A natural EDF Ansatz for homogeneous nuclear matter: The shortest path to nuclei

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A nuclear energy-density functional inspired by an effective field theory

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An *ab initio* energy density functional (?)

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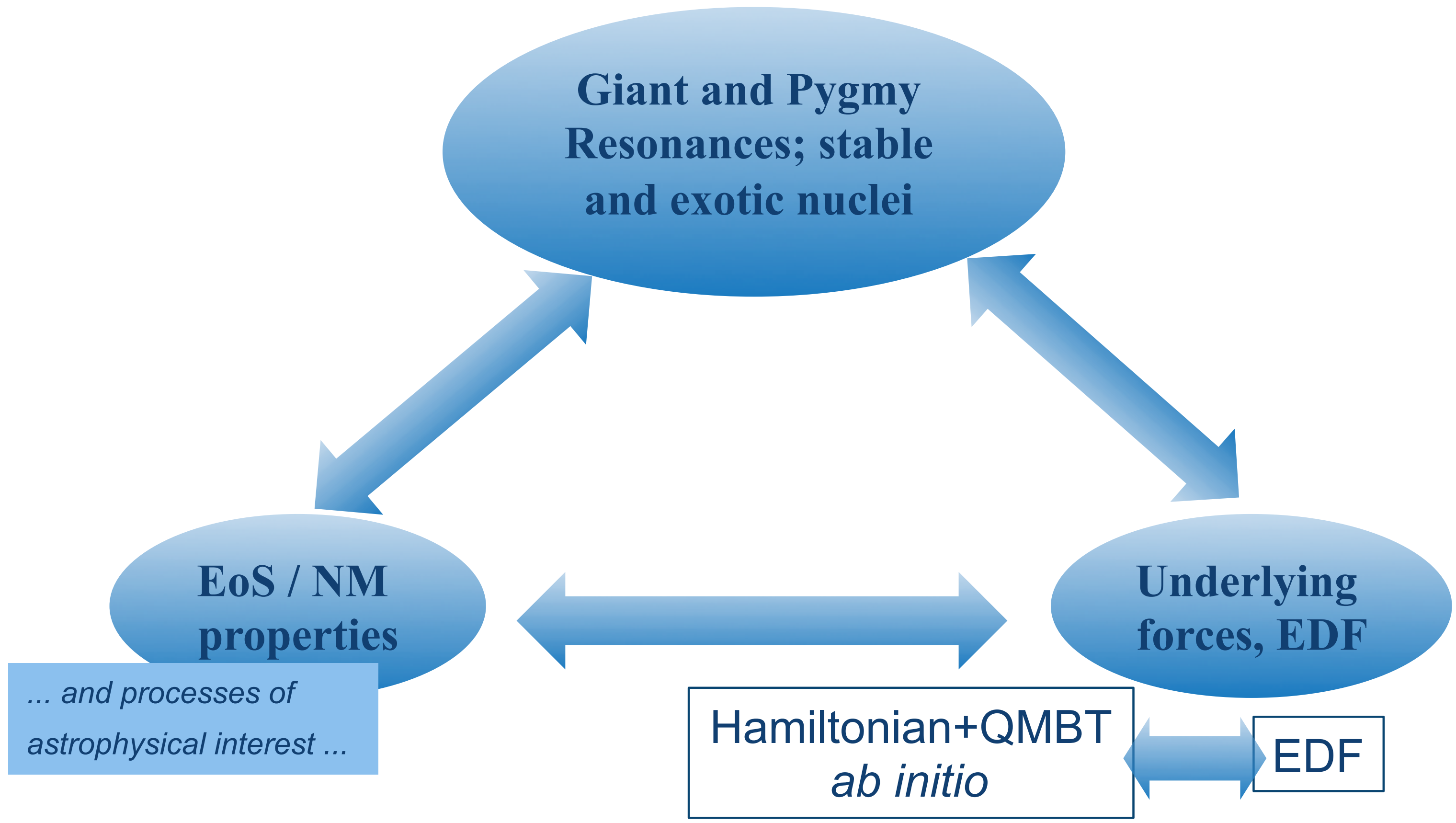
A triangle of interests



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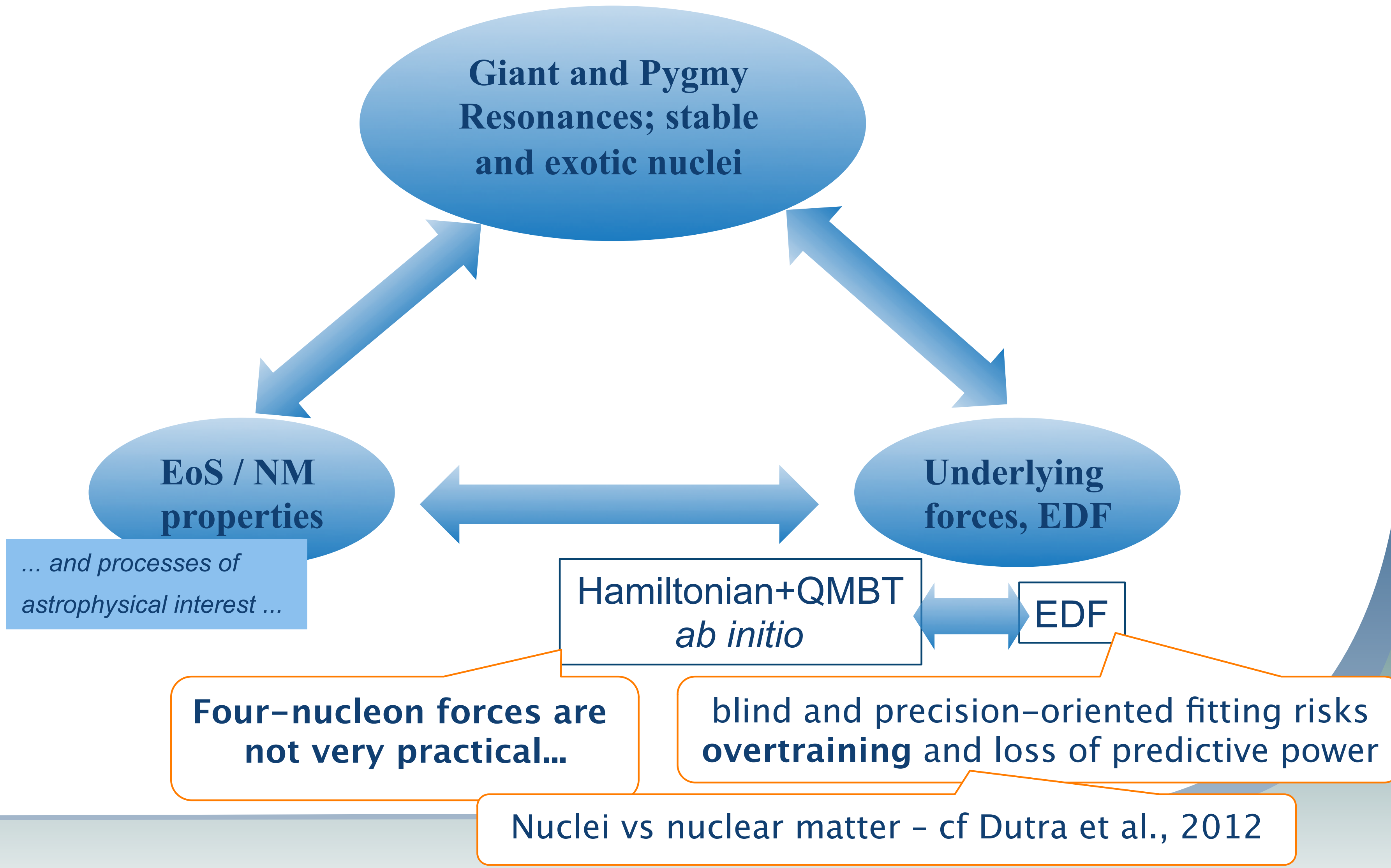

People

- ❖ Chang Ho Hyun, **D**aegu University
- ❖ Tae-Sun Park, **S**ungkyunkwan University
- ❖ Yeunhwan Lim, IBS (now in Texas)

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  - **I**BS (that’s me and YHL)
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  - **S**ungkyunkwan
- Hana Gil, Kyungpook National University
- Yongseok Oh, Kyungpook National University
- Gilho Ahn, University of Athens, Greece

Overview

- About those density-dependent “interactions”
- Motivation for the KIDS Ansatz
  - A textbook example
  - EFT of dilute matter
- Fitting in homogeneous matter
  - APR pseudodata
  - Hierarchy of terms?
  - Naturalness
- Mapping onto a Skyrme functional and applications in nuclei
  - *With no refitting*
- Many prospects and open questions → Overtime

Density-dependent "interaction"

- Original Ansatz by Skyrme [Nucl.Phys.9(1958)615]:

$$T = \sum_{i<j} t_{ij} + \sum_{i<j<k} t_{ijk}$$

$$t_{12} = \delta(\mathbf{r}_1 - \mathbf{r}_2) t(\mathbf{k}', \mathbf{k})$$

$$t_{123} = \delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_3 - \mathbf{r}_1) t_3$$

$$\begin{aligned} t(\mathbf{k}', \mathbf{k}) = {} & t_0(1+x_0 P^\sigma) + \tfrac{1}{2}t_1(1+x_1 P^\sigma)(\mathbf{k}'^2+\mathbf{k}^2) \\ & + t_2[1+x_2(P^\sigma - \tfrac{4}{5})]\mathbf{k}' \cdot \mathbf{k} \\ & + \tfrac{1}{2}T[\boldsymbol{\sigma}_1 \cdot \mathbf{k}\boldsymbol{\sigma}_2 \cdot \mathbf{k} - \tfrac{1}{3}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \mathbf{k}^2 + \text{conj.}] \\ & + \tfrac{1}{2}U[\boldsymbol{\sigma}_1 \cdot \mathbf{k}'\boldsymbol{\sigma}_2 \cdot \mathbf{k} - \tfrac{1}{3}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \mathbf{k}' \cdot \mathbf{k} + \text{conj.}] \\ & + V[i(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{k}' \times \mathbf{k}], \end{aligned}$$

- t_{123} term equivalent to a density-dependent t_{12} term

$$\frac{t}{6}(1+P_\sigma)\rho[(\mathbf{r}_1+\mathbf{r}_2)/2]\delta(\mathbf{r}_1-\mathbf{r}_2)$$ [Vautherin&Brink,PRC5(1972)]

- Extension: fractional-power density dependence

$$\frac{t}{6}(1+P_\sigma)\rho^\alpha[(\mathbf{r}_1+\mathbf{r}_2)/2]\delta(\mathbf{r}_1-\mathbf{r}_2)$$

  - Explosion of activity!
  - Gogny-type forces: similar term

Density-dependent "interaction"

- From local-density approximation: $\sim\rho^{2/3}$
  - Bethe,PR167(1968); Moszkowski, PRC2(1970)
  - In Skyrme-type forces, from momentum dependence
- From empirical arguments: $\sim\rho^{1/3}$
  - Zamick,PL45B(1973)
- And more generally $\sim\rho^{\alpha}$ with $\alpha\leq 2/3$
  - Krivine et al.,NPA336(1980) – esp. for compressibility
  - And many since
- In current use: 1/2,1/3,1/6,fitted...
- Also: more than one density-dependent term
  - Agrawal et al., Xiong et al., Zhang et al., ...

