-nucleon interaction

single-particle external potential

Problem setup

Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \sum_i 2v_q \cos(\mathbf{q}\cdot\mathbf{r}_i)$$

Trial wave function

$$|\Psi_T\rangle = \prod_{i<j} f(r_{ij})\ \mathcal{A}\left[\prod_i |\phi_i, s_i\rangle\right]$$

Problem setup

Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \sum_i 2v_q \cos(\mathbf{q}\cdot\mathbf{r}_i)$$

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single-particle orbitals:
- plane waves
- Mathieu functions

Problem setup

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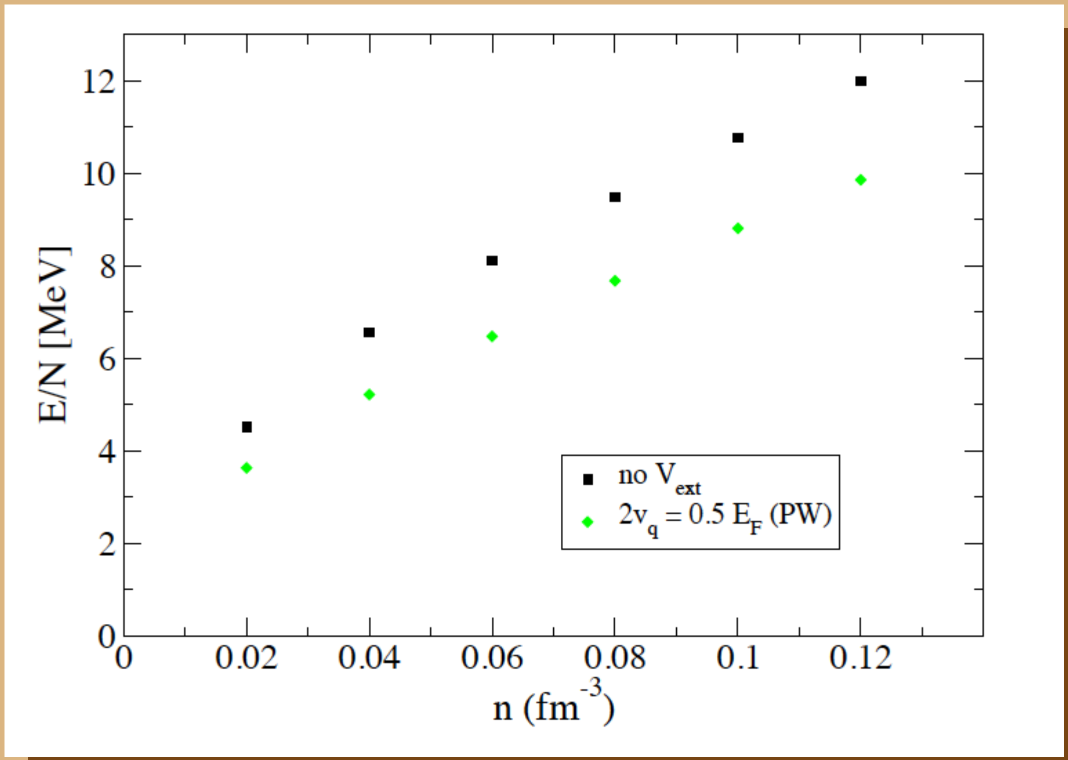
single-particle orbitals:
- plane waves
- Mathieu functions

Approach: Carry out microscopic QMC calculations for ~100 particles

One periodicity, one strength

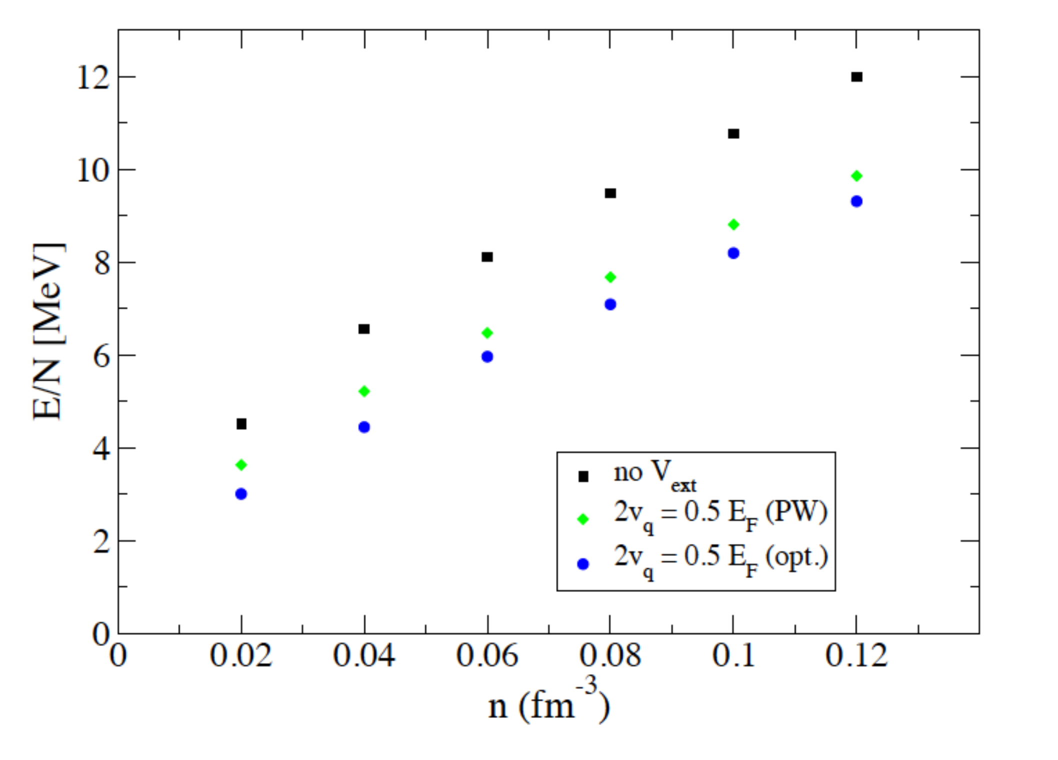


- Periodic potential in addition to nuclear forces
- Energy trivially decreased

M. Buraczynski and A. Gezerlis,
Phys. Rev. Lett. **116**, 152501 (2016)

NEUTRONS

One periodicity, one strength



- Periodic potential in addition to nuclear forces
- Energy trivially decreased
- Considerable dependence on wave function (physics input)
- Microscopic input for energy-density functionals

M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)

NEUTRONS

Background on DFT

Standard functional in PNM

$$\mathcal{E} = \frac{\hbar^2}{2m}\tau + s_1 n^2 + s_2 n^{\sigma+2} + s_3 n\tau + s_4(\nabla n)^2$$

