Quantum droplets of ultracold bosons

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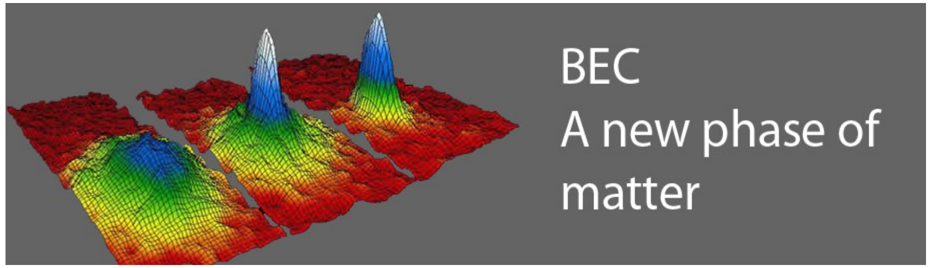
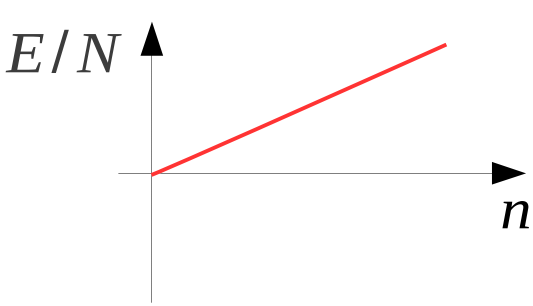
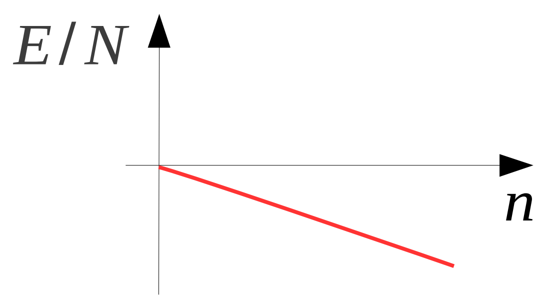
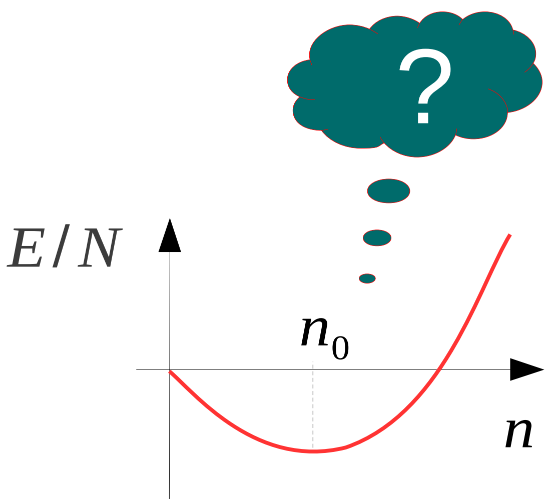


GAS requires trapping or collapses

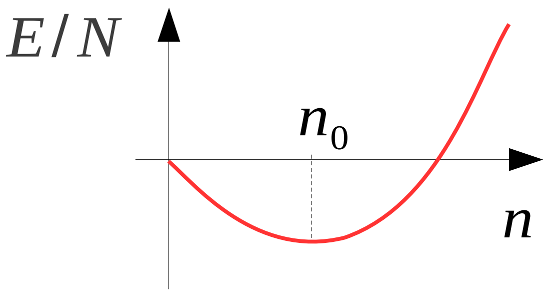






$$E/N \propto g_2 n + g_{\alpha+1} n^{\alpha}, \quad \alpha > 1$$

The gas should remain dilute, otherwise short lifetime!



$$E/N \propto a\,n + L^{3\alpha-2} n^{\alpha}, \quad \alpha>1$$

$$n_0 \sim \frac{1}{L^3}\left(\frac{a}{L}\right)^{\frac{1}{\alpha-1}}$$

Dilute = simultaneously small a and large L
and prefer small α

3-body $(\alpha=2)$

- Resonant (Efimov) 3-body force (Bulgac'02)

  lossy :(

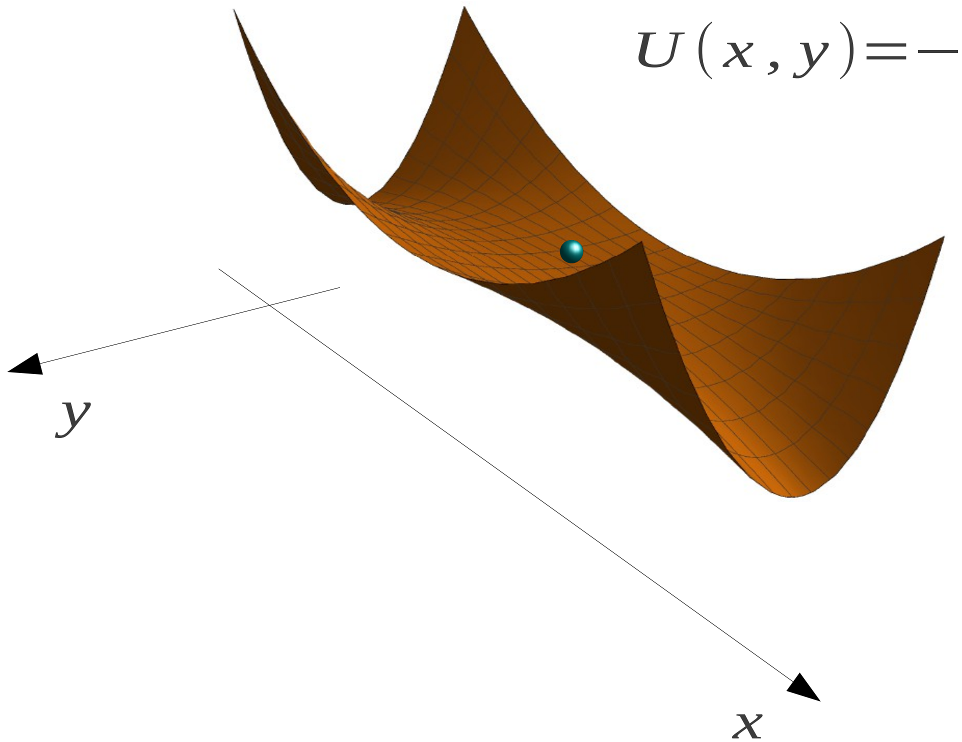
- Non-resonant 3-body force (DP'14)

  technically complicated :(

Lee-Huang-Yang $(\alpha=3/2)$

- Beyond-mean-field QUANTUM MECHANISM!
- Realized with dipoles and bosonic mixtures
  Dy (Stuttgart), Er (Innsbruck), K (Barcelona)
- LHY corection depends on dimensionality, properties of Bogoliubov spectrum (gapped/nongapped)... ⇒ lots of prospects