Density-dependent “interaction”

- Many questions:
  - **What should the fraction be?**
    - Precise value often chosen arbitrarily
  - Do we need more than one density-dependent couplings?
  - More terms always provide better fits... but they still risk loss of predictive power
  - Is there any guidance *before* we start cumbersome fitting?

Our answer so far:

- Low-order powers of $\boldsymbol{\rho}^{1/3}$
- More than one powers necessary
- **SNM and PNM have different “preferences”**

Interaction vs energy-density functional

- The elementary entity is the energy density (or energy per particle) as a unique functional of the density
  - Mapping as per Hohenberg-Kohn
  - The function E[ρ] is a **black box**
- The "interaction" which, in an orbital basis, yields the correct E[ρ] is an auxiliary entity with no immediate connection to an on-shell interaction
- Density-dependent couplings in the "interaction" **arise even in the absence of three-nucleon interactions** – fundamental requirement

Very dilute Fermi system

To our knowledge, the most complete low-density expansion for the ground-state energy per particle of such a system is [13–16]:

$$\frac{E}{N} = \frac{k_F^2}{2M}\Bigg[\frac{3}{5} + (g-1)\Bigg\{\frac{2}{3\pi}(k_F a_s) + \frac{4}{35\pi^2}(11 - 2\ln 2)(k_F a_s)^2 + \frac{1}{10\pi}(k_F r_s)(k_F a_s)^2 + \big(0.076 + 0.057(g-3)\big)(k_F a_s)^3\Bigg\} + (g+1)\frac{1}{5\pi}(k_F a_p)^3 + (g-1)(g-2)\frac{16}{27\pi^3}\big(4\pi - 3\sqrt{3}\big)(k_F a_s)^4 \ln(k_F a_s) + \cdots\Bigg]. \quad (1)$$

In Eq. (1), a_s and r_s are the s-wave scattering length and effective range, and a_p is the p-wave scattering length. The spin degeneracy is denoted by g. For a natural system, this is an expansion in Fermi momentum k_F over the scale Λ. The mean-field correction of $\mathcal{O}(k_F^3)$ dates from 1929 [17], the $\mathcal{O}(k_F^4)$ correction from the 1950's [18,19], while the $\mathcal{O}(k_F^5)$ corrections and the logarithm were found in the 1960's [20]. The complete expression in Eq. (1) has been derived using the method of correlation functions [13,14], by expanding Goldstone diagrams [15,16], and by expanding Feynman diagrams [16]. Here we rederive and illuminate this result using EFT methods.

H.-W. Hammer, R.J. Furnstahl / Nuclear Physics A 678 (2000) 277–294

Functional vs "interaction"

- Any term of E/A ~ ρ^{1+a} can be generted by a density-dependent zero-range "interaction" ~$\rho^a\delta(r_{12})$
- More generally, any term of E/A ~ f(ρ) can be generted by a density-dependent "interaction" ~$[f(\rho)/\rho]\delta(r_{12})$
- Plus asymmetry depenence: exchange term

- We will determine an Ansatz for EDF
- We will fix everything in homogeneous matter
  - Statistical analysis: how many terms do we need?
- Nuclei will give us the two unconstrained parameters:
  - Effective mass and spin-orbit force

Guessing the EDF form - part I: Brueckner theory

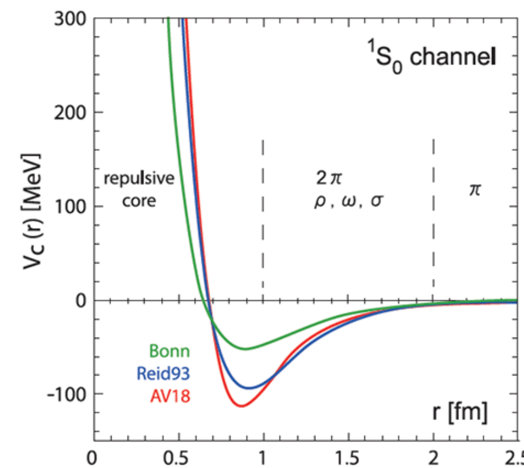
Fetter and Walecka, "Quantum theory of many-particle systems"

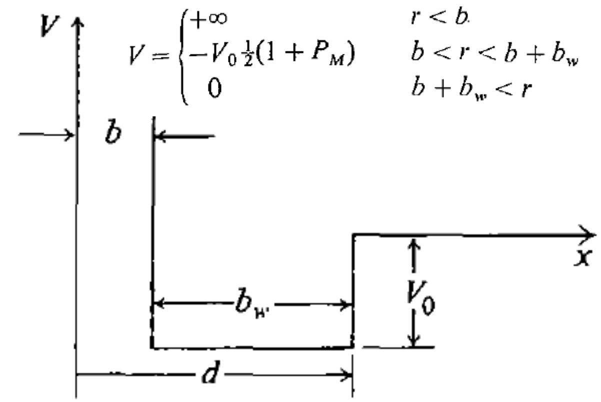
- ❖ Realistic potential: strong repulsive core plus attraction at longer range
- ❖ Apply Brueckner methodology in the calculation of nuclear matter energy

➔ Result: k_F^2, k_F^3, k_F^4, k_F^5, k_F^6, … , converging

- ◆ Even powers: from repulsive part
- ◆ Odd powers: from both

➔ The Fermi momentum is the relevant variable : **powers of $\rho^{1/3}$**





Guessing the EDF form - part II: effective field theory

- Saturation density is low...
  - with respect to (effective) boson exchange range (?)
    - one-pion exchange: vanishing expectation value
    - next boson: rho with m_ρ~775MeV~4fm^{-1}
  - Effective Lagrangian in powers of $\mathbf{k_F/m_\rho}$
- Expansion of E/A in powers of k_F
  - ... which means, again, powers of $\rho^{1/3}$
  - The Fermi momentum as the relevant variable
  - k_F^3 and k_F^4 (i.e., coupling~$\rho^{1/3}$) known to be important for obtaining saturation [Kaiser et al.,NPA697(2002)]
- Dilute Fermi gas: plus logarithmic terms

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ENERGY DENSITY FUNCTIONAL FOR KIDS

The Ansatz

Explore and fix homogeneous matter first

Map to a Skyrme interaction for nuclei

EDF Ansatz

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho.\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3} + c_{\ln}(\delta)\rho^2 \ln[\rho/(1\mathrm{fm}^3)]$$

kinetic energy: $\mathcal{T} = \mathcal{T}_p + \mathcal{T}_n\,;\; \mathcal{T}_{p,n} = \frac{3}{5}\frac{\hbar^2}{2m_{p,n}} x_{p,n}^{5/3}(3\pi^2\rho)^{2/3}\,;\; x_{p,n} \equiv \rho_{p,n}/\rho$

asymmetry: $\delta = (\rho_n - \rho_p)/\rho$

correspondence with Skyrme

Nuclear potential	Order	KIDS parameter	Skyrme parameter
$\mathcal{E}_0$	k_F^3	$c_0(\delta)$	(t_0, x_0)
$\mathcal{E}_1$	k_F^4	$c_1(\delta)$	$(t_3, x_3), \alpha = 1/3$
$\mathcal{E}_2$	k_F^5	$c_2(\delta)$	$(t_1, x_1), (t_2, x_2), (t_3', x_3'), \alpha' = 2/3$
$\mathcal{E}_3$	k_F^6	$c_3(\delta)$	$(t_3'', x_3''), \alpha'' = 1$

Objectives and procedures

What terms are most important for describing homogeneous matter? Is there a low-order expansion?

- We will fit all possible combinations of 1,2,3,4,5 terms to pseudodata and analyse the fits

Once we choose a robust set, verify:

- Are the parameters natural?

same order of magnitude?

$$\mathcal{E}_i(\rho,\delta) = c_i(\delta)\rho^{1+i/3} = \left[\left(\frac{\nu}{6\pi^2}\right)^{1+i/3} c_i(\delta) m_\rho^{2+i}\right] m_\rho \left(\frac{k_F}{m_\rho}\right)^{3+i}$$

- Can we use them in nuclei without refitting?
  - Under what conditions?

APR pseudodata and cost function

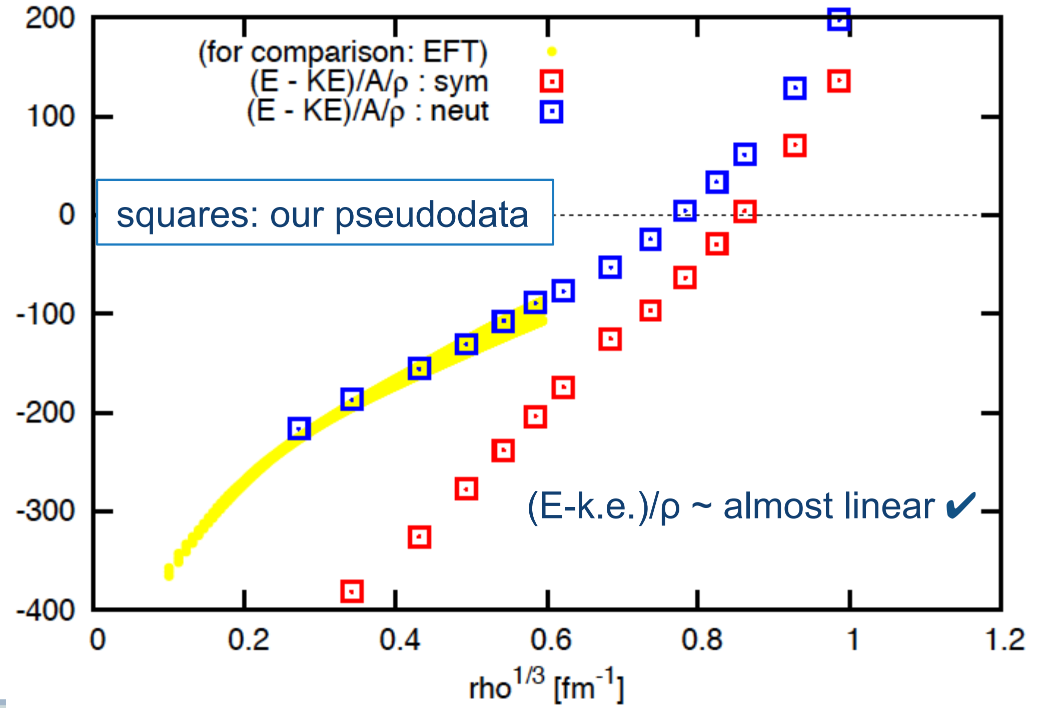
Shown: **E/A[MeV]** from
- Akmal,Pandharipande,Ravenhall, Phys. Rev. C 58,1804: AV18+Urbanna
- Drischler, Soma, Schwenk, Phys. Rev. C 89, 025806: chiral EFT (asymmetric matter)

cost function:

$$\chi^2(\delta) = \sum_j \exp\{-\beta\rho_j/\varrho_0\} \left(\frac{\mathcal{E}_j - D_j}{T_j}\right)^2 \; ; \quad \beta \geq 0$$

normalized:

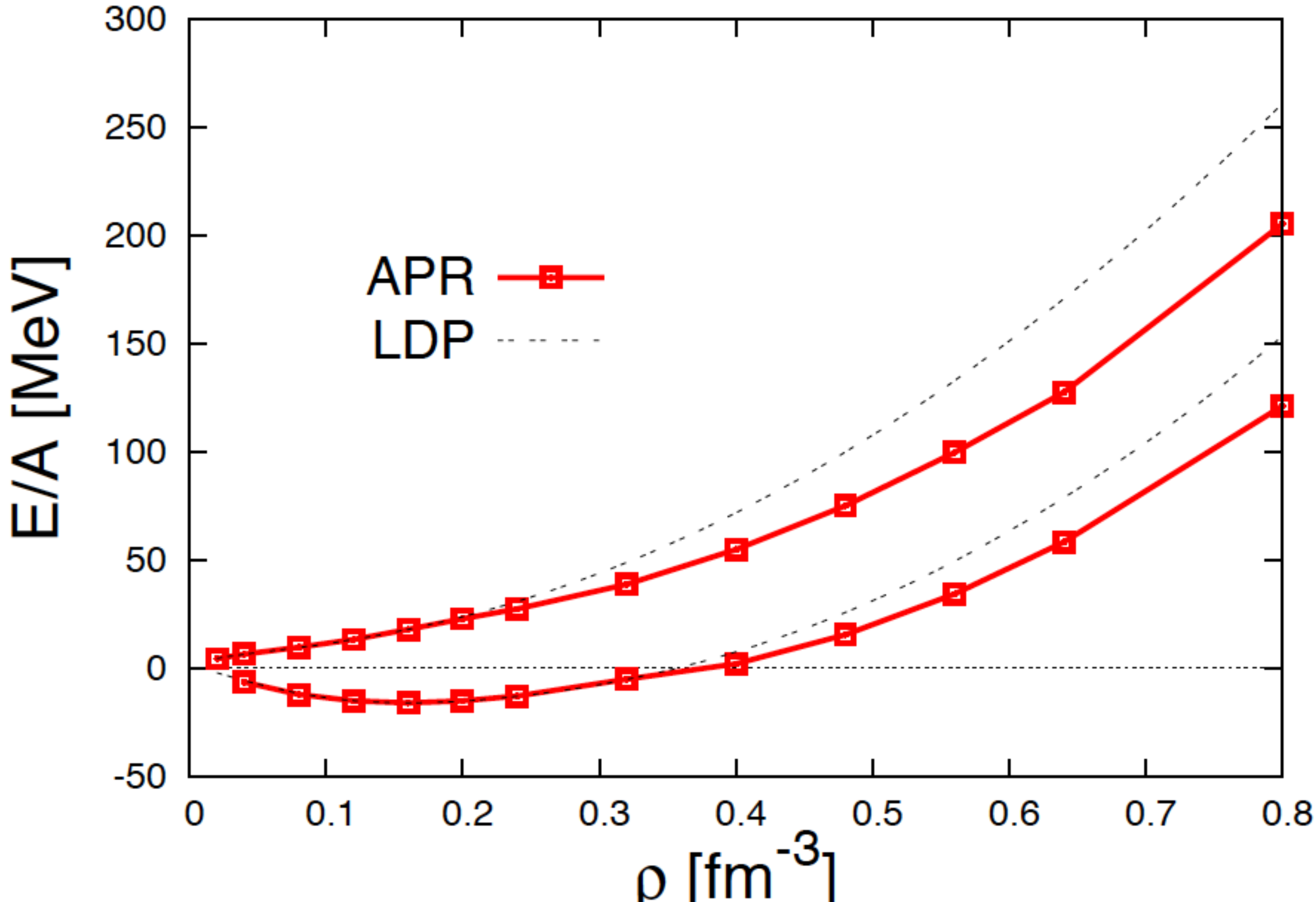
$$\chi^2_n(\delta) = \chi^2(\delta) \left[\sum_j \exp\{-\beta\rho_j/\varrho_0\}\right]^{-1}$$



31 total combinations of:
- 1 term only
- 2 terms
- ...
- 5 terms

31 fits for PNM and 31 for SNM

APR pseudodata



Hierarchy of powers ✔

PP,Park,Lim,Hyun,arXiv:1606.04219

For an equal number of terms (2,3,...), a combination of lower-power terms gives a better fit than a compination of higher-power terms

	$\beta = 0$		$\beta = \frac{1}{2}$		$\beta = 1$	
	SNM	PNM	SNM	PNM	SNM	PNM
$k = 0$	1.595335	0.397036	0.930742	0.171609	0.490650	0.071632
$k = 1$	1.801776	0.346198	1.527834	0.223333	1.089477	0.138133
$k = 0, 1$	0.013044	0.022028	0.003866	0.007482	0.001151	0.001566
$k = 0, 2$	0.009356	0.005804	0.012267	0.001864	0.009435	0.000719
$k = 0, 3$	0.041156	0.002160	0.047771	0.003059	0.035831	0.003220
$k = 1, 2$	0.085297	0.005936	0.108696	0.009991	0.090303	0.010973
$k = 1, 3$	0.175982	0.014031	0.216418	0.022334	0.183405	0.023312
$k = 2, 3$	0.342376	0.031821	0.440564	0.048252	0.398009	0.050970
$k = 0, 1, 2$	0.005009	0.003287	0.002588	0.001781	0.001016	0.000529
$k = 0, 2, 3$	0.006453	0.002055	0.004070	0.001540	0.001284	0.000636
$k = 1, 2, 3$	0.021528	0.005183	0.018591	0.005162	0.008571	0.003018
$k = 0, 1, \ln$	0.007486	0.007088	0.003000	0.002874	0.001025	0.000696
$k = 0, 3, \ln$	0.009117	0.002129	0.006681	0.001878	0.002380	0.000930
$k = 0, 1, 2, 3$	0.001616	0.000163	0.001731	0.000188	0.001015	0.000138
$k = 0, 1, 2, (\frac{7}{3})$	0.001420	0.000115	0.001597	0.000136	0.001016	0.000112
$k = 0, 1, 2, \ln$	0.001314	0.000094	0.001510	0.000107	0.001011	0.000092
$k = 0, 1, 2, (\frac{1}{6})$	0.002277	0.000462	0.002072	0.000415	0.000977	0.000221

Shown here: chi-square values

Details in the preprint

PP,Park,Lim,Hyun,arXiv:1606.04219

A hierarchy of terms, where the lower-order ones are more important than the higher-order ones, is inferred from the present results:

- Generally speaking, for a given number of non-zero parameters, the sets which include the $k = 0$ term give better fits than those which do not. There are a few exceptions and mostly for low β.
- In the majority of cases, if we replace the $k = 1$ term with the $k = 3$ term we get noticeably higher χ^2_n values. This result is in concordance with the preference for Skyrme functionals with a fractional-power, rather than linear, density dependence.
- If we use only two parameters, the sets of two low-order parameters produce better fits than the sets of two higher-order parameters. For example for $\beta = 1$ one may arrange the sets from the best to worst as follows: For SNM, $k = (0,1)$, $(0,2)$, $(0,3)$, $(1,2)$, $(1,3)$, $(2,3)$, $(3,\ln)$.[1] For PNM the order is the same except that $k = (0,2)$ is better than $(0,1)$.
- In fact for smaller β the $k = 3$ term in PNM seems more efficient. The inclusion of a linear dependence in Skyrme functionals might be recommended especially for dense-matter applications. We note that the discontinuity of the data may contaminate the systematics of the low-β fits.
- For three parameters, we found that the smallest χ^2_n are generally obtained without the logarithmic term.

Hierarchy of powers ✔

Replacing $\rho^{1/3}$ with linear dependence – e.g.

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Hierarchy of powers ✔

Fit quality almost indifferent to choice of 4th term.
Interesting exception: $\rho^{1/6}$ (somewhat worse fits, generally)

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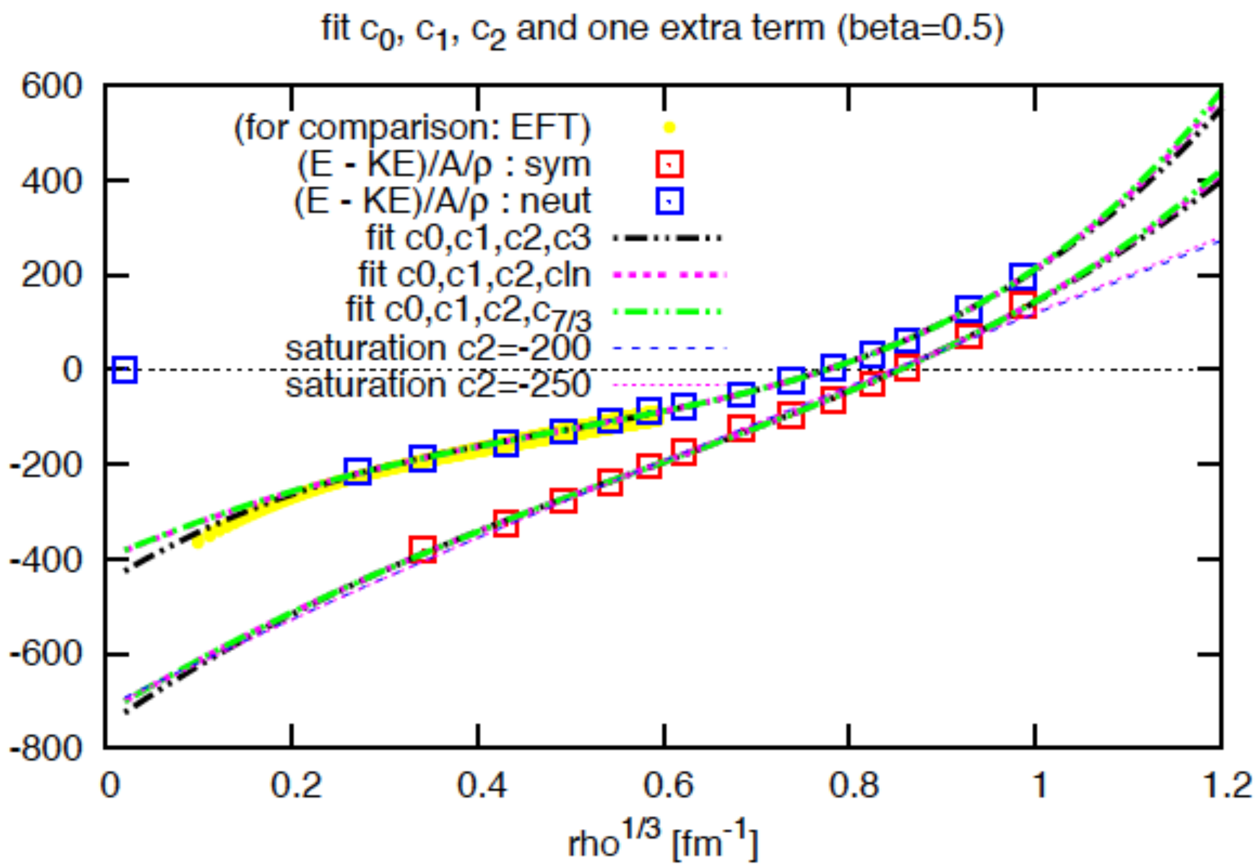
No 5th term needed ✔



