Nuclear structure with regularized EDF generators

Why one more effective interaction ?!

K. Bennaceur[1,2,3], J. Dobaczewski[2,3,4,5], Y. Gao[2], M. Kortelainen[2,3], A. Idini[2,6], F. Raimondi[6]

[1]IPNL, CNRS/IN2P3, Université Claude Bernard Lyon 1, France,
[2]Department of Physics, University of Jyväskylä, Finland,
[3]Helsinki Institute of Physics, Finland,
[4]Faculty of Physics, University of Warsaw, Poland,
[5]Department of Physics, University of York, UK,
[6]Department of Physics, University of Surrey, Guilford, UK

October 2nd, 2017

Outline

Mean-field methods and nuclear structure

- Time-independent Schrödinger equation for A particles

$$\hat{H}\Psi = \left(\hat{T} + \hat{V}_2 + \hat{V}_3 + \ldots \right) \Psi = E_0 \Psi$$

- Mean-field approximation, Hartree-Fock(-Bogolyubov) equations

$$E = \langle \Phi | \hat{H}_{\mathrm{eff}} | \Phi \rangle \simeq E_0 = \langle \Psi | \hat{H} | \Psi \rangle$$

- Effective interaction $\hat{H}_{\mathrm{eff}} = \hat{T} + \hat{V}_{\mathrm{eff}}$

$$\hat{V}_{\mathrm{eff}} = \hat{V}_{\mathrm{eff}}(\mathbf{p}) \,, \quad \mathbf{p} \in \mathbb{R}^n \,, \quad n \lesssim 10$$

Standard form of the Skyrme interaction $\hat{V}_{\mathrm{eff}} = \hat{V}_2 + \hat{V}_3$

- Two-body term (with $x \equiv \mathbf{r}, s, q$) $\simeq$ SV interaction

$$
\begin{aligned}
\hat{V}_2(x_1, x_2; x_3, x_4) = \Big[& t_0 \left(\boldsymbol{\delta}^s + x_0\, \mathbf{P}^s\right) \\
& + \tfrac{1}{2}\, t_1 \left(\boldsymbol{\delta}^s + x_1\, \mathbf{P}^s\right) \left(\hat{\mathbf{k}}_{12}^{*2} + \hat{\mathbf{k}}_{34}^{2}\right) \\
& + t_2 \left(\boldsymbol{\delta}^s + x_2\, \mathbf{P}^s\right) \hat{\mathbf{k}}_{12}^{*} \cdot \hat{\mathbf{k}}_{34} \\
& + \mathrm{i}\, W_0\, \boldsymbol{\delta}^s \left(\hat{\boldsymbol{\sigma}}_{13} + \hat{\boldsymbol{\sigma}}_{24}\right) \cdot \left(\hat{\mathbf{k}}_{12}^{*} \times \hat{\mathbf{k}}_{34}\right) \Big] \\
& \times \delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_2 - \mathbf{r}_4)\delta(\mathbf{r}_1 - \mathbf{r}_2)
\end{aligned}
$$

☺ ρ_{sat}, ☺ E/A, ☹ m^*/m, ☹ K_∞

- Three-body term

$$
\hat{V}_3(x_1, x_2, x_3; x_4, x_5, x_6) = t_3\, \delta_{x_1 x_4} \delta_{x_2 x_5} \delta_{x_3 x_6} \delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_1 - \mathbf{r}_3)
$$

☺ ρ_{sat}, ☺ E/A, ☺ m^*/m, ☹ K_∞

Standard form of the Skyrme interaction $\hat{V}_{\mathrm{eff}} = \hat{V}_2 + \hat{V}_3$

