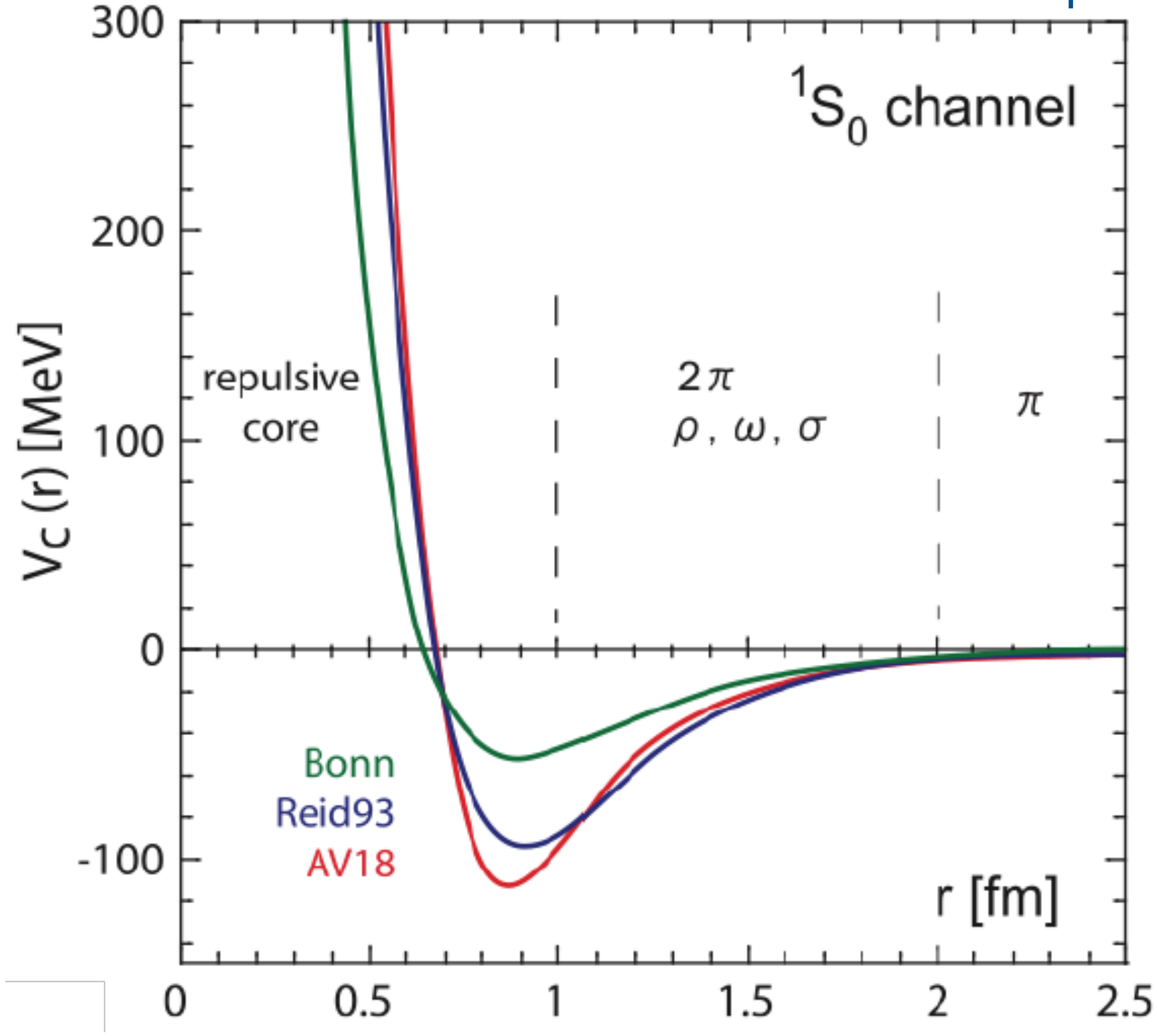
Green's functions for dense matter & mean-field connections

Arnau Rios Huguet
Lecturer in Nuclear Theory
Department of Physics
University of Surrey

IPN workshop 3rd October 2017

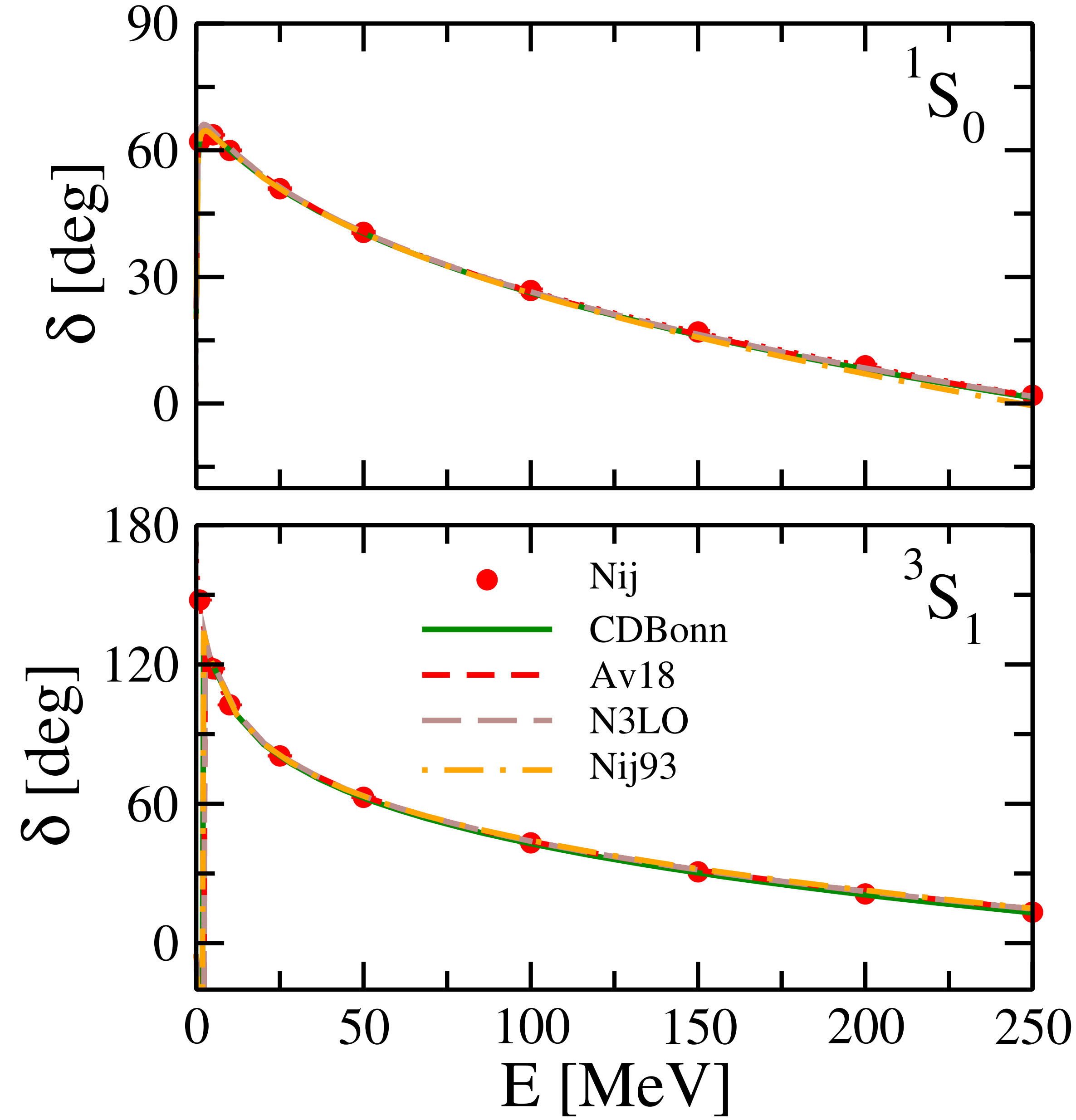
Complications

NN interaction is not unique



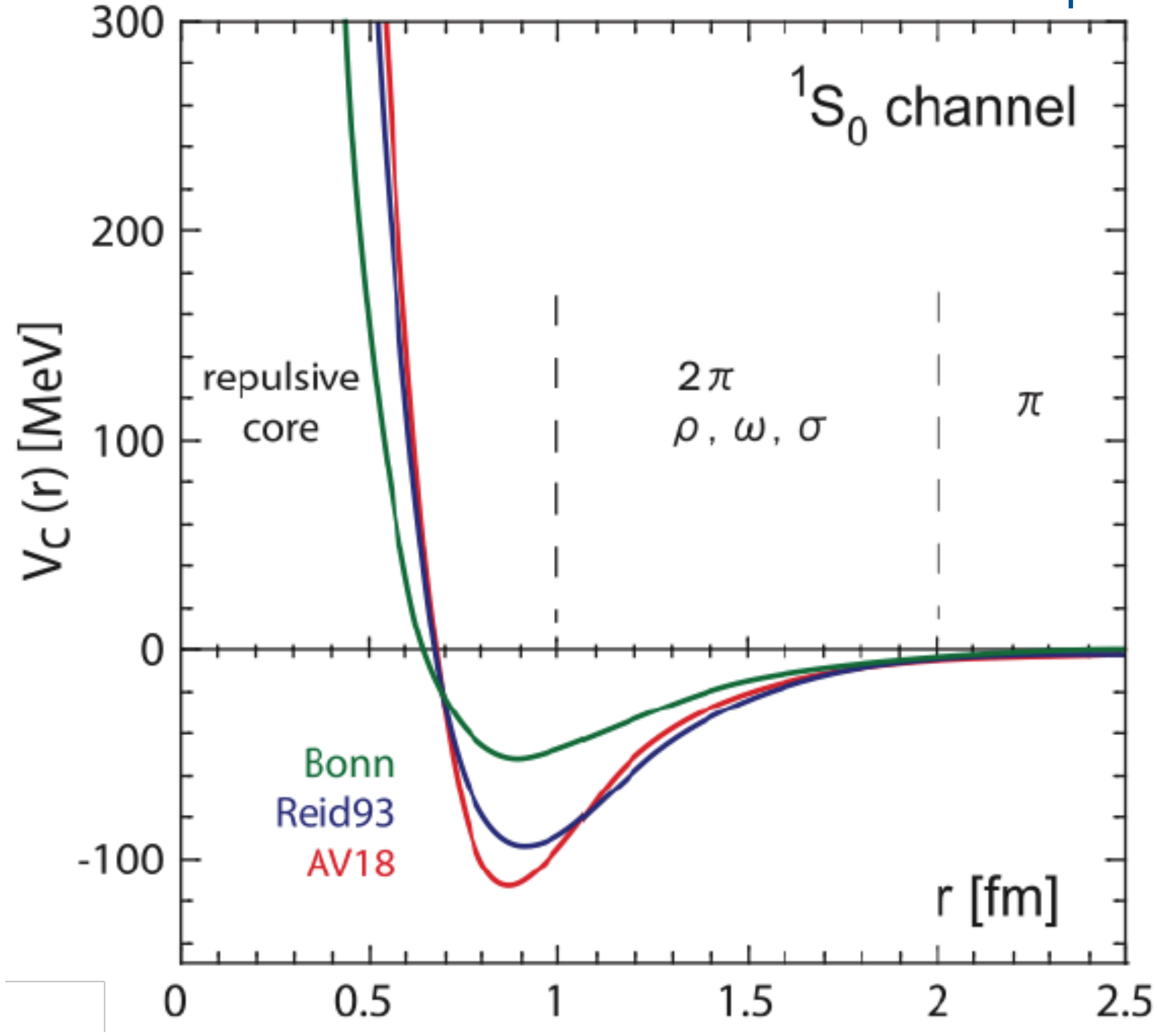
S. Aoki, et al. *Comput. Sci. Dis.* **1** 015009 (2008)

...but phase-shift equivalent!



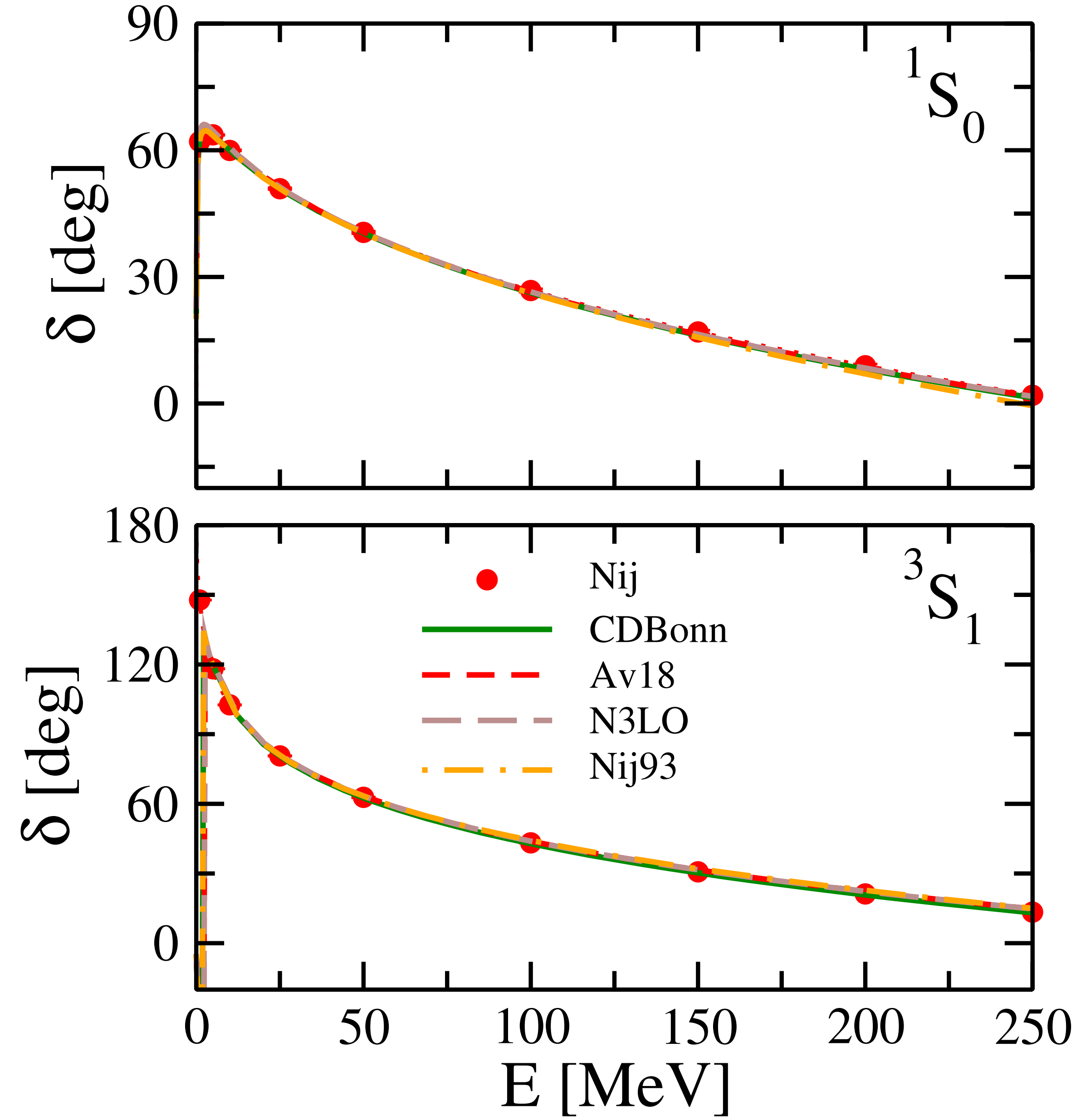
Complications

NN interaction is not unique



S. Aoki, et al. *Comput. Sci. Dis.* **1** 015009 (2008)

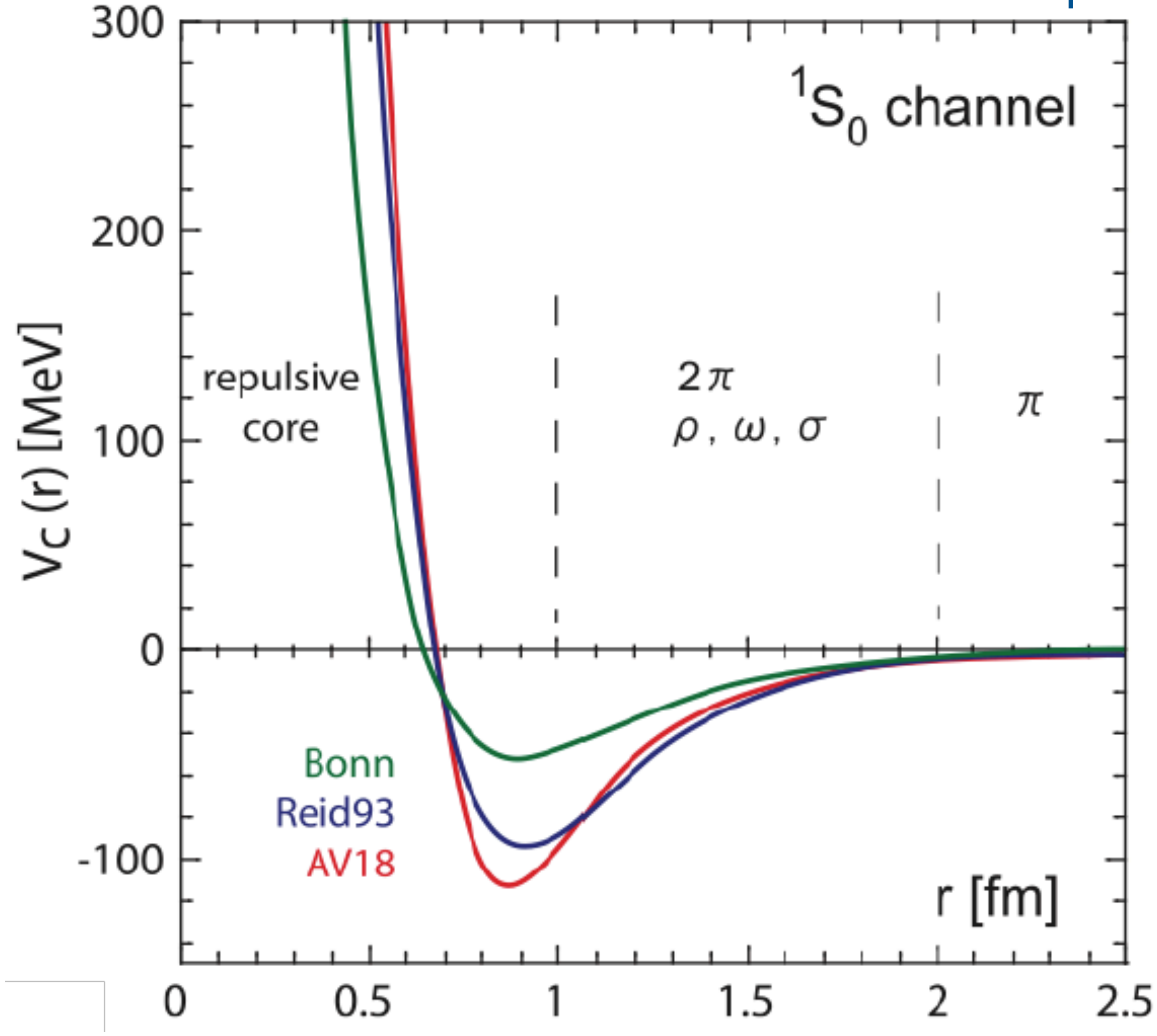
...but phase-shift equivalent!



•Non-uniqueness of nucleon forces ✘

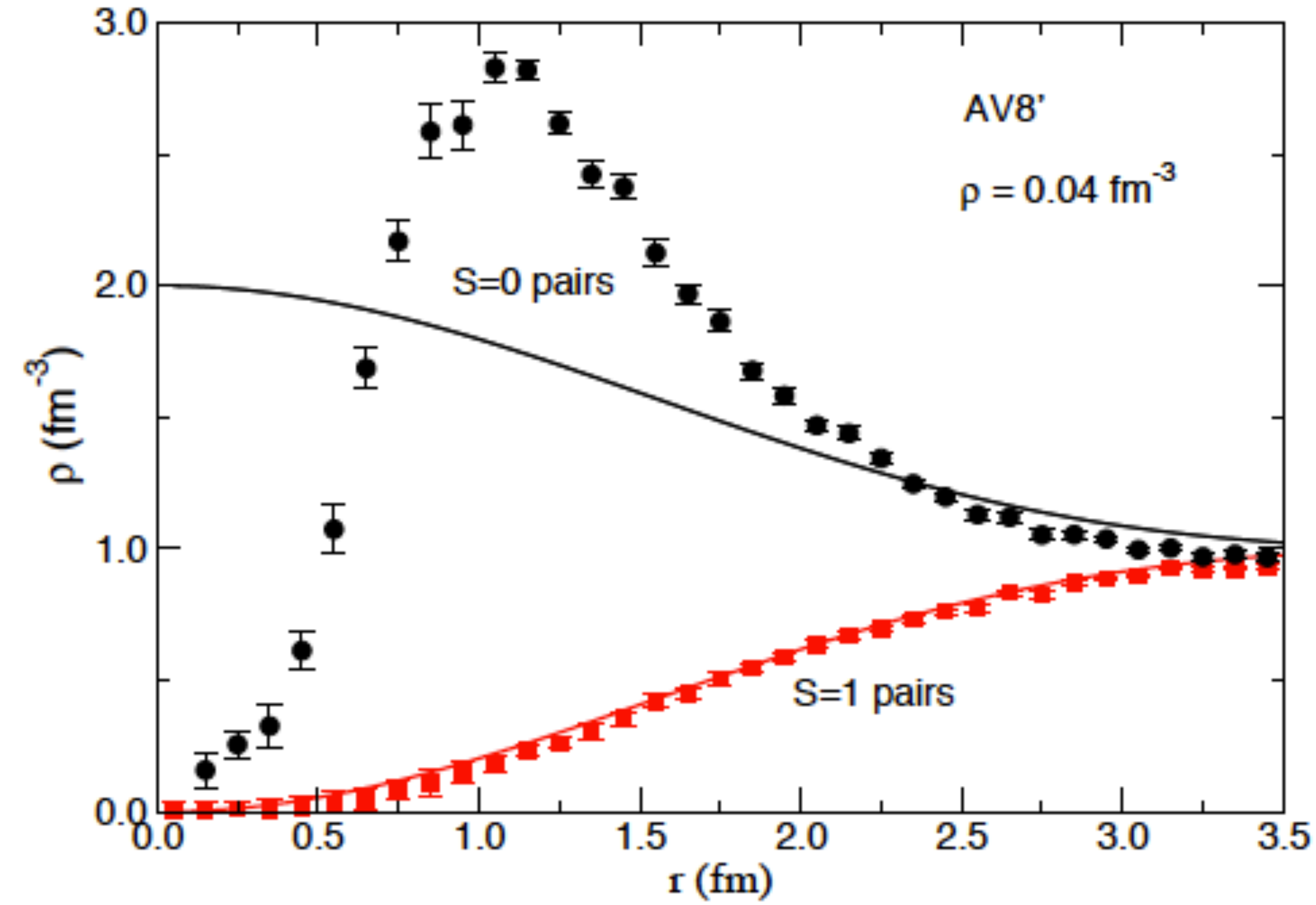
Complications

NN interaction is not unique



S. Aoki, et al. *Comput. Sci. Dis.* **1** 015009 (2008)

Strong short-range correlations

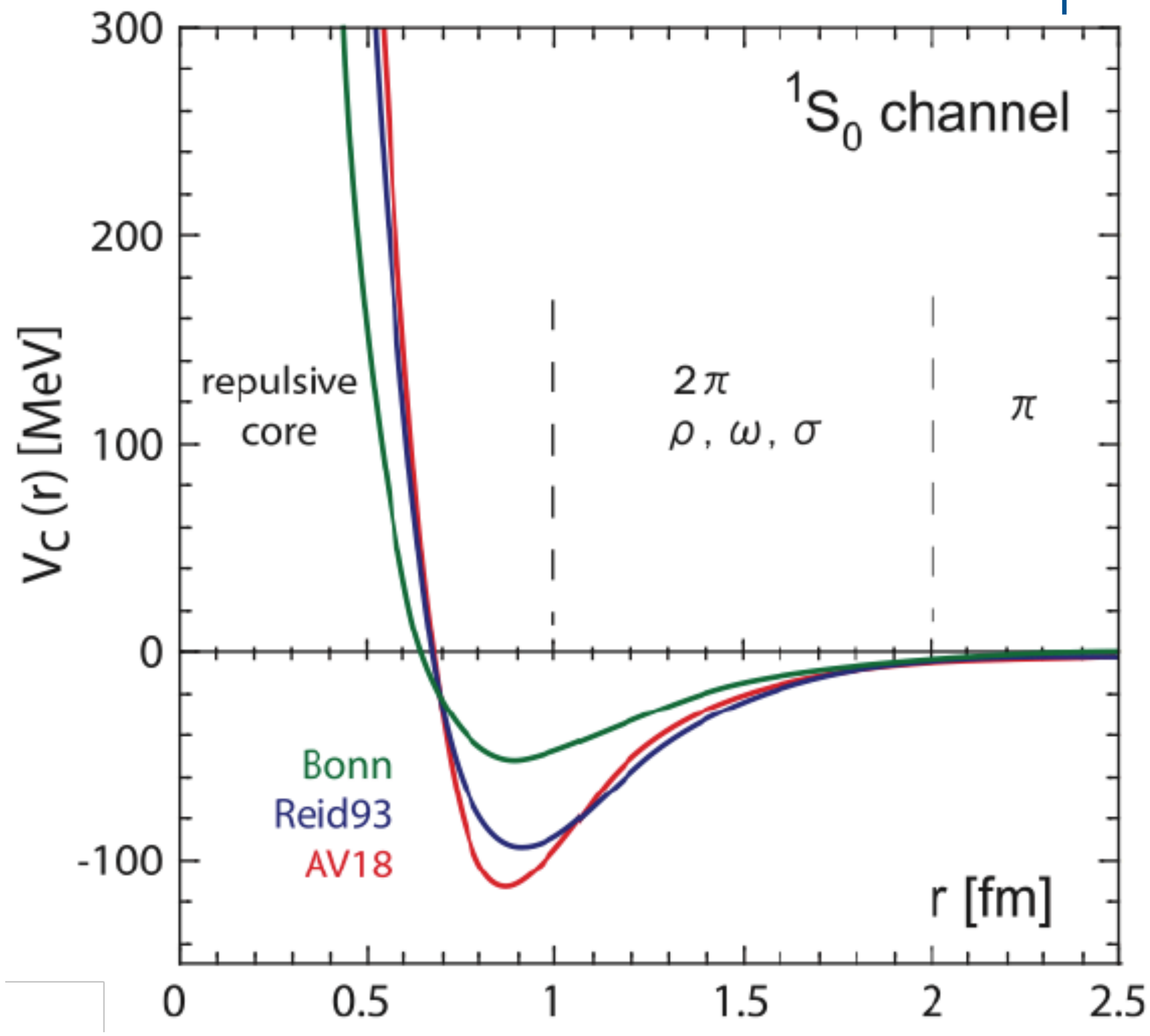


Carlson et al., *Phys. Rev. C* **68** 025802 (2003)