Fitting results

PP,Park,Lim,Hyun,arXiv:1606.04219

SNM from fits

SNM from ρ_0, E_0, K_{inf}, (ad-1: and m*)

β	Matter	c_0	c_1	c_2	c_3	ϱ_0	$\mathcal{E}_0$ / J	K_∞ / L
0	SNM	−863.36	1945.05	−2060.20	1129.96	0.178	-15.4	215
	PNM	−483.96	1433.54	−2119.68	1385.22		34.2	55.9
$\frac{1}{2}$	SNM	−753.98	1389.20	−1171.03	678.87	0.177	-15.8	234
	PNM	−451.91	1254.32	−1812.62	1221.33		34.4	56.0
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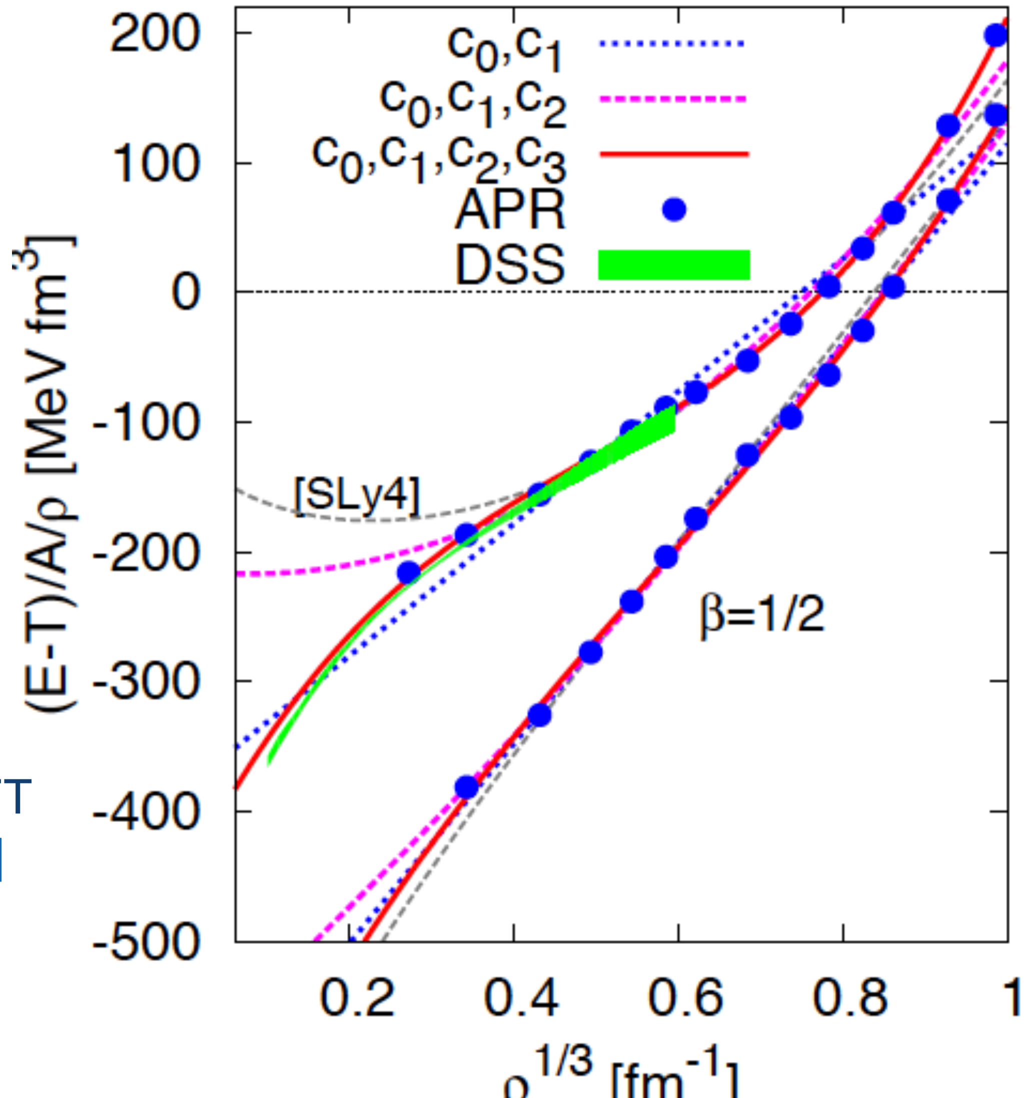
	PNM	−408.56	991.76	−1323.81	937.96		34.0	54.9
ad-1	SNM	−648.72	676.25	200.92	−98.73	0.160	-16.0	240
	PNM	−451.91	1254.32	−1812.62	1221.33		32.8	47.9
ad-2	SNM	−664.52	763.55	40.13	0.00	0.160	-16.0	240
	PNM	−411.13	1007.78	−1354.64	956.47		33.5	50.5

- c_0,c_1 robust
- For PNM, also c_2, c_3

Fitting results

The data do show a roughly linear dependence on $\rho^{1/3}$

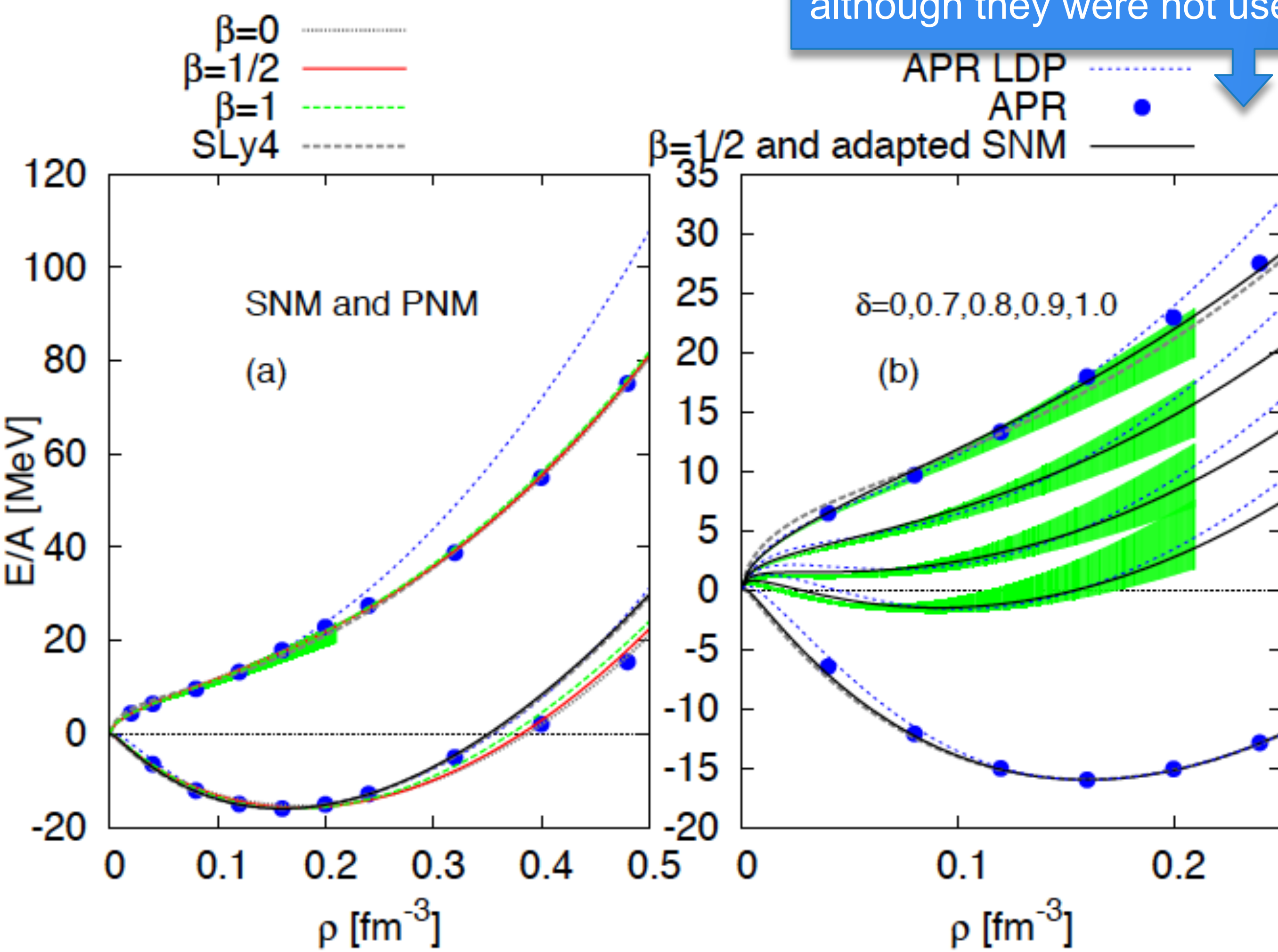


"DSS":
Comparison (not fitting) to χEFT
[Drischler et al., PRC89(2014)]