- Two-body term (with $x \equiv \mathbf{r}, s, q$) $\simeq$ SV interaction

$$\begin{aligned}\hat{V}_2(x_1,x_2;x_3,x_4) = \Big[& t_0\,(\boldsymbol{\delta}^s + x_0\,\mathbf{P}^s) \\ & + \tfrac{1}{2}\,t_1\,(\boldsymbol{\delta}^s + x_1\,\mathbf{P}^s)\left(\hat{\mathbf{k}}^{*2}_{12} + \hat{\mathbf{k}}^{2}_{34}\right) \\ & + t_2\,(\boldsymbol{\delta}^s + x_2\,\mathbf{P}^s)\,\hat{\mathbf{k}}^{*}_{12}\cdot\hat{\mathbf{k}}_{34} \\ & + \mathrm{i}\,W_0\,\boldsymbol{\delta}^s\,(\hat{\boldsymbol{\sigma}}_{13} + \hat{\boldsymbol{\sigma}}_{24})\cdot\left(\hat{\mathbf{k}}^{*}_{12}\times\hat{\mathbf{k}}_{34}\right)\Big] \\ & \times\,\delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_2 - \mathbf{r}_4)\delta(\mathbf{r}_1 - \mathbf{r}_2)\end{aligned}$$

☺ ρ_{sat}, ☺ E/A, ☹ m^*/m, ☹ K_∞

- Two-body density dependent term $\simeq$ SIII interaction

$$\hat{V}_3(x_1,x_2;x_3,x_4) = \tfrac{1}{6}\,t_3\,(\boldsymbol{\delta}^s + x_3\,\mathbf{P}^s)\,\rho_0(\mathbf{r}_1)\delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_2 - \mathbf{r}_4)\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

☺ ρ_{sat}, ☺ E/A, ☺ m^*/m, ☹ K_∞

Standard form of the Skyrme interaction $\hat{V}_{\rm eff} = \hat{V}_2 + \hat{V}_3$

- Two-body term (with $x \equiv \mathbf{r}, s, q$) $\simeq$ SV interaction

$$\begin{aligned}\hat{V}_2(x_1,x_2;x_3,x_4) = \Big[& t_0\left(\boldsymbol{\delta}^s + x_0\,\mathbf{P}^s\right) \\ & + \tfrac{1}{2}\,t_1\left(\boldsymbol{\delta}^s + x_1\,\mathbf{P}^s\right)\left(\hat{\mathbf{k}}^{*2}_{12} + \hat{\mathbf{k}}^{2}_{34}\right) \\ & + t_2\left(\boldsymbol{\delta}^s + x_2\,\mathbf{P}^s\right)\hat{\mathbf{k}}^{*}_{12}\cdot\hat{\mathbf{k}}_{34} \\ & + \mathrm{i}\,W_0\,\boldsymbol{\delta}^s\left(\hat{\boldsymbol{\sigma}}_{13} + \hat{\boldsymbol{\sigma}}_{24}\right)\cdot\left(\hat{\mathbf{k}}^{*}_{12}\times\hat{\mathbf{k}}_{34}\right)\Big] \\ & \times\,\delta(\mathbf{r}_1-\mathbf{r}_3)\,\delta(\mathbf{r}_2-\mathbf{r}_4)\,\delta(\mathbf{r}_1-\mathbf{r}_2)\end{aligned}$$

☺ $\rho_{\rm sat}$, ☺ E/A, ☹ m^*/m, ☹ K_∞

- Two-body term depending on a fractional power of the density $\simeq$ SLy

$$\hat{V}_3(x_1,x_2;x_3,x_4) = \tfrac{1}{6}\,t_3\left(\boldsymbol{\delta}^s + x_3\,\mathbf{P}^s\right)\rho_0^{\alpha}(\mathbf{r}_1)\,\delta(\mathbf{r}_1-\mathbf{r}_3)\,\delta(\mathbf{r}_2-\mathbf{r}_4)\,\delta(\mathbf{r}_1-\mathbf{r}_2)$$

☺ $\rho_{\rm sat}$, ☺ E/A, ☺ m^*/m, ☺ K_∞

Gogny effective interaction

J. Dechargé and D. Gogny, Phys. Rev. C 21 (1980) 1568

- Two-body finite-range term

$$\hat{V}_2(x_1,x_2;x_3,x_4) = \Big[\sum_{i=1,2} \left(W_i\, \boldsymbol{\delta}^s \boldsymbol{\delta}^q + B_i\, \mathbf{P}^s \boldsymbol{\delta}^q - H_i\, \boldsymbol{\delta}^q \mathbf{P}^q - M_i\, \mathbf{P}^s \mathbf{P}^q\right) \mathrm{e}^{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\mu_i{}^2}}$$

$$+ \mathrm{i}\, W_0\, \boldsymbol{\delta}^s \left(\hat{\boldsymbol{\sigma}}_{13} + \hat{\boldsymbol{\sigma}}_{24}\right) \cdot \left(\hat{\mathbf{k}}^*_{12} \times \hat{\mathbf{k}}_{34}\right) \delta(\mathbf{r}_1 - \mathbf{r}_2)\Big]$$

$$\times\, \delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_2 - \mathbf{r}_4)$$