- Non-uniqueness of nucleon forces ✗
- Short-range core needs many-body treatment ✗

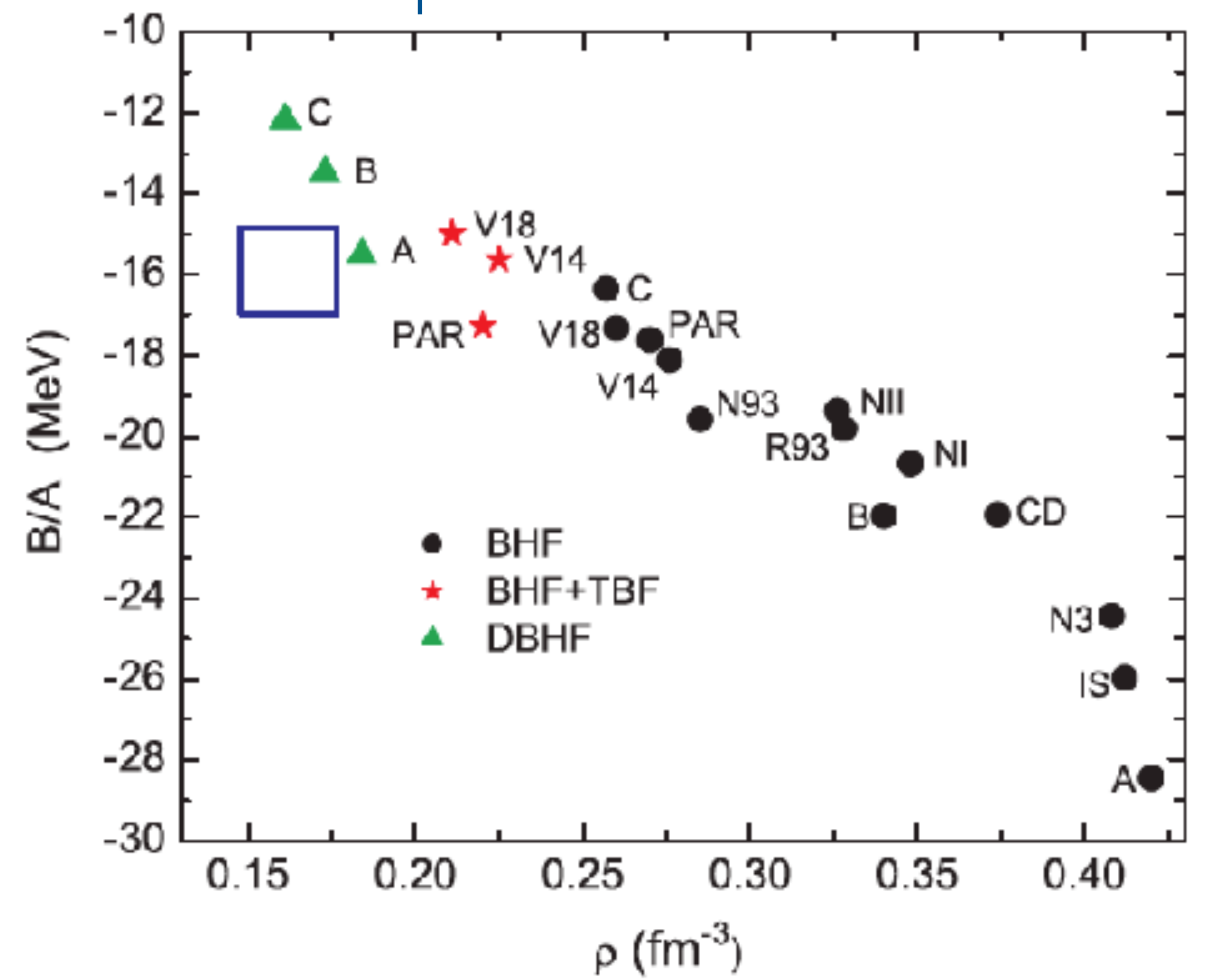
Complications

NN interaction is not unique



S. Aoki, et al. *Comput. Sci. Dis.* **1** 015009 (2008)

Saturation point of nuclear matter

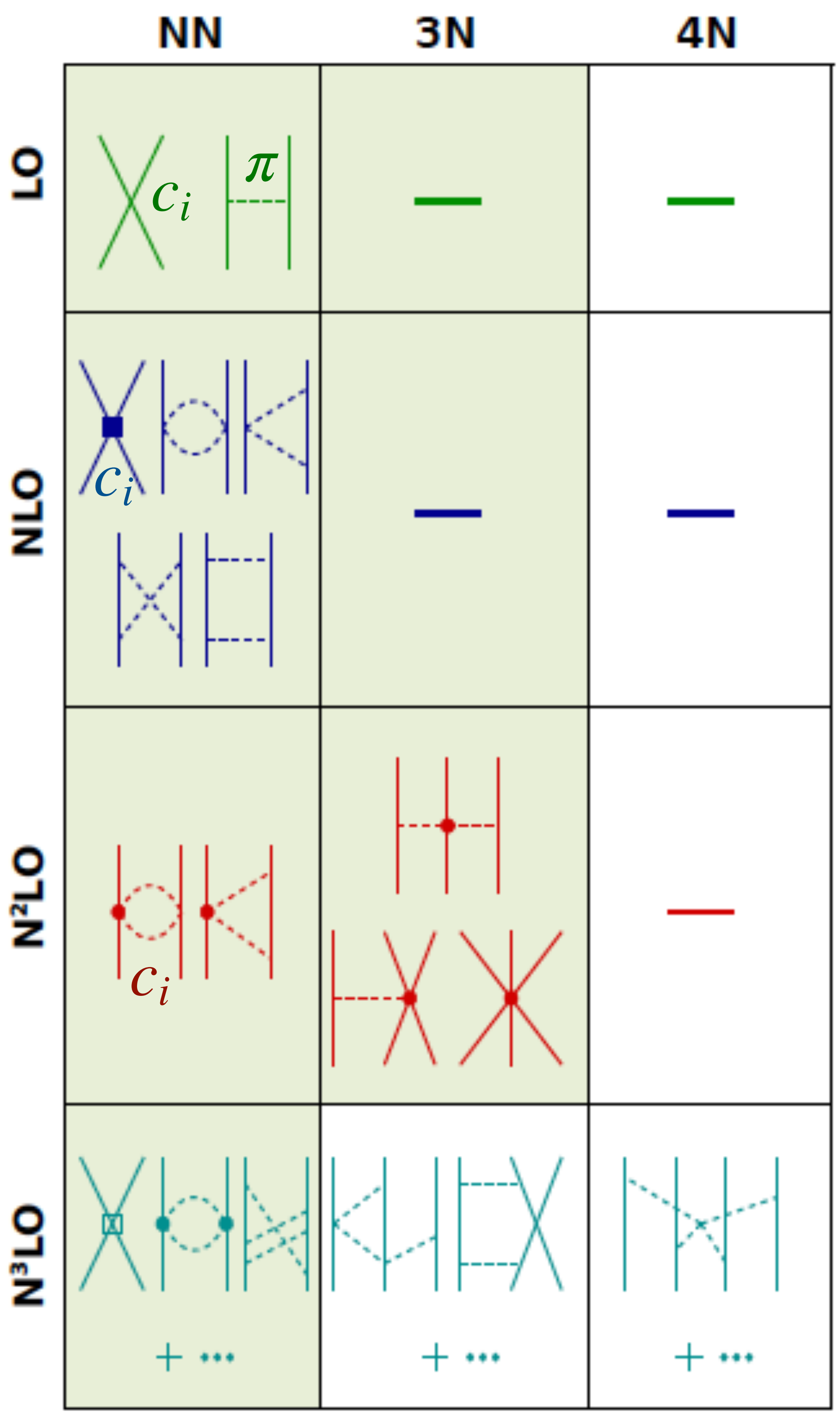


Li, Lombardo, Schulze et al. *PRC* **74** 047304 (2006)

- Non-uniqueness of nucleon forces ✘
- Short-range core needs many-body treatment ✘
- Three-body forces needed for saturation ✘

NN forces from EFTs of QCD

UNIVERSITY OF SURREY



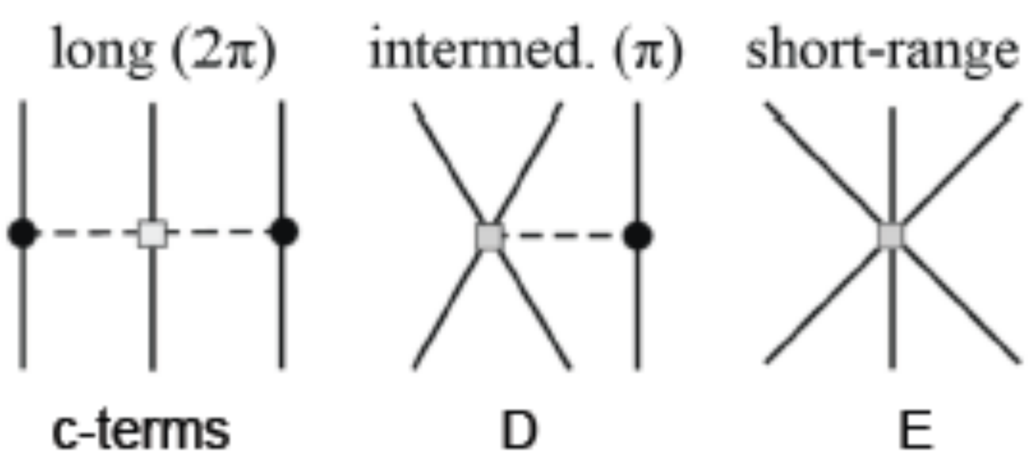
Robert Roth – TU Darmstadt – 04/2013

Chiral perturbation theory

- π and N as dof
- Systematic expansion
- 2N at N³LO - LECs from πN, NN
- 3N at N²LO - 2 more LECs
- (Often further renormalized)

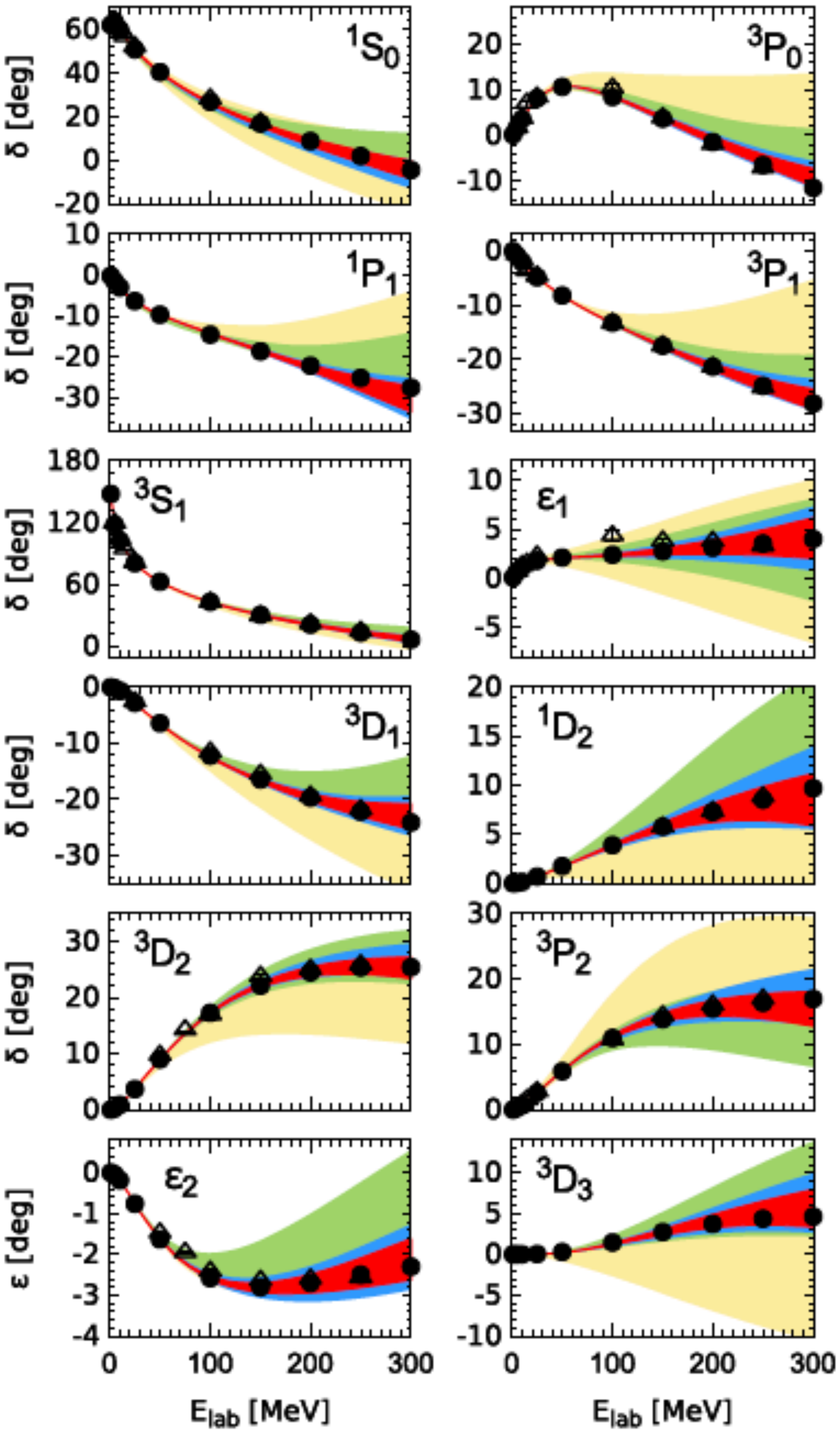
$$\mathcal{O}\left(\frac{Q}{\Lambda}\right)$$

$$\Lambda \sim 1\ \mathrm{GeV}$$



Weinberg, *Phys. Lett. B* **251** 288 (1990), *NPB* **363** 3 (1991)
Entem & Machleidt, *PRC* **68**, 041001(R) (2003)
Tews, Schwenk *et al.*, *PRL* **110**, 032504 (2013)
Epelbaum, Frebs & Meissner, *PRL* **115**, 122301 (2015)

NN forces from EFTs of QCD

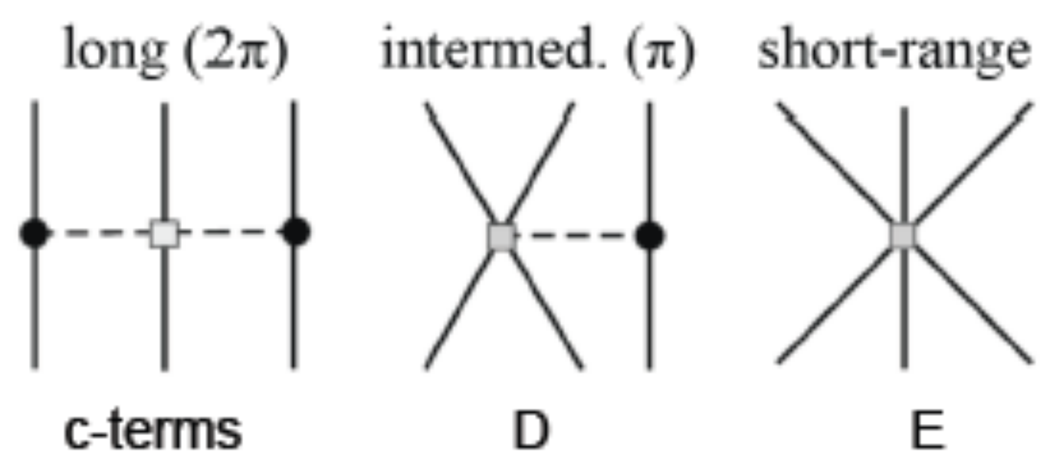


Chiral perturbation theory

- π and N as dof
- Systematic expansion
- 2N at N^{3}LO - LECs from πN, NN
- 3N at N^{2}LO - 2 more LECs
- (Often further renormalized)

$$\mathcal{O}\left(\frac{Q}{\Lambda}\right)$$

$$\Lambda \sim 1 \text{ GeV}$$

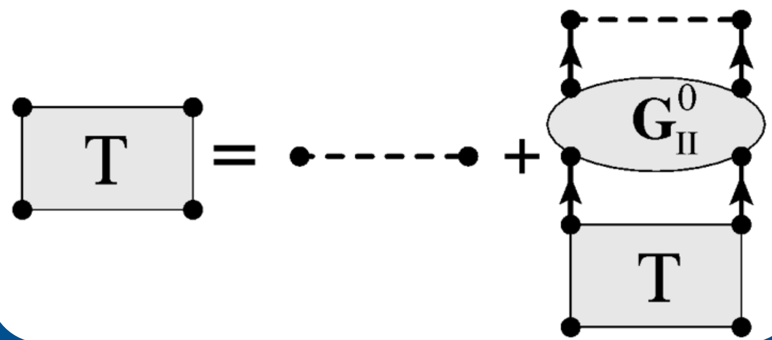


Weinberg, *Phys. Lett. B* **251** 288 (1990), *NPB* **363** 3 (1991)
Entem & Machleidt, *PRC* **68**, 041001(R) (2003)
Tews, Schwenk *et al.*, *PRL* **110**, 032504 (2013)
Epelbaum, Frebs & Meissner, *PRL* **115**, 122301 (2015)

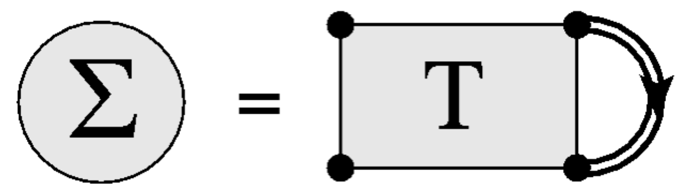
SCGF Ladder approximation

UNIVERSITY OF SURREY

In-medium interaction

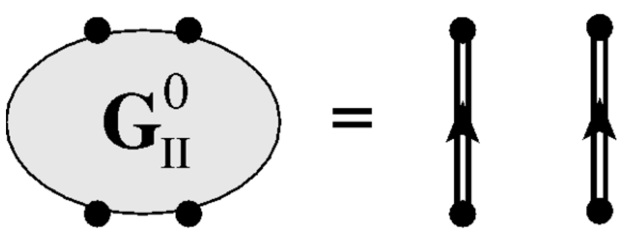


Ladder self-energy

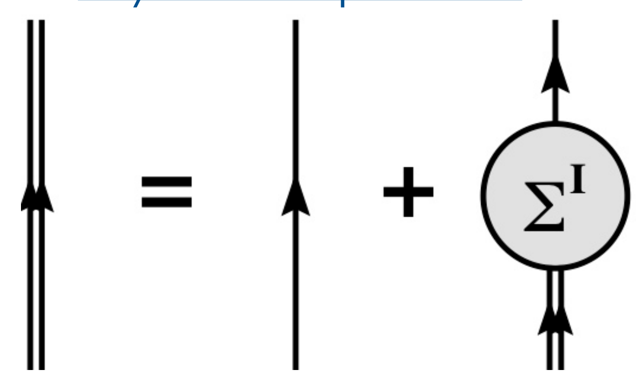


- Self-consistent resummation
- Energy and momentum integral
- @Finite T (Matsubara)

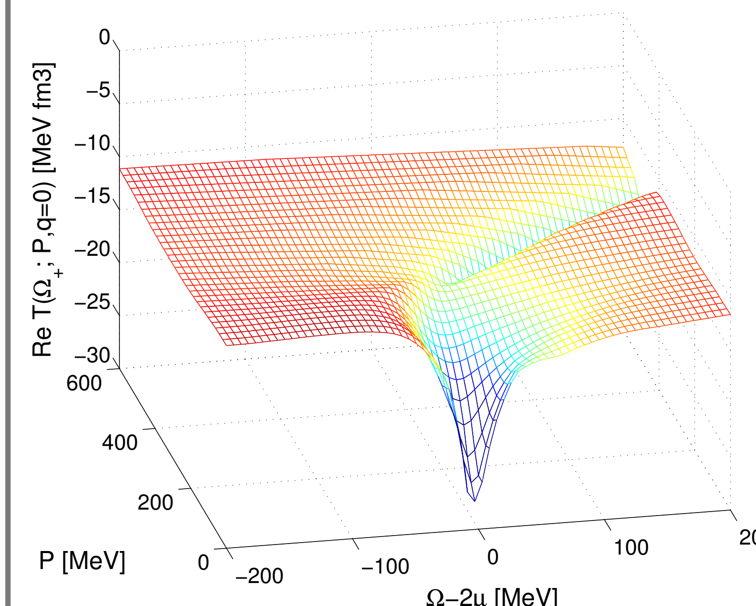
pp & hh Pauli blocking



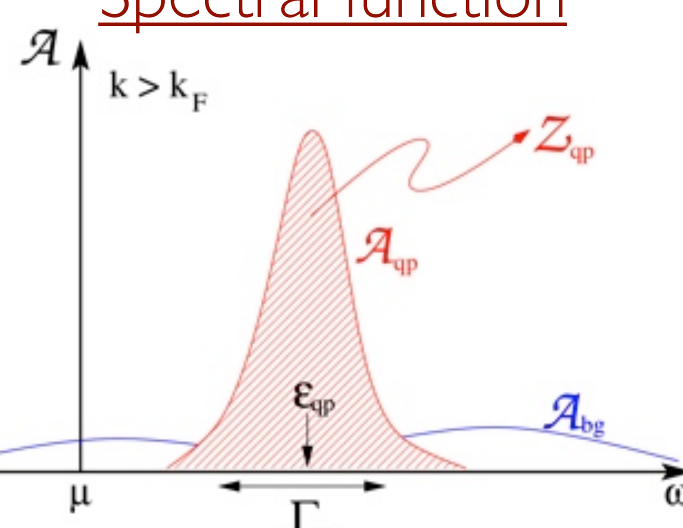
Dyson equation



T-matrix at T=5 MeV



Spectral function



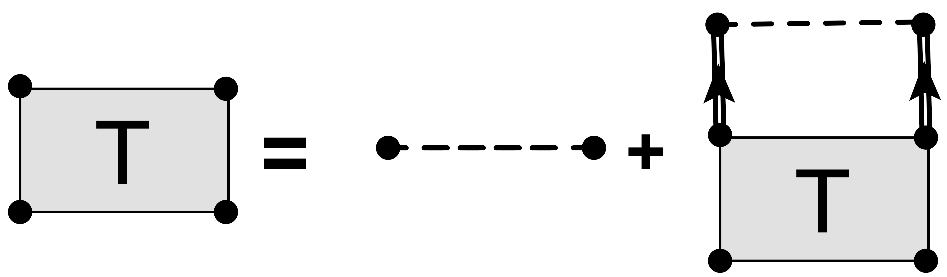
One-body properties
Momentum distribution
Thermodynamics & EoS
Transport

Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)
Rios & Soma PRL **108** 012501 (2012)