Background on DFT

Standard functional in PNM

$$\mathcal{E} = \frac{\hbar^2}{2m}\tau + s_1 n^2 + s_2 n^{\sigma+2} + s_3 n\tau + s_4(\nabla n)^2$$

Skyrme functional in isospin representation

$$\mathcal{E}_{\mathrm{Skyrme}} = \sum_{T=0,1}\left[(C_T^{n,a} + C_T^{n,b} n_0^{\sigma}) n_T^2 + C_T^{\Delta n}(\nabla n_T)^2 + C_T^{\tau} n_T \tau_T\right]$$

Background on DFT

Standard functional in PNM

$$\mathcal{E} = \frac{\hbar^2}{2m}\tau + s_1 n^2 + s_2 n^{\sigma+2} + s_3 n\tau + s_4(\nabla n)^2$$

Skyrme functional in isospin representation

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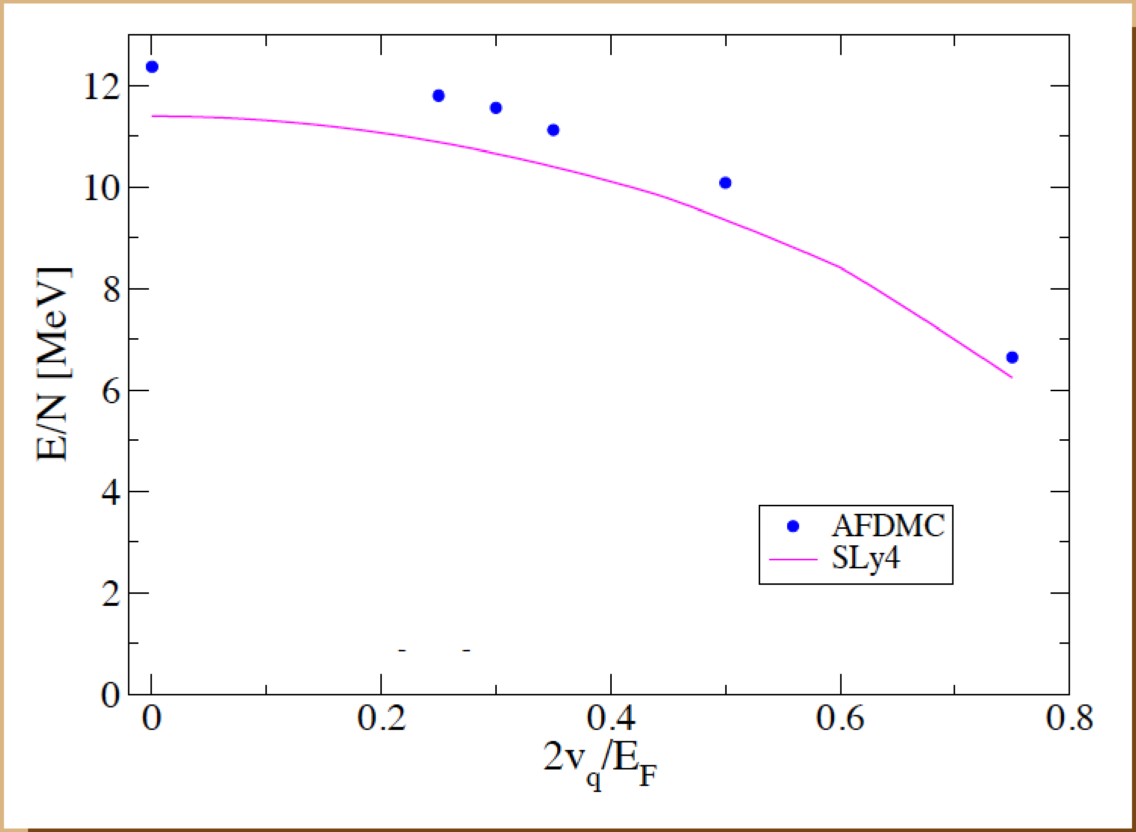
Approach: Use QMC results to constrain DFT gradient term(s)
(which then apply to terrestrial nuclei and neutron-stars more broadly)

One periodicity, many strengths



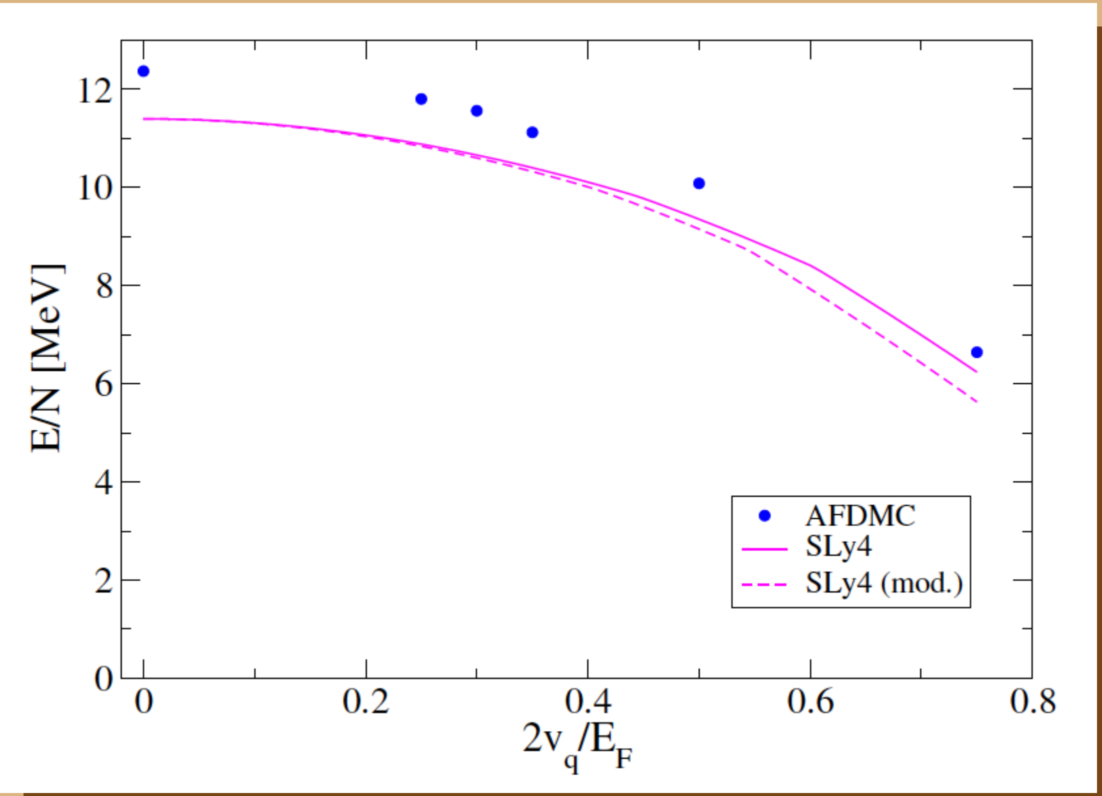
M. Buraczynski and A. Gezerlis,
Phys. Rev. Lett. **116**, 152501 (2016)