- Two-body zero-range term depending on $\rho_0^{1/3}$

$$\hat{V}_3(x_1,x_2;x_3,x_4) = t_3\left(\boldsymbol{\delta}^s + \mathbf{P}^s\right) \rho_0^{1/3}(\mathbf{r}_1)\delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_2 - \mathbf{r}_4)\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

☺ ρ_{sat}, ☺ E/A, ☺ m^*/m, ☺ K_∞

Functional derived from an *effective* (Skyrme) interaction

For a spherical nucleus at the HF approximation

$$E = \langle \hat{T} + \hat{V}_{\mathrm{eff}} \rangle = \int \frac{\hbar^2}{2m} \tau_0 \, \mathrm{d}^3 r + \sum_{t=0,1} \int \mathcal{E}_t \, \mathrm{d}^3 r$$

with

$$\mathcal{E}_t = C_t^{\rho}[\rho_0]\rho_t^2 + C_t^{\tau} \rho_t \tau_t + C_t^{\Delta\rho} \rho_t \Delta \rho_t + C_t^{\nabla J} \rho_t \nabla \cdot \mathbf{J}_t + \frac{1}{2} C_t^{J} \mathbf{J}_t^2$$

The coupling constants of the functional $C_t^{\rho}[\rho_0]$, C_t^{τ}, etc., are entirely determined by the parameters $\mathbf{p}$ of the interaction

Skyrme interaction and Skyrme functional: $E \neq \langle \hat{T} + \hat{V}_{\rm eff} \rangle$

- Interaction:
  - All the terms of the functional determined by the parameters of the interaction
  - Tricky to obtain satisfactory properties in all channels
  - Some terms of the functional are difficult to constrain
- Functional: more versatile
  - Complicated, poorly determined or "dangerous" terms, *i.e.* $\mathbb{J}^2$, $\rho_1 \Delta \rho_1$, $\mathbf{s}_0 \Delta \mathbf{s}_0$, $\mathbf{s}_1 \Delta \mathbf{s}_1$, ... omitted or separately adjusted
  - A different interaction can be used in the pairing channel
  - Slater approximation can be used for the Coulomb exhange term

$\Rightarrow$ Very efficient at the mean-field level

  - SLyn ($n = 4, 5, 6, 7$),
    Nucl. Phys. **A 627** (1997) 710 et **A 635** (1998) 231
  - UNEDFn' ($n' = 0, 1, 2$)
    Phys. Rev. **C 82**, 024313, **C 85**, 024304 et **C 89**, 054314

Constraints on the effective interactions

- Mean-field approximation
  - sometimes inadequate to describe the ground states of nuclei
  - does not provide excited states energies and good quantum numbers

- Beyond mean-field approaches
  - Use of symmetry breaking / symmetry restoration mecanisms
  - Configuration mixing along collective coordinates (GCM)
  - Need to calculate energy $\mathcal{E}[q, q']$ and overlap $\mathcal{N}[q, q']$ kernels to evaluate E with correlations
  - For example

$$E^N = \int_0^{2\pi} \mathrm{d}\varphi \, \mathcal{E}[0, \varphi] \mathcal{N}[0, \varphi]$$

depends on transition densities between an HFB state $|\Phi_0\rangle$ and a *rotated* state $|\Phi_\varphi\rangle$: $\rho^{0\varphi}$, $\kappa^{0\varphi}$ et $\kappa^{\varphi 0 *}$

Pitfalls with functionals not derived from an interaction

- Skyrme functional are (most of the time) not strictly derived from an interaction
- E^N will show divergences each time a single particle state goes through the Fermi energy

Cf. M. Anguiano *et al.*, NPA 696, 467
J. Dobaczewski *et al.*, PRC 76, 054315
D. Lacroix et al., PRC 79, 044318