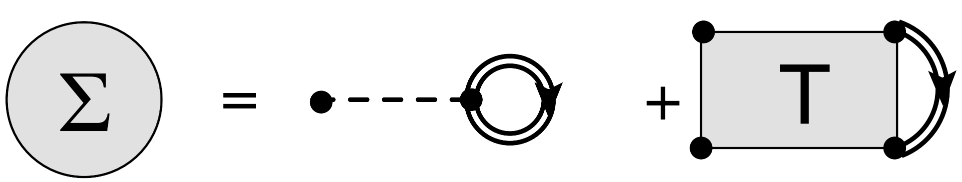
Ladder approximation with 3BF

Two-body interaction

In-medium T-matrix

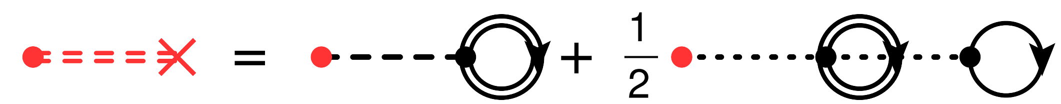


Self-energy

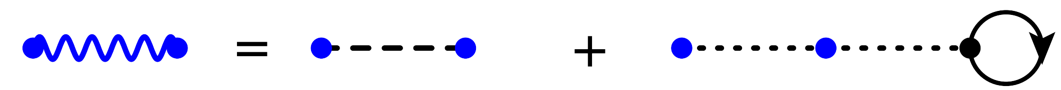


Effective interactions

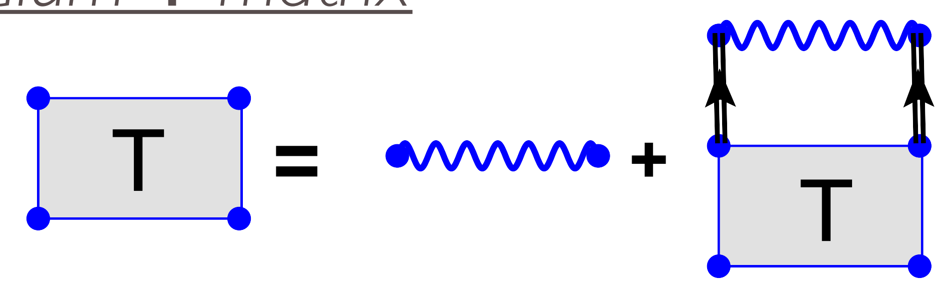
Effective one-body force



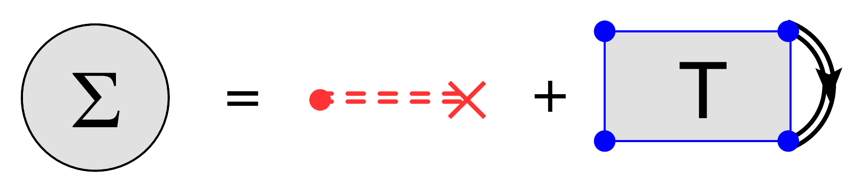
Effective two-body force

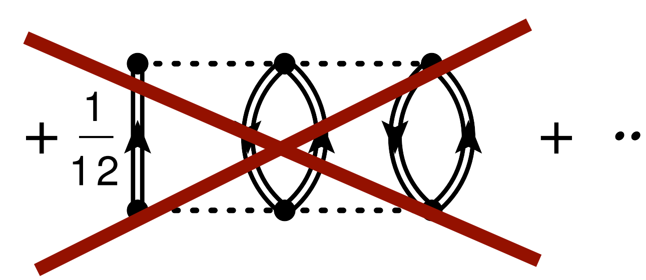


In-medium T-matrix

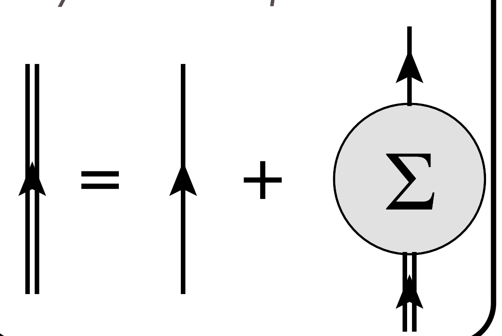


Self-energy





Dyson equation



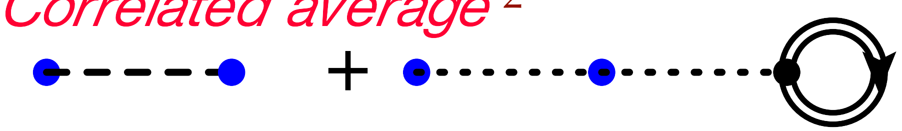
Carbone *et al., PRC* **88** 054326 (2013)
A. **Carbone**, *PhD Thesis*

Density-dependent interaction

Two-body N3LO

Uncorrelated average [1]

Correlated average [2]



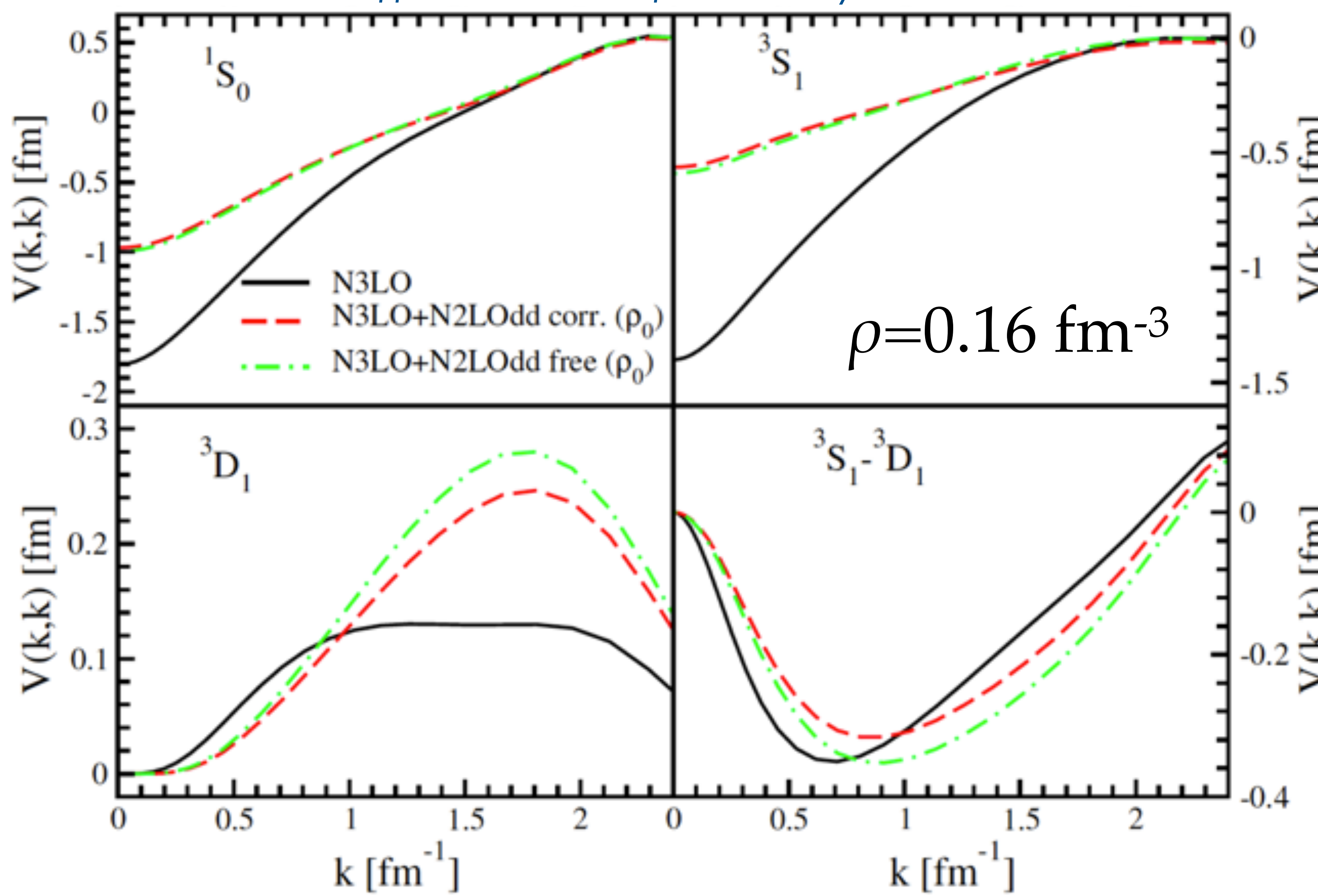
LECs

c_D=-1.11

c_E=-0.66

$$k \neq k' \Rightarrow \frac{1}{2}(k+k')$$

Chiral NN effective 2B forces: symmetric matter



- 3NF bring repulsion: correlated & uncorrelated averages are similar
- Correlated average brings small corrections to 1/2 of terms
- Diagonal *k*=*k*' matrix elements computed
- Off-diagonal extrapolated & regulated

[1]Holt *et al.* *Phys. Rev. C* **81** 024002 (2010)

[2] Carbone, Polls & Rios, *PRC* **90**, 054322 (2014); A. Carbone, PhD thesis

Symmetric matter

Theoretical uncertainties: Chiral expansion

Two-body N3LO

Uncorrelated average

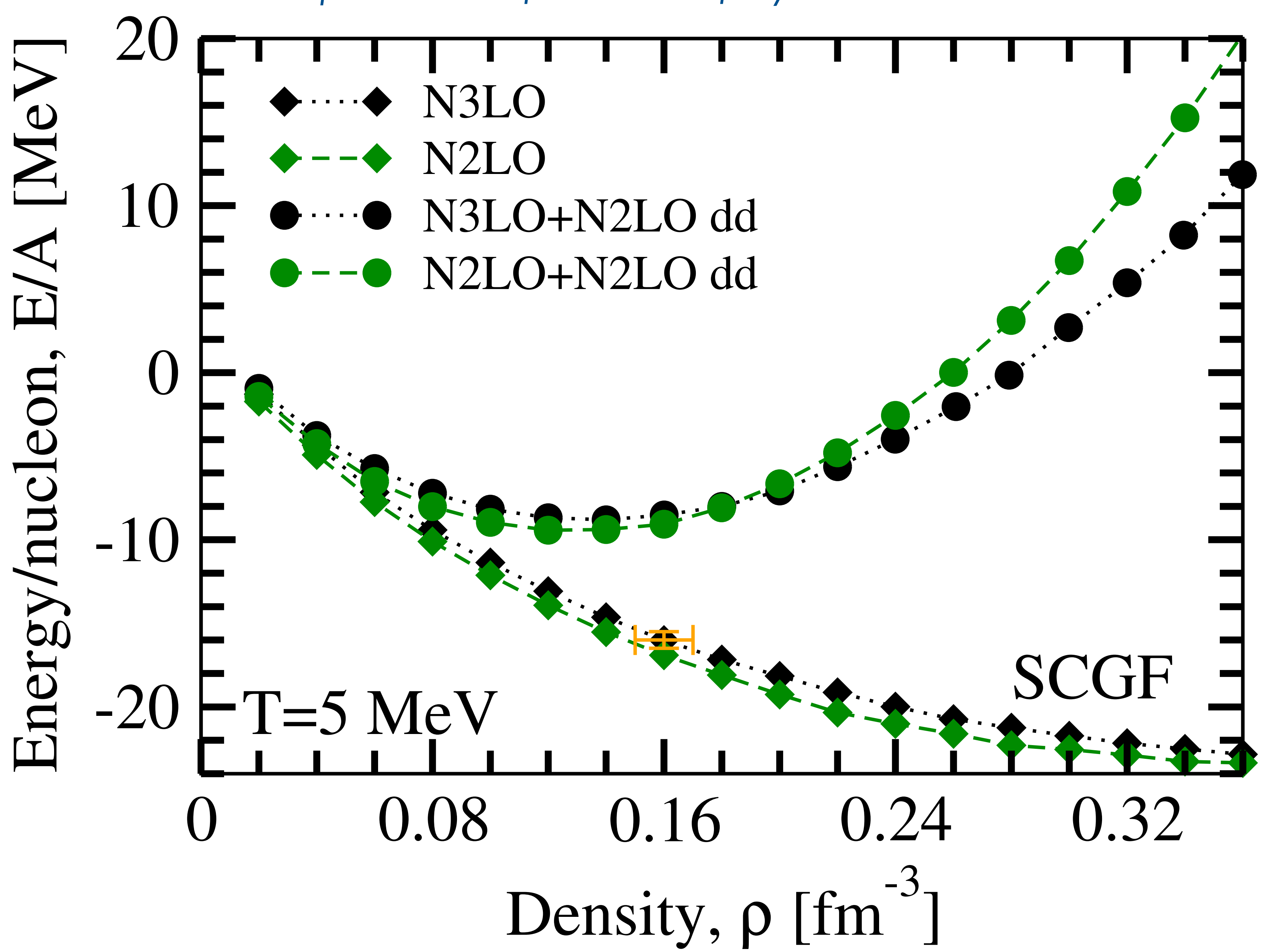
<u>LECs</u>

c_D=-1.11

c_E=-0.66

K_0~60 MeV

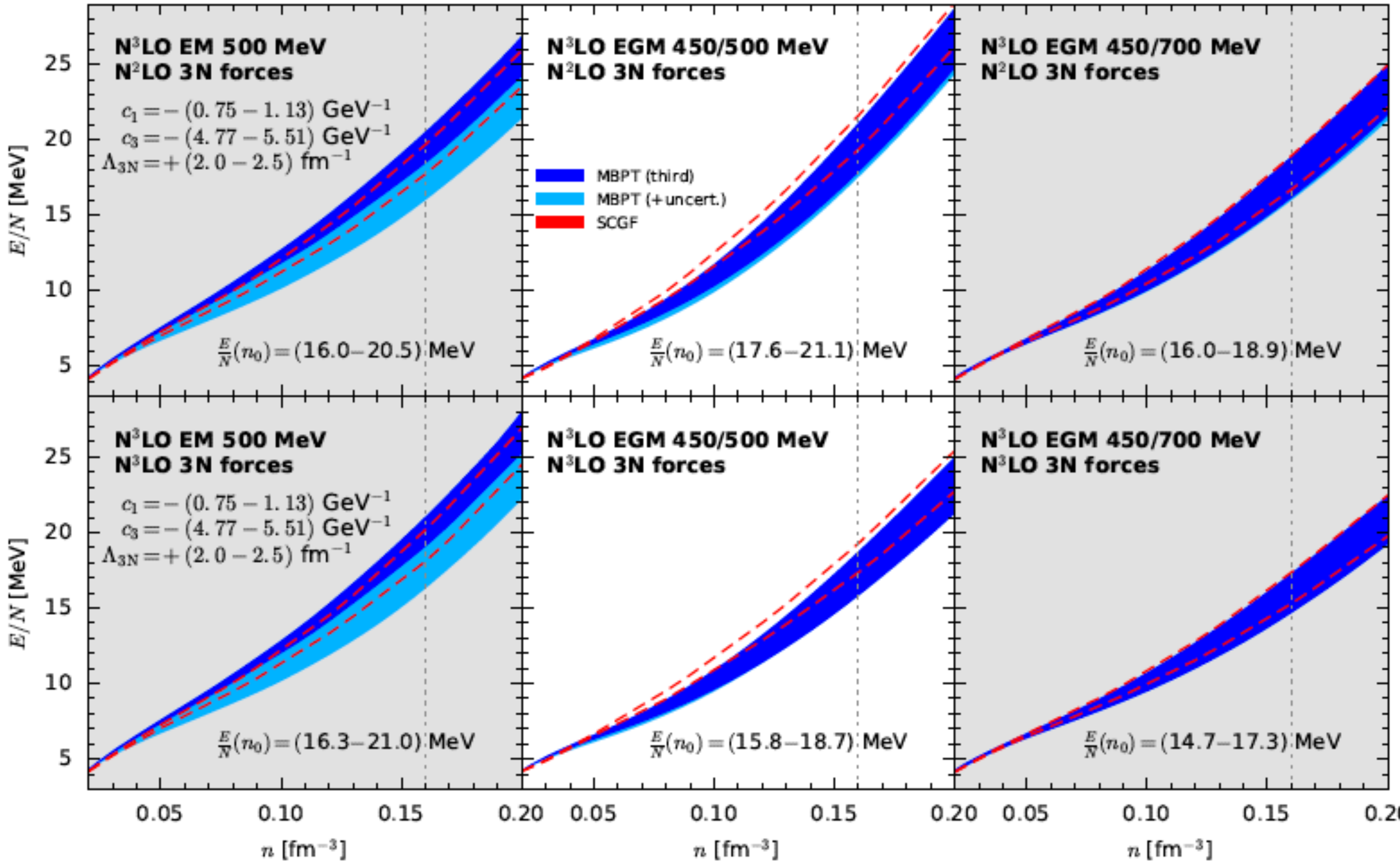
<u>*Equation of state of symmetric matter*</u>



*Carbone, Polls & Rios, PRC **88** 044302 (2014)*

- 3NF result is still *<u>underbound</u>*
- Small difference in infinite matter for N3LO & N2LO...
- In contrast to finite nuclei!

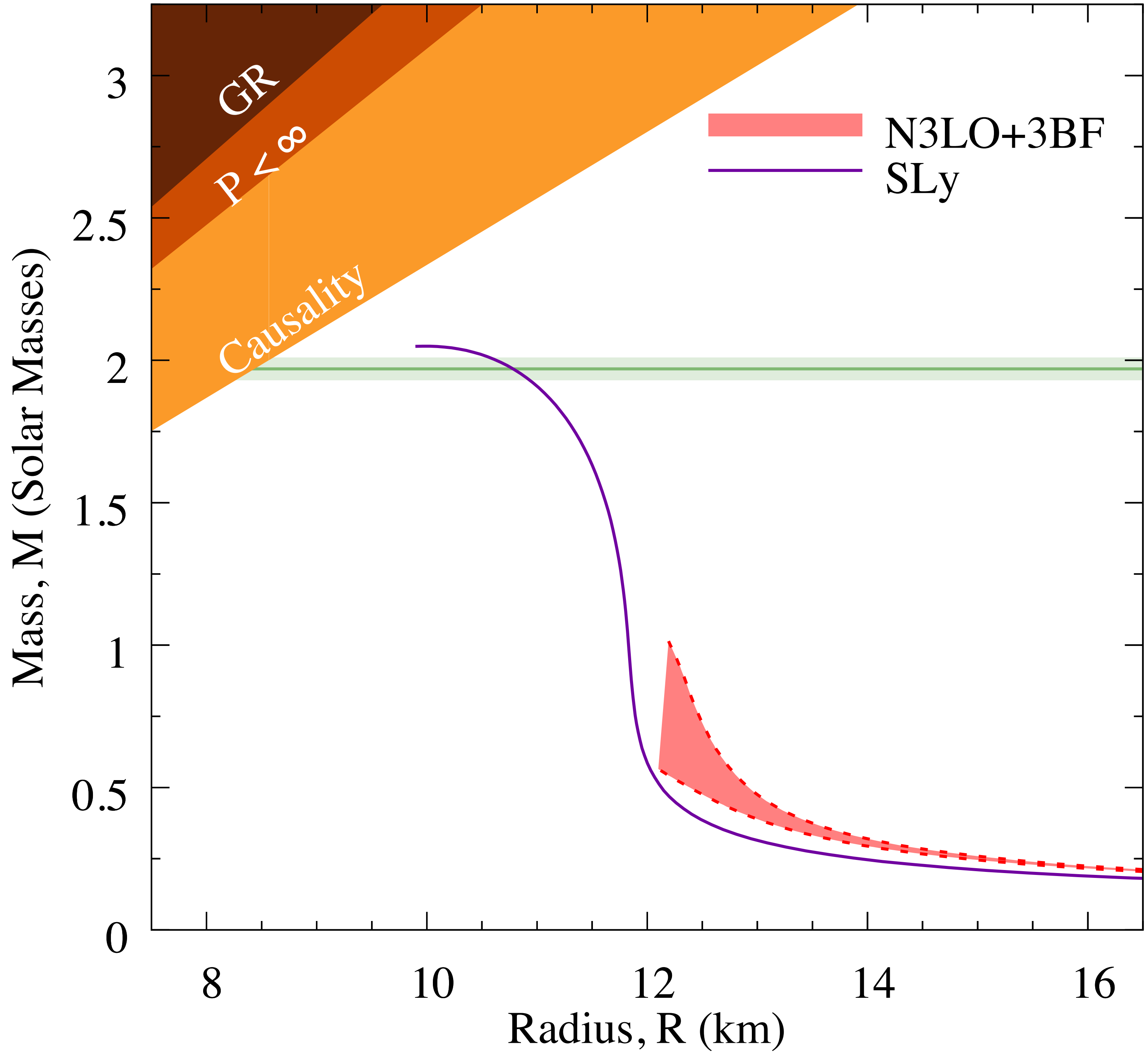
Neutron matter



Drischler, **Carbone**, Hebeler, Schwenk *PRC* **94** 054307 (2016)

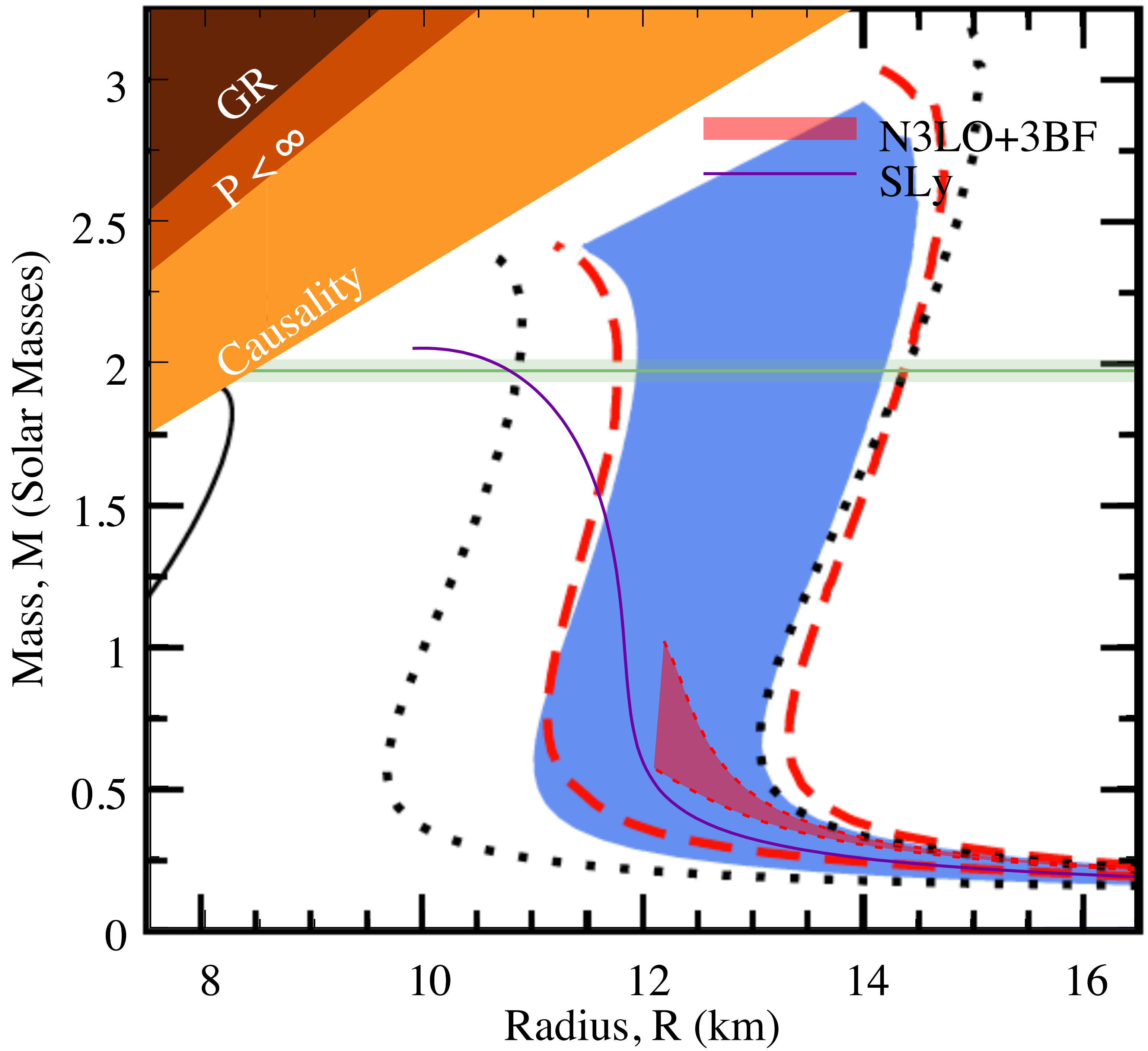
- Uncertainty band from unknown ChPT LECs + cutoff + MBPT
- Finite temperature available too

Neutron matter



- Mass-Radius relation from SCGF calculations
- Cut-off variation (N3LO) and/or SRG evolution

