Skyrme N2LO functionals: first results on finite nuclei

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- N2LO/N3LO extensions : physical motivation
- Results in infinite matter
- Extension of Gogny interaction
- Application in astrophysics
- Application to finite nuclei: first results
- Conclusion

N2LO/N3LO extensions : physical motivation

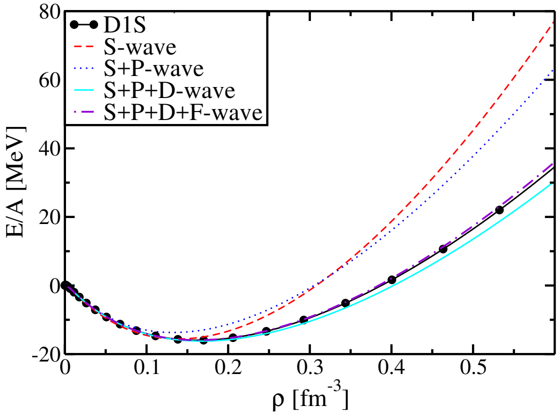
- Construction of new effective interactions necessary!
- Instabilities experienced with popular interactions (Skyrme, Gogny)

- Initial idea (Skyrme) : expansion in powers of momentum (k^2) $\rightarrow$ systematic expansion up to k^n ... which n???
- N2LO : $n = 2$; N3LO : $n = 3$; ...
- Gogny: e^{-r^2/μ^2}, M3Y : $e^{-\mu r}/\mu r$, ... : SAME kind of expansion

  [F. Raimondi et al., Phys.Rev. C84 (2011) 064303]

N2LO/N3LO extensions : physical motivation

Finite-range interaction D1S: infinite sum of partial waves.



Only S, P, D and F ($\ell < 4$) waves necessary → **N3LO good enough**

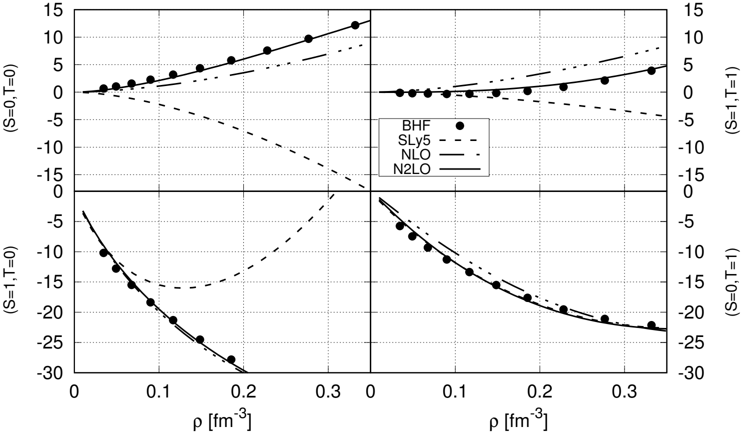
Skyrme pseudo-potential N2LO/N3LO

$$\begin{aligned}\mathcal{V}(\mathbf{r_1},\mathbf{r_2}) = \; & t_0\,(1+x_0\,P_\sigma) + \frac{1}{6}\,t_3\,(1+x_3 P_\sigma)\rho^\alpha(R) \\ & + \frac{1}{2}\,t_1\,(1+x_1\,P_\sigma)\left[\mathbf{k}'^2+\mathbf{k}^2\right] + t_2\,(1+x_2\,P_\sigma)\,\mathbf{k}'\cdot\mathbf{k} \quad \Big\}\ \text{Skyrme (N1LO)} \\ & + \frac{1}{4}\,t_1^{(4)}(1+x_1^{(4)}P_\sigma)\left[(\mathbf{k}^2+\mathbf{k}'^2)^2+4(\mathbf{k}'\cdot\mathbf{k})^2\right] \\ & + t_2^{(4)}(1+x_2^{(4)}P_\sigma)(\mathbf{k}'\cdot\mathbf{k})(\mathbf{k}^2+\mathbf{k}'^2) \quad \Big\}\ \text{Skyrme N2LO} \\ & + \frac{1}{2}\,t_1^{(6)}\left(1+x_1^{(6)}P_\sigma\right)(\mathbf{k}'^2+\mathbf{k}^2)\left[(\mathbf{k}'^2+\mathbf{k}^2)^2+12(\mathbf{k}'\cdot\mathbf{k})^2\right] \\ & + t_2^{(6)}\left(1+x_2^{(6)}P_\sigma\right)(\mathbf{k}'\cdot\mathbf{k})\left[3(\mathbf{k}'^2+\mathbf{k}^2)^2+4(\mathbf{k}'\cdot\mathbf{k})^2\right] \quad \Big\}\ \text{Skyrme N3LO}\end{aligned}$$

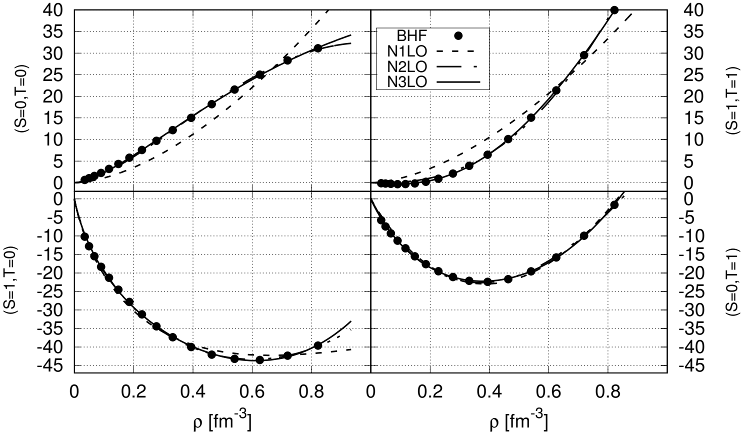
- D and F partial waves included
- Gauge invariance
- Also includes:
  - spin-orbit term W_0
  - tensor terms