n = 0.10 fm^{-3}

- Try to disentangle bulk from isovector gradient contribution

NEUTRONS

One periodicity, many strengths



M. Buraczynski and A. Gezerlis,
Phys. Rev. Lett. **116**, 152501 (2016)

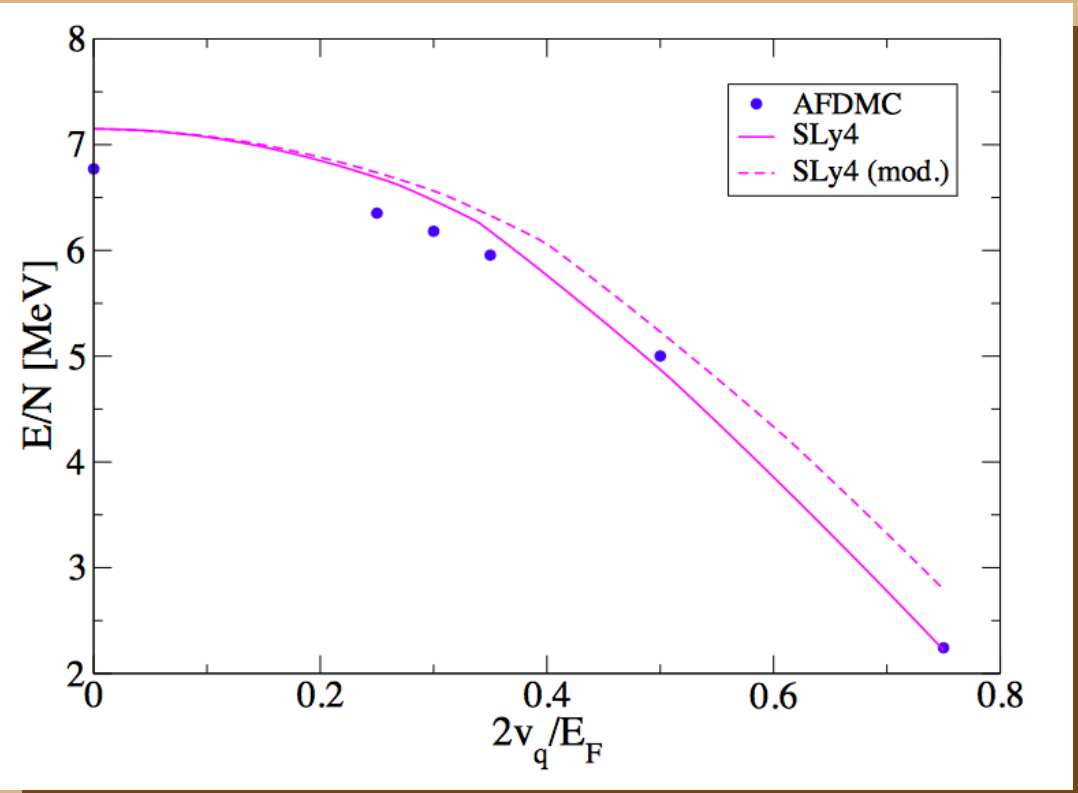
n = 0.10 fm^{-3}

- Try to disentangle bulk from isovector gradient contribution (homogeneous EOSs also differ)

NEUTRONS

One periodicity, many strengths

$n = 0.04\ \text{fm}^{-3}$



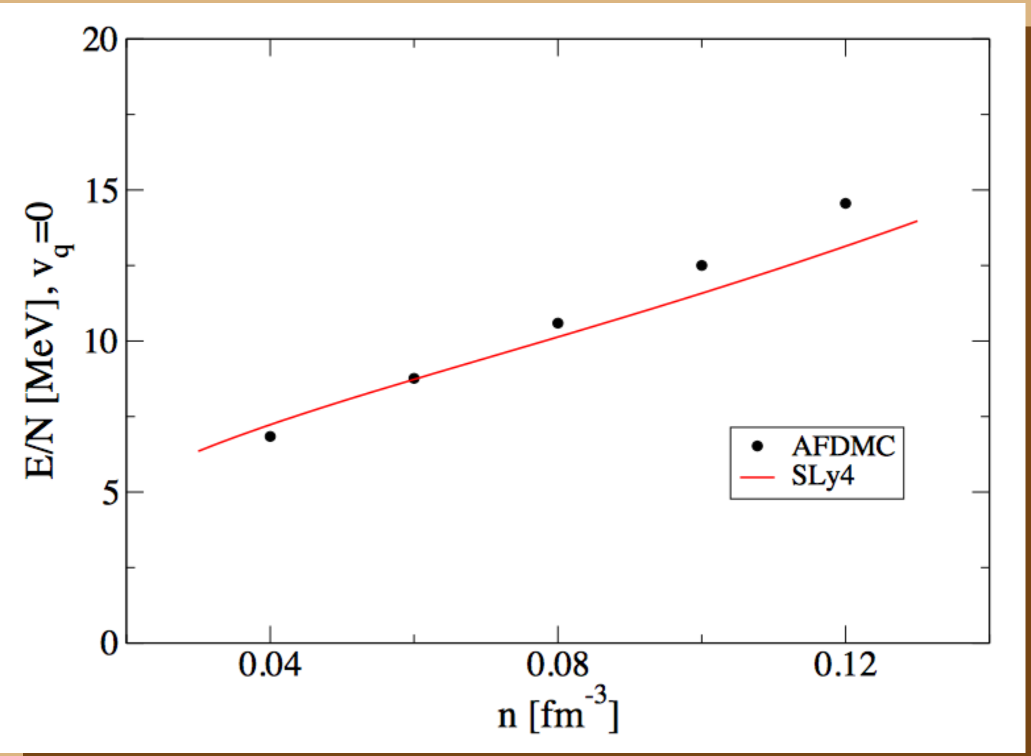
- Repeat exercise at lower density
- Homogeneous relation is reversed
- Same holds for inhomogeneous case, for not-too-large strengths