Quantum stabilization idea



$$U(x,y) = -\alpha x^2 + (\Omega^2 + \epsilon x^2) y^2$$

Stable for sufficiently large ϵ / α

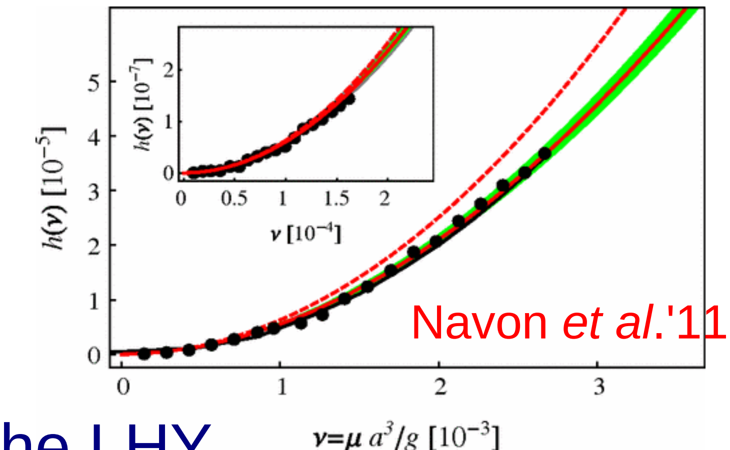
Classically unstable degree of freedom stabilized by quantum fluctuations in another degree of freedom!

Lee-Huang-Yang term

For spinless BEC: $$\frac{E}{\text{Volume}} = \frac{g_2 n^2}{2}\left(1 + \frac{128}{15}\sqrt{\frac{na^3}{\pi}} + \ldots\right)$$

LHY correction is UNIVERSAL (depends only on the scattering length) and QUANTUM (zero-point energy of Bogoliubov phonons)!

Observed in ultracold gases where the scattering length is tunable by using Feshbach resonances (Innsbruck, MIT, ENS, JILA, Rice)



Unfortunately, the effect is perturbative and the LHY term is smaller than the mean-field one!

Bose-Bose mixture, mean field

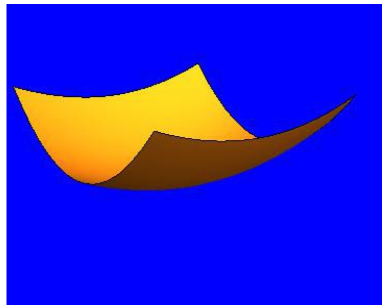
Mean-field energy density: $$\frac{E_{MF}}{\mathrm{Volume}} = \frac{g_{11} n_1^2 + g_{22} n_2^2 + 2 g_{12} n_1 n_2}{2}$$

g_{12}

$g_{12} > \sqrt{g_{11} g_{22}}$ phase separation

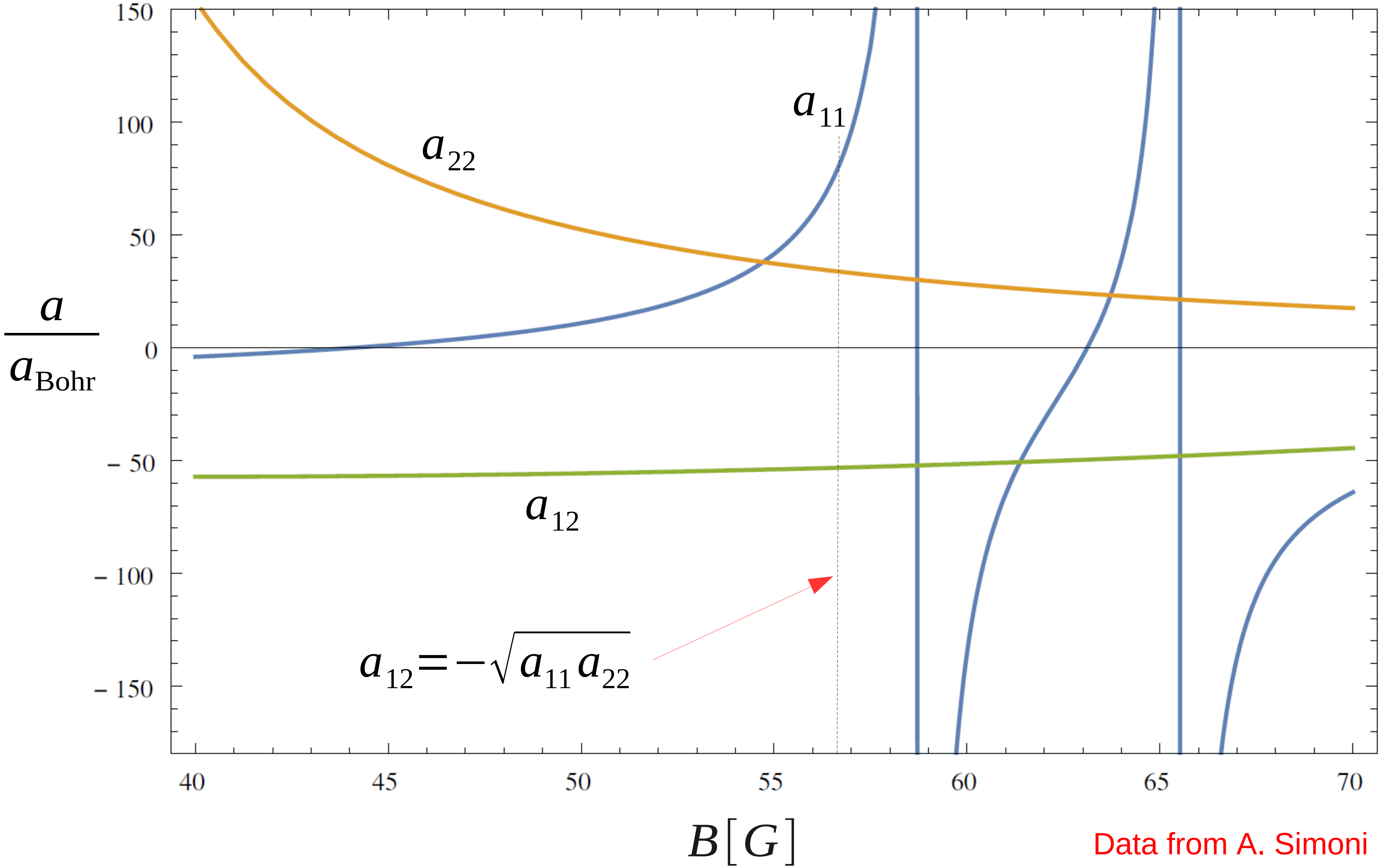
mean-field stability

$g_{11} > 0, \quad g_{22} > 0, \quad \text{and} \quad g_{12}^2 < g_{11} g_{22}$



$g_{12} < -\sqrt{g_{11} g_{22}}$ collapse

^{39}K: |F=1,m$_F$=0> and |F=1,m$_F$=-1>



Data from A. Simoni

LHY correction

Bogoliubov theory

$$E_{\pm}(k)=\sqrt{c_{\pm}^2 k^2+k^4/4}; \quad c_{\pm}^2=\frac{g_{11}n_1+g_{22}n_2\pm\sqrt{(g_{11}n_1-g_{22}n_2)^2+4g_{12}^2 n_1 n_2}}{2}$$

$$\frac{E}{\text{Volume}}=\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}+\frac{1}{2}\sum_{\pm}\sum_{k}[E_{\pm}(k)-k^2/2-c_{\pm}^2]=$$

$$=\underbrace{\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}}_{\text{MF}\propto g n^2}+\underbrace{\frac{8}{15\pi^2}(c_+^5+c_-^5)}_{\text{LHY}\propto (g n)^{5/2}}$$