Infinite matter: (S,T) channels N2LO

- Used as a preliminary test before dealing with finite nuclei
- First step: (S,T) channels
- Results compared to BHF calculations from Baldo *and al.* (1997)

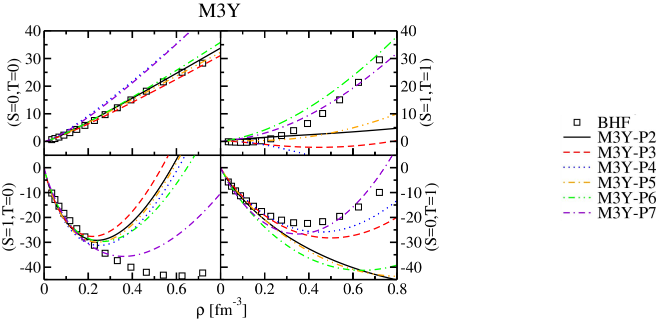


Infinite matter: (S,T) channels N3LO



- Agreement up to $\rho = 0.8$ fm^{-3}
- Exploration of a new parameter space

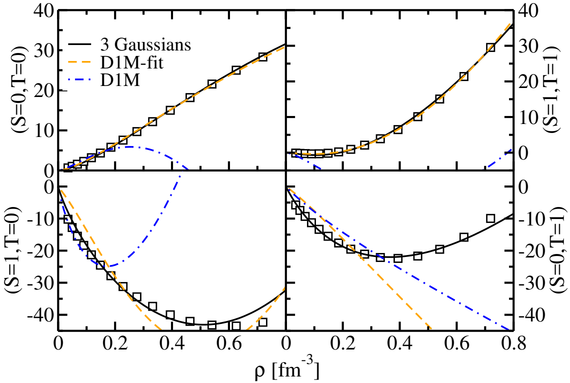
Infinite matter: (S,T) channels M3Y



- M3Y takes into account nuclei **and** (S,T) channels: both are not incompatibles

Infinite matter: (S,T) channels Gogny

Not possible...

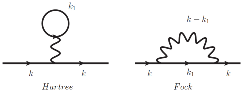


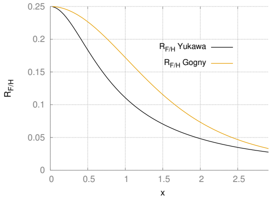
... except with a third gaussian

Determination of the three ranges

Physical meaning of a range :

- Yukawa potential: related to masses (770, 490, 140 MeV)
- Gaussian potential??? → definition via the self-energy

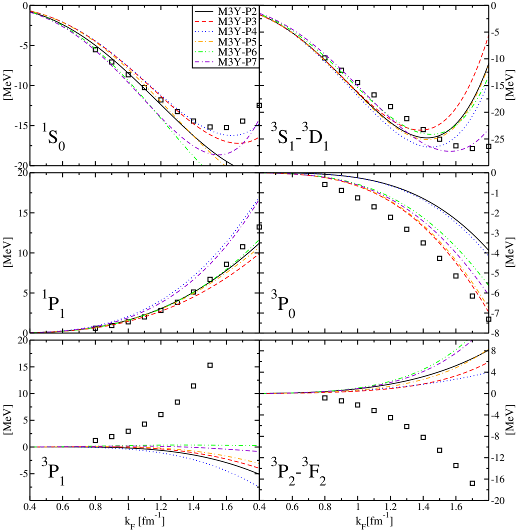




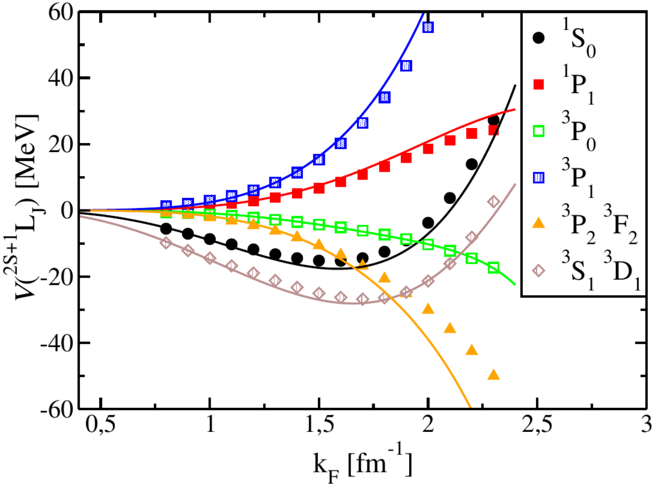
Example:
m_ρ = 770 MeV
$\rightarrow \mu_Y^{-1}$=0.256 fm
$\rightarrow R(\mu_Y^{-1})$=0.228 =$R(\mu_G)$
$\rightarrow \mu_G$ = 0.475 fm

$\rightarrow$ ranges: μ_1 = 0.475 fm, μ_2 = 0.746 fm , μ_3 = 1.964 fm

Partial waves $^{2S+1}L_J$ with M3Y



Partial waves with Skyrme N3LO



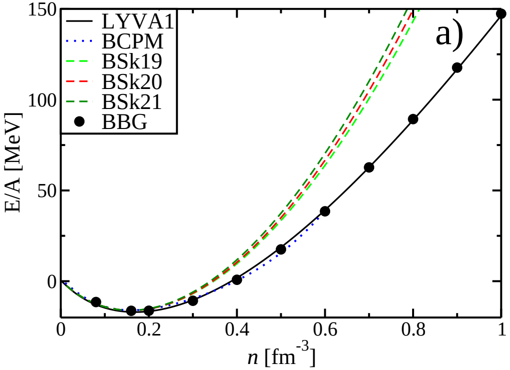
High degree of flexibility!