NEUTRONS

M. Buraczynski and A. Gezerlis, Phys. Rev. C **95**, 034012 (2017)

One periodicity, many strengths

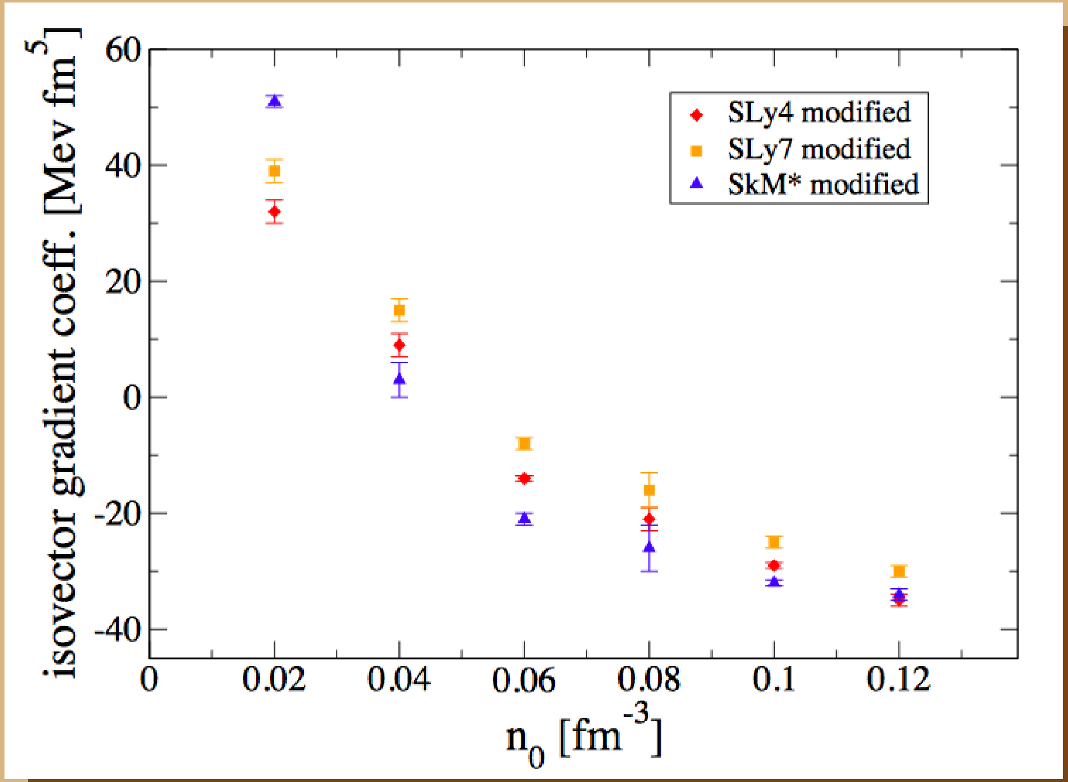
Relationship between homogeneous EOSs depends on the density



M. Buraczynski and A. Gezerlis, Phys. Rev. C **95**, 034012 (2017)

NEUTRONS

One periodicity, many strengths



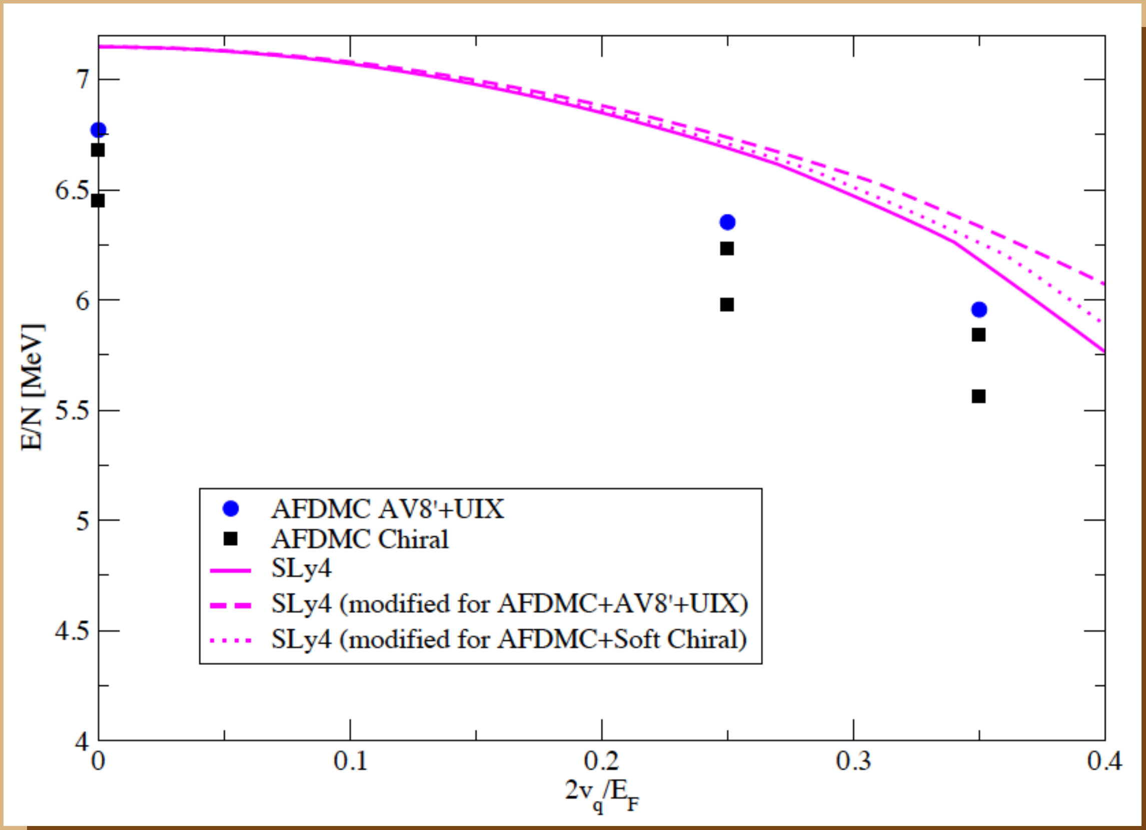
Many densities

- Repeat exercise at lower density
- Homogeneous relation is reversed
- Find density-dependent isovector coefficient, analogously to what is seen with DME (Holt, Kaiser)