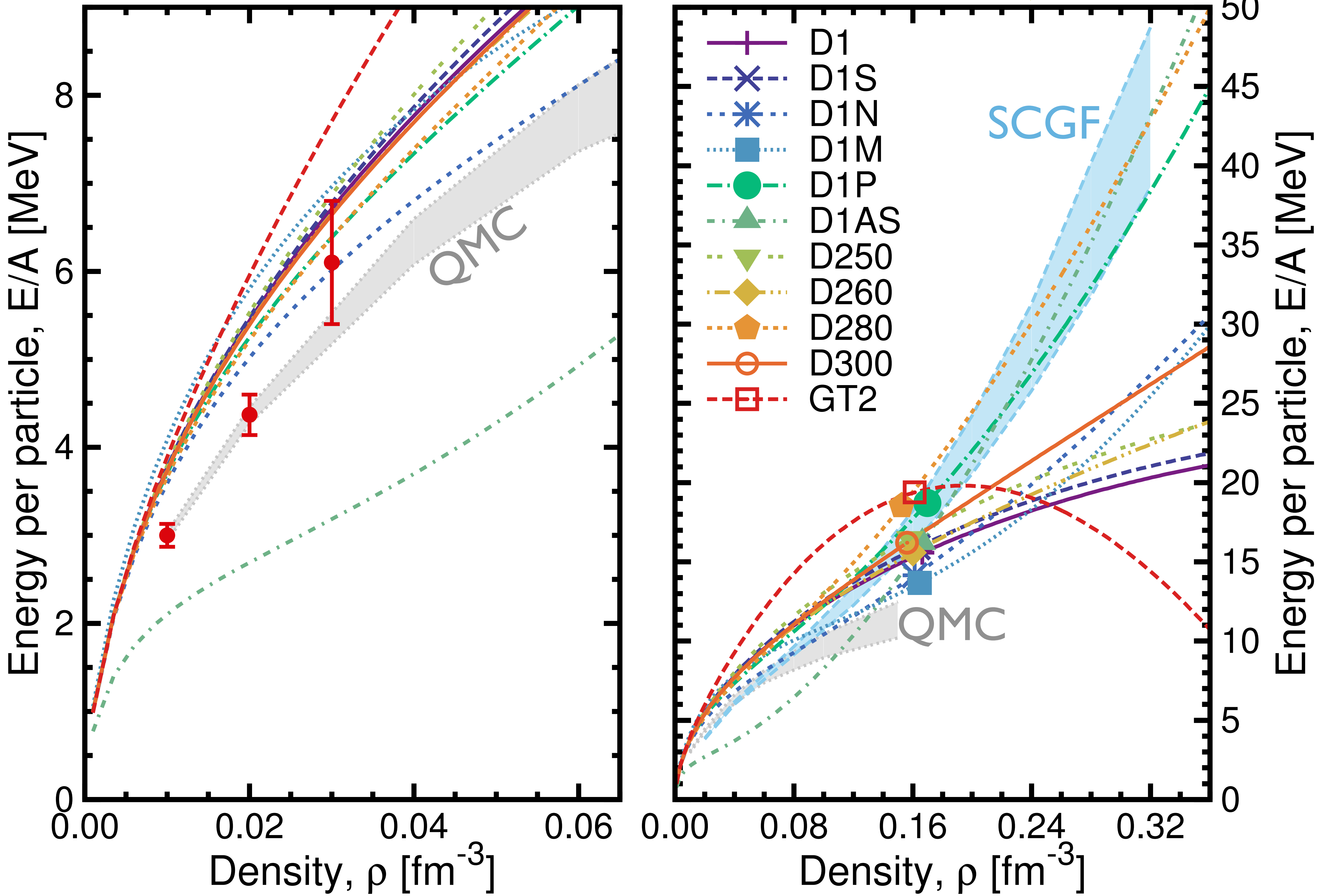
Neutron matter



Hebeler, Lattimer, Pethick, Schwenk *ApJ* **773** 11 (2013)

- Mass-Radius relation from SCGF calculations
- Cut-off variation (N3LO) and/or SRG evolution

Neutron matter

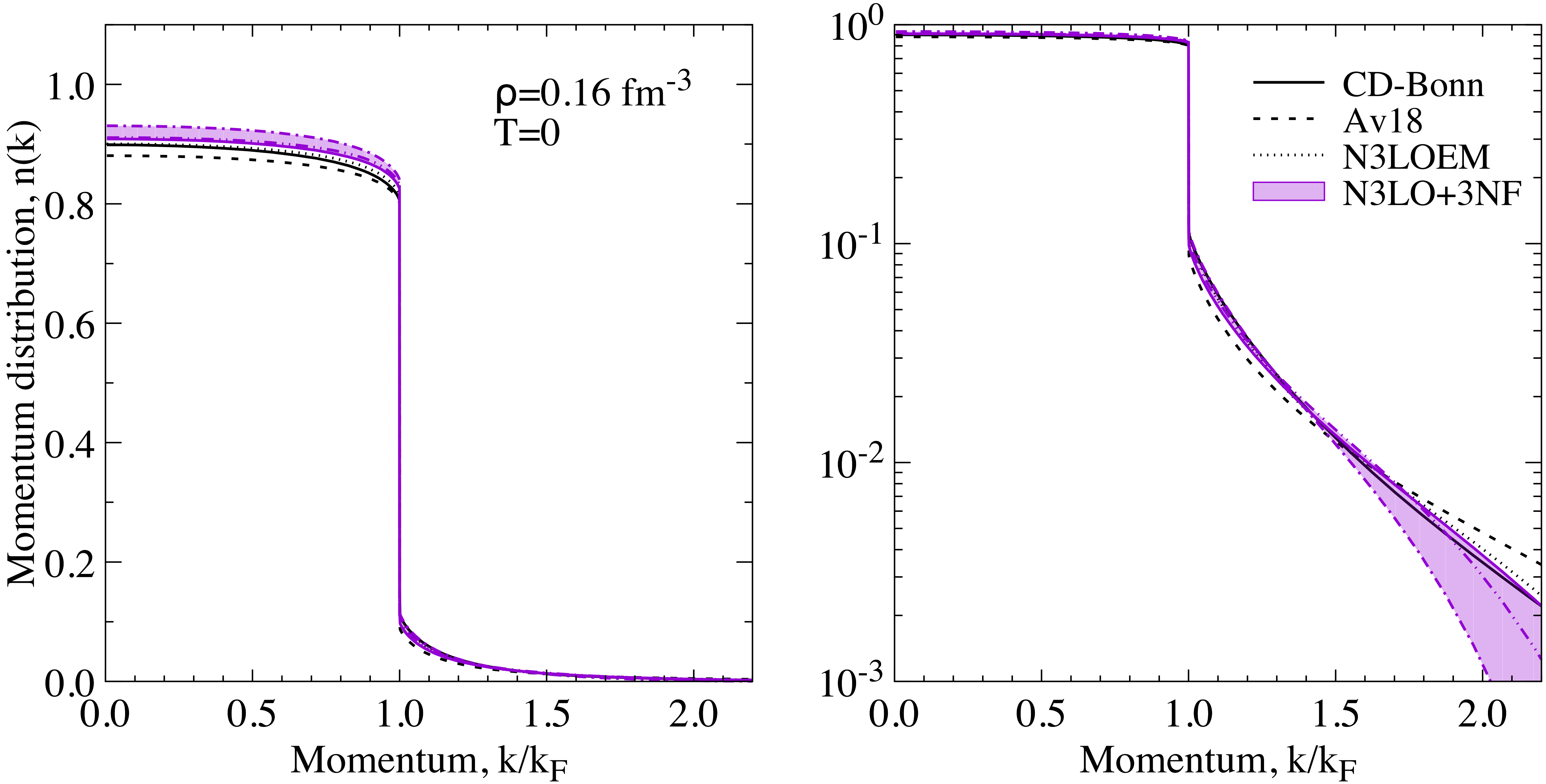


Red points: Brown & Schwenk PRC **89** 011307(R) (2014
R. Sellahewa & A. Rios, *PRC* **90**, 054327 (2014)

Momentum distribution
Single-particle occupation

$$n(k) = \left\langle a_k^\dagger a_k \right\rangle \qquad \nu \int \frac{d^3k}{(2\pi)^3} n(k) = \rho$$

Symmetric matter



- Dependence on NN interaction **understood**
- N3LO+3NF = N3LO 2NF + N2LO 3NF @ Λ=414-500 MeV (cutoff variation only)

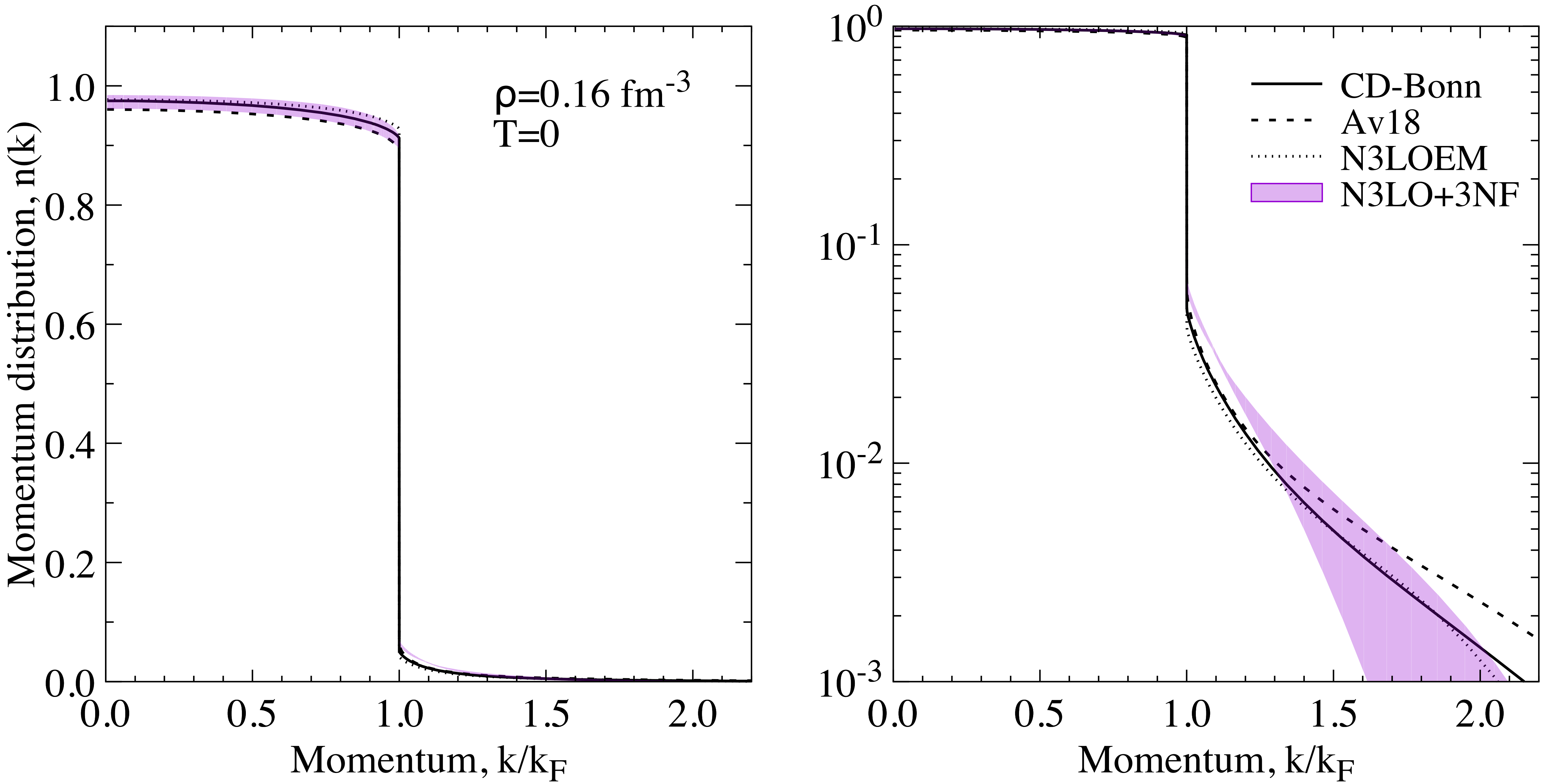
Coraggio, Holt, et al. PRC **89** 044321 (2014)

Momentum distribution

Single-particle occupation

$$n(k) = \left\langle a_k^\dagger a_k \right\rangle \qquad \nu \int \frac{d^3k}{(2\pi)^3} n(k) = \rho$$

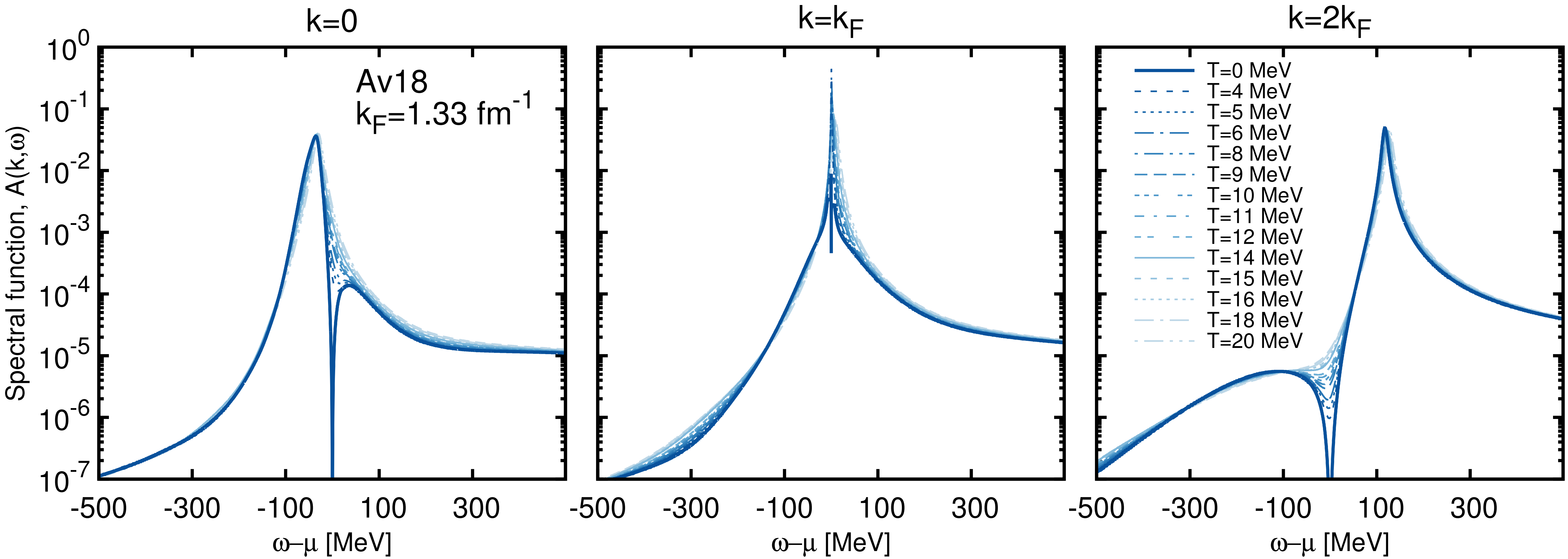
Neutron matter

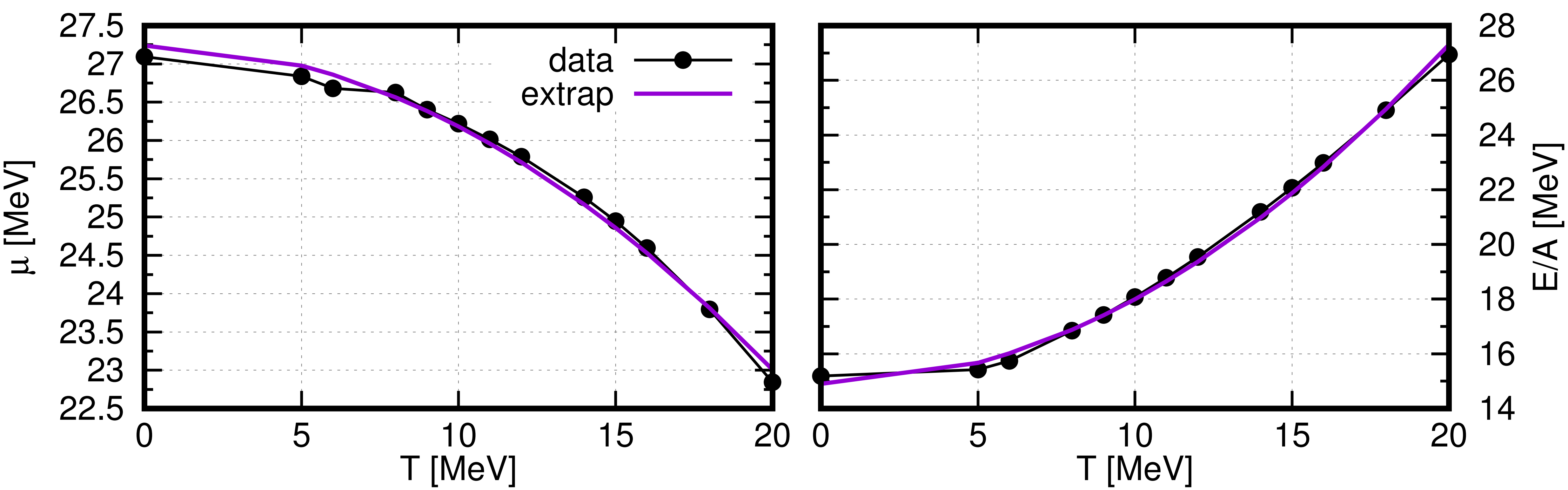


- Dependence on NN interaction **understood**
- N3LO+3NF = N3LO 2NF + N2LO 3NF @ Λ=414-500 MeV (cutoff variation only)

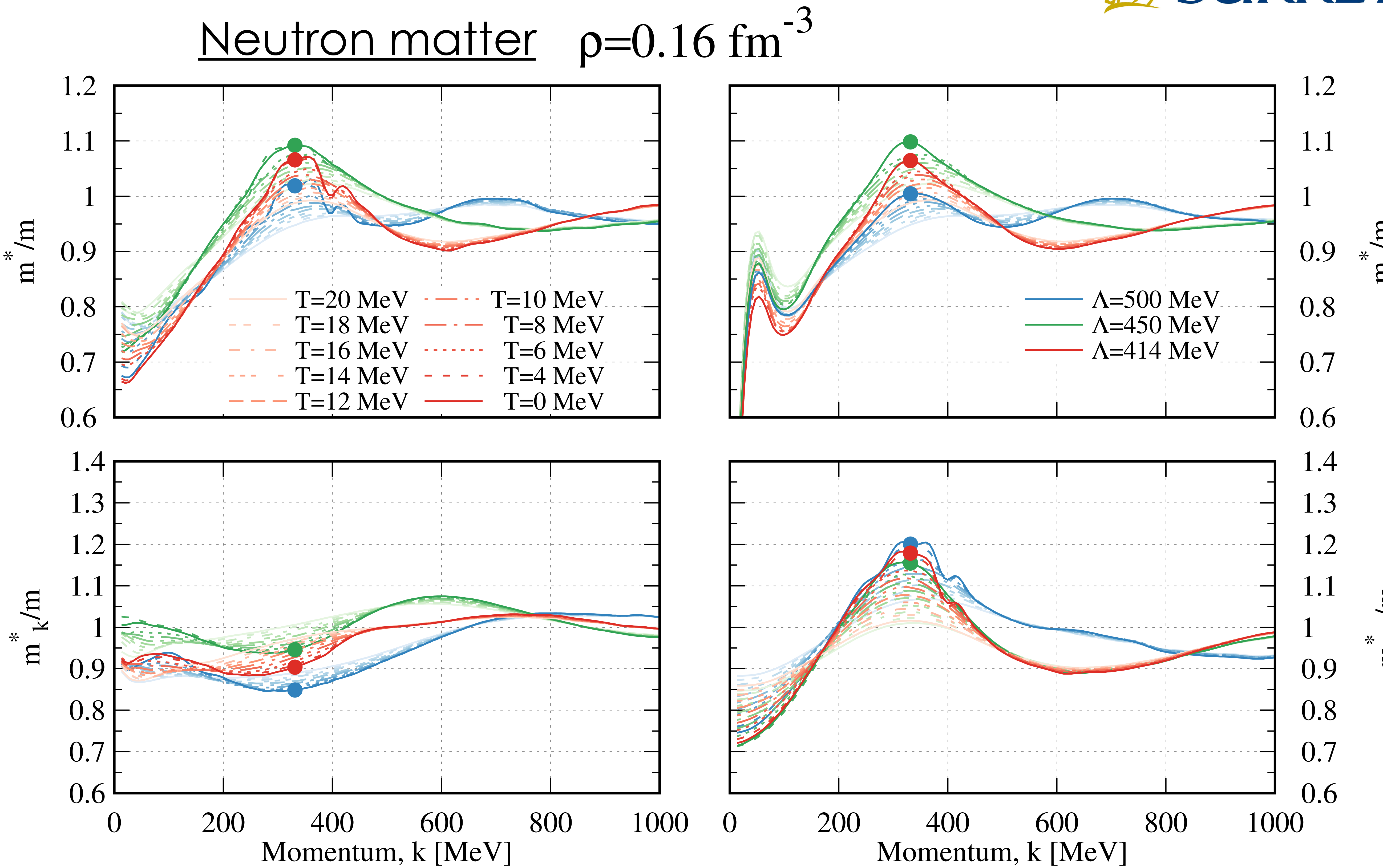
Coraggio, Holt, et al. PRC **89** 044321 (2014)

T=0 extrapolations





Effective masses

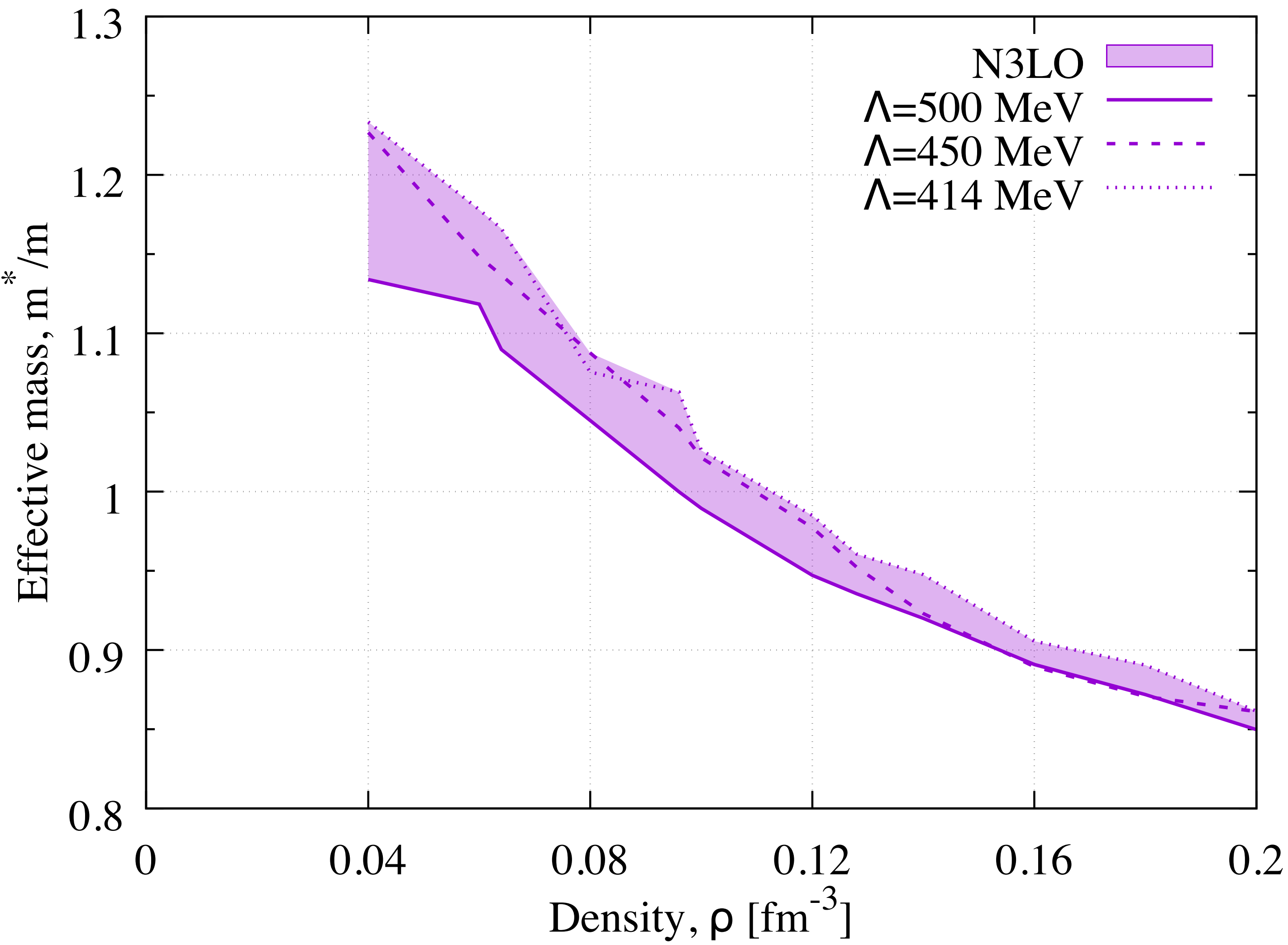


Coraggio, Holt, et al. PRC **89** 044321 (2014)