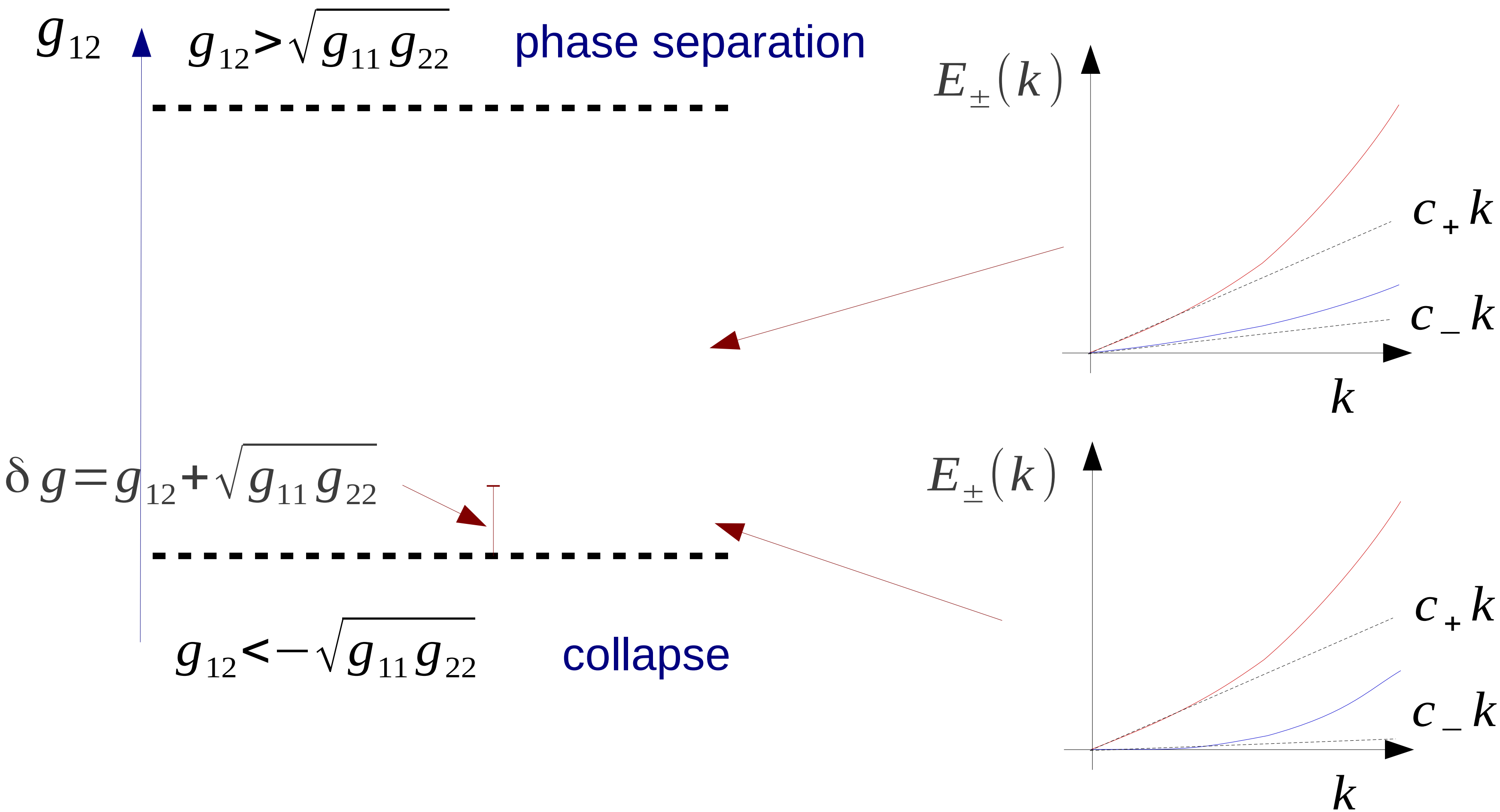
(Larsen'63)

In contrast to one-component case, MF and LHY depend on (different) combinations of $g_{\sigma\sigma'}n_{\sigma}$

...and, thus, can be independently controlled!

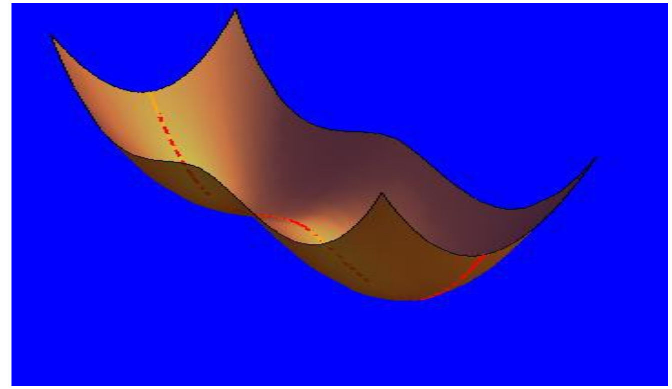
LHY correction

$$\frac{E}{\text{Volume}} = \frac{g_{11} n_1^2 + g_{22} n_2^2 + 2 g_{12} n_1 n_2}{2} + \frac{8}{15\pi^2} \frac{m^4}{\hbar^3} \left(c_+^5 + c_-^5 \right) + \ldots$$



Quantum stabilization

$$\delta g = g_{12} + \sqrt{g_{11} g_{22}} \ll \sqrt{g_{11} g_{22}} = g$$



The mean-field term "locks" the ratio $\frac{n_2}{n_1} = \sqrt{\frac{g_{11}}{g_{22}}}$

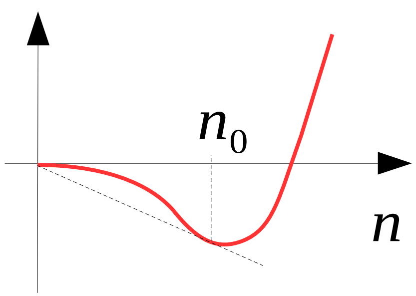
Softening of lower Bog. mode $c_- \ll c_+ \propto \sqrt{gn/m}$

only the ``stiff'' + branch contributes to the LHY term

The structure of the energy-density functional:

$$\frac{E}{\text{Volume}} = A_1 \delta g n^2 + A_2 (m/\hbar^2)^{3/2} (gn)^{5/2}$$

$\delta g < 0$ ⇨



⇨ Gas exists in equilibrium with vacuum. Saturation density

$$n_0 \propto \frac{1}{a^3}\left(\frac{\delta g}{g}\right)^2$$

Density is tunable by modifying interaction parameters!