- Even if the functional is derived from an interaction, the density dependent term ρ_0^α requires a particular treatment
  - transition (or mixed) density: $\rho^{0\varphi} = \langle \Phi_0 | \hat{\rho} | \Phi\varphi \rangle$
  - average density: $\bar{\rho}^\alpha = \frac{1}{2}\left(\langle \Phi_0 | \hat{\rho} | \Phi_0 \rangle^\alpha + \langle \Phi_\varphi | \hat{\rho} | \Phi_\varphi \rangle^\alpha\right)$
  - correlated density: $\rho^N = \int \mathrm{d}\varphi \, \rho^{0\varphi} \, \mathcal{N}[0, \varphi]$

Pitfalls with functionals not derived from an interaction

- Skyrme functional are (most of the time) not strictly derived from an interaction
- E^N will show divergences each time a single particle state goes through the Fermi energy

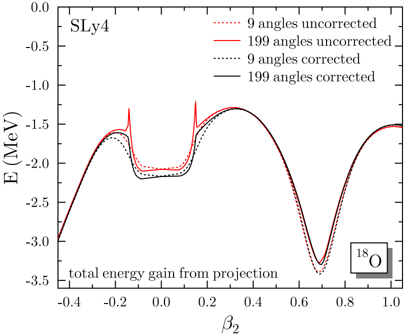
Cf. M. Anguiano *et al.*, NPA 696, 467
J. Dobaczewski *et al.*, PRC 76, 054315
D. Lacroix et al., PRC 79, 044318

- Even if the functional is derived from an interaction, the density dependent term ρ_0^α requires a particular treatment
  - transition (or mixed) density: $\rho^{0\varphi} = \langle \Phi_0 | \hat{\rho} | \Phi\varphi \rangle \in \mathbb{C}$
  - average density: ~~$\bar{\rho}^\alpha = \frac{1}{2} \left(\langle \Phi_0 | \hat{\rho} | \Phi_0 \rangle^\alpha \langle \Phi_\varphi | \hat{\rho} | \Phi_\varphi \rangle^\alpha \right)$~~
  - correlated density: ~~$\rho^N = \int \mathrm{d}\varphi \, \rho^{0\varphi} \, \mathcal{N}[0, \varphi]$~~

L. M. Robledo, JPG 37, 064020

Fractional power of the density[1]

- The energy kermel $\mathcal{E}[q, q']$ must be extented in $\mathbb{C}$
- $\rho_0^\alpha \;\Rightarrow\; \mathcal{E}[q, q']$ is a multivalued function in the complexe plane



Problem analyzed by J. Dobaczewski *et al.*, PRC 76, 054315:

... with solutions that might not be usable with all symmetry restorations

[1] T. Duguet, M. Bender, K.B., D. Lacroix, T. Lesinski, PRC 79, 044320

Functional for beyond mean-field calculations

- Functional not derived from an effective interaction
  → Divergences of the energie
  - The considered regularization methods might be difficult to implement, seem to be *ad hoc* and are not proven to be usable in all circumstances
- Effective interaction with density dependent term ρ_0^α
  → Steps in the energy
  - No solution proven to be usable in all situations (yet)

Drastic solution

The functional has to be strictly derived from an interaction with no density dependent term
(what we call a *pseudopotentiel*)

Finite-range two-body pseudopotentials[2]

- In a nutshell:

  take a Skyrme interaction and replace $\delta(\mathbf{r})$ with $g_a(\mathbf{r}) = \frac{e^{-\frac{r^2}{a^2}}}{(a\sqrt{\pi})^3}$

- Pseudopotential at “NLO”

$$
\begin{aligned}
v &= \tilde{v}_0(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)\, t_0 \left(1_{\sigma q} + x_0 1_q \hat{P}^\sigma - y_0 1_\sigma \hat{P}^q - z_0 \hat{P}^\sigma \hat{P}^q\right) \\
&+ \tilde{v}_1(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)\, t_1 \left(1_{\sigma q} + x_1 1_q \hat{P}^\sigma - y_1 1_\sigma \hat{P}^q - z_1 \hat{P}^\sigma \hat{P}^q\right) \\
&+ \tilde{v}_2(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4)\, t_2 \left(1_{\sigma q} + x_2 1_q \hat{P}^\sigma - y_2 1_\sigma \hat{P}^q - z_2 \hat{P}^\sigma \hat{P}^q\right)
\end{aligned}
$$