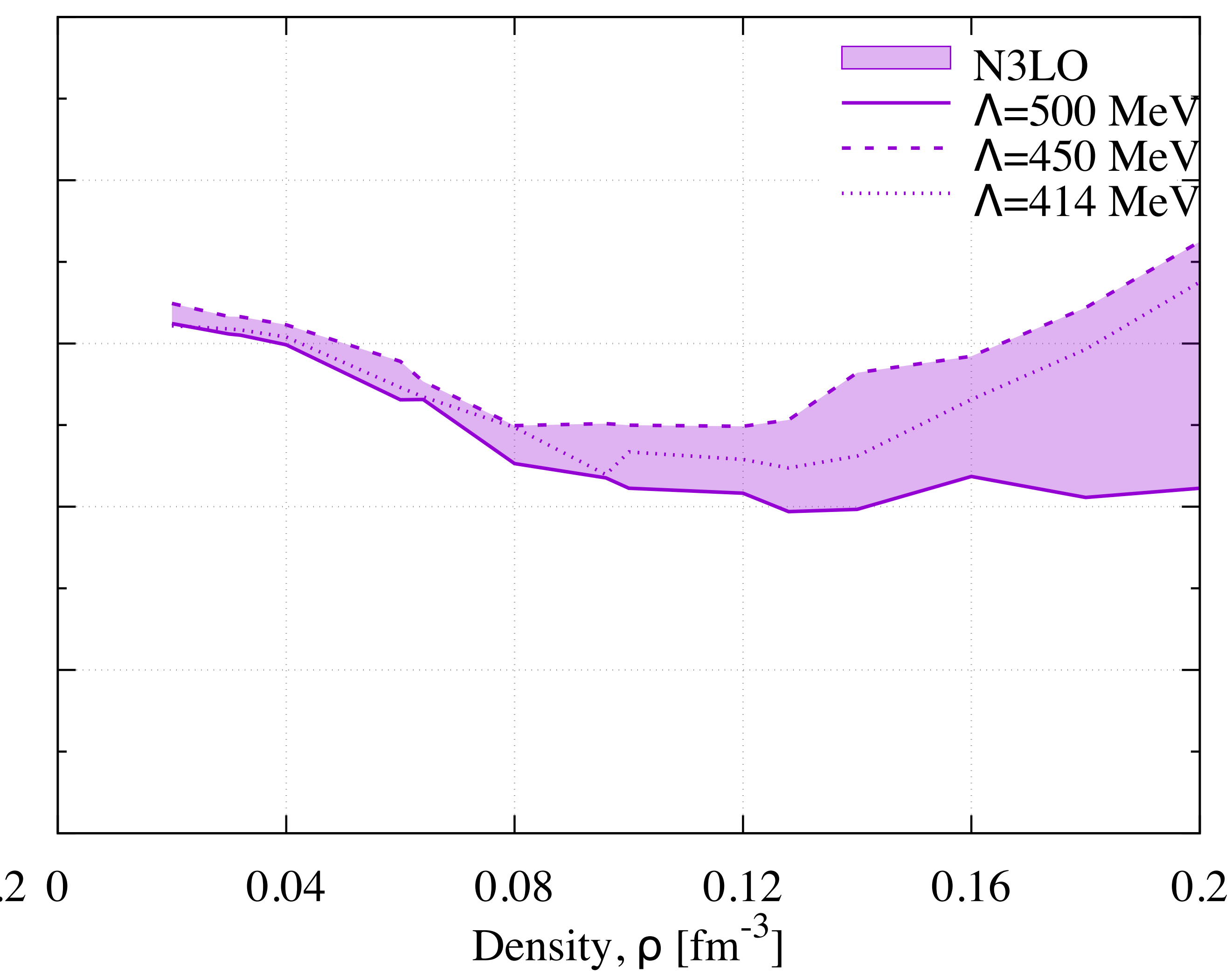
In preparation, will be publicly available

Density dependence of m*

Symmetric matter



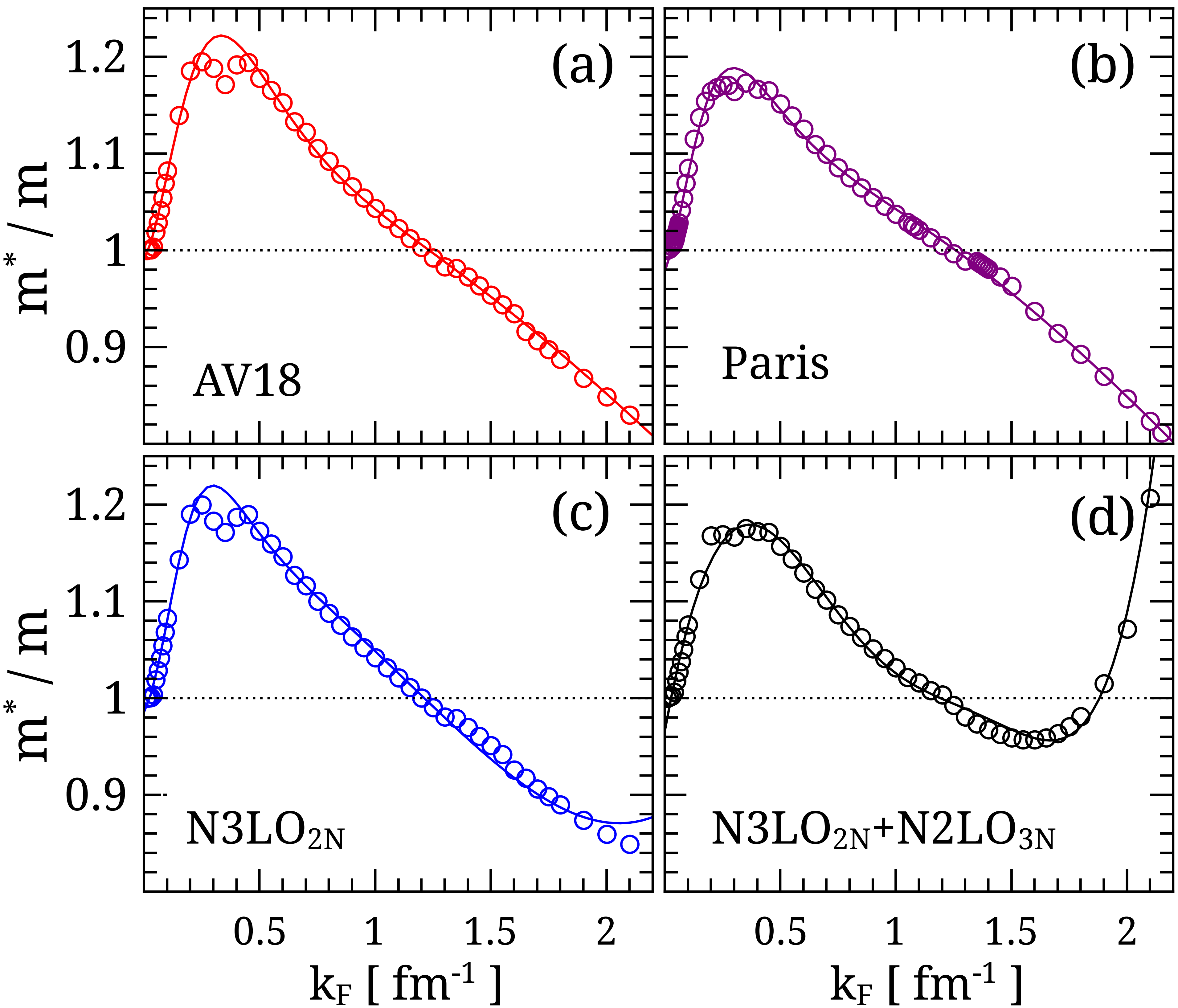
Neutron matter



A. Rios, JW Holt, *preliminary*

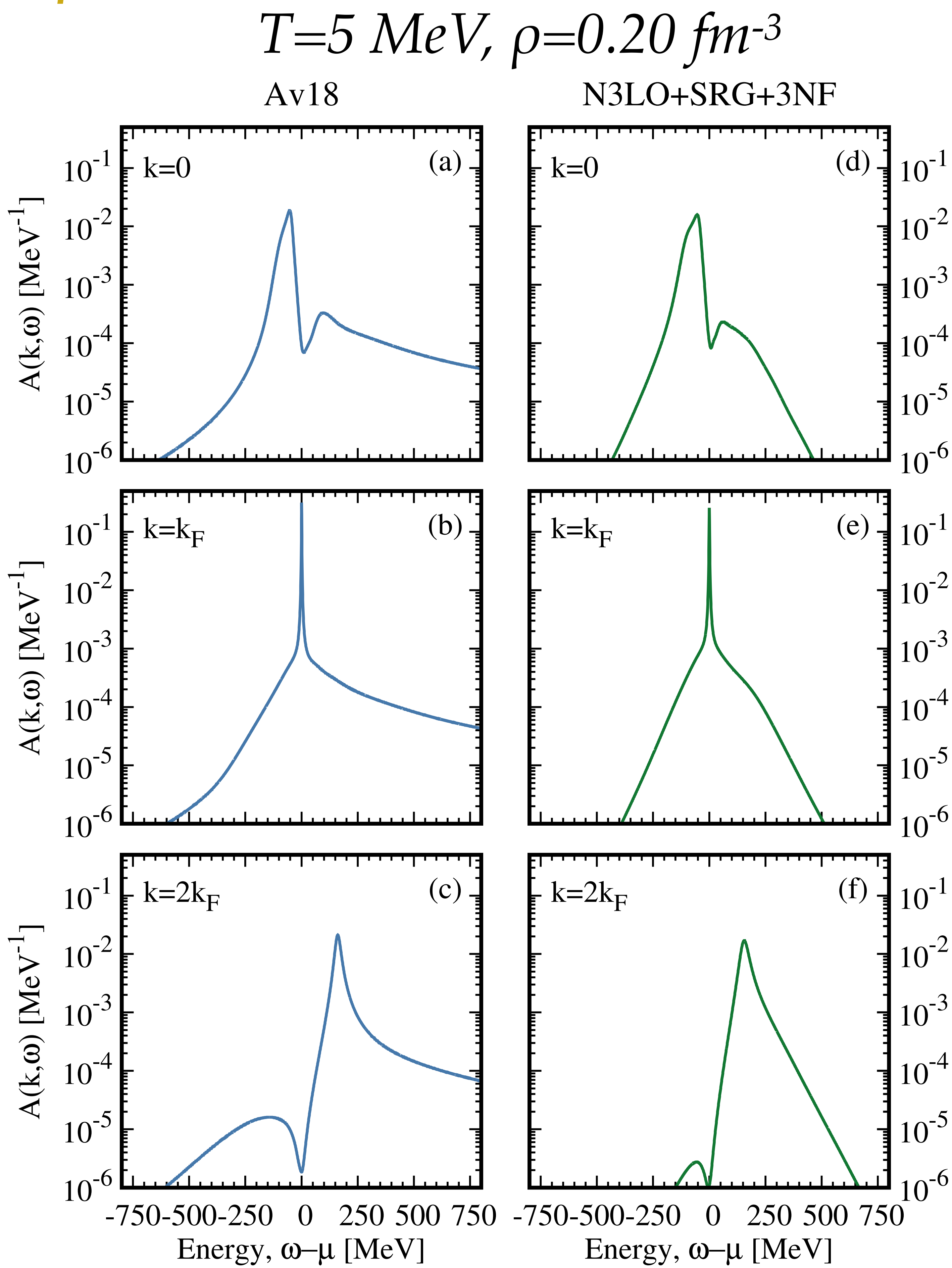
Density dependence of m*

UNIVERSITY OF SURREY



F. Isaule, H. Arellano & A. Rios, *PRC* ***94*** *034004 (2016)*

Spectral function



Can we **quantify** **differences**?

Energy weighted sum rules

$$m_k^{(0)} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{A}_k(\omega) = 1$$

$$m_k^{(1)} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega \mathcal{A}_k(\omega) = \frac{\hbar^2 k^2}{2m} + \Sigma_k^{\infty}$$

$$m_k^{(2)} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega^2 \mathcal{A}_k(\omega) = \left[m_k^{(1)} \right]^2 + \sigma_k^2$$

Ventura, Polls et al, PRC **49** 3050 (1994)
Frick, Mother & Polls, PRC **69** 054305 (2004)
Duguet & Hagen, PRC **85** 034330 (2012)
Duguet, Herbert et al, PRC **92** 034313 (2015)
Rios, Carbone, Polls, PRC **96** 014003 (2017)

Effective sp energies

Centroid energies

$$m_k^{(1)} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega \mathcal{A}_k(\omega) = \frac{\hbar^2 k^2}{2m} + \Sigma_k^{\infty}$$

$$\Sigma_k^{\infty} = \int \frac{d^3k_1}{(2\pi)^3} \langle \vec{k}\vec{k}_1 | V | \vec{k}\vec{k}_1 \rangle_a n_{k_1} + \frac{1}{2} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \langle \vec{k}\vec{k}_1\vec{k}_2 | W | \vec{k}\vec{k}_1\vec{k}_2 \rangle_a n_{k_1} n_{k_2}$$

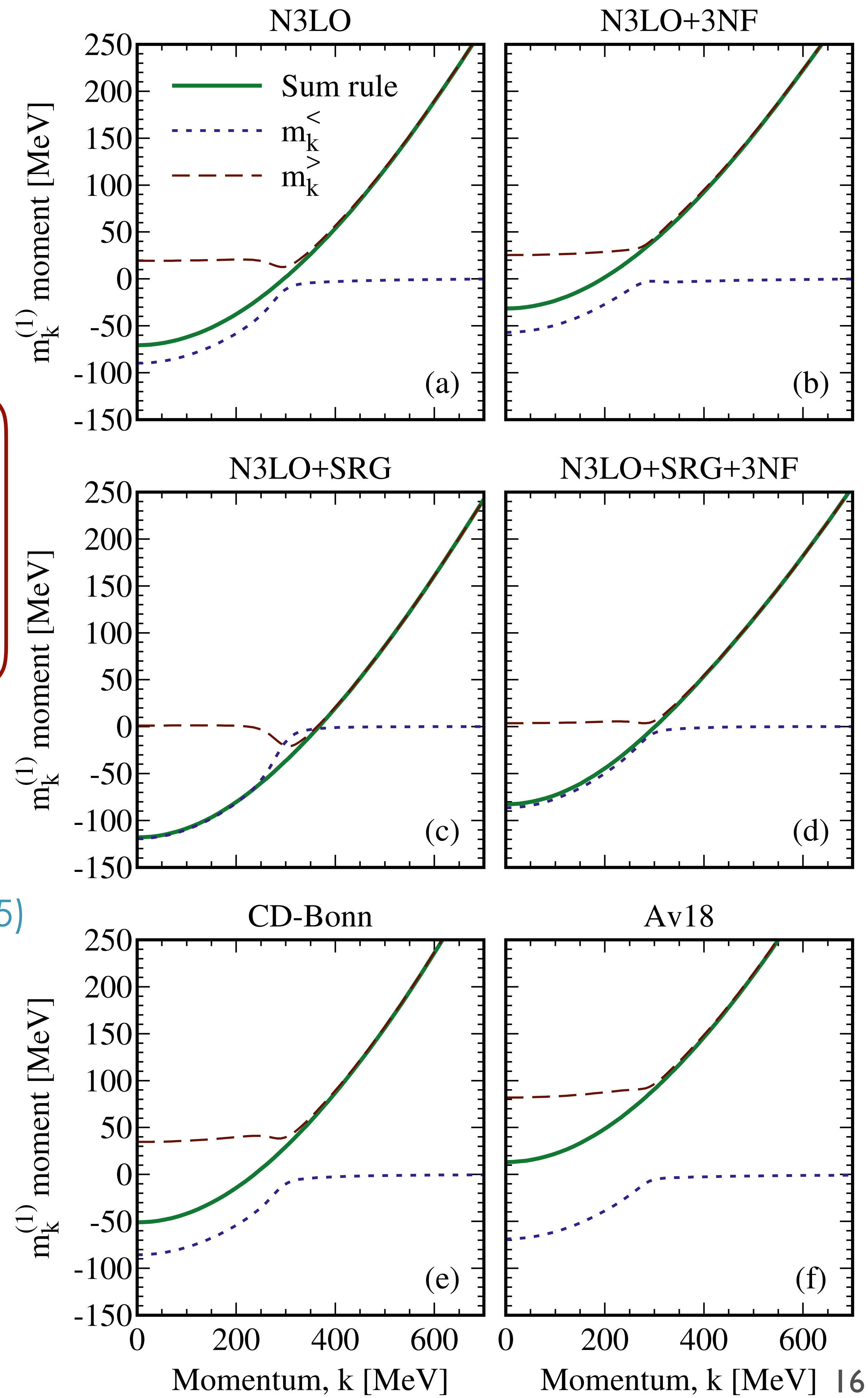
Cutoff evolution

Duguet, Herbert et al, PRC **92** 034313 (2015)

$$\frac{d}{d\lambda} O(\lambda) \equiv [\eta(\lambda), O(\lambda)],$$

$$\frac{d}{d\lambda} M_{pq}^{(0)}(\lambda) = 0,$$

$$\frac{d}{d\lambda} M_{pq}^{(1)}(\lambda) = -\langle \Psi_0^A(\lambda) | \{[[\eta(\lambda), a_p], H(\lambda)], a_q^\dagger\} + \{[a_p, H(\lambda)], [\eta(\lambda), a_q^\dagger]\} | \Psi_0^A(\lambda) \rangle.$$

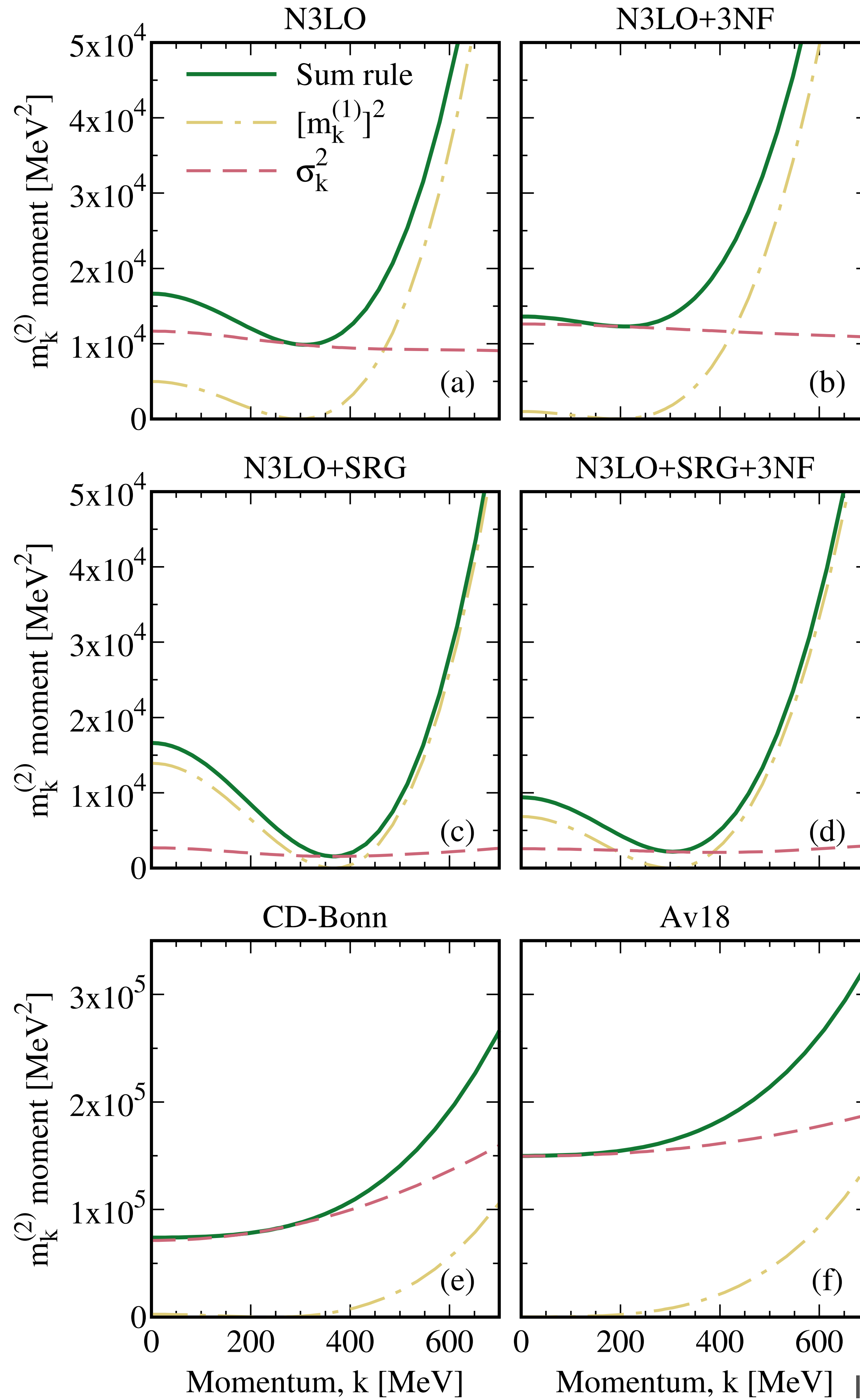


Second sum rule

sf variance

$$\sigma_k^2 = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} [\omega - m_k^{(1)}]^2 \mathcal{A}_k(\omega)$$

$$\sigma_k^2 = -\int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \, \mathrm{Im} \; \Sigma_k(\omega)$$