NEUTRONS

M. Buraczynski and A. Gezerlis, Phys. Rev. C **95**, 034012 (2017)

One periodicity, many strengths



preliminary

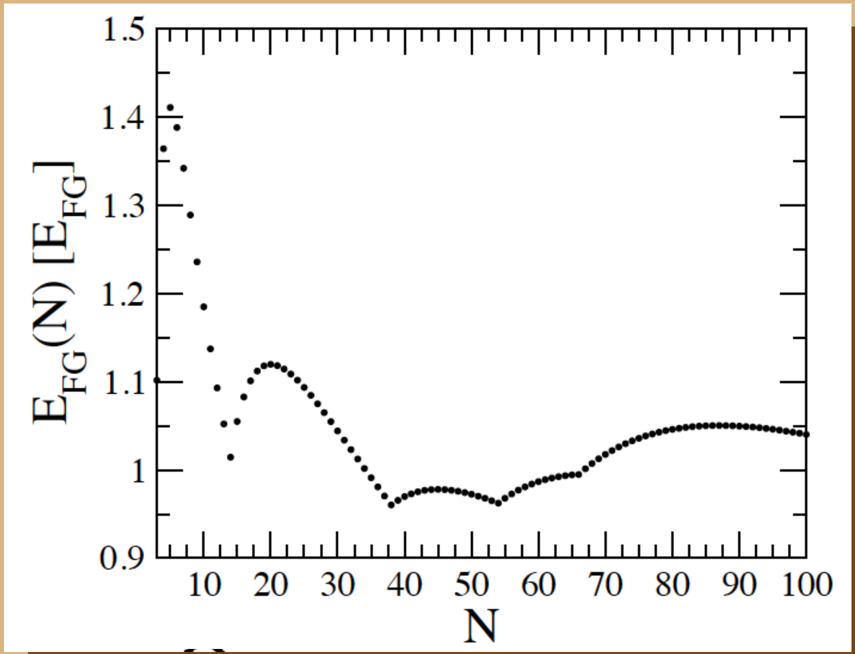
n = 0.10 fm^{-3}

- New results, using chiral EFT interactions as input to AFDMC (and from there to the Skyrme fitting)

NEUTRONS

Finite-size effects

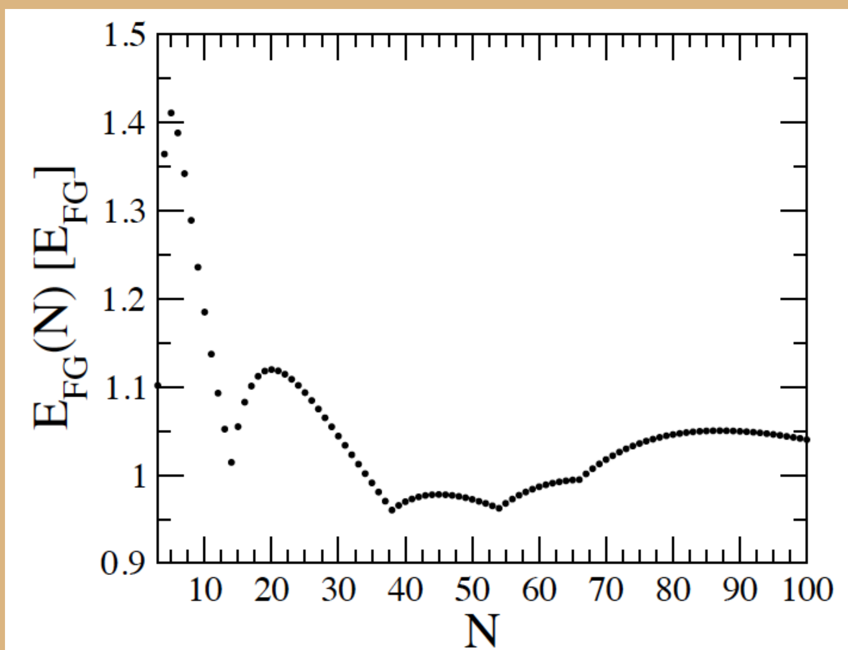
Free non-interacting gas



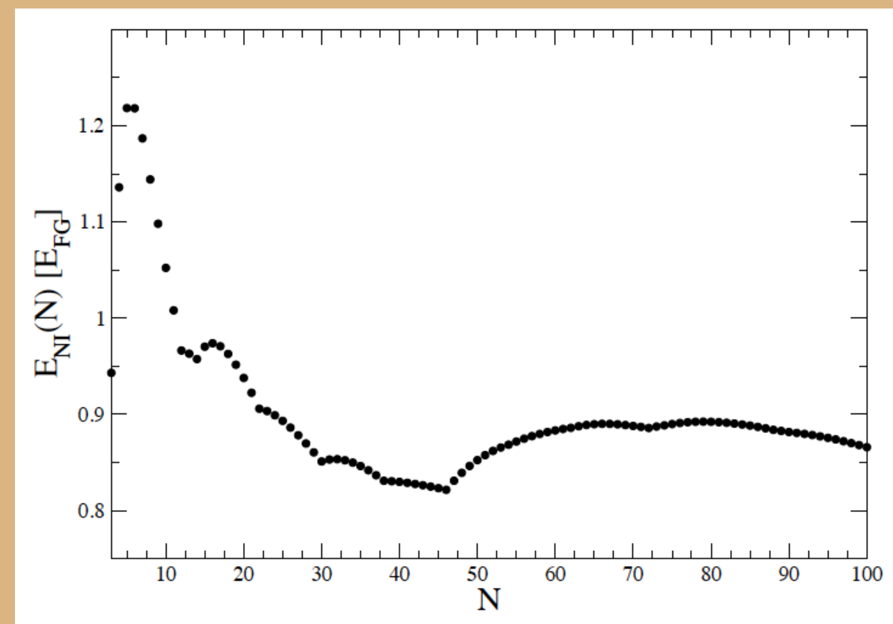
M. Buraczynski and A. Gezerlis, Phys. Rev. C **95**, 034012 (2017)

Finite-size effects

Free non-interacting gas



Modulated non-interacting gas



M. Buraczynski and A. Gezerlis, Phys. Rev. C **95**, 034012 (2017)

Neutron matter density response

Non-interacting gas: Lindhard function

$$\chi_L = -\frac{mq_F}{2\pi^2\hbar^2}\left[1 + \frac{q_F}{q}\left(1 - \left(\frac{q}{2q_F}\right)^2\right)\ln\left|\frac{q+2q_F}{q-2q_F}\right|\right]$$

Neutron matter density response

Non-interacting gas: Lindhard function

$$\chi_L = -\frac{mq_F}{2\pi^2\hbar^2}\left[1 + \frac{q_F}{q}\left(1 - \left(\frac{q}{2q_F}\right)^2\right)\ln\left|\frac{q+2q_F}{q-2q_F}\right|\right]$$

$$\frac{E_{\rm tot}}{N} = \frac{E_0}{N} + \frac{\chi(q)}{n_0}v_q^2 + C_4 v_q^4 + \cdots$$