with

$$
\begin{aligned}
\tilde{v}_0(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4) &= \delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_2-\mathbf{r}_4) g_a(\mathbf{r}_1-\mathbf{r}_2) \\
\tilde{v}_1(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4) &= \delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_2-\mathbf{r}_4) g_a(\mathbf{r}_1-\mathbf{r}_2)\, \frac{1}{2}\left[\mathbf{k}_{12}^{*2} + \mathbf{k}_{34}^2\right] \\
\tilde{v}_2(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_3,\mathbf{r}_4) &= \delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_2-\mathbf{r}_4) g_a(\mathbf{r}_1-\mathbf{r}_2)\, \mathbf{k}_{12}^{*} \cdot \mathbf{k}_{34}
\end{aligned}
$$

- Thanks to the finite range: $\hat{P}^\sigma \hat{P}^q \equiv -\hat{P}^x \not\equiv \pm 1$
- Can be generalized at N^{2}LO, N^{3}LO, ...

[2] F. Raimondi, K.B., J. Dobaczewski, J. Phys. G 41, 055112

Finite-range two-body local pseudopotentials

- The conditions

$$t_1 = -t_2\,,\quad x_1 = x_2\,,\quad y_1 = y_2\,,\quad z_1 = z_2$$

(and same for higher order terms) make the pseudopotential local

- This is a **severe** restriction on the flexibility of the functional
- ... but it greatly simplifies the implementation in computer codes
- and it limits the number of free parameters
(and that's all we have so far anyway...)
- Use of a standard two-body zero-range spin-orbit interaction

Preliminary fits of the parameters

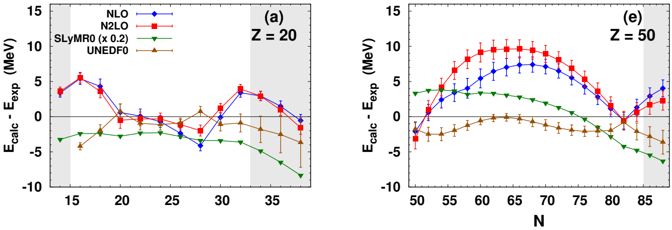
- Two-body finite-range local pseudopotentials at NLO and N^{2}LO

Details on the fits:
K.B., A. Idini, J. Dobaczewski, P. Dobaczewski, M. Kortelainen, F. Raimondi, J. Phys. G 44, 045106 (2017)
A. Idini, K.B., J. Dobaczewski, J. Phys. G 44, 064004 (2017)
K.B., J. Dobaczewski, Y. Gao, arXiv:1701.08062

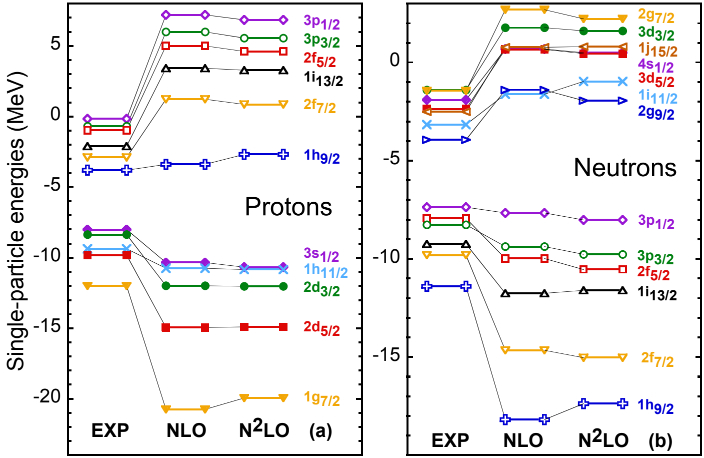
- Infinite nuclear matter properties

	$\rho_{\rm sat}$ (fm^{-3})	B (MeV)	K_∞ (MeV)	m^*/m	J (MeV)	L (MeV)
NLO	0.1599	-16.17	229.8	0.4076	31.96	64.04
N^{2}LO	0.1601	-16.09	230.0	0.4061	31.95	64.68

- Binding energies of semi-magic nuclei



Single particle energies with a low effective mass...