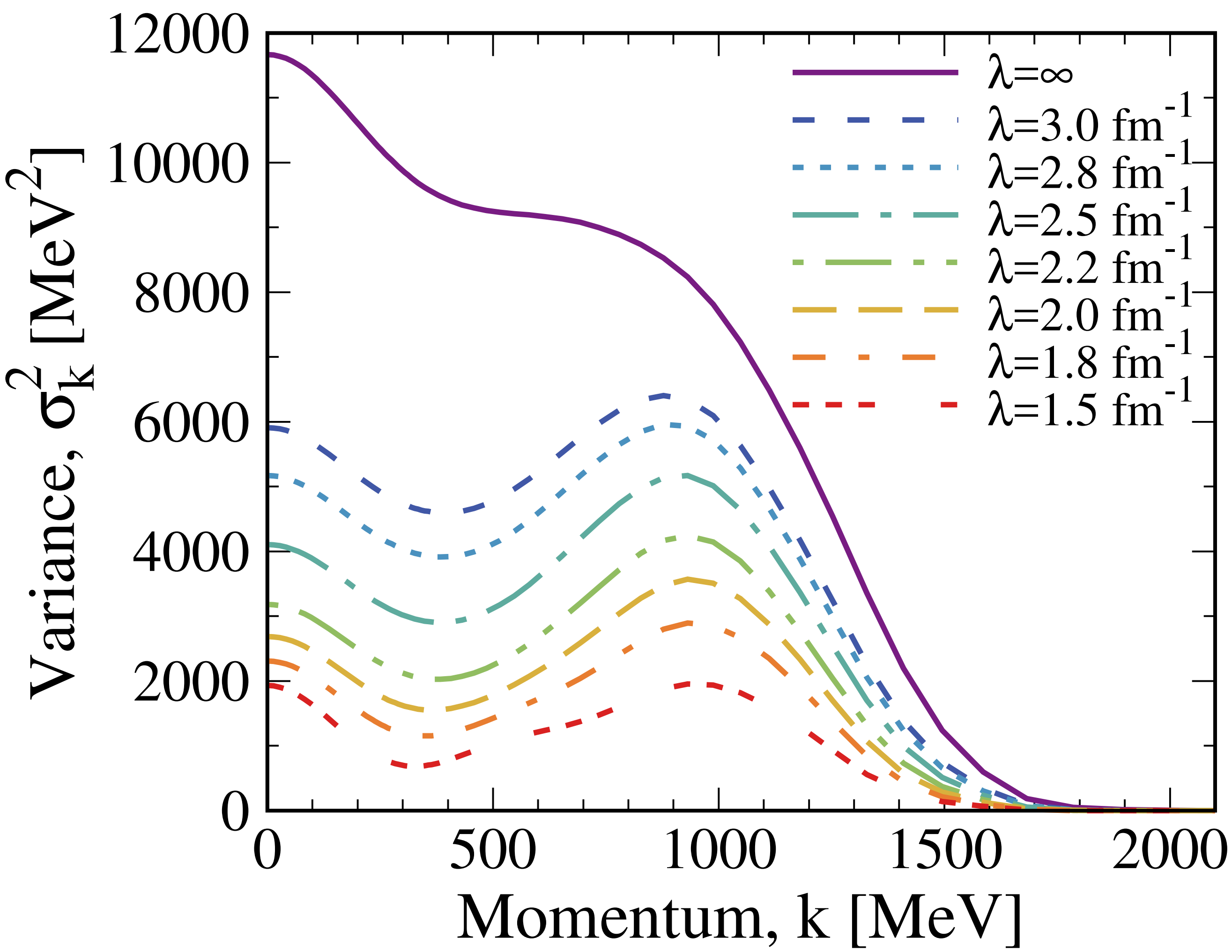


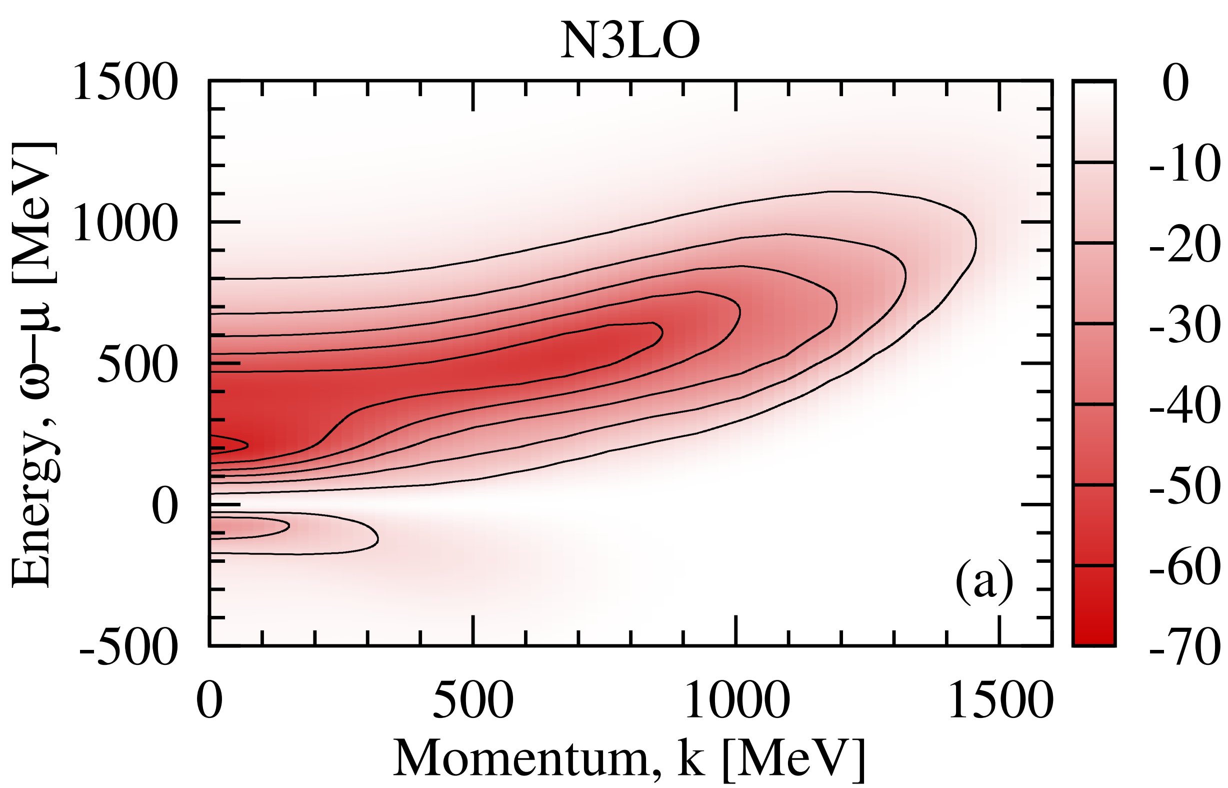
Second sum rule

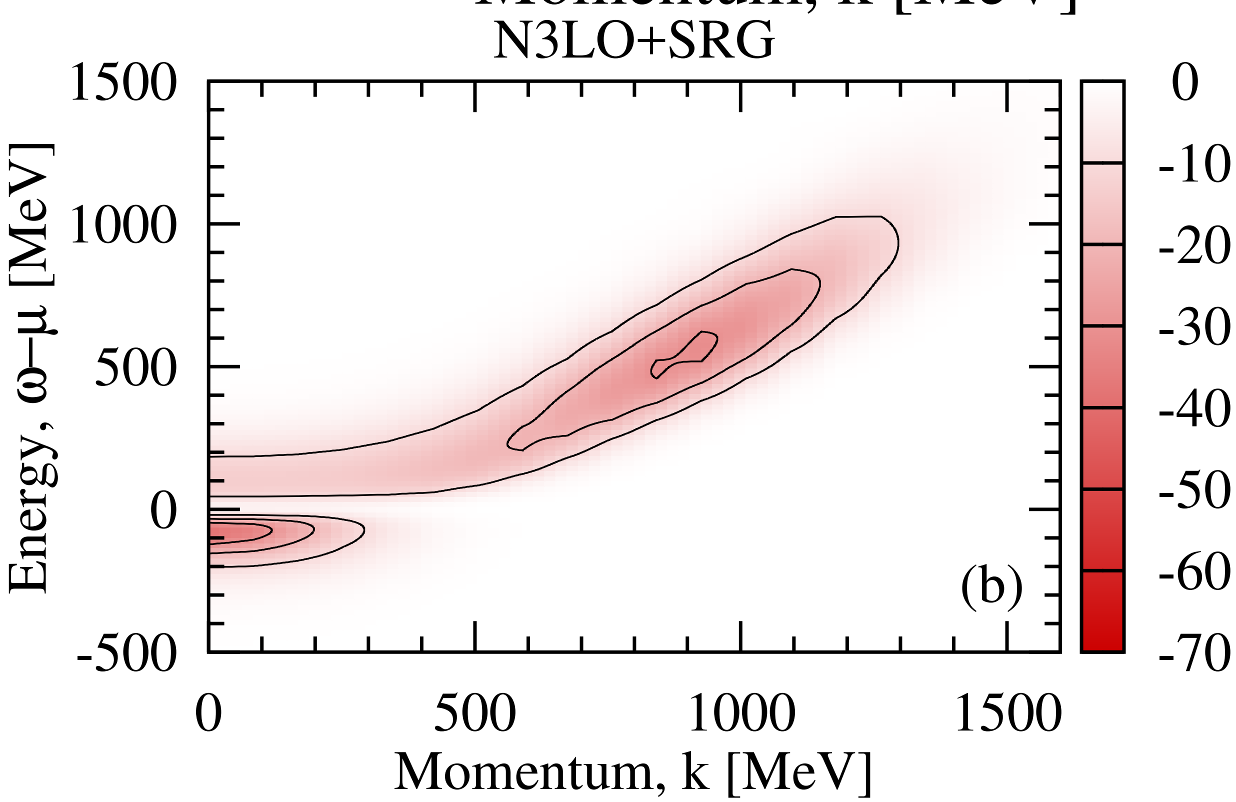
sf variance

$$\sigma_k^2 = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} [\omega - m_k^{(1)}]^2 \mathcal{A}_k(\omega)$$

$$\sigma_k^2 = -\int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \text{Im } \Sigma_k(\omega)$$

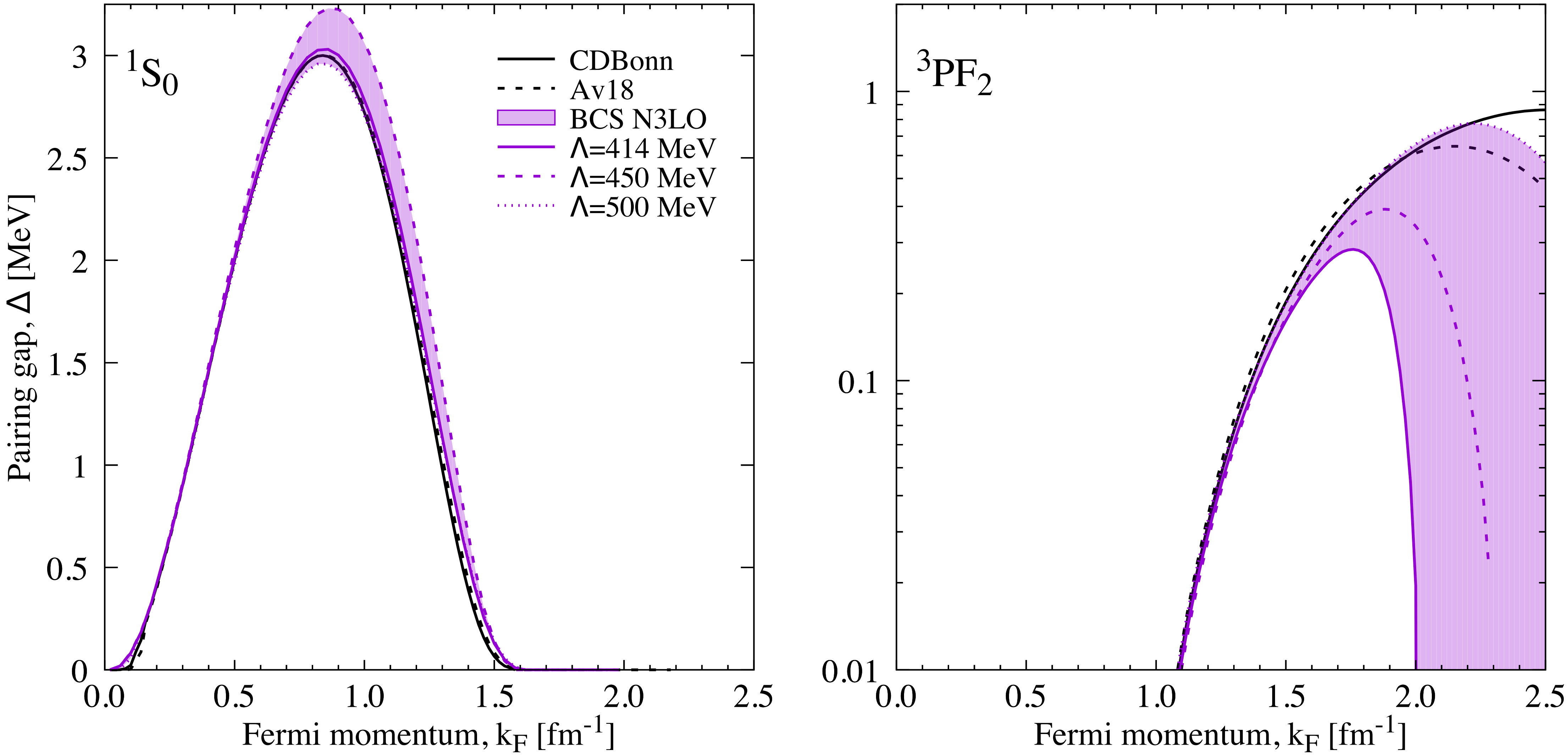






Bardeen-Cooper-Schrieffer pairing

UNIVERSITY OF SURREY

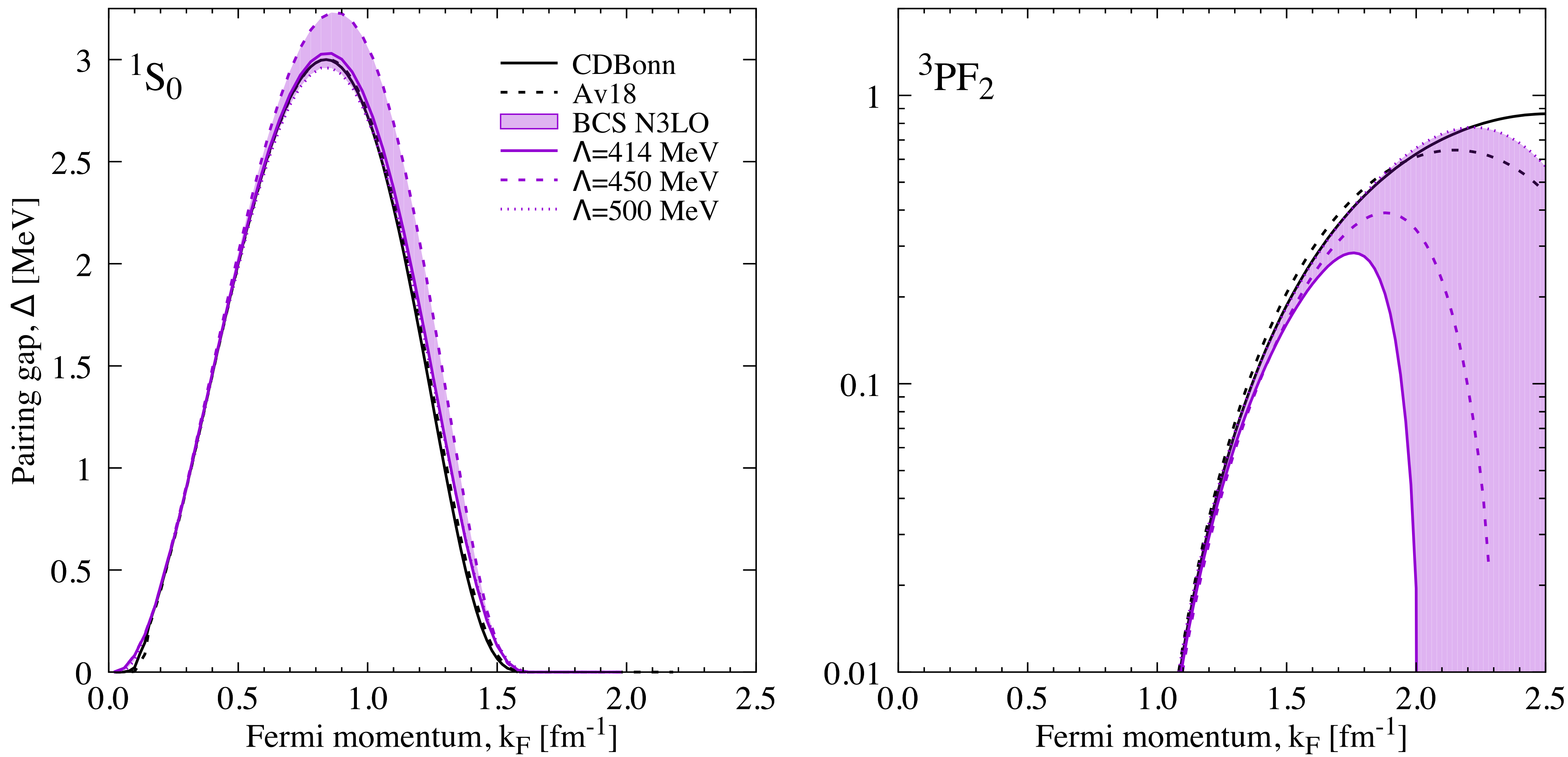


BCS equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \chi_k = \varepsilon_k - \mu$$

- Single-particle spectrum choice: $\varepsilon_k = \frac{k^2}{2m} + U(k) - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2$

Bardeen-Cooper-Schrieffer pairing

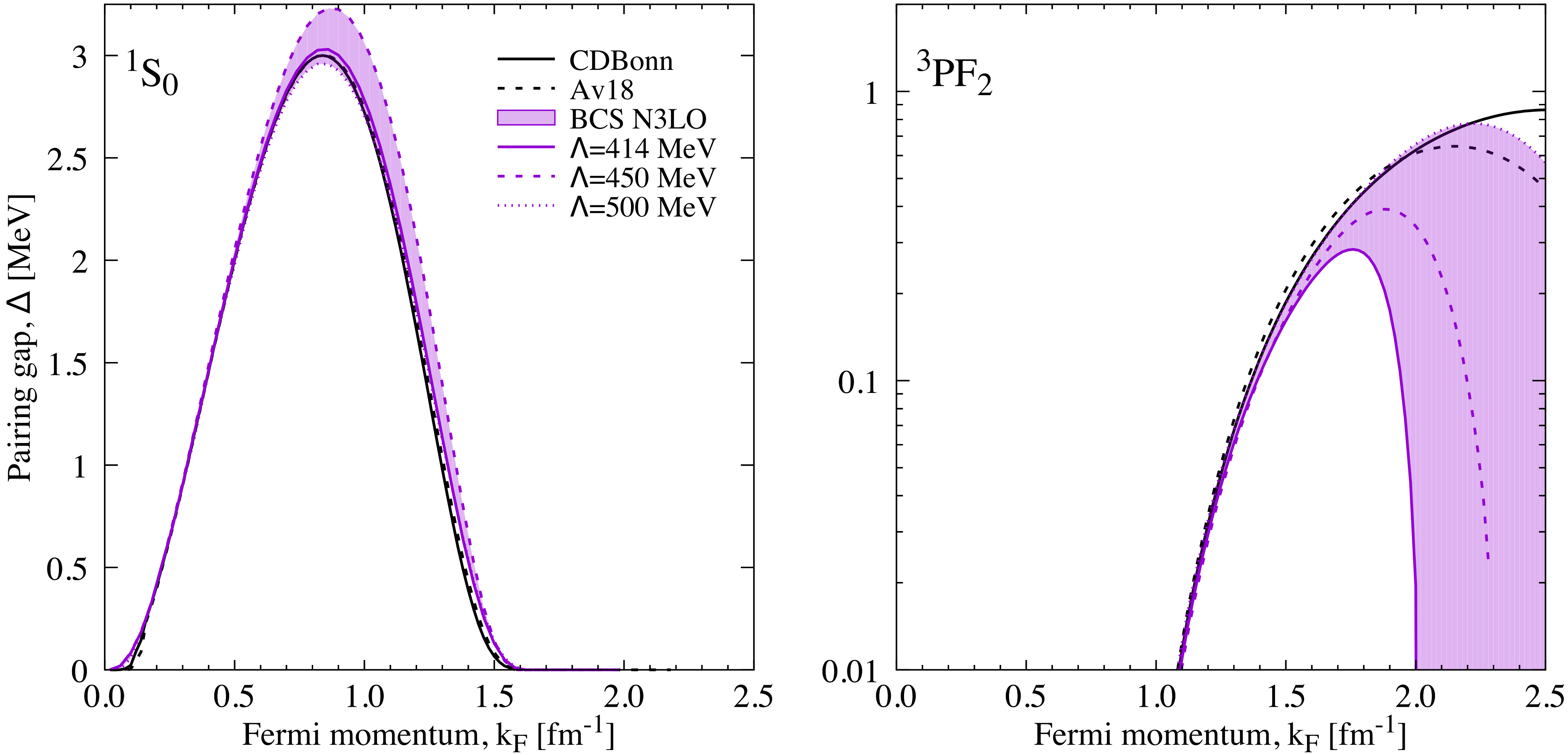


BCS equation

$$\boxed{\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \chi_k = \varepsilon_k - \mu}$$

- Single-particle spectrum choice: $\varepsilon_k = \frac{k^2}{2m} + \cancel{U(k)} - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2$

Bardeen-Cooper-Schrieffer pairing

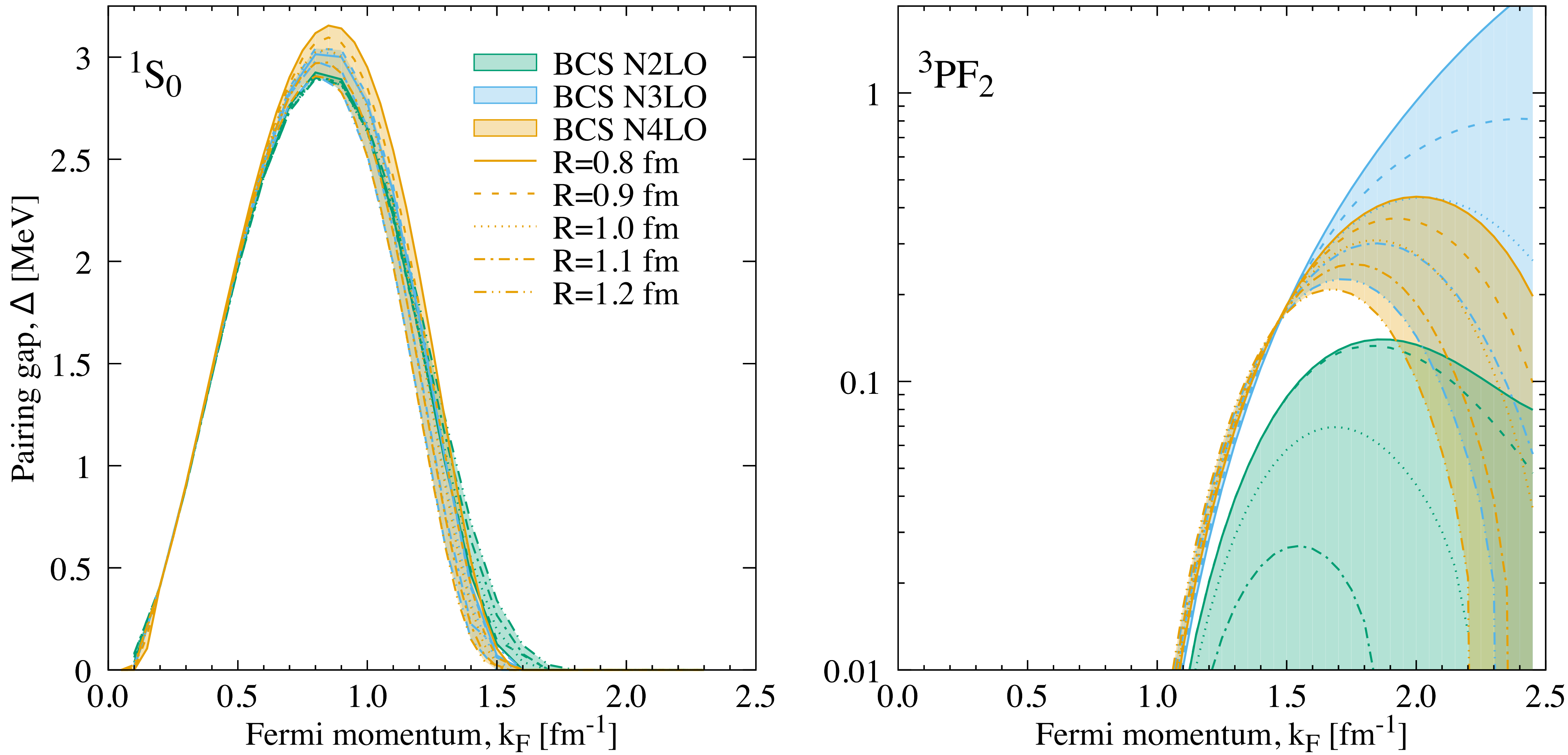


BCS equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \chi_k = \varepsilon_k - \mu$$

- Single-particle spectrum choice: $\varepsilon_k = \frac{k^2}{2m} + \cancel{U(k)} - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2 \approx |\Delta_k^L|^2$

Bardeen-Cooper-Schrieffer pairing



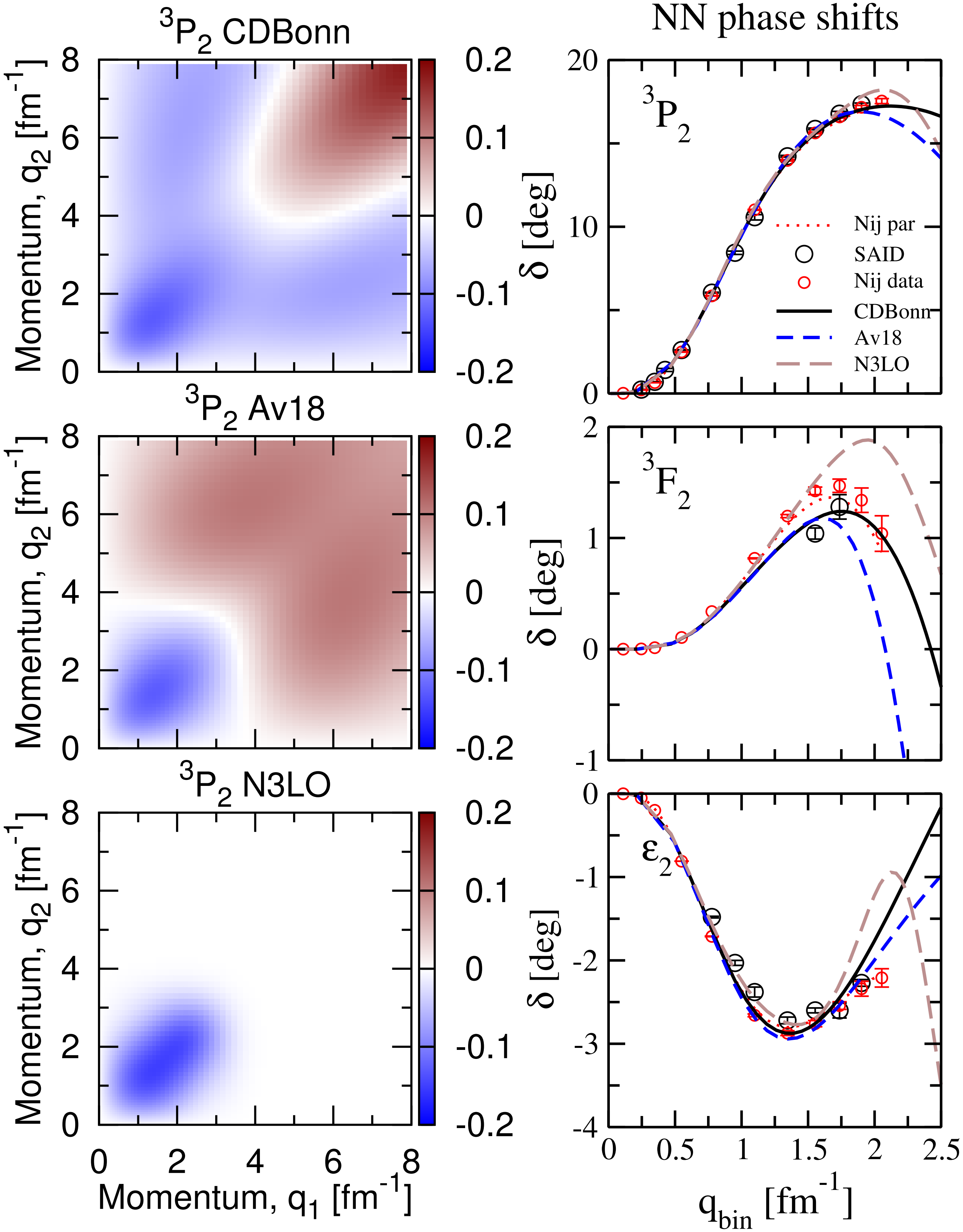
BCS equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \chi_k = \varepsilon_k - \mu$$