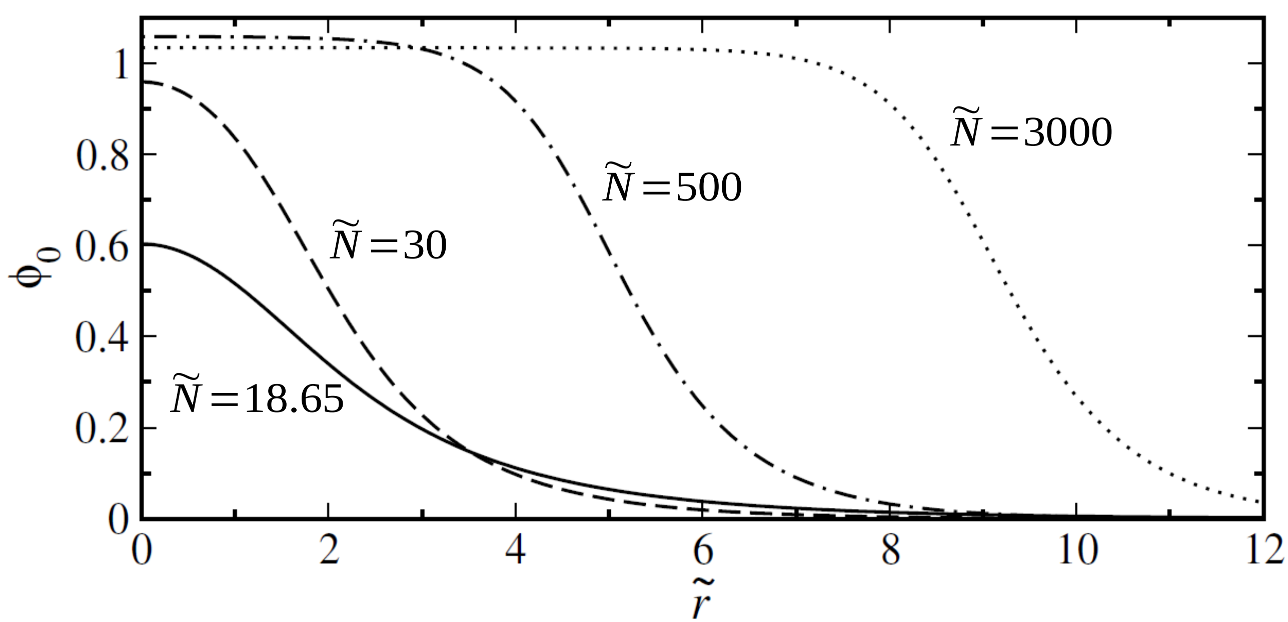
Gross-Pitaevskii eq., droplet shape

Rescaling $\vec{r} = \xi \vec{\tilde{r}}, \; t = \tau \tilde{t}, \; N = n \xi^3 \tilde{N}$, where $\xi \propto 1/\sqrt{m|\delta g|n}, \; \tau \propto 1/|\delta g| n$

$$i \partial_{\tilde{t}} \varphi = \left(-\nabla_{\vec{\tilde{r}}}^{\,2}/2 - 3|\varphi|^2 + 5|\varphi|^3/2 - \tilde{\mu}\right)\varphi$$

$$\tilde{N} = \int |\varphi|^2 d^3 \tilde{r}$$

Modified Gross-Pitaevskii equation cubic-quartic nonlinearities



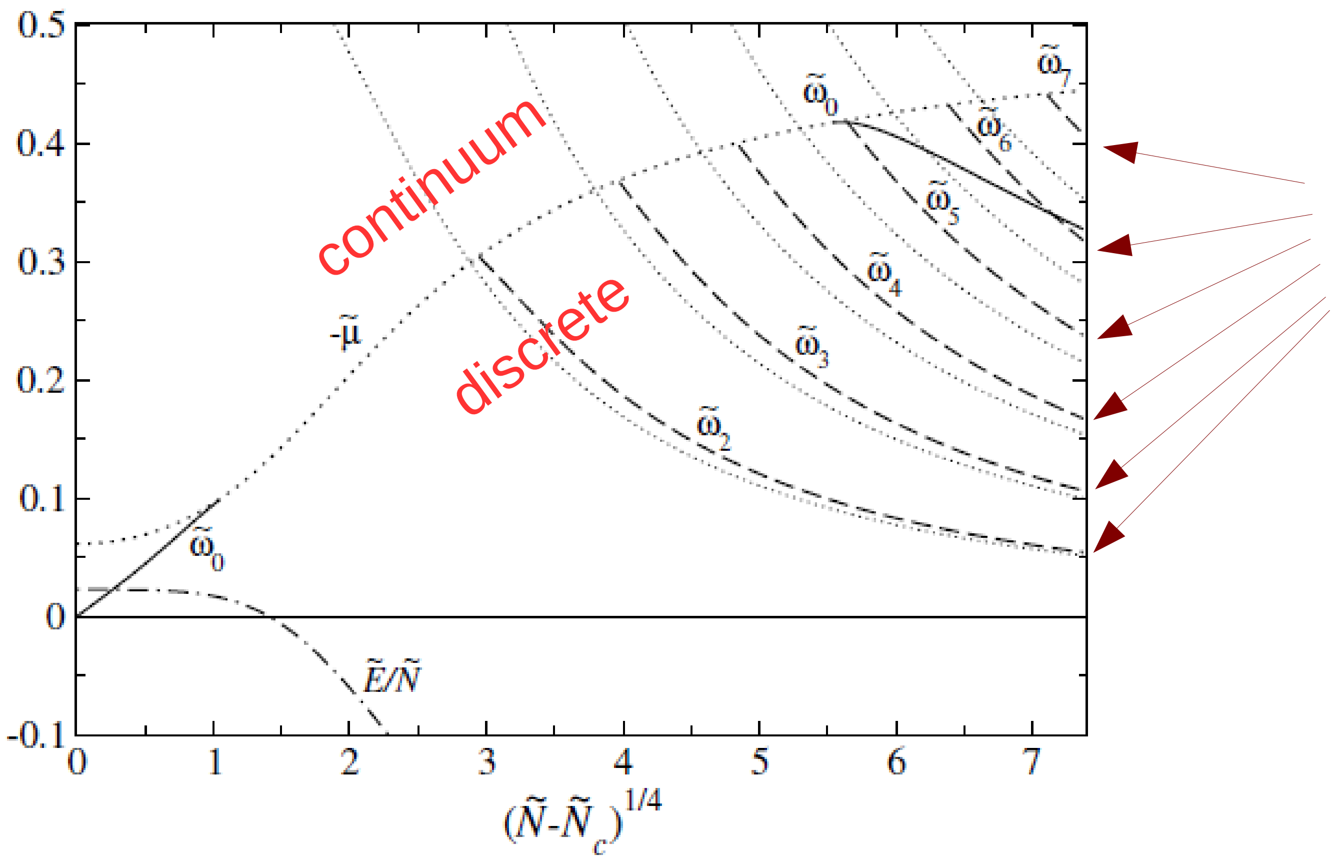
Bogoliubov-de Gennes eqs., excitations

$$\varphi(\tilde{t},\vec{\tilde{r}})=\varphi_0(\vec{\tilde{r}})+\delta\varphi(\tilde{t},\vec{\tilde{r}})$$