How to increase the effective mass ?

A three-body interaction seems to be the only way...

- Finite-range three-body: not doable in 3D codes
- Semi-contact three-body: not doable either in 3D codes ☹

  D. Lacroix, K.B., Phys. Rev. C 91, 011302(R) (2015)
- Zero-range contact interaction: too repulsive in the pairing channel
- Non-local zero-range contact 3-body interaction
  (*i.e.* Finite-range 2-body and 3-body with grandients):

  Original idea:
  N. Onishi and J. Negele, Nuclear Physics A 301 (1978) 336

Three-body terms with grandients

Same as in J. Sadoudi *et al.*, Phys. Rev. C 88 (2013) 064326

- Symmetrized expression built from

$$v_3(x_1,x_2,x_3;x_4,x_5,x_6) = \left[v_{30} + v_{31} + v_{32}\right] \delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_3) \times \delta(\mathbf{r}_1-\mathbf{r}_4)\delta(\mathbf{r}_2-\mathbf{r}_5)\delta(\mathbf{r}_3-\mathbf{r}_6)\delta_{q_1 q_4}\delta_{q_2 q_5}\delta_{q_3 q_6}$$

with

$$\begin{aligned}
v_{30} &= u_0\, \delta_{s_1 s_4}\delta_{s_2 s_5}\delta_{s_3 s_6} \\
v_{31} &= \frac{u_1}{2}\left(\delta_{s_1 s_4}\delta_{s_2 s_5} + y_1\, \delta_{s_1 s_5}\delta_{s_2 s_4}\right)\delta_{s_3 s_6}\left(\hat{\mathbf{k}}^{*2}_{12} + \hat{\mathbf{k}}^{2}_{45}\right) \\
v_{32} &= u_2\left[\delta_{s_1 s_4}\delta_{s_2 s_5}\delta_{s_3 s_6} + y_{21}\, \delta_{s_1 s_5}\delta_{s_2 s_4}\delta_{s_3 s_6}\right. \\
&\qquad \left. + y_{22}\left(\delta_{s_1 s_6}\delta_{s_2 s_5}\delta_{s_3 s_4} + \delta_{s_1 s_4}\delta_{s_2 s_6}\delta_{s_3 s_5}\right)\right]\hat{\mathbf{k}}^{*}_{12}\cdot\hat{\mathbf{k}}_{45}
\end{aligned}$$

- 6 additional parameters...

Fit of the parameters and infinite nuclear matter properties

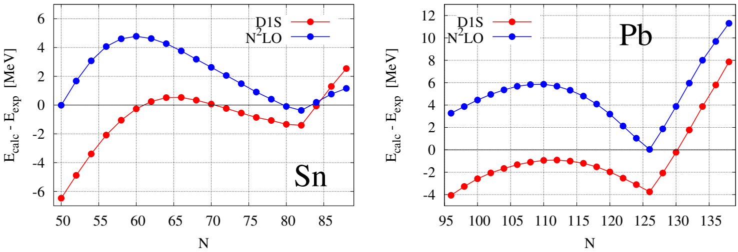
- Finite-range local terms + spin-orbit + 3-body → 19 parameters...
- Setting ρ_{sat}, m^*/m and J to the empirical values leaves 16 free parameters
- Infinite nuclear matter

	ρ_{sat} (fm^{-3})	B (MeV)	K_∞ (MeV)	m^*/m	J (MeV)	L (MeV)
NLO	0.1599	-16.17	229.8	0.4076	31.96	64.04
N^2LO	0.1601	-16.09	230.0	0.4061	31.95	64.68
N^2LO + 3B	0.1600	-16.02	258.6	0.7000	32.00	35.94

- The three-body terms with gradients allows to increase the effective mass and *seems* to give attractive pairing

Results

- Equations of states are OK...
- Pairing strong enough...
- Binding energies of spherical nuclei



Very encouraging results, but...

Yet another illustration of Murphy's law

- Calculations of spherical nuclei with a spherical code give nice results
- Calculations for the **same** spherical nuclei with a code allowing deformation give calamitous results
  - Collapse of the local part of the pairing density $\tilde{\rho}(\mathbf{r},\mathbf{r}) = 0$
  - Huge pairing energies
  - Unphysical binding energies
  - Although nuclei are perfectly spherical...
- No bug found so far...
- So what ?