Application to astrophysics

Goal: only one parametrisation for all quantities

BHF calculations as reference

LYVA1 = N3LO Skyrme parametrisation for astrophysics



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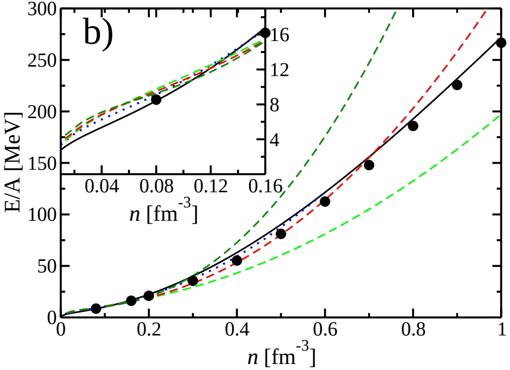
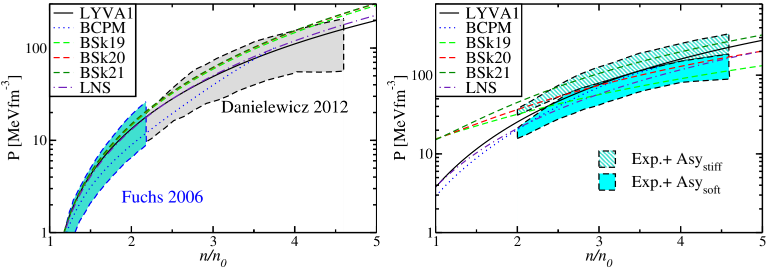


Application to astrophysics

$$P = n^2 \frac{\partial(E/A)}{\partial n}$$



Experimental constraints satisfied by LYVA1

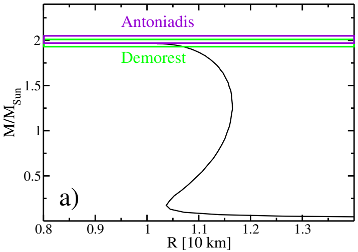
Application to astrophysics

LYVA1 also reproduces :

- INM properties
- Symmetry energy
- Causality
- Effective masses splitting
- ...

LYVA1 compatible with a neutron star of 2 $M_\odot$. Here, we have M=1.96 $M_\odot$.

TOV equations solved



Finite nuclei: N2LO mean-field equation

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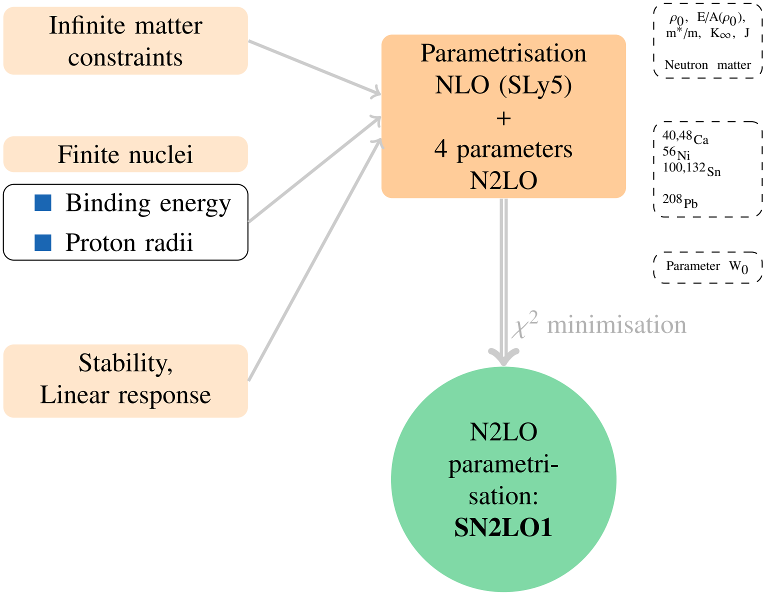
$$
\begin{aligned}
\epsilon\, R &= A_4 R^{(4)} + A_3 R^{(3)} + A_{2R} R^{(2)} + A_{1R} R' + A_{0R} R \\
&+ \frac{\ell(\ell+1)}{r^2}\left[A_{2C} R^{(2)} + A_{1C} R' + A_{0C} R + \frac{\ell(\ell+1)}{r^2} A_{0CC} R\right] \\
&+ C_{jls}\left[W_{2R} R^{(2)} + W_{1R} R' + W_{0R} R + \frac{\ell(\ell+1)}{r^2} W_{0C} R\right]
\end{aligned}
$$

- 14 parameters, **4 new**
- New spin-orbit contributions: W_{2R}, W_{1R}, $\frac{\ell(\ell+1)}{r^2} W_{0C}$
- No particular behavior at origin (same as Skyrme)
- New term: $\left(\frac{\ell(\ell+1)}{r^2}\right)^2$ (possible applications)

Fitting protocol



First results: infinite matter

	SN2LO1	SLy5*	Constraint	Error	Result
ρ_0 [fm^{-3}]	0.162	0.161	0.16	0.02	✓
E/A(ρ_0) [MeV]	-15.95	-16.02	-16.0	0.1	✓
K_∞ [MeV]	221.9	229.8	230	20	✓
m^*/m	0.709	0.696	0.7	0.02	✓
J [MeV]	31.95	32.03	32.01	2.0	✓