PP,Park,Lim,Hyun,arXiv:1606.04219

Fitting results

Calculations with chiral interactions reproduced, although they were not used for fitting



asymmetric matter:
quadratic interpolation

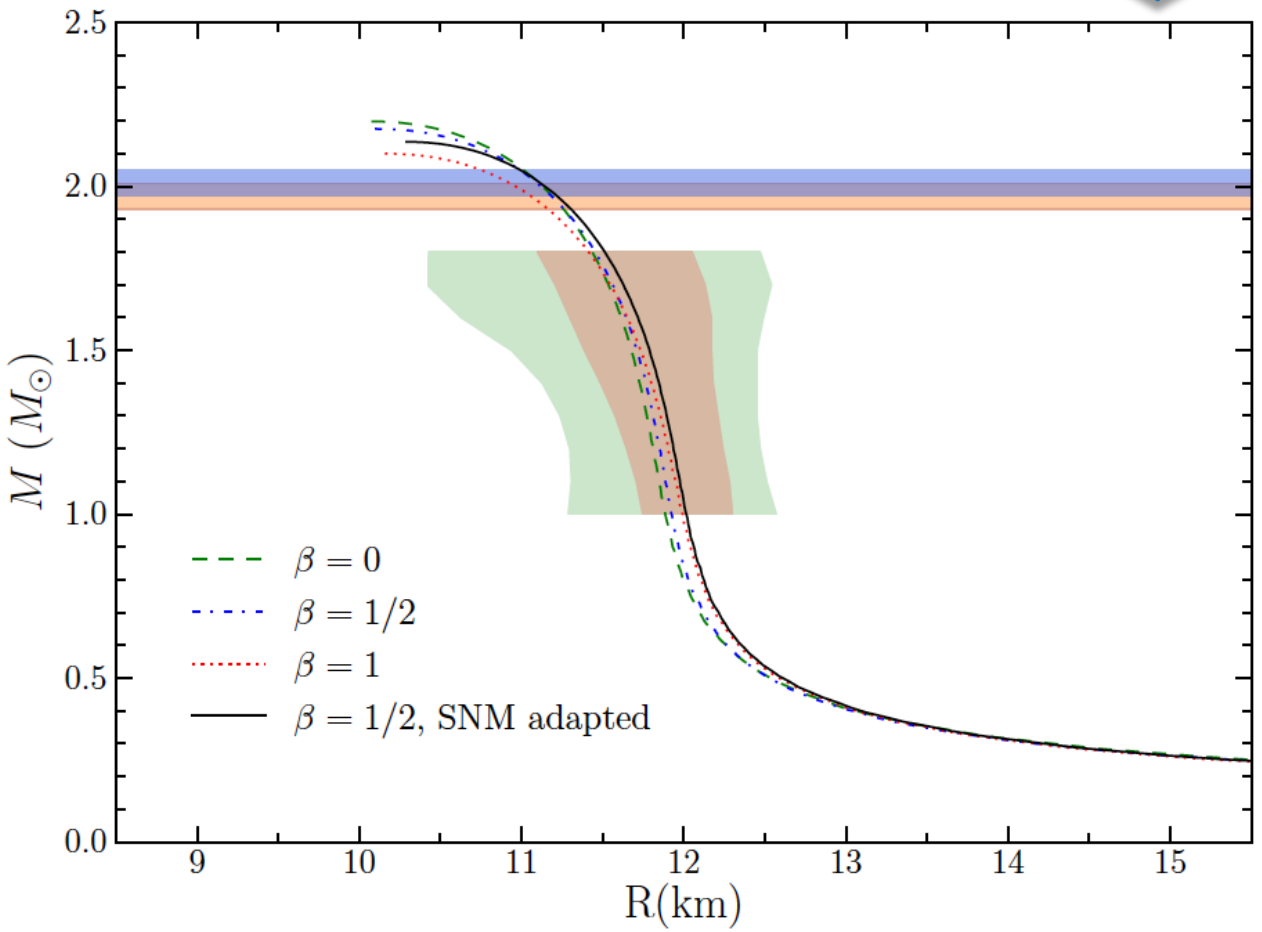
$$c_k(\delta) = \alpha_k + \delta^2 \beta_k$$

PP,Park,Lim,Hyun,arXiv:1606.04219

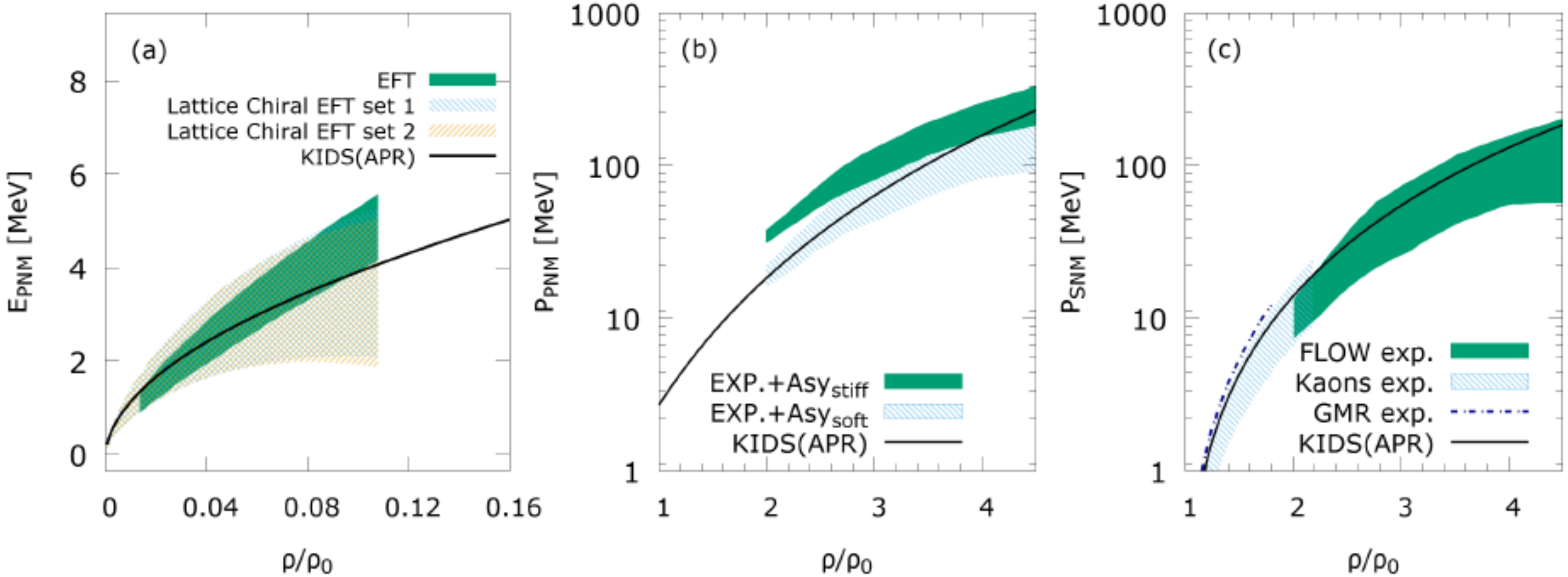
Dense matter: neutron stars

Agreement with observational data

PP,Park,Lim,Hyun,arXiv:1606.04219



Other constraints



From soft to stiff

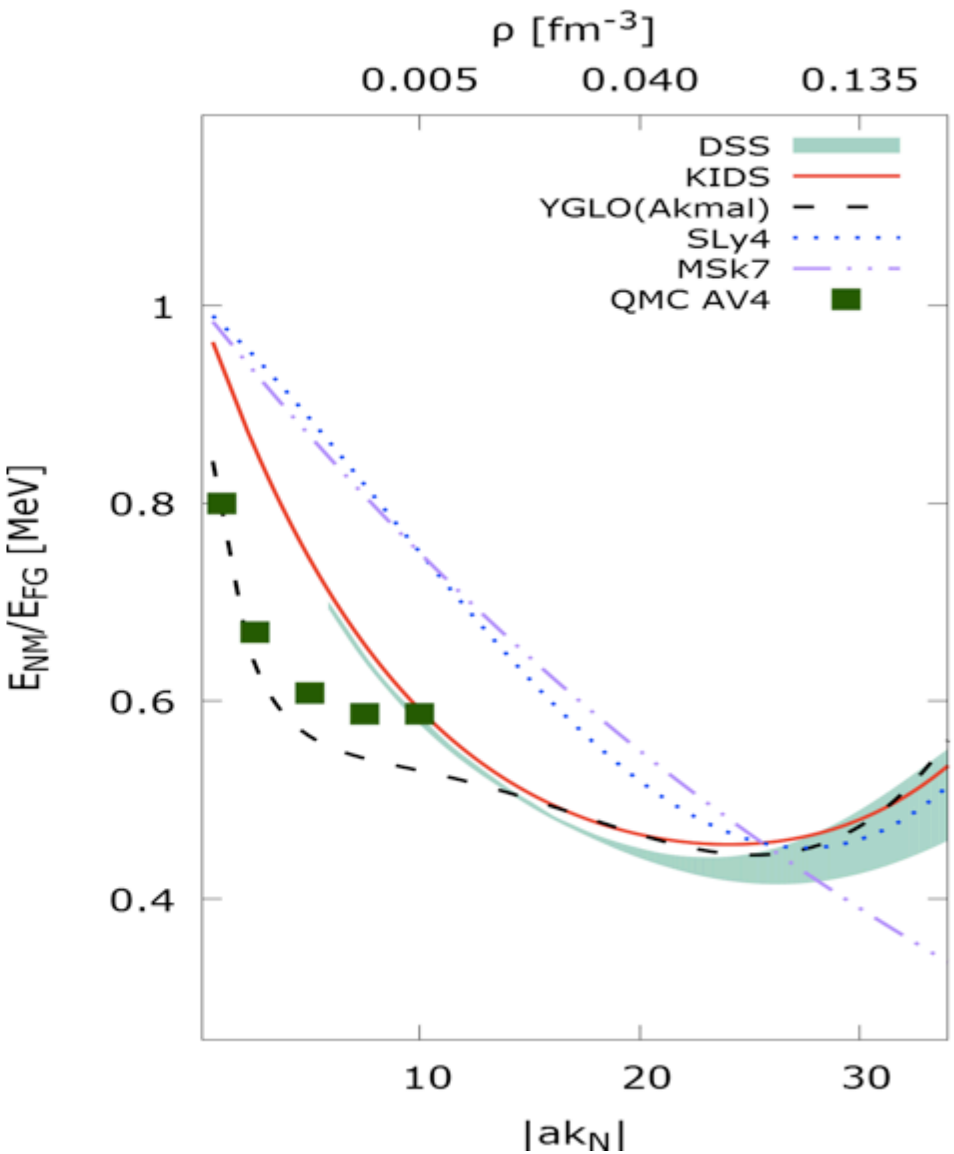
Other constraints

Model	K_0 [MeV]	$-Q_0$ [MeV]	J [MeV]	L [MeV]	$K_{\tau,v}$ [MeV]	$S(\rho_0/2)/J$	$3P_{PNM}/(L\rho_0)$
KIDS	240.00	372.65	32.75	49.10	-375.06	0.667	1.03
GSkI	230.27	405.70	32.03	63.45	-364.24	0.620	1.02
GSkII	234.14	400.15	34.22	67.08	-409.23	0.616	1.07
SSk	228.40	373.81	33.46	52.74	-349.08	0.673	1.03
LNS	210.84	382.65	33.43	61.46	-384.60	0.631	1.06
MSk7	231.23	385.37	27.95	9.40	-315.39	0.786	1.13
SLy4	229.82	362.94	32.00	45.96	-322.86	0.691	1.03
SkM*	216.61	386.08	30.03	45.78	-349.01	0.662	1.10
Exp.	$200 \sim 260$	$200 \sim 1200$	$30 \sim 35$	$40 \sim 76$	$-760 \sim -372$	$0.57 \sim 0.86$	$0.90 \sim 1.10$