⇩

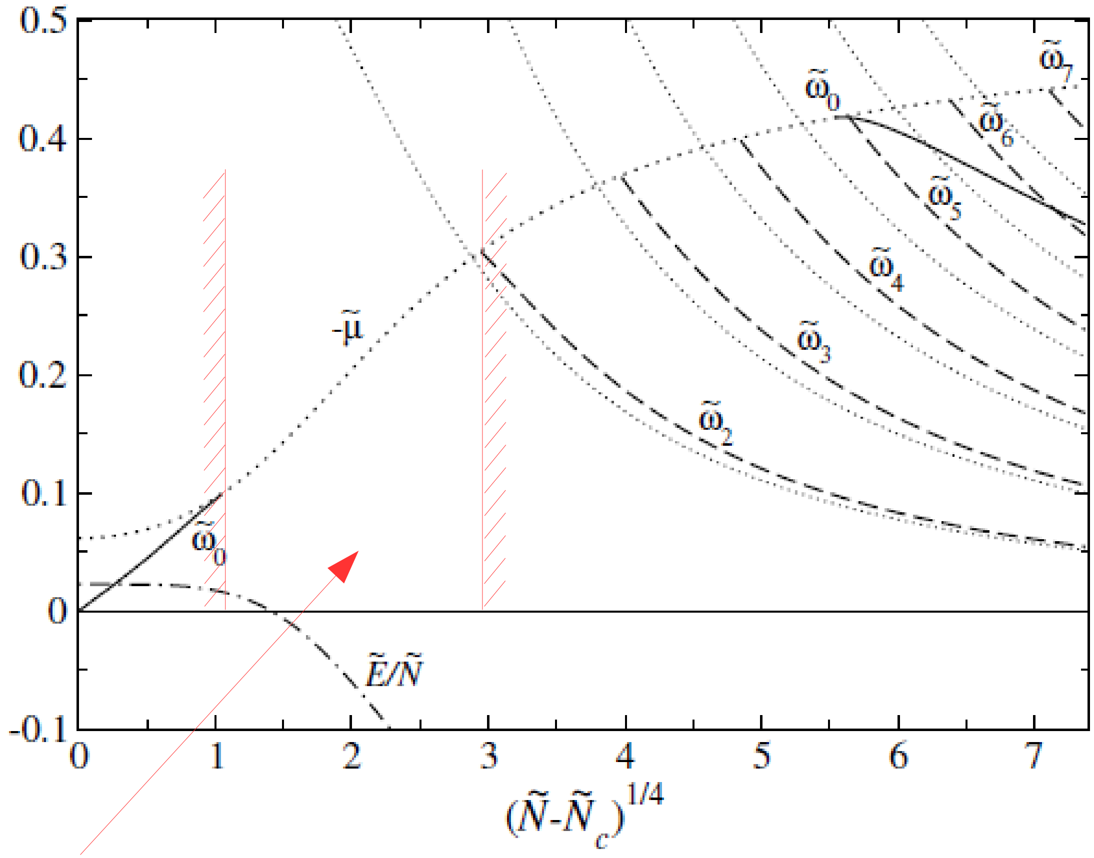
linearize $i\,\partial_{\tilde{t}}\varphi=(-\nabla_{\vec{\tilde{r}}}{}^2/2-3|\varphi|^2+5|\varphi|^3/2-\tilde{\mu})\varphi$ with respect to small $\delta\varphi(\tilde{t},\vec{\tilde{r}})$

⇩



Surface modes

Zero-temperature object



No discrete modes → the droplet evaporates itself to zero T!

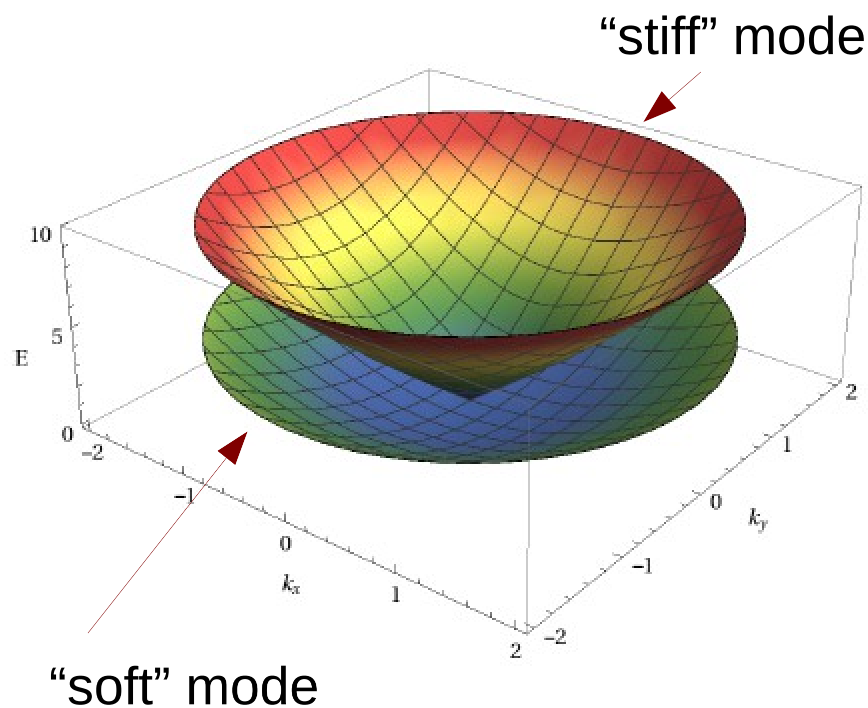
(by contrast, ^{4}He droplets always have discrete modes)

Macroscopic zero-temperature object:

- is interesting by itself
- can be used for sympathetic cooling of other systems