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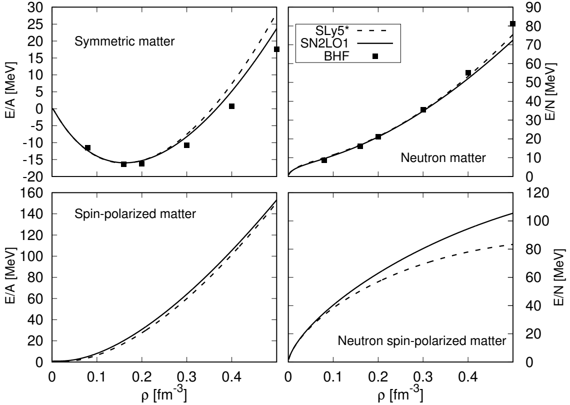
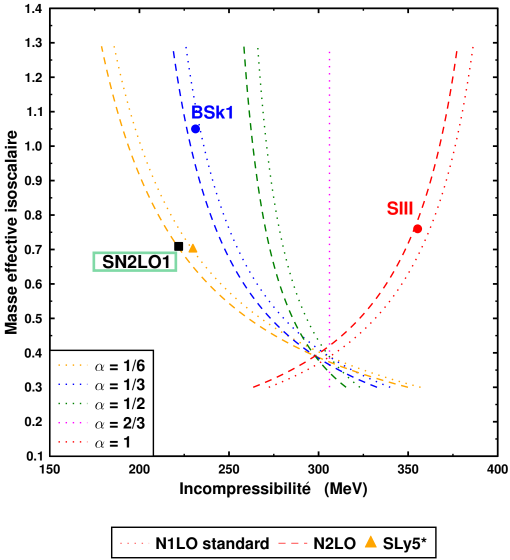
Equation of state of SN2LO1



N2LO interaction close to ab initio calculations

No ferro-magnetic transition

Correlation incompressibility/effective mass

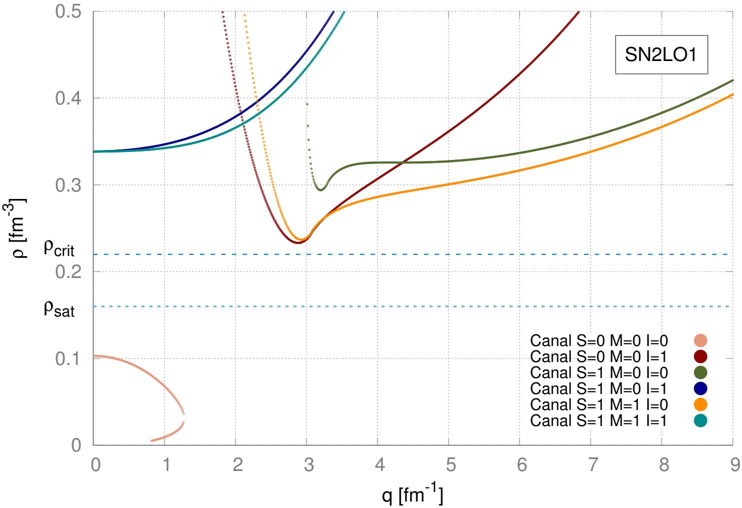


$$K_\infty = A\frac{\mathbf{m}}{\mathbf{m}^*} + \mathrm{B}$$

where

$$\mathrm{B} = B_{N1LO} + B_{N2LO}$$

Stability SN2LO1

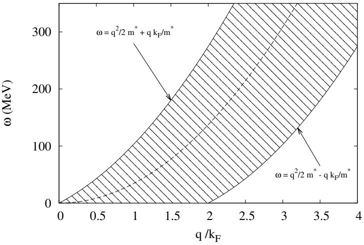


$\rho_{crit} = 1{,}4\ \rho_{sat}$

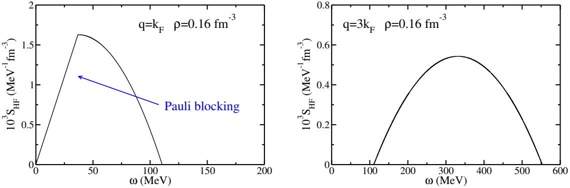
Interaction N2LO stable

Response function - Formalism

Domain for allowed excitations :



Response function of a free Fermi gas (at zero temperature) :



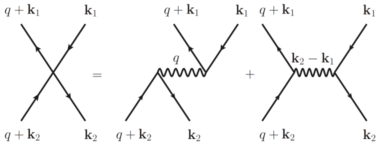
Response function - Formalism

Response function of an interacting gas of nucleons....
The RPA propagator is the solution of Bethe-Salpeter equation :

$$
\begin{aligned}
G^{(\mathrm{SMI})}_{RPA}(q,\omega,\mathbf{k}_1) \quad &= \quad G_{HF}(q,\omega,\mathbf{k}_1) \\
&+ G_{HF}(q,\omega,\mathbf{k}_1) \sum_{(\mathrm{S'M'I'})} \int \frac{d^3k_2}{(2\pi)^3} V^{(\mathrm{SMI;S'M'I'})}_{ph}(q,\mathbf{k}_1,\mathbf{k}_2) G^{(\mathrm{S'M'I'})}_{RPA}(q,\omega,\mathbf{k}_2)
\end{aligned}
$$

with : $V^{(\alpha,\alpha')}_{ph}(q,\mathbf{k}_1,\mathbf{k}_2) = \langle \mathbf{q}+\mathbf{k}_1, \mathbf{k}_1^{-1}, (\alpha) | V_{eff} | \mathbf{q}+\mathbf{k}_2, \mathbf{k}_2^{-1}, (\alpha') \rangle$

Excitation : $\sum_j \exp^{i\mathbf{qr}} \Theta^j_\alpha \quad \Theta^j_\alpha = 1, \sigma^j, \hat{\tau}^j, \sigma^j \hat{\tau}^j$