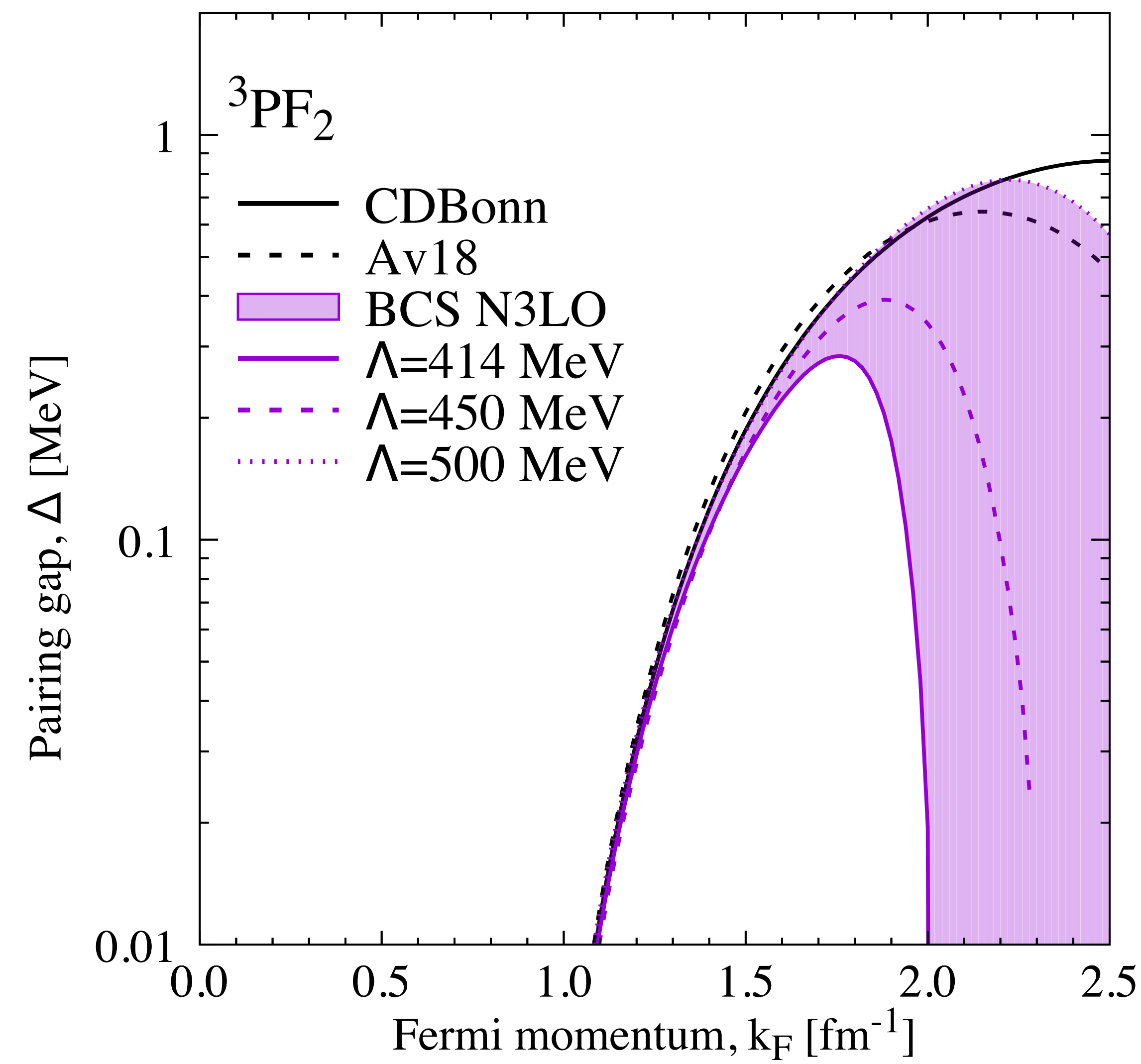
- Single-particle spectrum choice: $\varepsilon_k = \frac{k^2}{2m} + \cancel{U(k)} - \mu$
- Angular gap dependence: $|\Delta_k|^2 = \sum_L |\Delta_k^L|^2 \approx |\Delta_k^L|^2$

$^3PF_2$ pairing: phase shift equivalence

UNIVERSITY OF SURREY

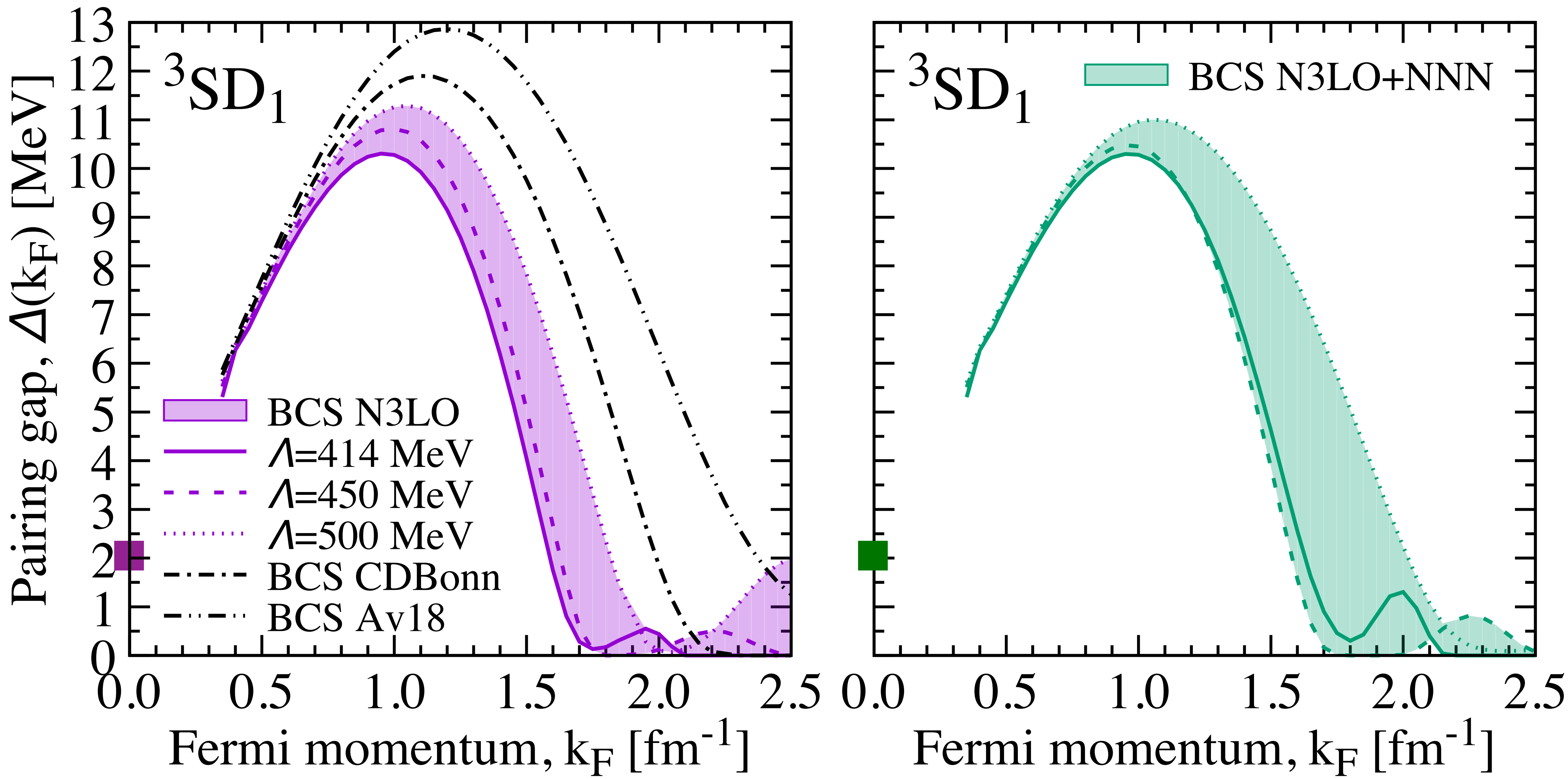


Neutron matter BCS gaps



Srinivias & Ramanan, *PRC* **94** 064303 (2016)

Beyond BCS 101: $^3SD_1$
Symmetric matter

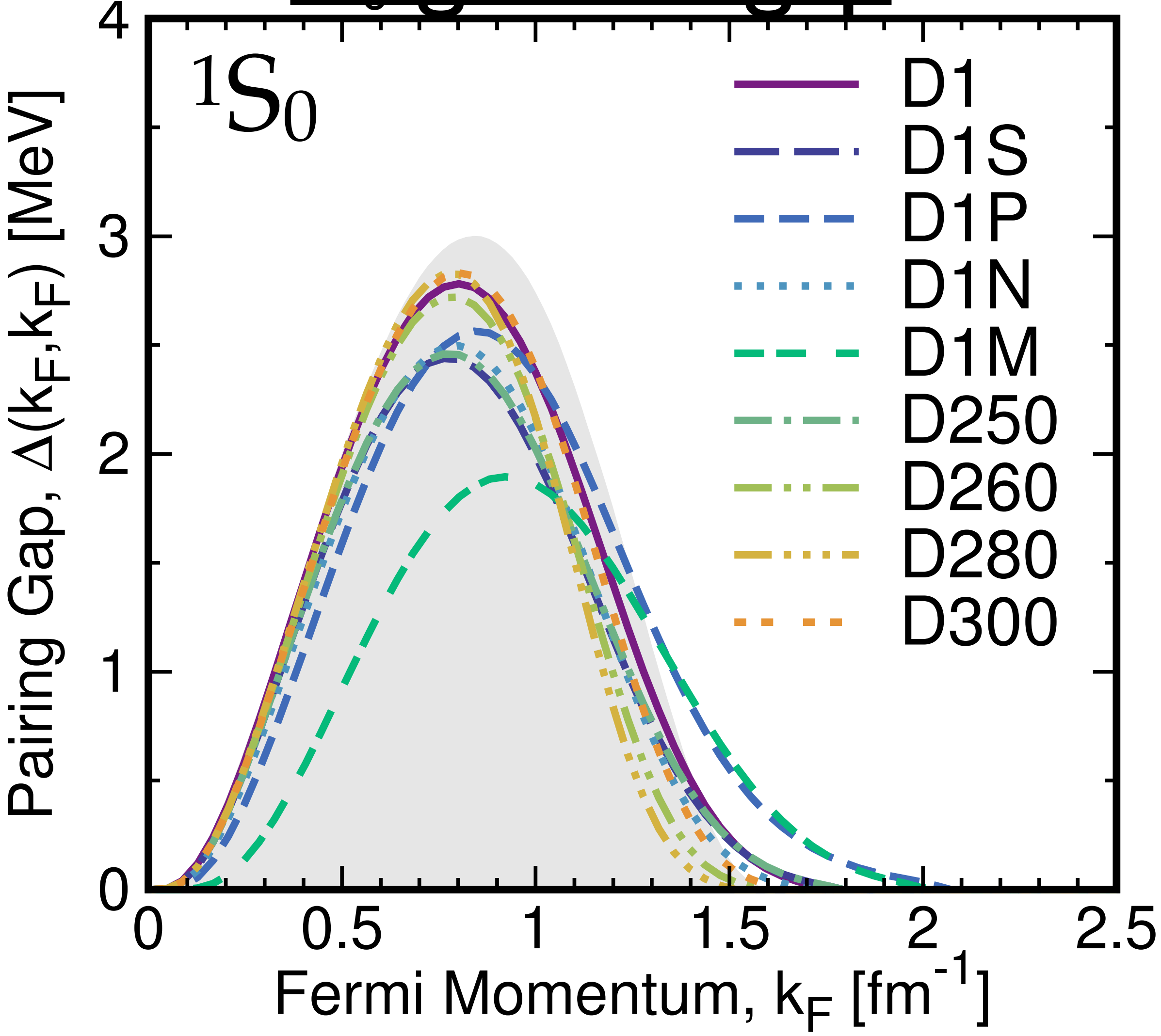


Muether & Dickhoff, *PRC* **72** 054313 (2005)
Rios, Polls & Dickhoff, arXiv:1707.04140
Maurizio, Holt & Finelli, *PRC* **90**, *044003* (2014)

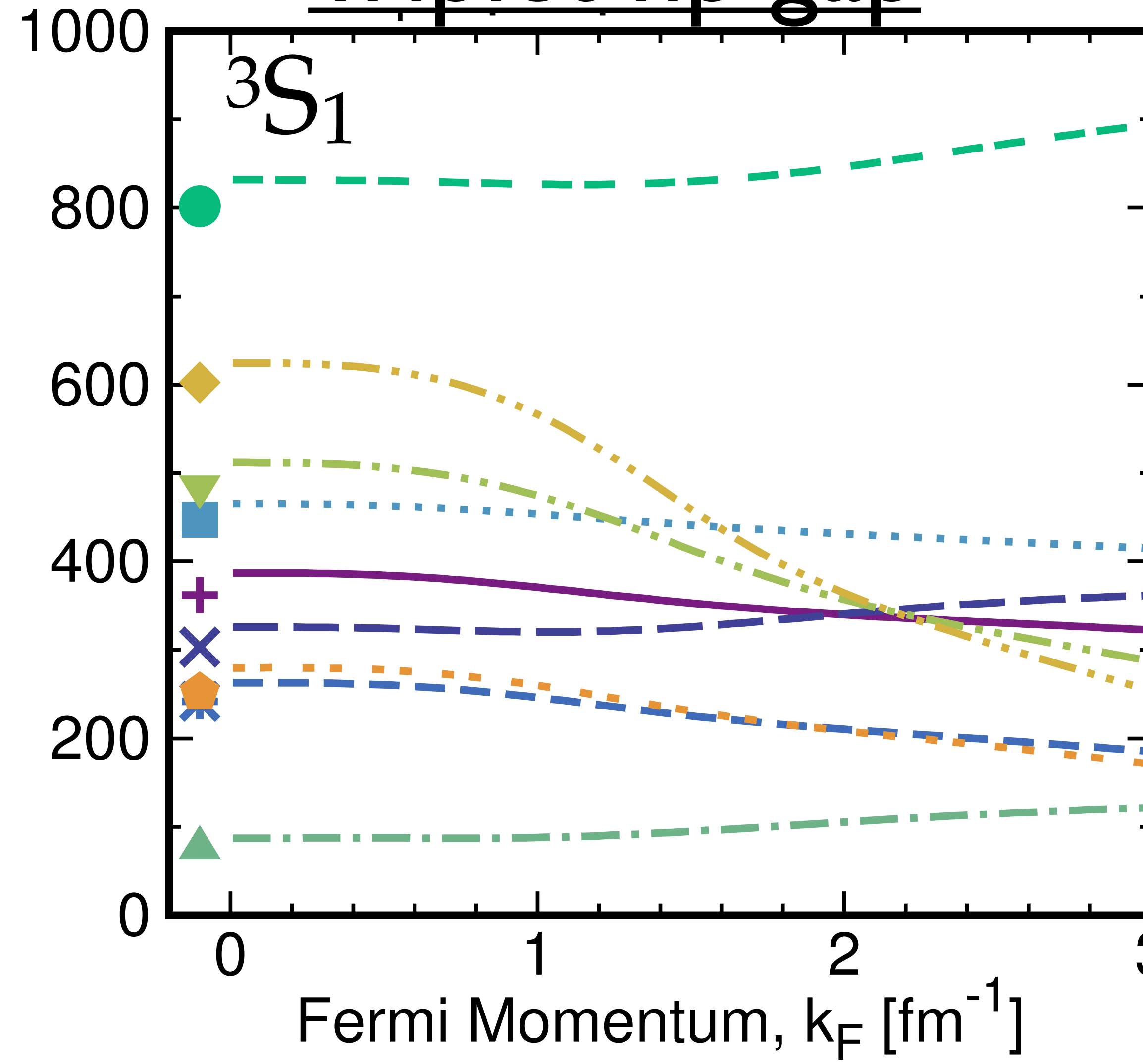
- **Massive** gaps $^3SD_1$ channel but…
- **No** evidence of strong *np* nuclear pairing
- 3NF do **not** alter picture **significantly**
- Short-range correlations **deplete** gap

BCS-pairing with Gogny

Singlet nn gap



Triplet np gap



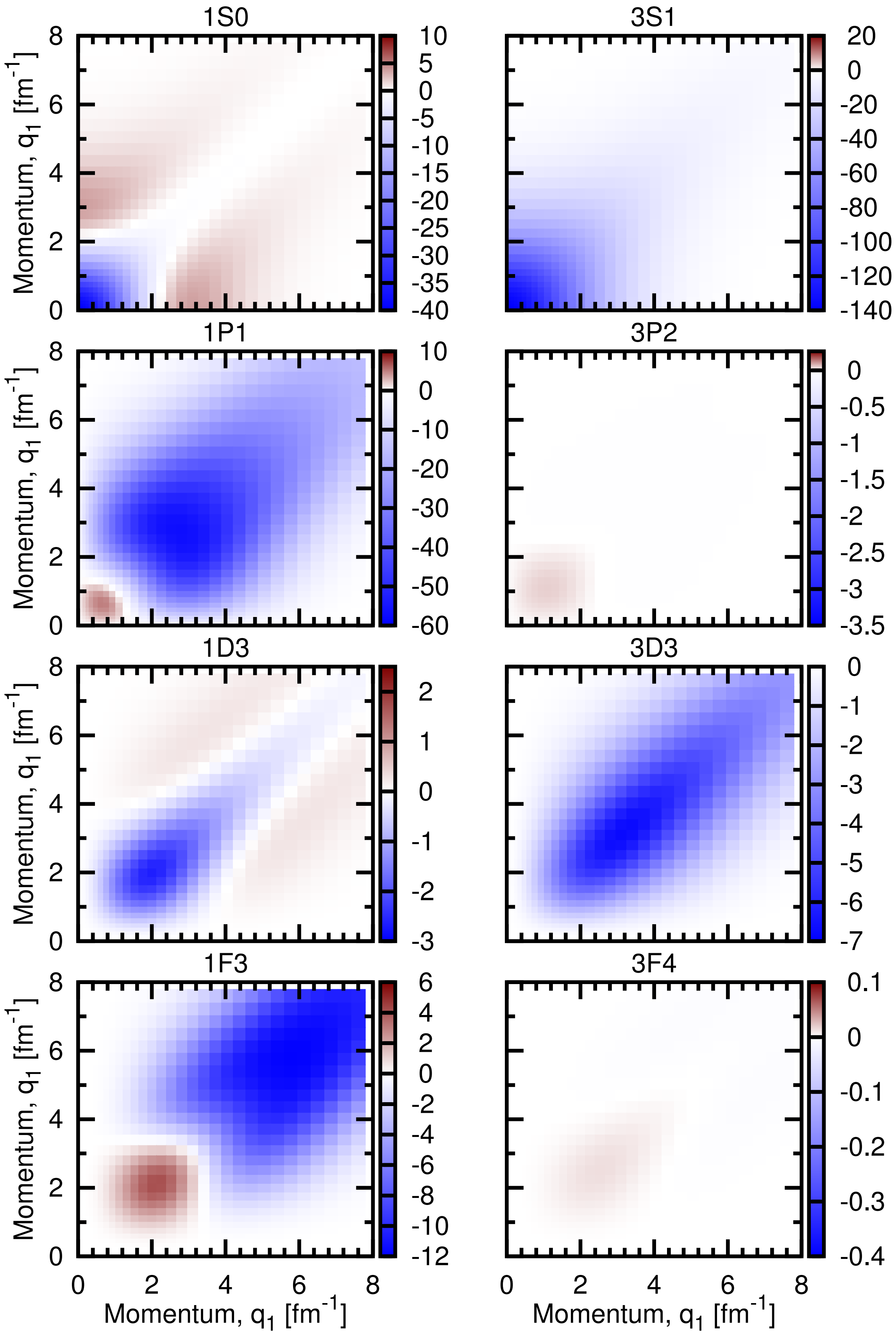
BCS equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad \mathbf{+} \quad \chi_k = \varepsilon_k - \mu$$

R. Sellahewa & A. Rios, *in preparation* + R. Sellahewa, *PhD thesis*

Matrix elements

D1S



$$V^{LST}(q,q') = \sum_i^2 \left[Z_i^{LS} + F_i^{ST} e^{\frac{-\mu_i^2\left(k^2+k'^2\right)}{4}} I_L\left(\frac{\mu_i^2 k k'}{2}\right)\right]$$

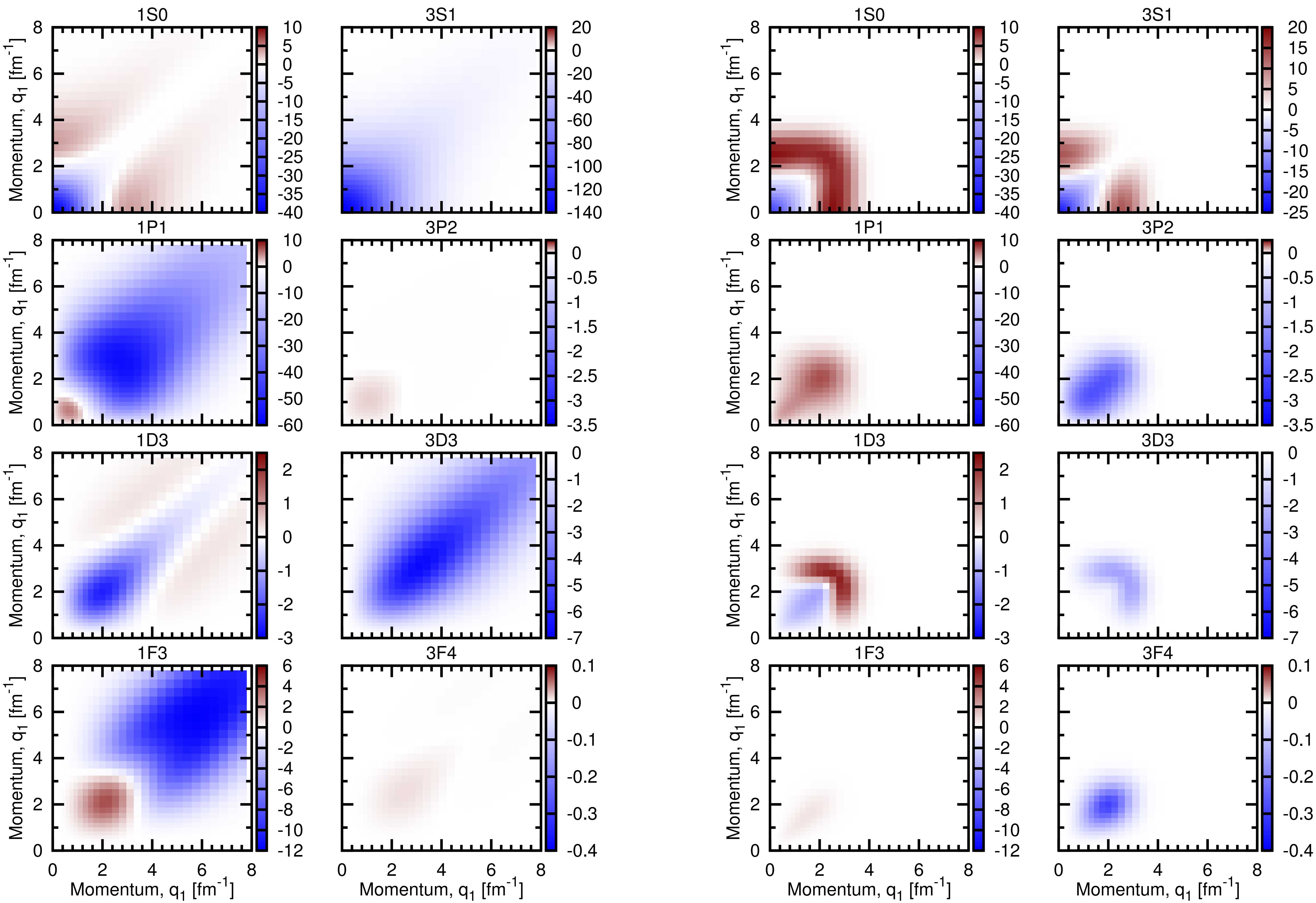
$$Z_i^{LS} = \frac{\delta_{L,0}}{4\pi^3}\left[t_0^i \rho^{\alpha_i}\left(1 - x_0^i\left(-\right)^S\right)\right]$$

$$F_i^{ST} = \frac{\mu_i^3}{2\pi^{\frac{1}{2}}}\left[W_i + B_i\left(-\right)^S - H_i\left(-\right)^T - M_i\left(-\right)^{S+T}\right]$$