Bose-Bose mixture vs Dy/Er

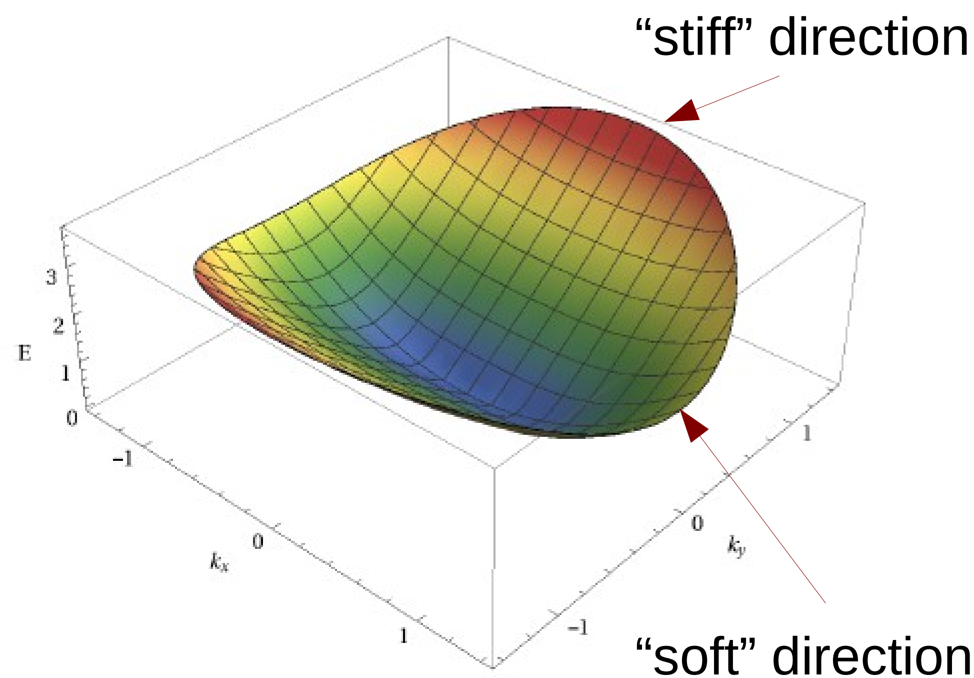
Bose-Bose mixture



$$\text{LHY} = \frac{8}{15\pi^2}\frac{m^4}{\hbar^3}\left(c_+^5 + c_-^5\right)$$

(Larsen'63)

dipolar Bose gas



$$\text{LHY} = \frac{8}{15\pi^2}\frac{m^4}{\hbar^3}\left\langle c^5(\hat{k})\right\rangle_{\hat{k}}$$

(Lima&Pelster'11)

LHY depends on ...

$$\frac{E}{\text{Volume}}=\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}+\frac{1}{2}\sum_{\pm}\sum_{k}\left[E_{\pm}(k)-k^2/2-c_{\pm}^2\right]=$$

number of components

density of states (dimension)

shape of the Bogoliubov spectrum
(anisotropy of the interaction,
driving the mixture, etc.)

...and life becomes ~~harder~~ more interesting in the inhomogeneous case
particularly if LDA is not valid

Quantum droplets in low-D

with Grisha Astrakharchik (UPC, Barcelona)

Bogoliubov theory – ``Mean field + LHY"

3D:

$$\frac{E_{3D}}{\text{Volume}} = \frac{1}{2}\sum_{\sigma\sigma'} g_{\sigma\sigma'} n_\sigma n_{\sigma'} + \frac{8}{15\pi^2}\sum_{\pm} c_\pm^5 \sim \delta g\, n^2 + (gn)^{5/2}$$

$$\sqrt{n g^3} \ll 1$$

2D:

$$\frac{E_{2D}}{\text{Surface}} = \frac{1}{2}\sum_{\sigma\sigma'} g_{\sigma\sigma'} n_\sigma n_{\sigma'} + \frac{1}{8\pi}\sum_{\pm} c_\pm^4 \ln\frac{c_\pm^2\sqrt{e}}{\kappa^2} \sim g^2 n^2 \ln\frac{n}{n_0}$$

$$g_{\sigma\sigma'} = 2\pi/\ln\left(2e^{-\gamma}/a_{\sigma\sigma'}\kappa\right) \ll 1$$

1D:

$$\frac{E_{1D}}{\text{Length}} = \frac{1}{2}\sum_{\sigma\sigma'} g_{\sigma\sigma'} n_\sigma n_{\sigma'} - \frac{2}{3\pi}\sum_{\pm} c_\pm^3 \sim \delta g\, n^2 - (gn)^{3/2}$$

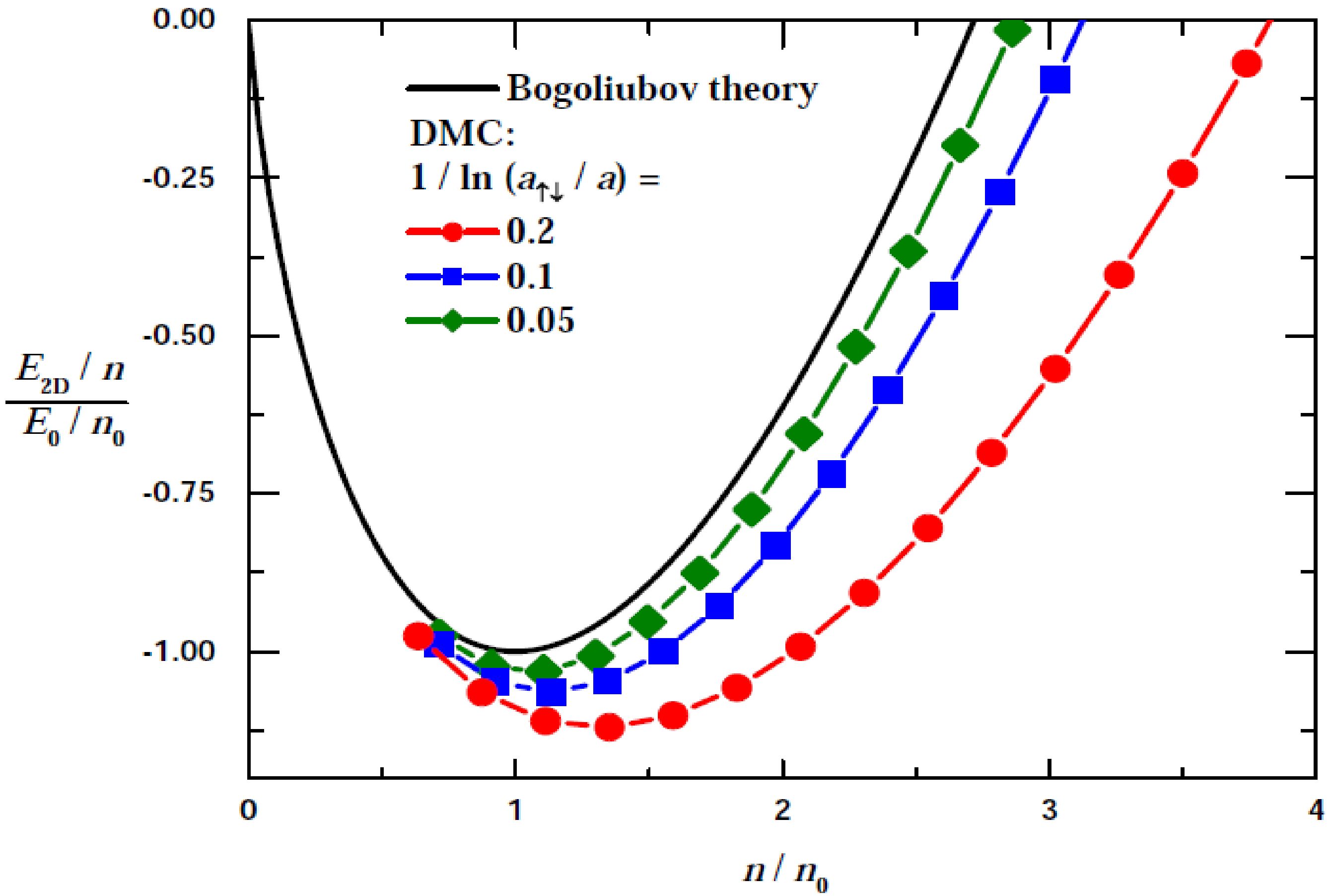
$$\sqrt{g/n} \ll 1$$

!