Response function - Formalism

Consider the residual interaction in the simplest case :

$$V_{ph}^{(\alpha,\alpha')}(\mathbf{k}_1,\mathbf{k}_2) = \delta(\alpha,\alpha')V_{ph}^{(\alpha)}(q,\omega)$$

$\rightarrow$ response function :

$$\chi_{RPA}^{(\alpha)}(q,\omega) = \frac{\chi_{HF}(q,\omega)}{1 - V_{ph}^{(\alpha)}(q,\omega)\chi_{HF}(q,\omega)}$$

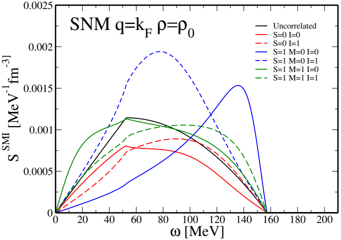
$\rightarrow \mathrm{Im}\chi_{RPA}(q,\omega) \propto \mathrm{Im}\chi_{HF}(q,\omega)$: same domain of definition as the free Fermi gas

$\rightarrow$ collective mode $1 - V_{ph}^{(\alpha)}\chi_{HF} = 0$ when $\mathrm{Im}\chi_{HF}(q,\omega) = 0$! (outside of the shaded domain!)

Response function - Formalism

Effective interaction : Skyrme

$$
\begin{aligned}
V_{\text{eff}} &= t_0\left(1+x_0\hat{P}_\sigma\right)+t_3\left(1+x_3\hat{P}_\sigma\right)\rho_0^\alpha && \text{local}\\
&+\tfrac{1}{2}t_1\left(1+x_1\hat{P}_\sigma\right)\left(\mathbf{k}'^2+\mathbf{k}^2\right)+t_2\left(1+x_2\hat{P}_\sigma\right)\mathbf{k}'\cdot\mathbf{k} && \text{non local}\\
&+\mathrm{i}\,W_0\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}'\times\mathbf{k}) && \text{spin-orbit}\\
&+\tfrac{1}{2}t_e\left\{\left[3\,(\boldsymbol{\sigma}_1\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k}')-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,\mathbf{k}'^2\right]+\text{h.c.}\right\} && \text{tensor}\\
&+\tfrac{1}{2}t_o\{[3\,(\boldsymbol{\sigma}_1\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k})-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,\mathbf{k}'\cdot\mathbf{k}]+\text{h.c.}\}
\end{aligned}
$$

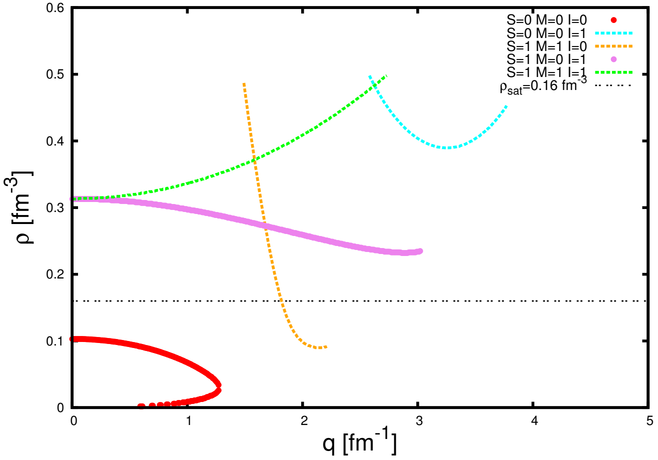


Calculations done : Skyrme/Landau residual interaction, SNM, ASM, PNM, zero and finite temperature

Instabilities

An instability in the functional causes an infinity in the response function :

$$1/\chi^{\mathrm{SMI}}(\omega = 0, q) = 0$$



Instabilities in the different spin/isospin channels (S,M,I) for T22.

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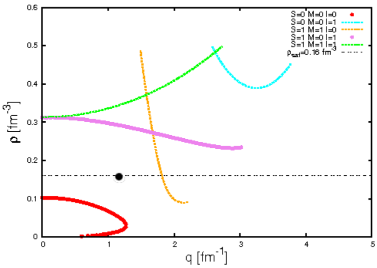

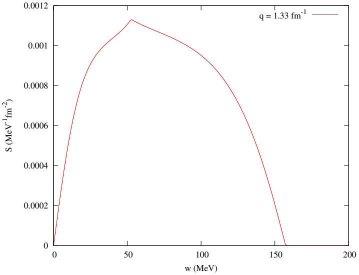

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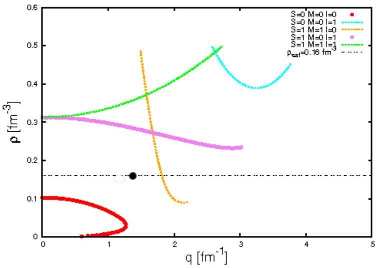

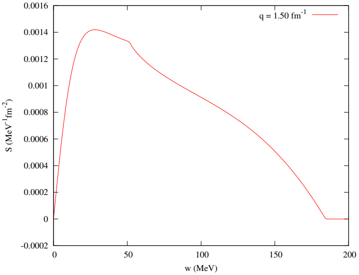

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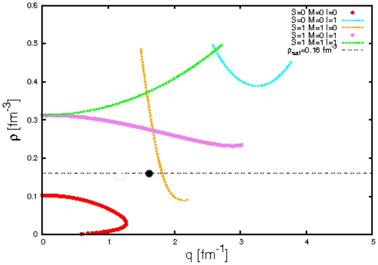

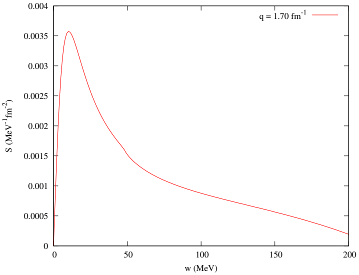

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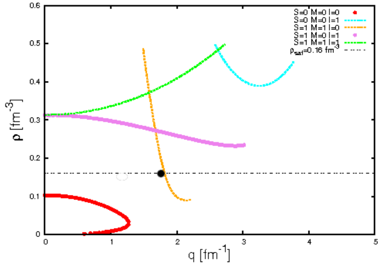