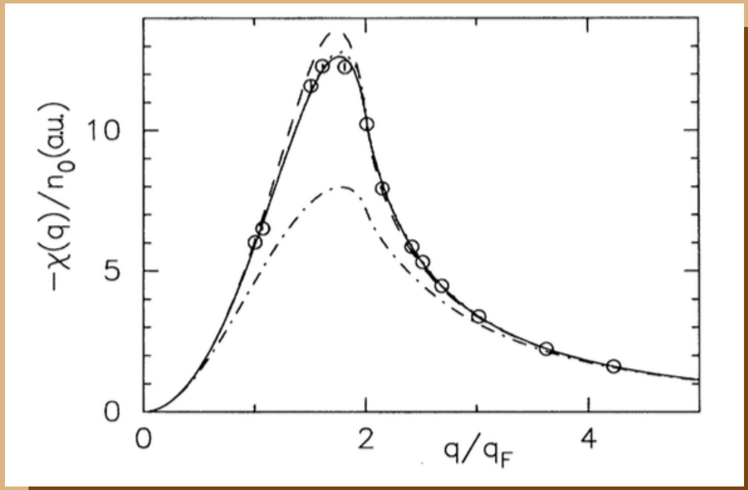
Neutron matter density response

Non-interacting gas: Lindhard function

$$\chi_L = -\frac{mq_F}{2\pi^2\hbar^2}\left[1 + \frac{q_F}{q}\left(1 - \left(\frac{q}{2q_F}\right)^2\right)\ln\left|\frac{q+2q_F}{q-2q_F}\right|\right]$$

Three-dimensional electron gas

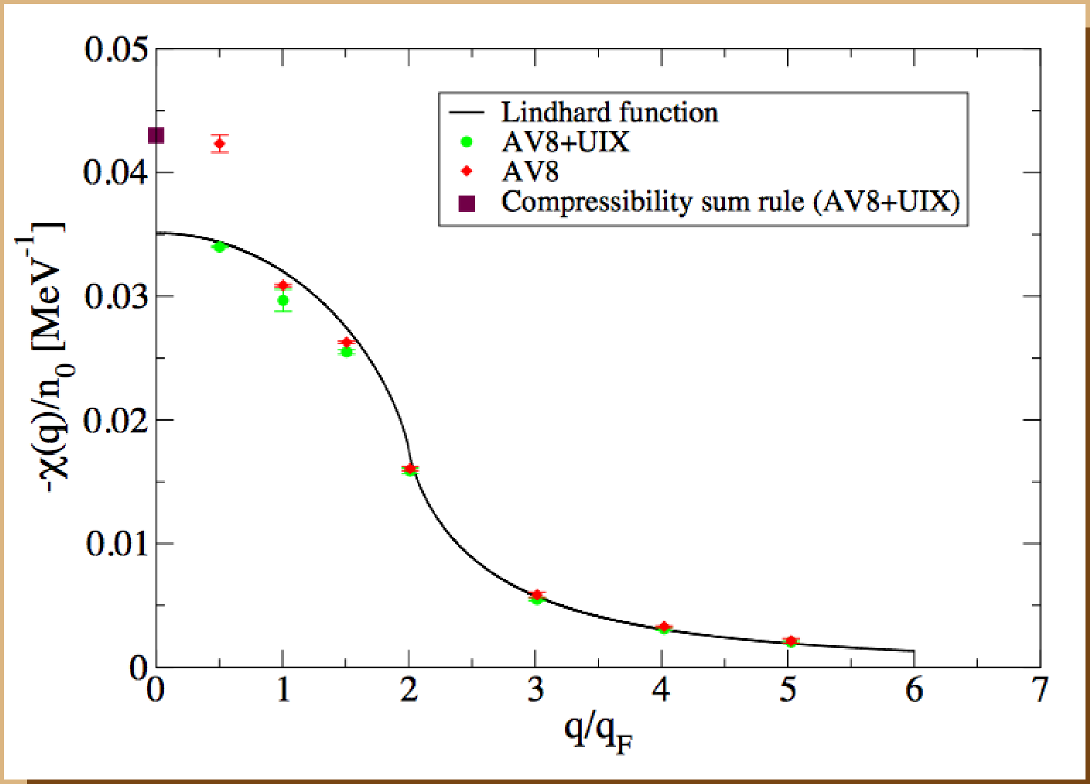
$$\frac{E_{\rm tot}}{N} = \frac{E_0}{N} + \frac{\chi(q)}{n_0}v_q^2 + C_4 v_q^4 + \cdots$$



S. Moroni, D. M. Ceperley, G. Senatore, Phys. Rev. Lett. **75**, 689 (1995)

Many periodicities, many strengths

***n* = 0.10 fm^{-3}**



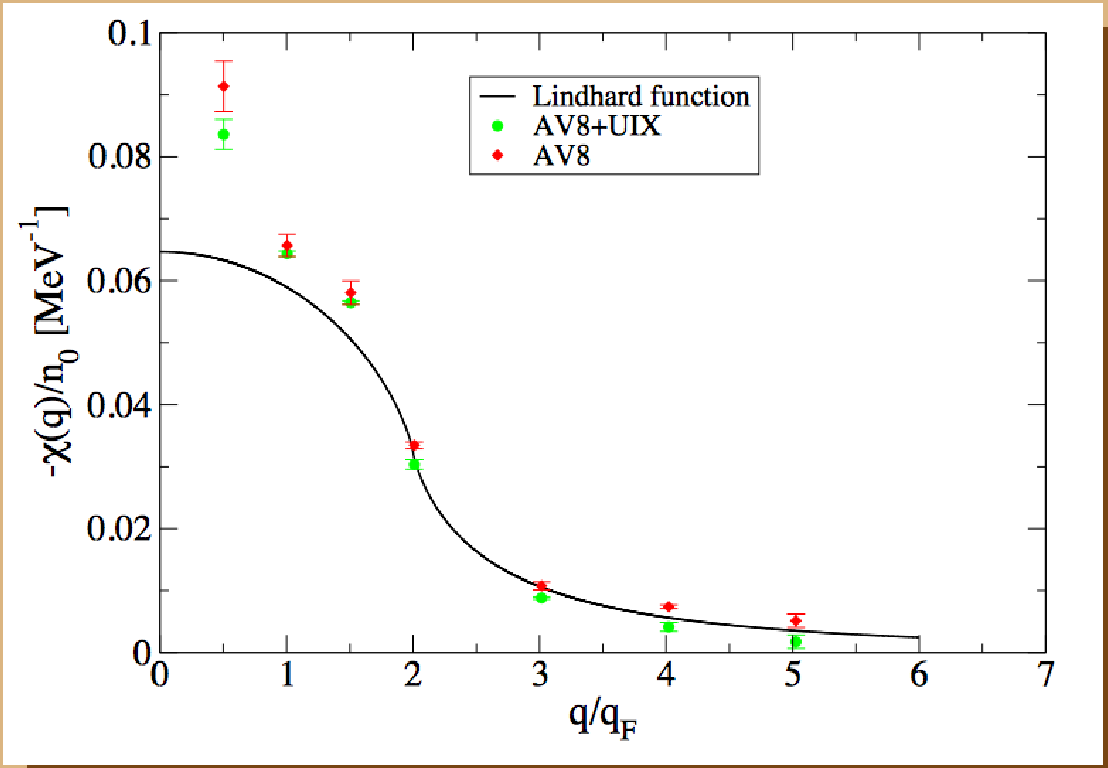
- First ever ab initio density-density response for neutron matter
- Neither Lindhard nor Coulomb
- Results on this plot derived from several strengths and periodicities