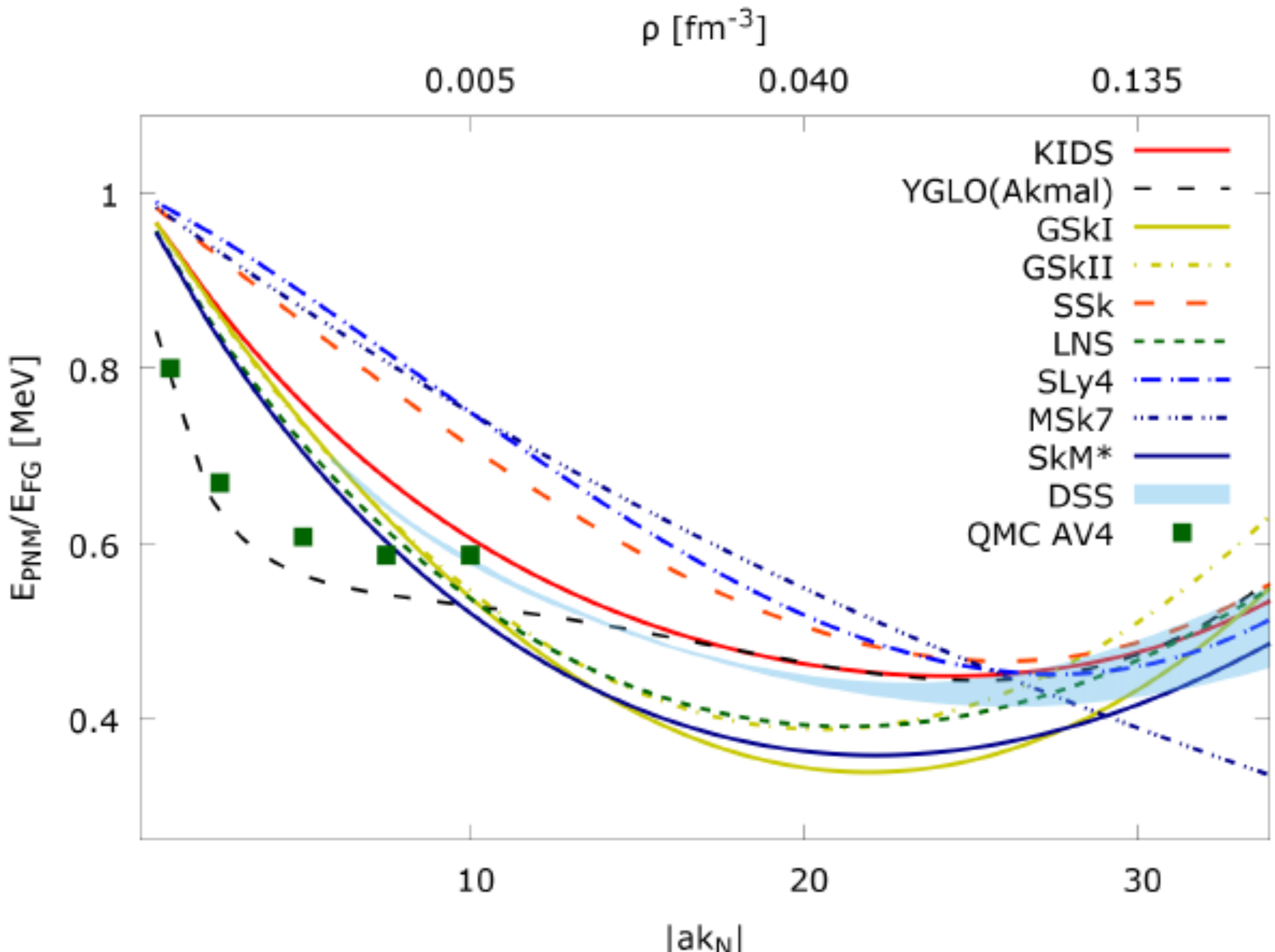
Dutra et al 2012

Gil,PP,Hyun,Oh, in preparation

Dilute neutron matter



Dilute neutron matter



Gil,PP,Hyun,Oh, in preparation

Objectives and procedures

What terms are most important for describing homogeneous matter?

- ❖ We will fit all possible combinations of 1,2,3,4,5 terms to the APR pseudodata and analyse the fits

Once we choose a robust set, verify:

- ❖ Are the parameters natural?

same order of magnitude?

$$\mathcal{E}_i(\rho,\delta) = c_i(\delta)\rho^{1+i/3} = \left[\left(\frac{\nu}{6\pi^2}\right)^{1+i/3} c_i(\delta) m_\rho^{2+i}\right] m_\rho \left(\frac{k_F}{m_\rho}\right)^{3+i}$$

- ❖ Can we use them in nuclei without refitting?
  - Under what conditions?

Power hierarchy and naturalness

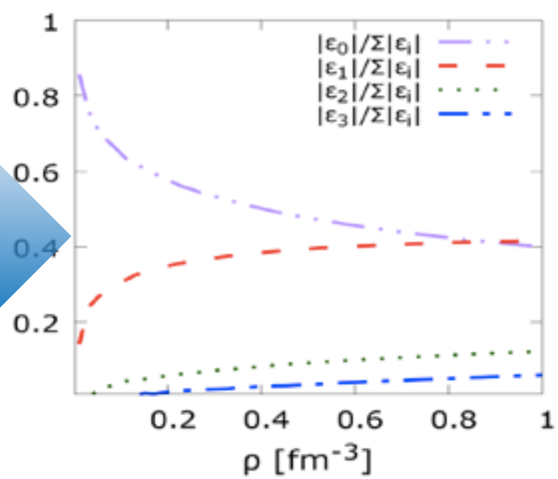
RAON

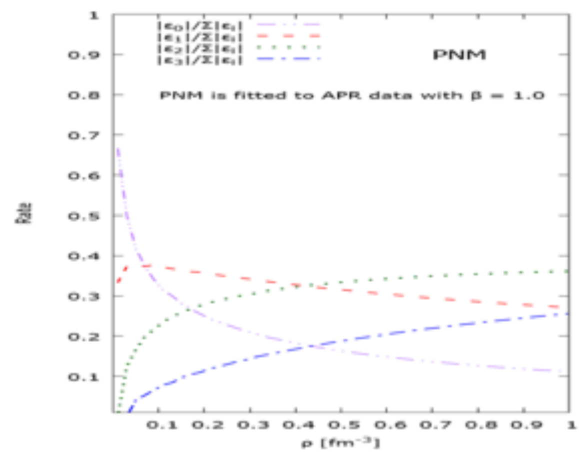
❖ Fermi momentum calculus and power hierarchy:

- ✓ $|E_0| > |E_1| > |E_2| > |E_3|$ within a large density range
- ✓ **For SNM up to ~1fm^{-3}**, for PNM up to **0.05**fm^{-3}

PNM vs SNM: Intriguing difference

E.g. from the β =1 fits to APR





❖ Naturalness?

adopted “ad-2” set

- ❖ SNM: $c_0^{dim} = -3.6, \quad c_1^{dim} = 6.6, \quad c_2^{dim} = 0.6$
- ❖ PNM: $c_0^{dim} = -1.1, \quad c_1^{dim} = 3.4, \quad c_2^{dim} = -5.9, \quad c_3^{dim} = 5.3$

"Natural" Ansatz

- At the very least: reproduce homogeneous matter (to the best of our knowledge)
- Better: based on a power expansion
  - Underlying EFT??
- Best: coefficients showing naturalness

Objectives and procedures

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- Can we use them in nuclei without refitting?
  - Under what conditions?

APPLICATIONS IN NUCLEI

Map to a Skyrme interaction for nuclei

First results appear in:
Gil,PP,Hyun,Park,Oh, Acta Phys.Pol.B48,305
Gil,Oh,Hyun,PP, Sae Mulli 67,456 (2017)

Fitting results

SNM from fits

SNM from ρ_0, E_0, K_{inf}, (ad-1: and m*)

β	Matter	c_0	c_1	c_2	c_3	ϱ_0	$\mathcal{E}_0$ / J	K_∞ / L
0	SNM	−863.36	1945.05	−2060.20	1129.96	0.178	-15.4	215
	PNM	−483.96	1433.54	−2119.68	1385.22		34.2	55.9
$\frac{1}{2}$	SNM	−753.98	1389.20	−1171.03	678.87	0.177	-15.8	234
	PNM	−451.91	1254.32	−1812.62	1221.33		34.4	56.0
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	PNM	−411.13	1007.78	−1354.64	956.47		33.5	50.5

adopted set for applications in nuclei

- SNM with canonical values of ρ_0, E_0, K_{inf}
- "Agnostic" w.r.t. m*/m

Skyrme-type interaction

$$
\begin{aligned}
v_{i,j}(\mathbf{k},\mathbf{k}') &= (t_0 + y_0 P_\sigma)\delta(\mathbf{r}_i - \mathbf{r}_j) \\
&+ \frac{1}{2}(t_1 + y_1 P_\sigma)[\delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{k}^2 + \mathbf{k}'^2\delta(\mathbf{r}_i - \mathbf{r}_j)] \\
&+ (t_2 + y_2 P_\sigma)\mathbf{k}' \cdot \delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{k} \\
&+ \frac{1}{6}\sum_{n=1}^{3}(t_{3n} + y_{3n} P_\sigma)\rho^{n/3}\delta(\mathbf{r}_i - \mathbf{r}_j) \\
&+ iW_0\,\mathbf{k}' \times \delta(\mathbf{r}_i - \mathbf{r}_j)\,\mathbf{k}\cdot(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j), \qquad (4)
\end{aligned}
$$

"$\mathbf{y}_i$" replaces "$t_i \mathbf{x}_i$" because $\mathbf{t}_i$ may be zero and the respective $\mathbf{y}_i$ may be finite. Cf. $c_3(0)$ and t_{33} of ad-2

Nuclear potential	Order	KIDS parameter	Skyrme parameter
$\mathcal{E}_0$	k_F^3	$c_0(\delta)$	(t_0, x_0)
$\mathcal{E}_1$	k_F^4	$c_1(\delta)$	$(t_3, x_3), \alpha = 1/3$
$\mathcal{E}_2$	k_F^5	$c_2(\delta)$	$(t_1, x_1), (t_2, x_2), (t_3', x_3'), \alpha' = 2/3$
$\mathcal{E}_3$	k_F^6	$c_3(\delta)$	$(t_3'', x_3''), \alpha'' = 1$