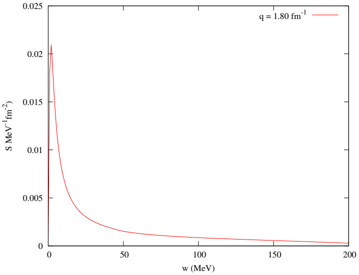

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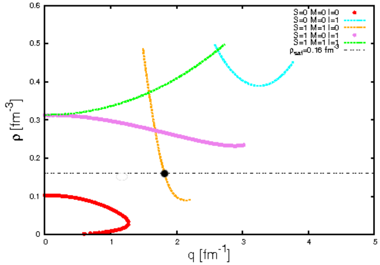
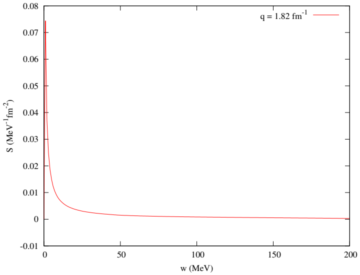
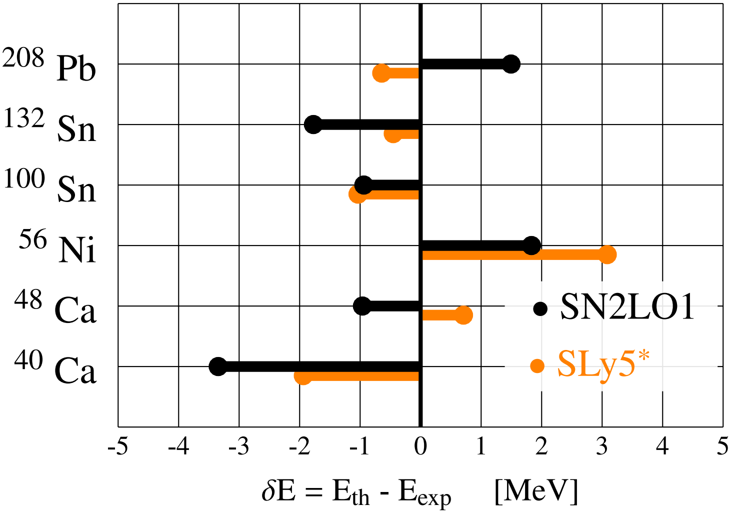


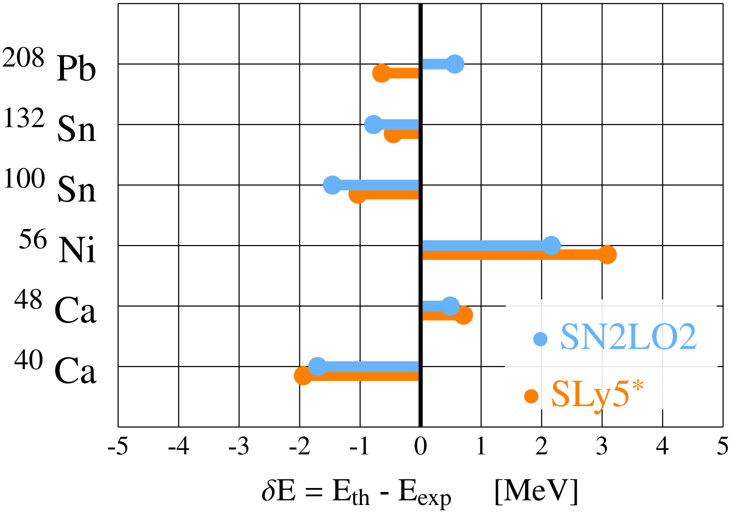


Instabilities in the different spin/isospin channels (S,M,I) for T22.