M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)

M. Buraczynski and A. Gezerlis, Phys. Rev. C **95**, 034012 (2017)

NEUTRONS

Many periodicities, many strengths



n = 0.04 fm^{-3}

- First ever ab initio density-density response for neutron matter
- Neither Lindhard nor Coulomb
- Results on this plot derived from several strengths and periodicities

NEUTRONS

M. Buraczynski and A. Gezerlis, Phys. Rev. Lett. **116**, 152501 (2016)

M. Buraczynski and A. Gezerlis, Phys. Rev. C **95**, 034012 (2017)

Conclusions

- Rich connections between physics of nuclei and that of compact stars
- Exciting time in terms of interplay between nuclear interactions, QCD, and many-body approaches
- Ab initio and phenomenology are mutually beneficial

Approach to collaborating

Quod si mea numina non sunt magna satis,
dubitem haud equidem implorare quod usquam est

But if my divine powers are not sufficient,
I won't hesitate to look for help wherever I find it

– Vergil
Aeneid, 7.261

Acknowledgments

Collaborators

Guelph

- Brendan Bulthuis
- Mateusz Buraczynski
- Hillary Dawkins
- Alexander Galea
- Ermal Rrapaj

LANL

- Joe Carlson
- Stefano Gandolfi

Darmstadt

- Hans-Werner Hammer
- Phillipp Klos
- Joel Lynn
- Achim Schwenk

INT

- George Bertsch
- Martin Hoferichter
- Ingo Tews

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MINISTÈRE DE LA RECHERCHE ET DE L'INNOVATION

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Extra slide 1

Big-picture questions

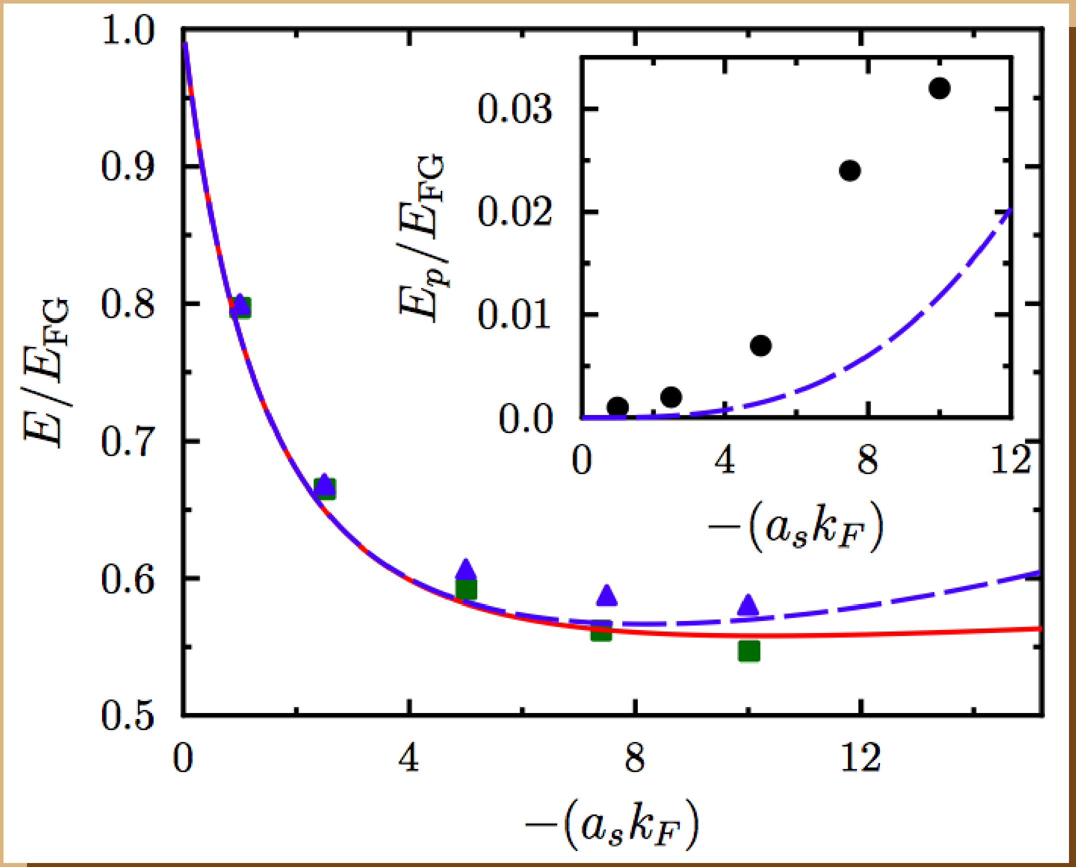
- Functionals tailored to neutron stars or universal density functional theory?
- Functional fit only to *ab initio* (as per Fayans and Orsay) or fit to any available data point?
- How will LIGO data constrain functionals? How will this propagate to *ab initio* and nuclear forces?