Application in nuclei

- Skyrme-type density-dependent “interaction”
- Skyrme-Hartree-Fock equations
- Parameters already known from homogeneous matter

- ... except those which do not contribute to the energy of homogeneous matter:
  - Contribution of momentum-dependent terms t_1-t_2 to $c_2(\delta)$; effective mass

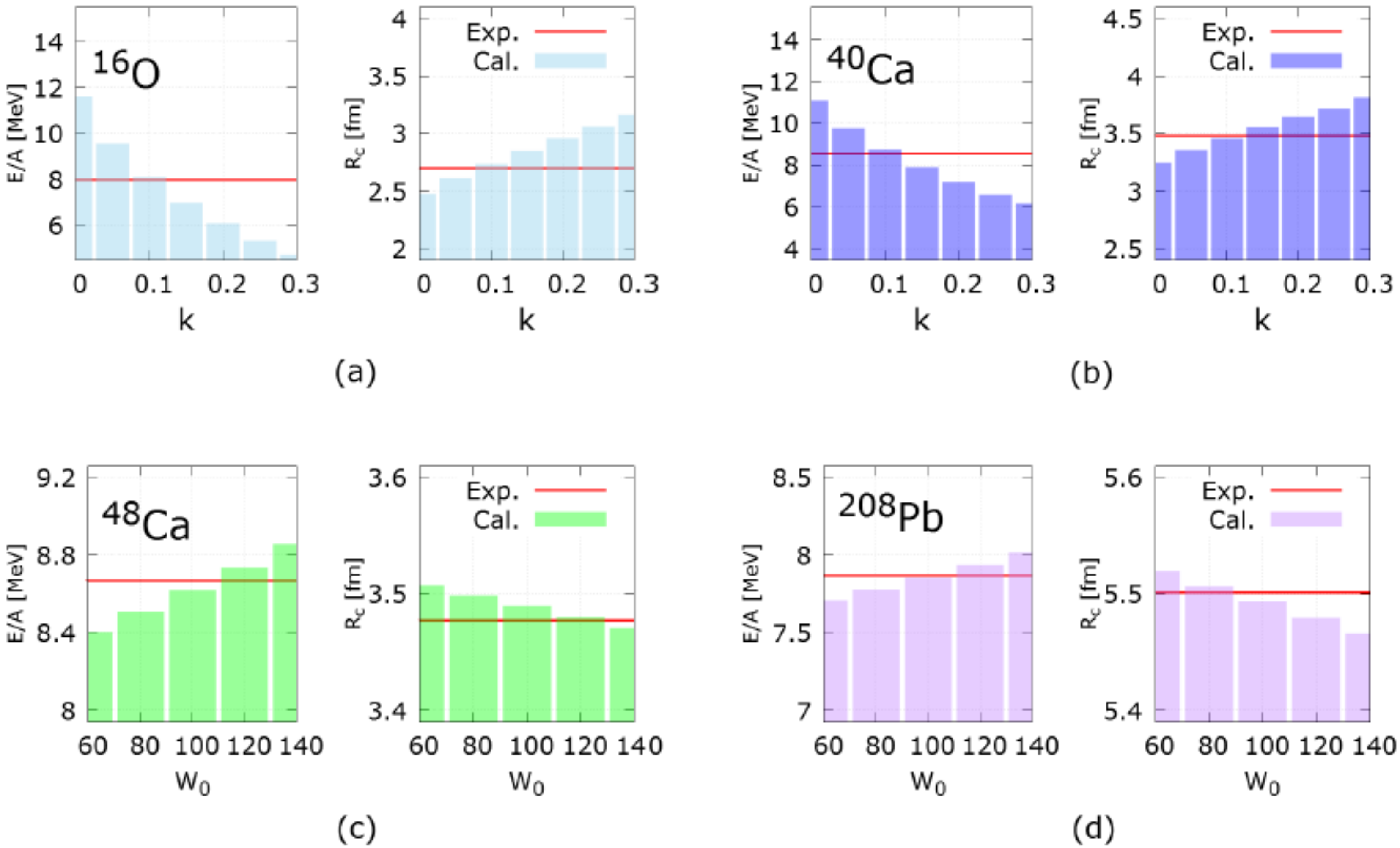
$$c_2(\delta) = k \cdot c_2(\delta) + (1-k) \cdot c_2(\delta) \equiv c_2^{t_3'=0}(\delta) + c_2^{t_1=t_2=0}(\delta)$$

Two free parameters:

- **portion** of momentum dependence k:
  From energy and radius of ^{16}O, ^{40}Ca
- spin-orbit strength W_0 :
  From energy and radius of ^{48}Ca, ^{208}Pb

Two parameters left

First results appeared in:
Gil,PP,Hyun,Park,Oh, Acta Phys.Pol.B48,305
Gil,Oh,Hyun,PP, Sae Mulli 67,456 (2017)



k~0.11 => m*/m ~0.995

Application in nuclei

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$$c_2(\delta) = k \cdot c_2(\delta) + (1-k) \cdot c_2(\delta) \equiv c_2^{t_3'=0}(\delta) + c_2^{t_1=t_2=0}(\delta)$$

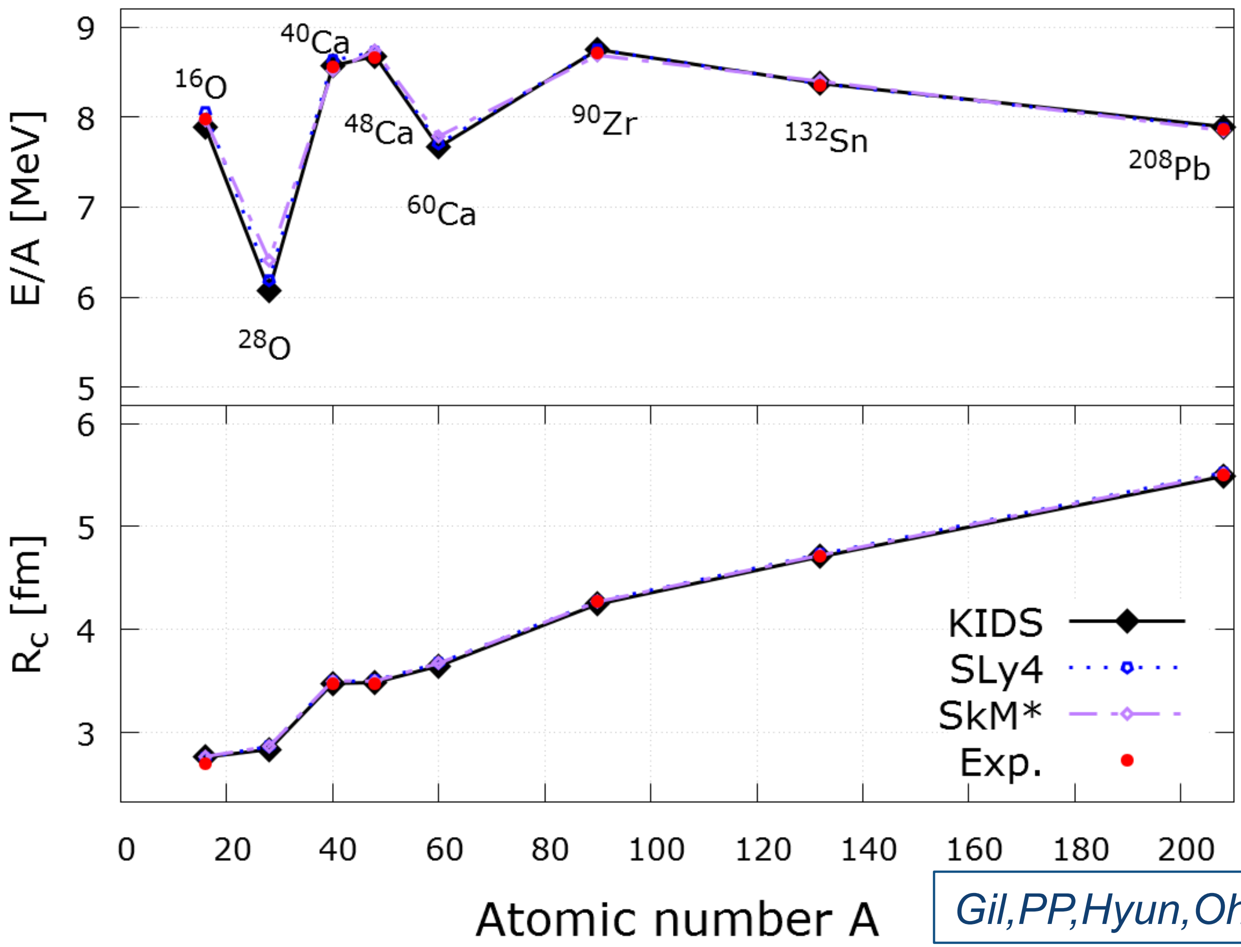
k~0.11 reproduces the energy and radius of ^{16}O, ^{40}Ca

  - spin-orbit strength W_0 :