2D symmetric case

$$a_{\uparrow\uparrow}=a_{\downarrow\downarrow}=a \;\Rightarrow\; n_{\uparrow}=n_{\downarrow}=n \;\Rightarrow\; \frac{E_{2D}}{\text{Surface}}=\frac{8\pi n^2}{\ln^2(a_{\uparrow\downarrow}/a)}\left[\ln(n/n_0)-1\right]$$

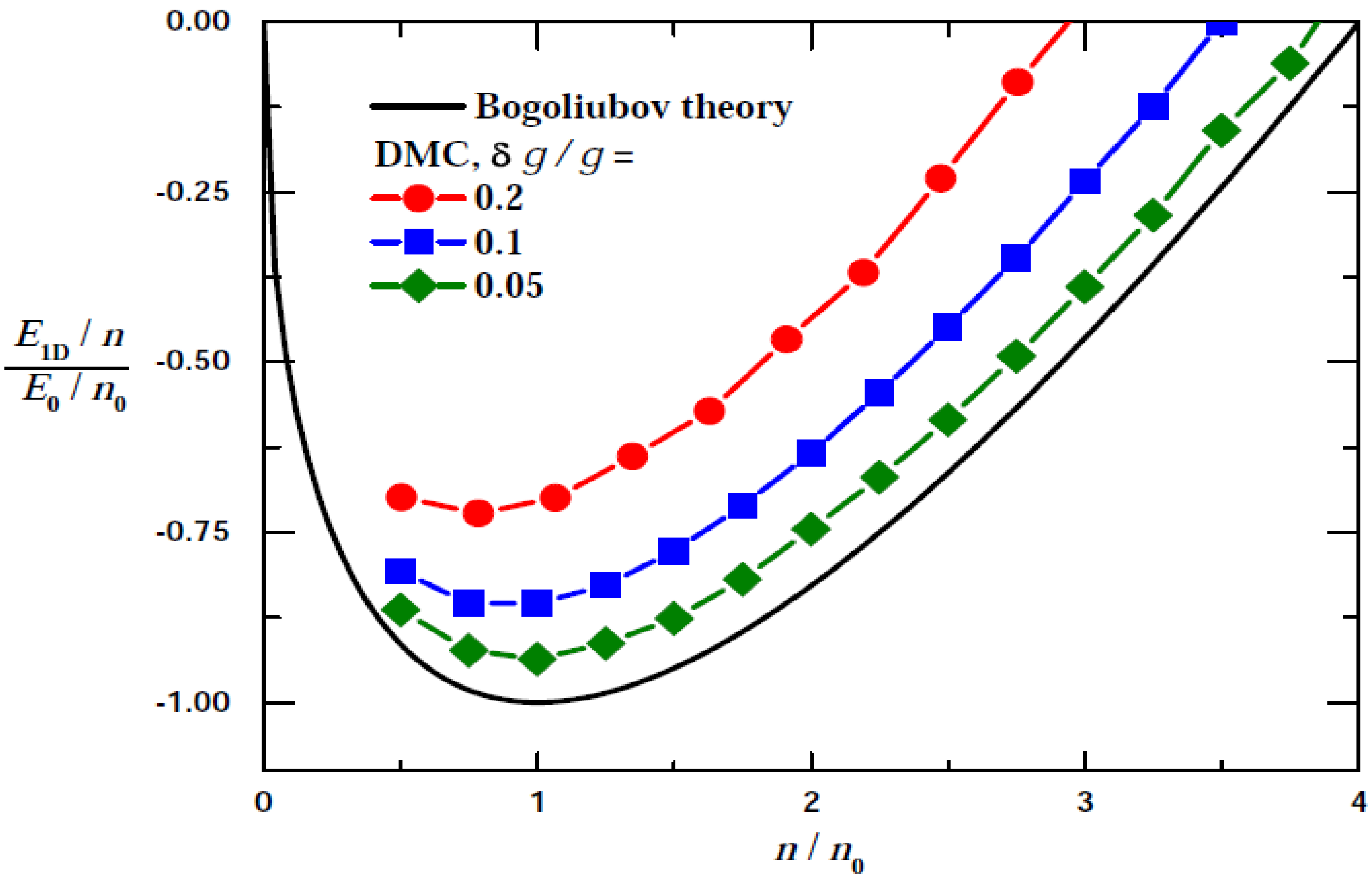
where $n_0=\frac{e^{-2\gamma-3/2}}{2\pi}\frac{\ln(a_{\uparrow\downarrow}/a)}{a a_{\uparrow\downarrow}}$



1D symmetric case

$$g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = g \Rightarrow n_{\uparrow} = n_{\downarrow} = n \Rightarrow \frac{E_{1D}}{\text{Length}} = \delta g\, n^2 - \frac{4\sqrt{2}}{3\pi}(gn)^{3/2}$$

Minimum for $\delta g > 0$ at $n_0 = \dfrac{8 g^3}{9\pi^2 \delta g^2}$



3D vs low-D

- 3D droplet disappears for $N<N_c$
- Low-D droplets are bound for any N

- 3D liquids are in the mean-field unstable regime ($\delta g<0$)
- 2D mixture liquefies for any weakly repulsive intra- and weakly attractive interspecies interaction
- 1D liquid is in the regime stable from the mean-field viewpoint ($\delta g>0$)

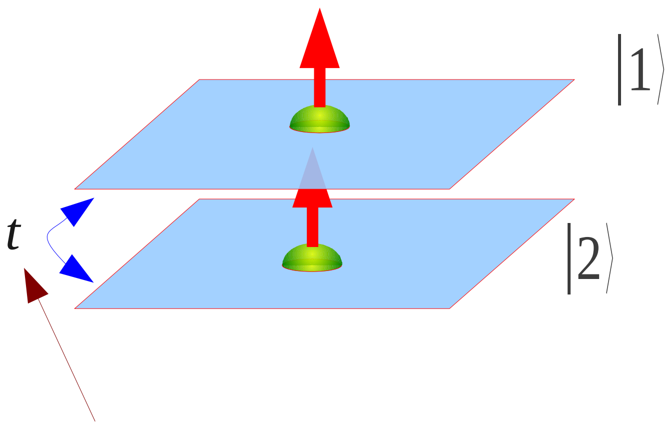
- The 1D modified stationary GP equation is solvable → full analytic solution for the shape of the droplet

$$\psi(x)=\frac{\sqrt{n_0}\,\mu/\mu_0}{1+\sqrt{1-\mu/\mu_0}\cosh\left(\sqrt{-2\mu}\,x\right)}$$