Extra slide 2

Little-picture questions

Extra slide 2a

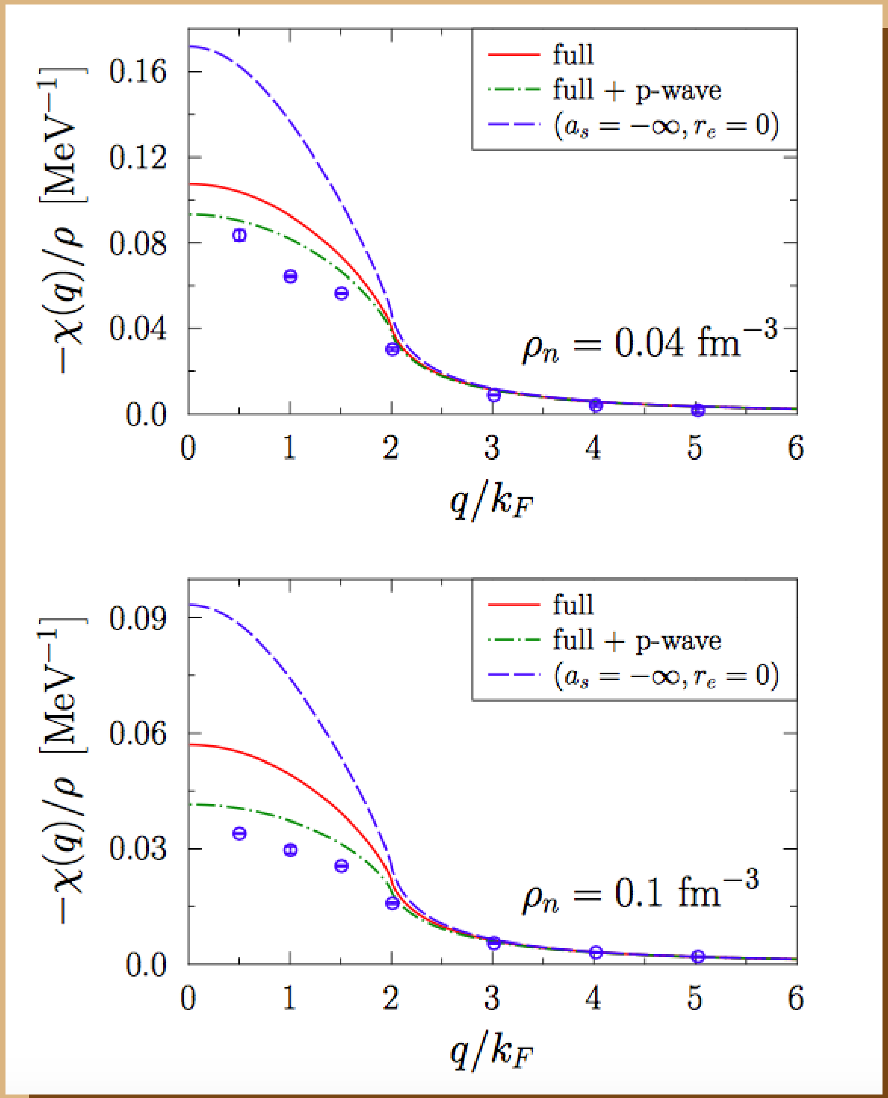
Superfluid properties within reach?



D. Lacroix, A. Boulet, M. Grasso, C.-J. Yang, Phys. Rev. C 95, 054306 (2017)

Extra slide 2b

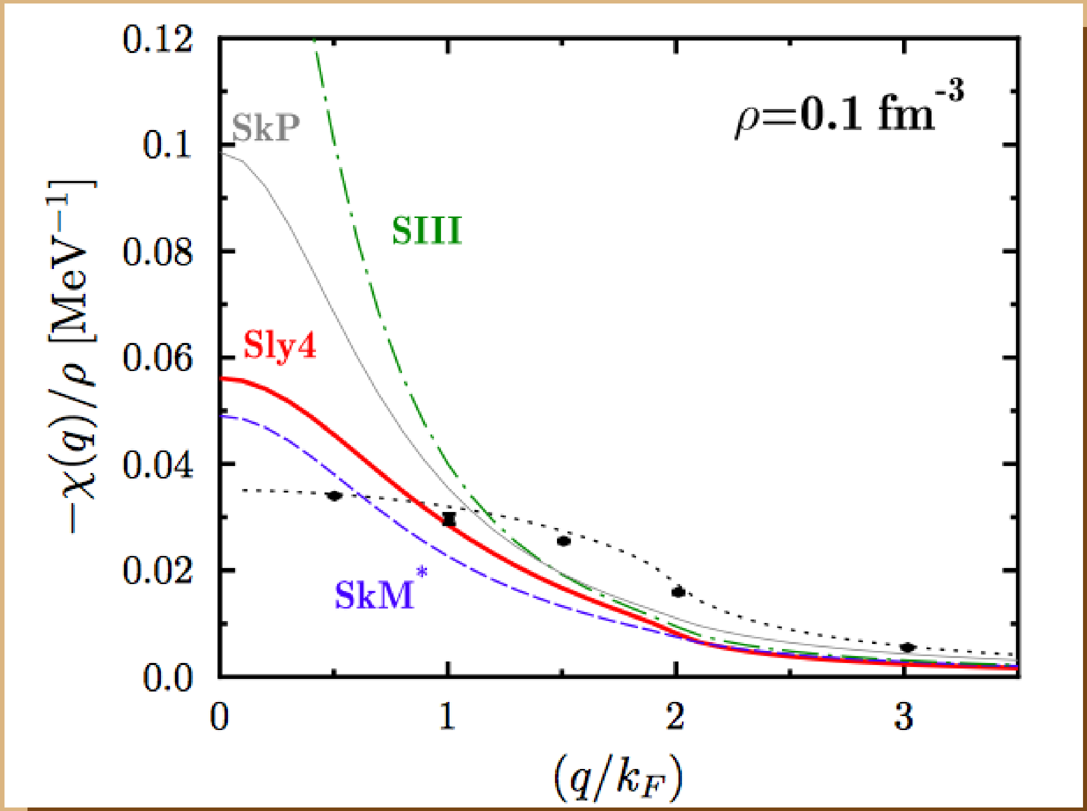
Response sensitive ro superfluidity? (i.e., what happens at low density?)



A. Boulet and D. Lacroix, arXiv:1709.05160

Extra slide 2c

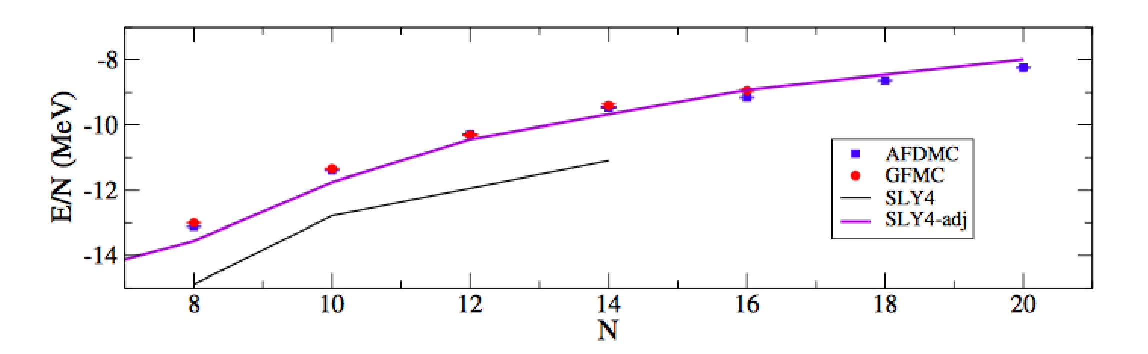
Something wrong with Skyrme response?



A. Boulet and D. Lacroix, arXiv:1709.05160

Extra slide 2d

Isovector coefficient density-dependent or not?



S. Gandolfi, J. Carlson, S. Pieper, Phys. Rev. Lett. 106, 012501 (2011)

M. Buraczynski and A. Gezerlis, Phys. Rev. C **95**, 034012 (2017)