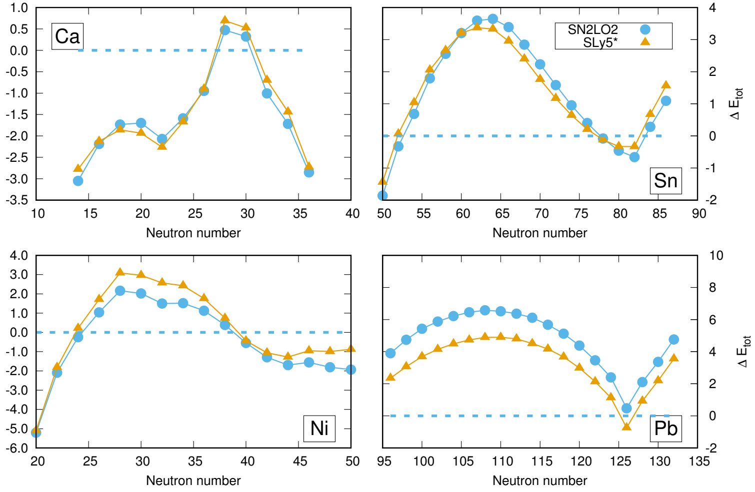
Binding energies SN2LO1



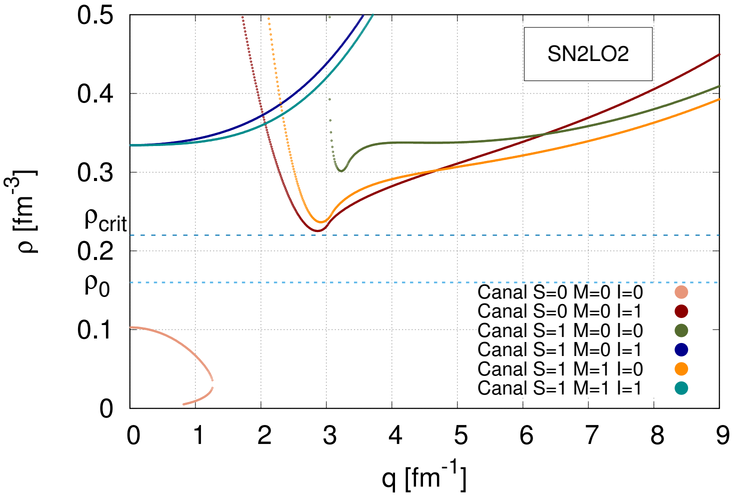
Binding energies SN2LO2



Isotopes with SN2LO2



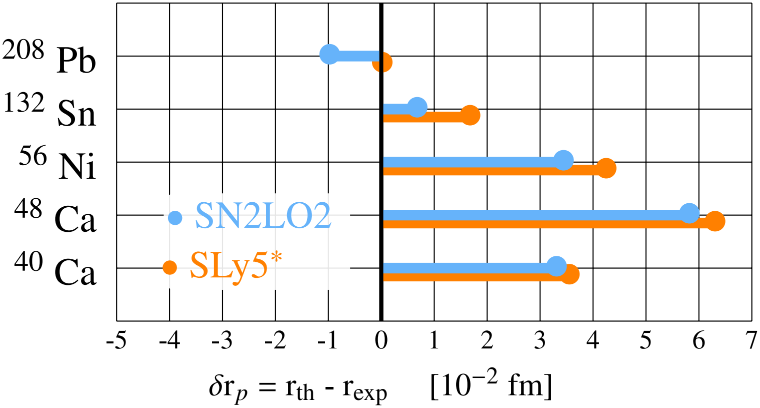
Stability SN2LO2



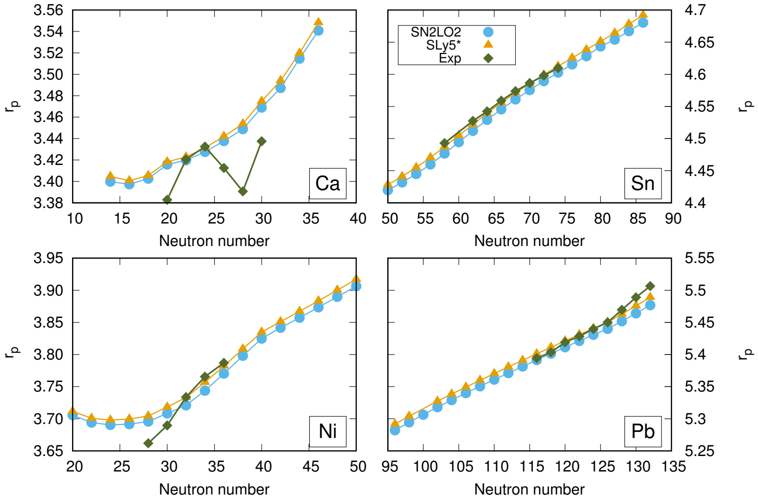
$\rho_{crit} = 1{,}4\ \rho_{sat}$

Interaction N2LO stable

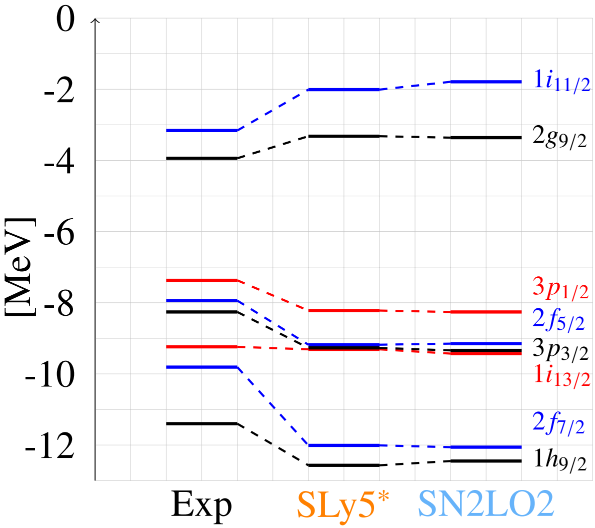
Proton radii SN2LO2



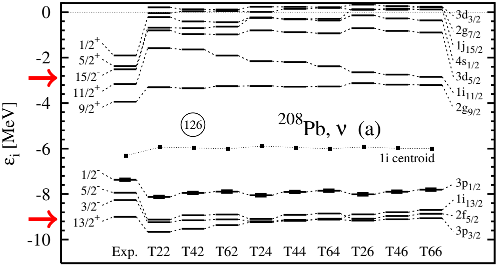
Protons radii SN2LO2



Neutron particle levels for ^{208}Pb



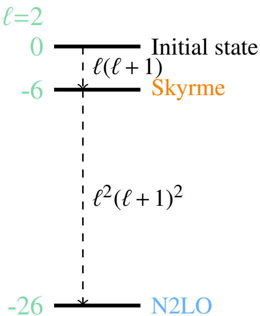
Intruder states problem



T. Lesinski, Phys.Rev. C76 (2007) 014312

N2LO:

$$C_1\ \ell(\ell+1) + C_2\ \ell^2(\ell+1)^2$$



Intruder states → Incorporate centroids in the fitting protocol

Conclusion

N2LO

- Numerical code for finite nuclei WHISKY
- Stable parametrisation SN2LO2
- Better results (compared to Skyrme)

Gogny

- Third gaussian
- Optimisation of the numerical part for the nuclei

Future prospects

- Centroids, kink for Pb
- Tensor terms
- N3LO
- Fitting protocol for finite-range potential (Gogny) with linear response