    W_0~ 110 from ^{48}Ca, ^{208}Pb

Results – energies and radii

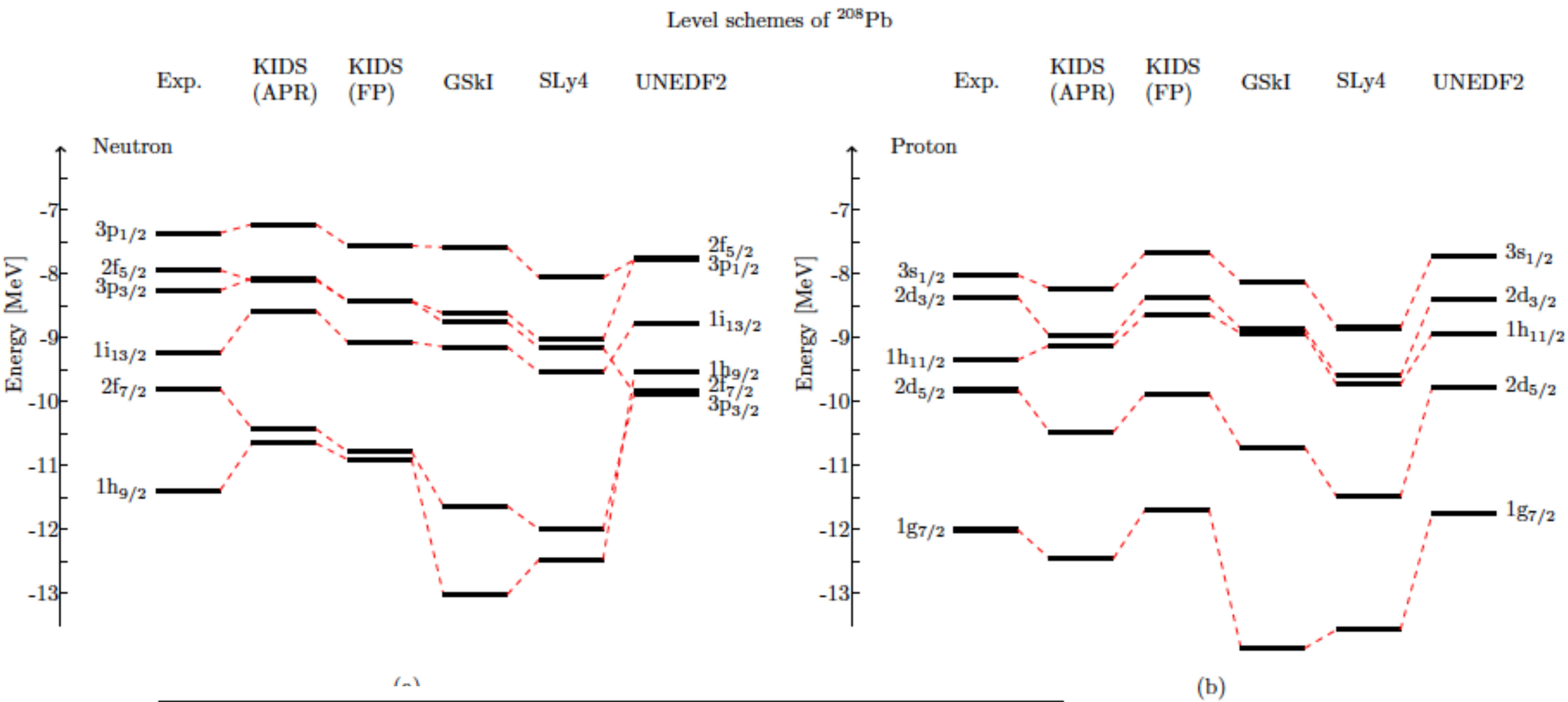
- All but two parameters from homogeneous matter
- Two parameters (k, W_0) from E and R_{ch} of ^{16}O, ^{40}Ca, ^{48}Ca, ^{208}Pb.
- Results of ^{28}O, ^{60}Ca, ^{90}Zr, ^{132}Sn are predictions.



Gil,PP,Hyun,Oh, in preparation

not fitted

Results – single-particle levels



	KIDS(APR)	KIDS(FP)	GSkI	SLy4	UNEDF2
neutron	4.34	4.50	9.3	11.6	8.3
proton	4.55	3.04	7.3	11.8	2.2

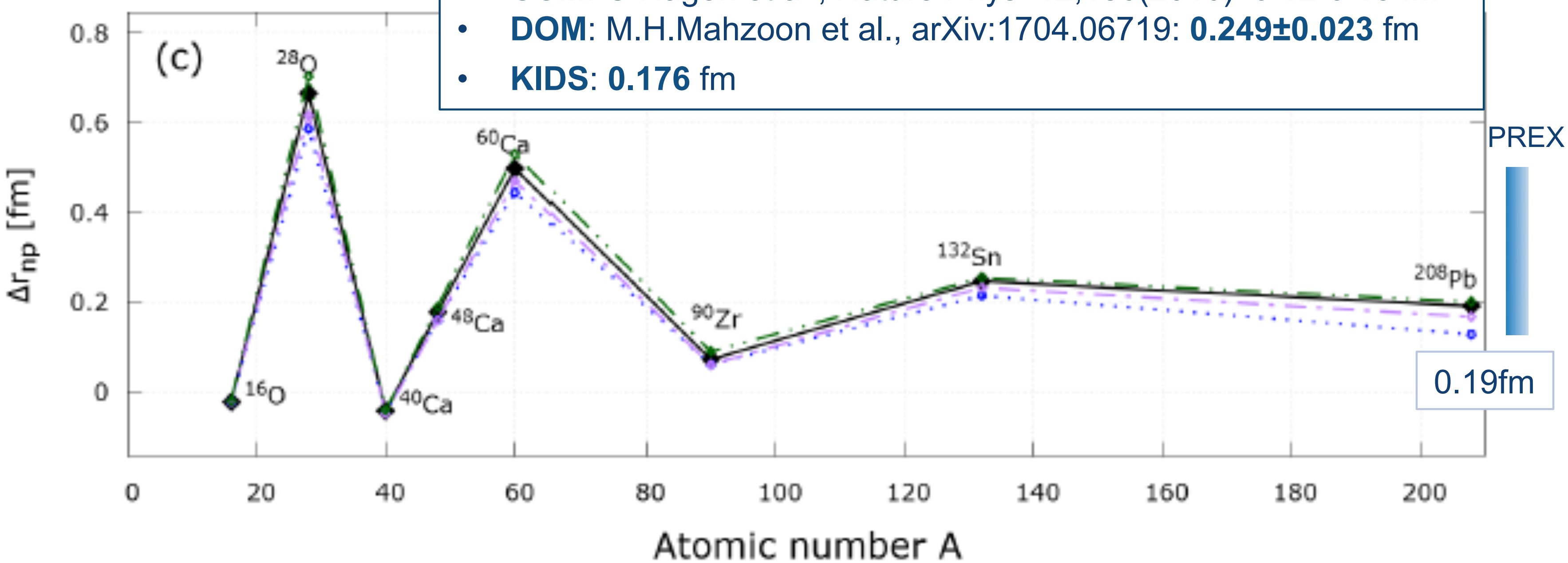
TABLE IV: Mean deviation D of the single particle levels in Fig. 5 in units of %.

m*/m ~0.995 (!)

Gil,PP,Hyun,Oh, in preparation

Results- predictions

<u>neutron skin of 48Ca:</u>
- **CCM**: G.Hagen et al., Nature Phys. 12,186(2016) **0.12-0.15** fm
- **DOM**: M.H.Mahzoon et al., arXiv:1704.06719: **0.249±0.023** fm
- **KIDS**: **0.176** fm



- **Prediction of ^{28}O and ^{60}Ca**

	^{28}O			^{60}Ca		
Model	E/A [MeV]	R_c [fm]	Δr_{np} [fm]	E/A [MeV]	R_c [fm]	Δr_{np} [fm]
SLy4	6.1925	2.8656	0.58476	7.703	3.6734	0.4435
SkM*	6.4114	2.8646	0.61631	7.7857	3.6713	0.4685
KIDS	6.0757	2.8353	0.66398	7.6652	3.6452	0.4960

AME2012: 5.9883

Prospects

- Symmetry energy : surface vs. volume, …
- Neutron skin with pygmy resonance, dipole polarizabilityMore high order terms and range of convergence
- Neutron systems (drops)
- Pairing, deformation, ...

Implications

- Fractional density dependence in Skyrme-type couplings is fundamentally justified through the k_F dependence of the dilute system
  - Or, generally, of the interacting-Fermion system
- Skyrme and Gogny "interactions" are not Hamiltonians
  - EDF a legitimate "black box"
- Given a functional: a good description of homogeneous matter leads to a surprisingly good description of finite nuclei ***as long as the $\rho^{5/3}$ term (i.e., the $\rho^{2/3}$ coupling) is not fully ascribed to momentum dependence*** (*)

(*) same procedure can be followed for higher-order terms

OVERTIME

Effective theories of SNM vs PNM

How about excitations? Linear response theory

1/3 vs 1/6

And why not density-matrix expansion?

(comments on the ensuing powers)

Implications&Questions

- Is saturated nuclear matter “dilute”? Why?
  - In-medium scattering lengths ??
- What terms are trully important for *neutron* matter? Why the difference with symmetric matter?
  - Currently examining higher-order terms, various signatures of convergence...
- Are there stringent *formal* answers?

Implications&Questions

- Linear response theory: HF=>RPA
  - residual interaction = derivative of mean field, OK

Roughly two approaches

- Given interaction + many-body method
  - Variational reference state + Equations of Motion
  - To lowest order, HF+RPA
  - Systematic inclusion of correlations / mp-mh until convergence

  - *"Wave-function approach"*
  - *Known Hamiltonian*

- Energy-density functionals + linear-response theory

  - *Kohn-Sham EDFT*
  - *E[ρ,...] known; Hamiltonian not necessarily known*
  - *"black box"*

  - The order of truncation depends on the application

Thoughts on: Beyond first order?

- E[ρ] : will give correct expectation values etc of one-body operators
  - Centroids of giant resonances?
  - *BUT: ✗ momentum distribution (sensitive to short-range correlations)*
- If you need higher-order effects perhaps you need an extended functional : **E[ρ_2]**
  - Incl. **the non-trivial part** of the two-body density matrix

$$g(\vec{r}_1,\vec{r}_2) = \rho_2(\vec{r}_1,\vec{r}_2) - \left[\rho_1(\vec{r}_1,\vec{r}_1)\rho_1(\vec{r}_2,\vec{r}_2) - \frac{1}{\nu}\rho_1(\vec{r}_1,\vec{r}_2)\rho_1(\vec{r}_2,\vec{r}_1)\right]$$

  - **A nuclear "energy - 2-body – density functional" ?**
  - Would need ab initio guidance! (Correlated system)
  - Cf geminal methods in quantum chemistry

Replacing $\rho^{1/3}$ with $\rho^{1/6}$

- Will examine 4 cases:
  - **3** terms $\{\rho, \rho^{1+\alpha}, \rho^{5/3}\}$ or **4** terms $\{\rho, \rho^{1+\alpha}, \rho^{5/3}, \rho^2\}$ in PNM
  - Power α = **1/3** or **1/6**
- SNM: 3 terms determined from E_0, ρ_0, K_{inf}
  - Power α = **1/3** or **1/6**, respectively
- Illustration: **effective mass**

	4 terms	**3 terms**
1/3	t_1=268 t_2=-157 m*/m=0.995	t_1=245 t_2=-172 m*/m=1.031
1/6		

robust...

Replacing $\rho^{1/3}$ with $\rho^{1/6}$

- Will examine 4 cases:
  - **3** terms $\{\rho, \rho^{1+\alpha}, \rho^{5/3}\}$ or **4** terms $\{\rho, \rho^{1+\alpha}, \rho^{5/3}, \rho^2\}$ in PNM
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- SNM: 3 terms determined from E_0, ρ_0, K_{inf}
  - Power α = **1/3** or **1/6**, respectively
- Illustration: **effective mass**

	4 terms	3 terms
1/3	t_1=268 t_2=-157 m*/m=0.995	t_1=245 t_2=-172 m*/m=1.031
1/6	t_1=317 t_2=-153 m*/m=0.957	t_1=880 t_2=67 m*/m=0.653

robust...

robust...

deviates

Thank you!

감사합니다~

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