From LHY to three-body force

with Alessio Recati (Trento&Munich)

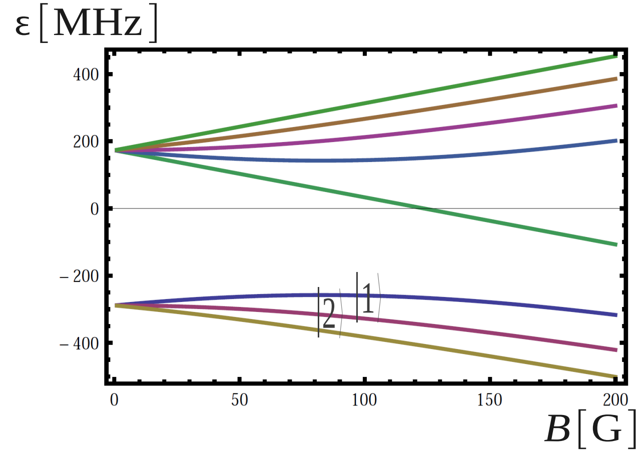
Driven Bose-Bose mixture



interlayer tunneling

OR

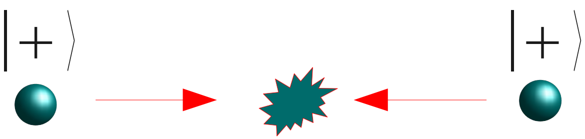
^{39}K: $|F=1,m_F=0\rangle$ and $|F=1,m_F=-1\rangle$



RF coupling with Rabi frequency $\Omega \sim \text{kHz}$

$$|1\rangle, |2\rangle \;\rightarrow\; |-\rangle = \frac{|1\rangle - |2\rangle}{\sqrt{2}}, \quad |+\rangle = \frac{|1\rangle + |2\rangle}{\sqrt{2}}$$

Ω ⇒ Effectively scalar Bose gas of |+> atoms



$$a_{++}(a_{11}, a_{22}, a_{12}, \Omega) \rightarrow 0 \;\Rightarrow\; \frac{E}{\text{Volume}} = g_3 \frac{n^3}{3!} \dots$$

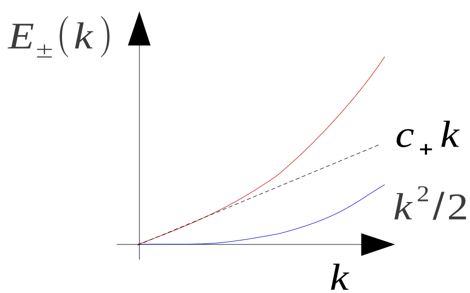
Three-body force, where is LHY?

Symmetric case close to collapse $g_{11}=g_{22}=-g_{12}=g$ and $n_1=n_2=n/2$

without coupling

$$E_+(k)=\sqrt{(k^2/2)(k^2/2+2\mathrm{gn})}$$

$$E_-(k)=k^2/2$$

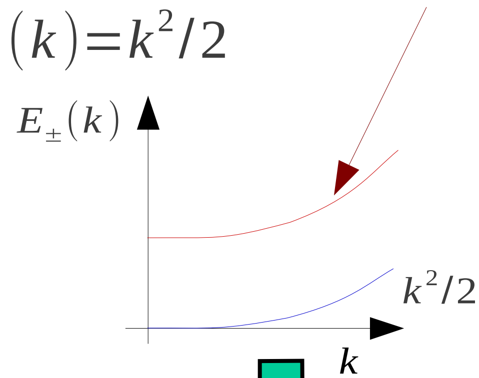


$$\mathrm{LHY}=\frac{8}{15\pi^2}(gn)^{5/2}$$

with coupling (Goldstein&Meystre'97)

$$E_+(k)=\sqrt{(k^2/2+\Omega)(k^2/2+\Omega+2\mathrm{gn})}$$

$$E_-(k)=k^2/2$$



$$\mathrm{LHY}=\frac{2}{\pi^2}(gn)^{5/2}\int_0^1\sqrt{x(1-x)\left(x+\frac{\Omega}{2\mathrm{gn}}\right)}\,dx$$

$gn \ll \Omega$

$$=\frac{\sqrt{\Omega}\,g^2}{2\sqrt{2}\pi}\frac{n^2}{2}+\frac{3g^3}{4\sqrt{2}\pi\sqrt{\Omega}}\frac{n^3}{3!}+\dots$$

Renormalization of two-body interaction

Effective three-body force

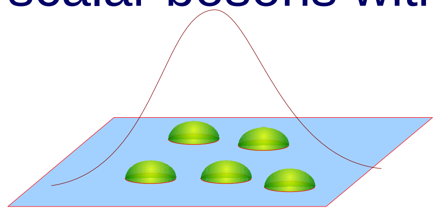
Useful conclusions

- Gapped stiff mode → LHY correction transforms into effective 3-body force
- Same in low dimensions (coincides with direct three-body calculations)
- Also checked the nonsymmetric case.
- Bogoliubov theory – powerful tool for calculating three-body force. More efficient than direct solution of the three-body problem.

Prospects

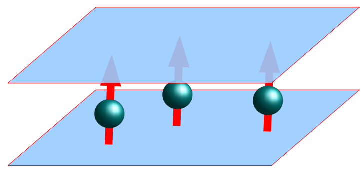
- Theory beyond local density approximation (LDA)

  - droplet of two-dimensional scalar bosons with zero-range interactions. Theory beyond Hammer&Son'04

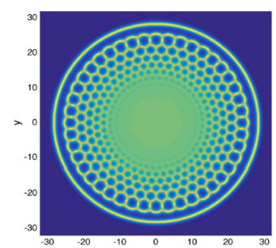


$$B_N / B_{N-1} \rightarrow 8.567$$

  - quasi-low-dimensional problems with short- and long-range interactions when the healing length of the mode responsible for the LHY correction is comparable to the size of the condensate



  - LHY correction to the supersolid phase?



- Get out of the weakly-interacting regime while staying dilute?

  - use sign-problem-free bosonic Monte Carlo

- Fermions?

Thank you!

- Three-body force, PRL 112, 103201 (2014)
- Multi-body interaction on lattice, PRA 90, 021601 (2014)
- 3D mixture + beyond mean field, PRL 115, 155302 (2015)
- Low-D mixture + beyond mean field, PRL 117, 100401 (2016)