References:

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  D.Davesne *et al*, Journal of Physics G, **41** 034001 (2015)
- *Extended Skyrme pseudo-potential deduced from infinite matter properties*
  P. Becker *et al*, Phys. Rev. C, **91** 064303 (2015)
- *Partial-wave decomposition of the finite-range effective tensor interaction*
  D.Davesne *et al*, Phys.Rev.C, **93** 064001 (2016)
- *Infinite matter properties and zero-range limit of non-relativistic finite-range interactions*
  D.Davesne *et al*,Annals Phys., **375** 288-312 (2016)
- *Does the Gogny interaction need a third Gaussian?*
  D.Davesne *et al*, Acta Phys.Polon., **B48** 265 (2017)
- *A numerical method for N2LO Hartree-Fock-Bogoliubov calculations*
  P.Becker *et al*, Accepted in Phys.Rev.C (2017)
- *Fit of the Gogny interaction with a third gaussian*
  P.Becker *et al*, in preparation, (2017)

Acknowledgments

M. Bender, K. Bennaceur, Y. Lallouet, J. Meyer, J. Navarro, A. Pastore, W. Ryssens.

Effective mass in nuclear effective theories

(K. Bennaceur, D. D., J. Meyer, J. Navarro, A. Pastore)

- Two-body : saturation, effective mass $\simeq 0.4$
- Density-dependent term : effective mass $\simeq 0.7$

Weisskopf's relation (1957): mean field U_i (for a state i) with a quadratic momentum dependance

$$U_i = U_0 + \frac{p_i^2}{p_F^2} U_1 \rightarrow \frac{m^*}{m} = 1 + \frac{U_1}{\varepsilon_F}$$

$$E/A = \frac{3}{5}\varepsilon_F + \frac{1}{2}U_0 + \frac{3}{10}U_1 \rightarrow \mathbf{\frac{m^*}{m} = \frac{3}{2} - \frac{5}{2}\frac{E/A}{\varepsilon_F}}$$

With $E/A = -16$ MeV and $k_F = 1.33$ fm^{-1}, one gets $m^*/m \simeq 0.4$. Example (SV interaction): $E/A = -16.06$ MeV, $k_F = 1.32$ fm^{-1} and $m^*/m = 0.38$. The relation gives $m^*/m = 0.383$.

N2LO/N3LO/finite-range, no density dependence

Exact relation up to N2LO (first correction: N3LO)

$$\frac{m}{m^*} = \frac{11}{8} + \frac{5}{72}\frac{K_\infty - 21\mathcal{E}_0}{\varepsilon_F} + \frac{1}{90}\frac{C_1^{(6)}\rho_0 k_F^6}{\varepsilon_F}$$

Typical values: $1.375 + \quad 1.033 \quad + \quad 0.012$

$\rightarrow m^*/m = 0.415$ (N2LO) or 0.413 (N3LO)

Finite-range potential $V(r/\mu)$:

$$\frac{m}{m^*} = \frac{11}{8} + \frac{5}{72}\frac{K_\infty - 21\mathcal{E}_0}{\varepsilon_F} + \frac{12}{\pi}\frac{C_E}{\varepsilon_F}\int \mathrm{d}z\, z^2\, V\left(\frac{z}{k_F\mu}\right) \times\left\{\mathcal{F}^m(z) + \frac{5}{72}\mathcal{F}^K(z) - \frac{105}{72}\mathcal{F}^{\mathcal{E}}(z)\right\}$$

N2LO/N3LO/finite-range, no density dependence

$$\mathcal{F}^{\mathcal{E}}(x) = \frac{2}{x^2} j_1^2(x) - \frac{2}{3x} j_0(x) j_1(x)$$
$$\mathcal{F}^{K}(x) = 2 j_0^2(x) - \frac{12}{x} j_0(x) j_1(x) + \left(\frac{18}{x^2} - 2\right) j_1^2(x)$$
$$\mathcal{F}^{m}(x) = \frac{1}{3} j_1^2(x)$$

$$\mathcal{F}^m(x) + \frac{5}{72}\mathcal{F}^{\mathcal{K}}(x) - \frac{105}{72}\mathcal{F}^{\mathcal{E}}(x) \simeq \frac{x^6}{127575} - \frac{8x^8}{9823275} + \frac{x^{10}}{25540515} + \dots$$

Exact cancellation up to x^4 as it should be!

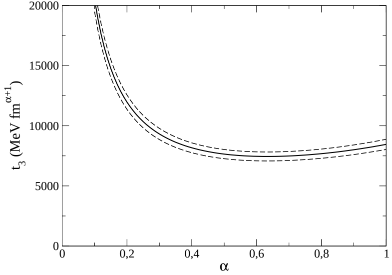
N3LO or finite-range correction to the exact relation: very small!!!

N2LO/N3LO/finite-range, density dependence

$$\frac{m}{m^*} = \frac{11}{8} + \frac{5}{72}\frac{K_\infty - 21\mathcal{E}_0}{\varepsilon_F} + \Delta_{\mathrm{FR}} - \frac{5}{384}\alpha(10+3\alpha)\frac{t_3\rho_0^{\alpha+1}}{\varepsilon_F}$$

D1: $1.375 + 1.049 + 0.010(\to 0.411) - 0.934(\to 0.667)$
D1S: $1.375 + 1.002 + 0.029(\to 0.416) - 0.963(\to 0.693)$
D1N: $1.375 + 1.067 + 0.033(\to 0.404) - 1.087(\to 0.720)$

For admitted values of $\frac{m}{m^*}, K_\infty, E/A$: relation between t_3 and α!



Curve almost flat: $t_3 \simeq 7500 - 8000$ Mev.fm$^{\alpha+1}$