Matrix elements

D1S

N3LO



Postdoc position opening imminently

+A. Carbone

+D. Ding, W. H. Dickhoff

+A. Polls

+C. Barbieri

+V. Somà

+H. Arellano, F. Isaule

TECHNISCHE UNIVERSITÄT DARMSTADT

Alexander von Humboldt Stiftung/Foundation

UNIVERSITAT DE BARCELONA

UNIVERSITY OF SURREY

cea Saclay

UNIVERSIDAD DE CHILE

comp star

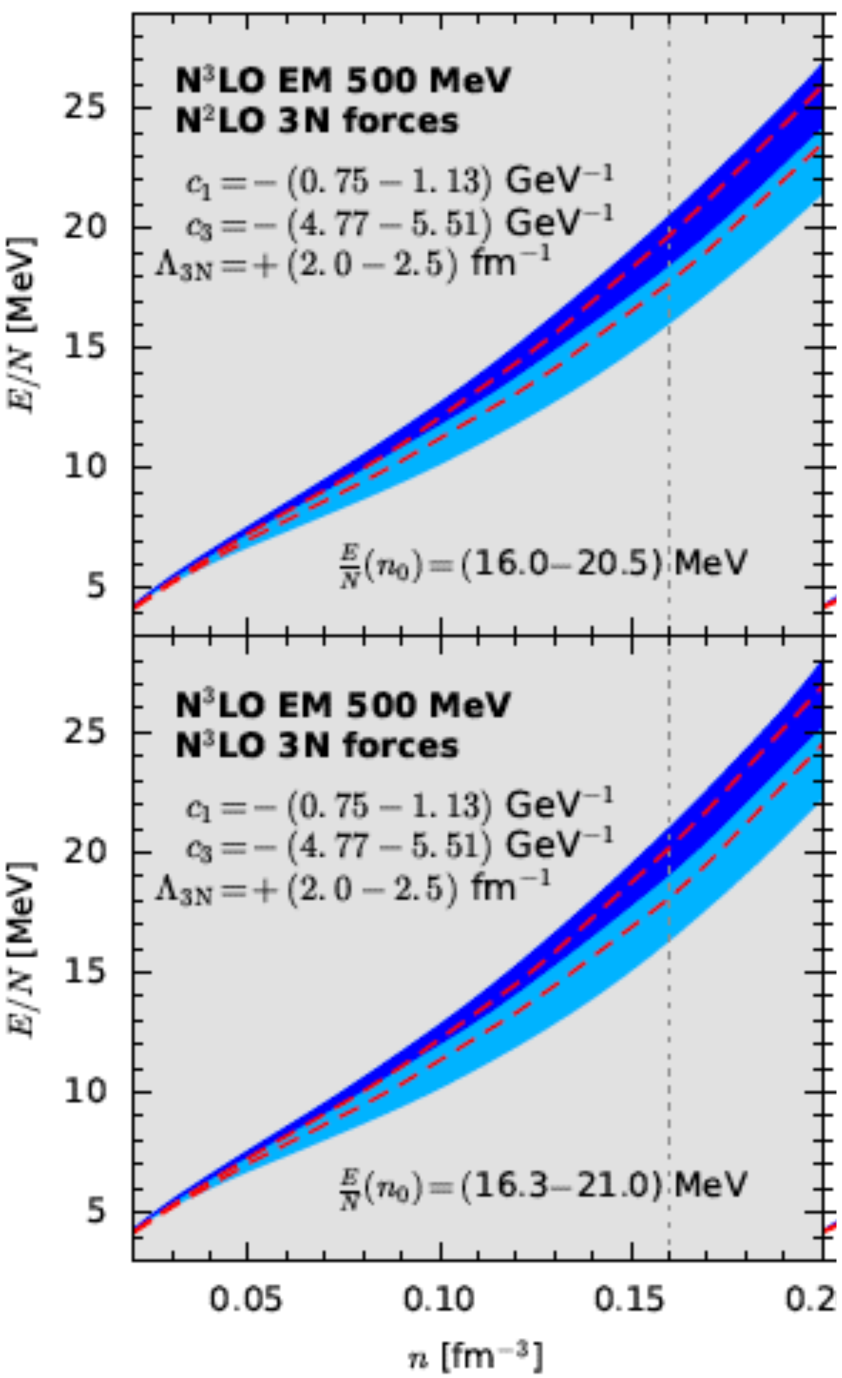
Science & Technology Facilities Council

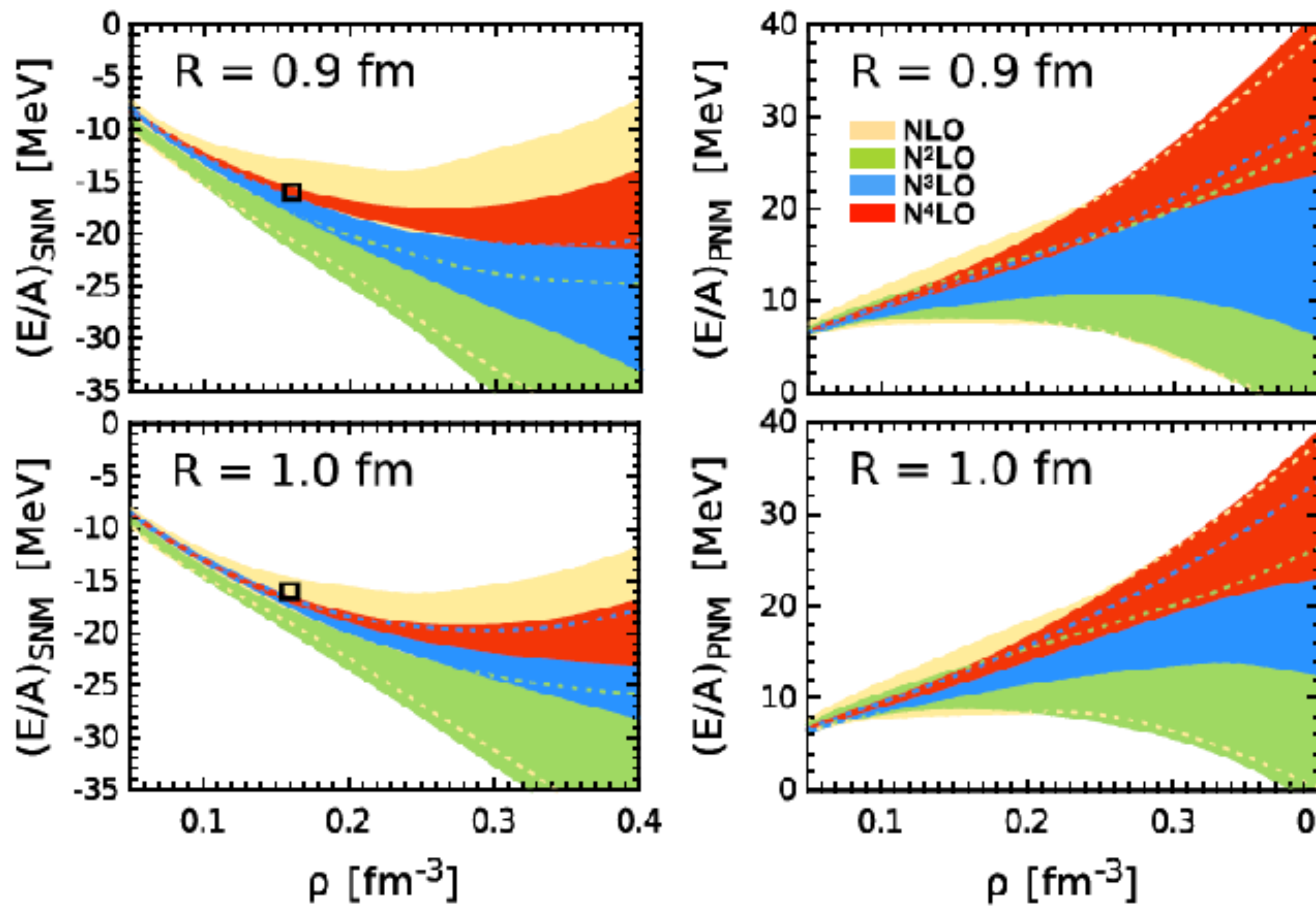
a.rios@surrey.ac.uk
@riosarnau

Open questions

Quantifiable uncertainties should be **propagated**

Not good to fit to a **single line** anymore!

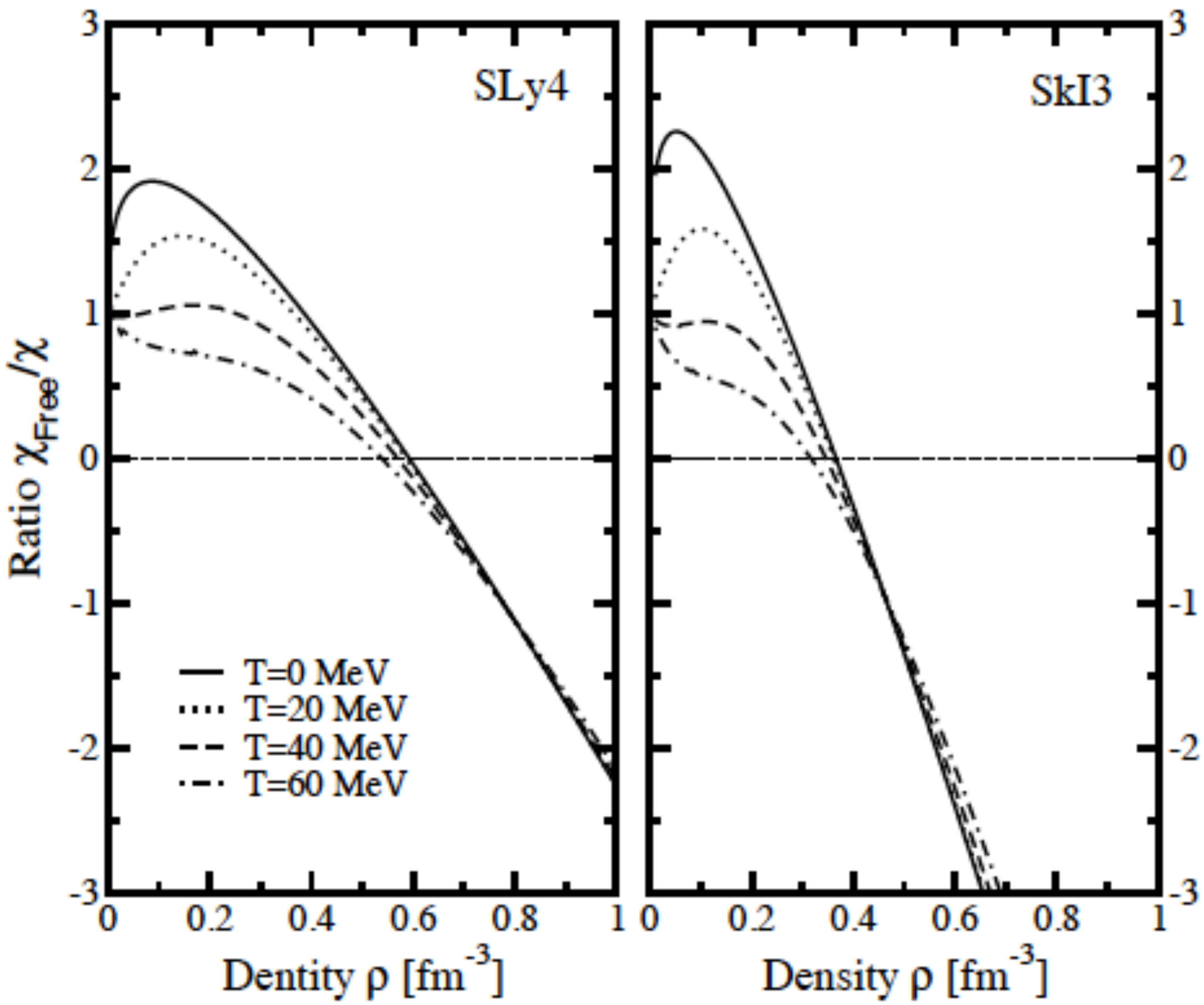




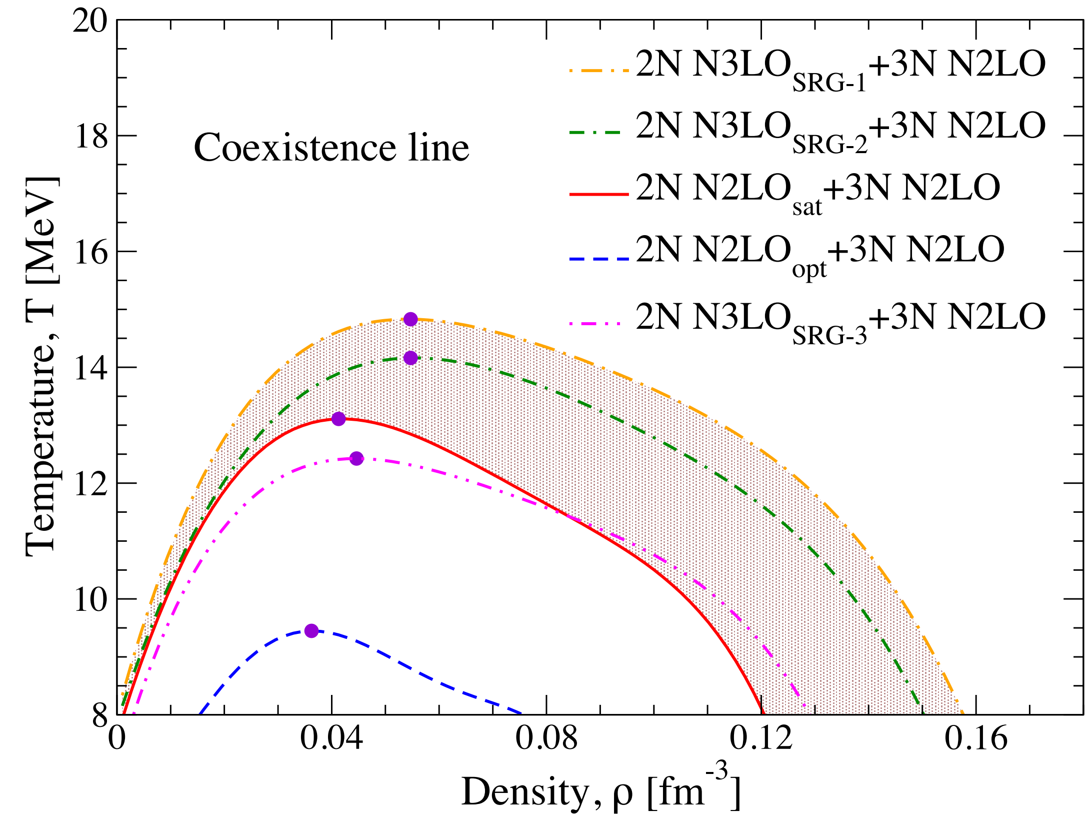
Drischler, **Carbone**, Hebeler, Schwenk *PRC* **94** 054307 (2016)
Hu, Zhang, Epelbaum, Meissner, Meng, *PRC* **96** 034307 (2017)

Open questions

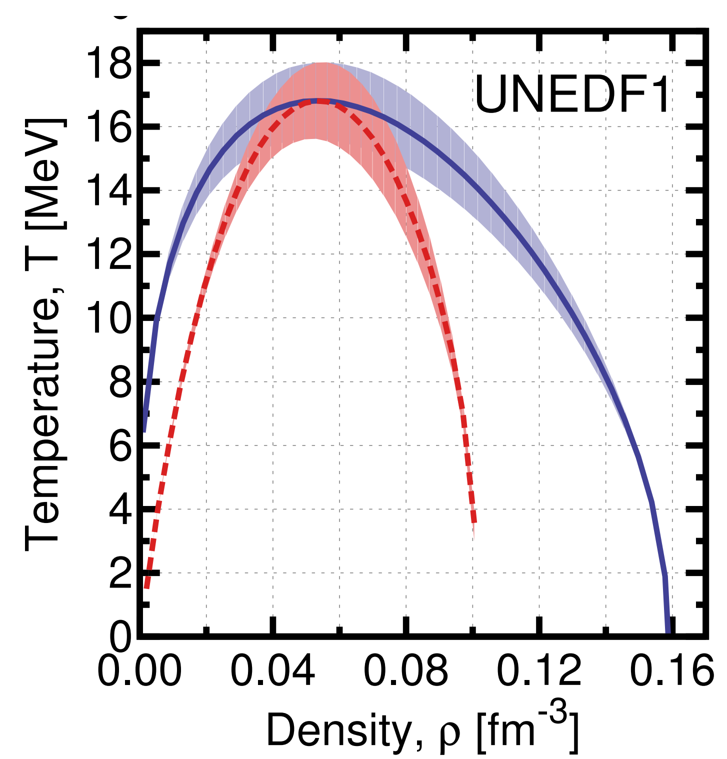
Polarzied matter & **finite temperature**



Rios, Vidana, Polls, PRC ***71*** *055802 (2005)*



Carbone, Rios, Polls, *work in progress*



Rios & Roca-Maza, *JPG* **42** 034005 (2015)

Open questions

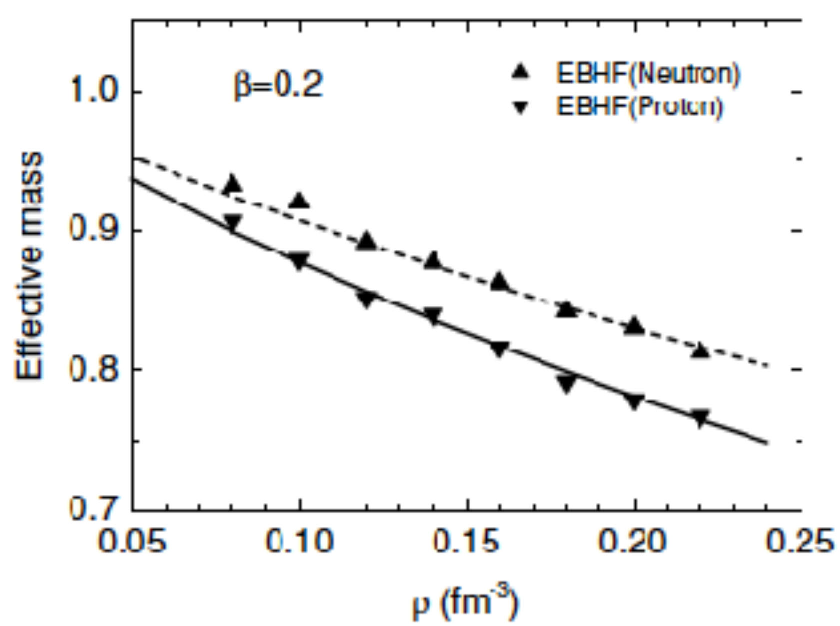
UNIVERSITY OF SURREY

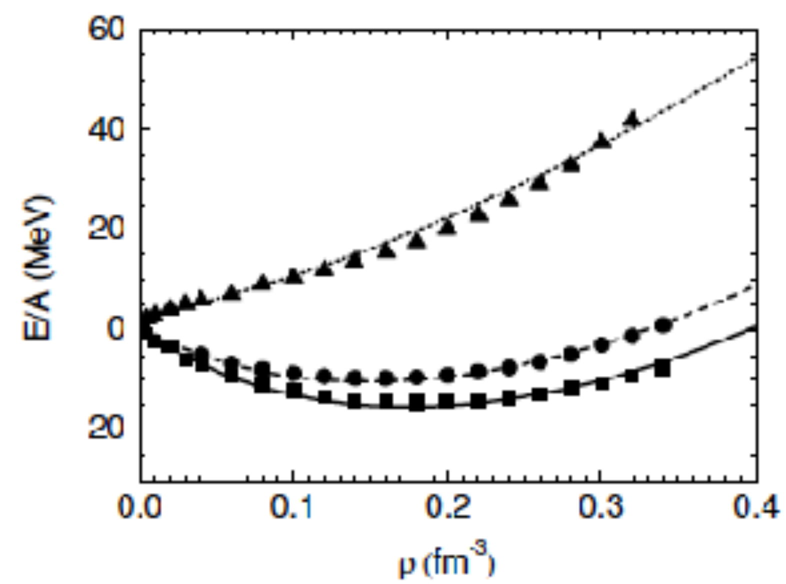
How can we get more **systematic** links?

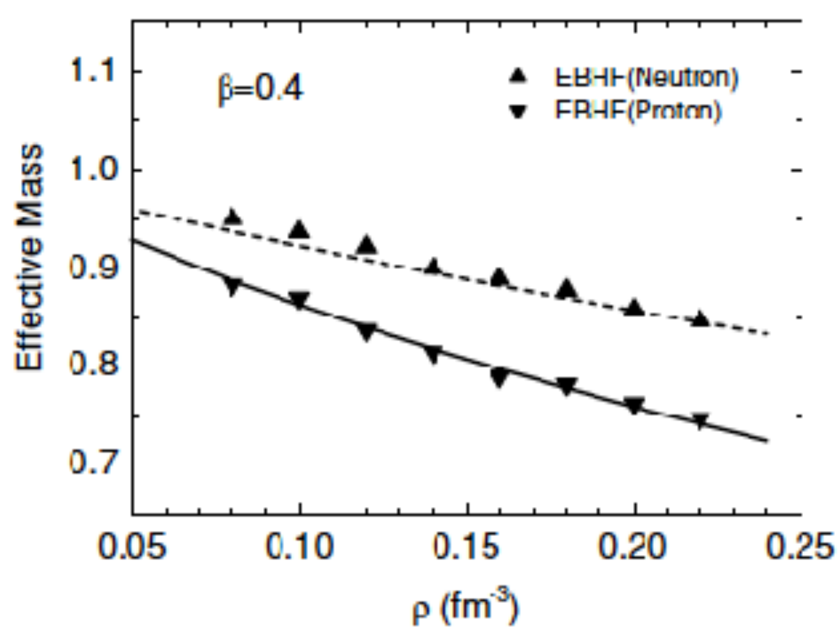
Is there a way to go beyond **subjective** choices?

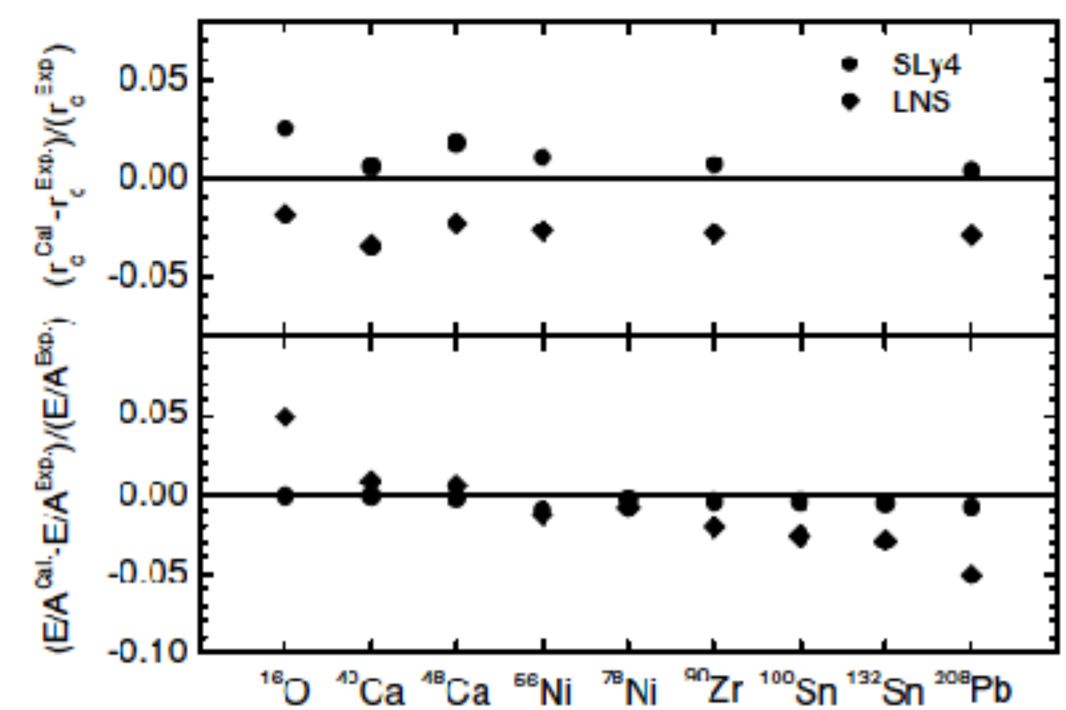
Landau parameters & response?

Skyrme LNS









Giai, Lombardo, Shen, Shui, *PRC* **73** 014313 (2006)