The culprit

The contact (local) 3-body term (mainly added to increase the effective mass) is always too repulsive in the pairing channel.

So, if it hurts, don't do it...

$$v_3(x_1, x_2, x_3; x_4, x_5, x_6) = [v_{30} + v_{31} + v_{32}]\, \delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_1 - \mathbf{r}_3) \times \delta(\mathbf{r}_1 - \mathbf{r}_4)\delta(\mathbf{r}_2 - \mathbf{r}_5)\delta(\mathbf{r}_3 - \mathbf{r}_6)\delta_{q_1 q_4}\delta_{q_2 q_5}\delta_{q_3 q_6}$$

with

$$v_{30} = 0$$

$$v_{31} = \frac{u_1}{2}\left(\delta_{s_1 s_4}\delta_{s_2 s_5} + y_1\, \delta_{s_1 s_5}\delta_{s_2 s_4}\right)\delta_{s_3 s_6}\left(\hat{\mathbf{k}}^{*2}_{12} + \hat{\mathbf{k}}^{2}_{45}\right)$$

$$v_{32} = u_2\left[\delta_{s_1 s_4}\delta_{s_2 s_5}\delta_{s_3 s_6} + y_{21}\, \delta_{s_1 s_5}\delta_{s_2 s_4}\delta_{s_3 s_6} + y_{22}\left(\delta_{s_1 s_6}\delta_{s_2 s_5}\delta_{s_3 s_4} + \delta_{s_1 s_4}\delta_{s_2 s_6}\delta_{s_3 s_5}\right)\right]\hat{\mathbf{k}}^{*}_{12} \cdot \hat{\mathbf{k}}_{45}$$

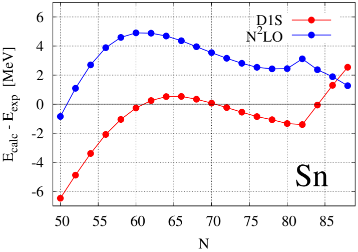
☺ One less parameter to fit !

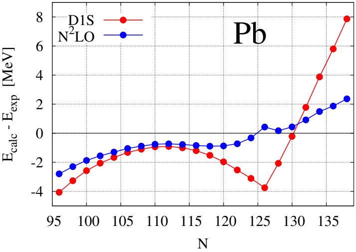
Fit of the parameters and infinite nuclear matter properties

- Preliminary fit with 4 spherical nuclei
- Saturation density ρ_{sat}, symmetry energy coefficient J and effective mass m^*/m fixed
- Pairing adjusted by tuning the contribution of the finite-range interaction in the pairing channel
- Empirical constraints to avoid finite-size instabilities

	ρ_{sat} (fm^{-3})	B (MeV)	K_∞ (MeV)	m^*/m	J (MeV)	L (MeV)
NLO	0.1599	-16.17	229.8	0.4076	31.96	64.04
N^2LO	0.1601	-16.09	230.0	0.4061	31.95	64.68
$N^2LO + 3B$	0.1600	-16.02	258.6	0.7000	32.00	35.94
$N^2LO + 3B$ ($U_0 = 0$)	0.1600	-16.42	276.3	0.5500	35.00	44.99

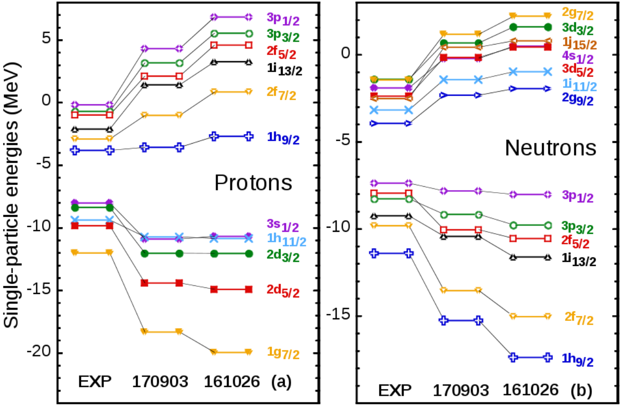
Sherical nuclei





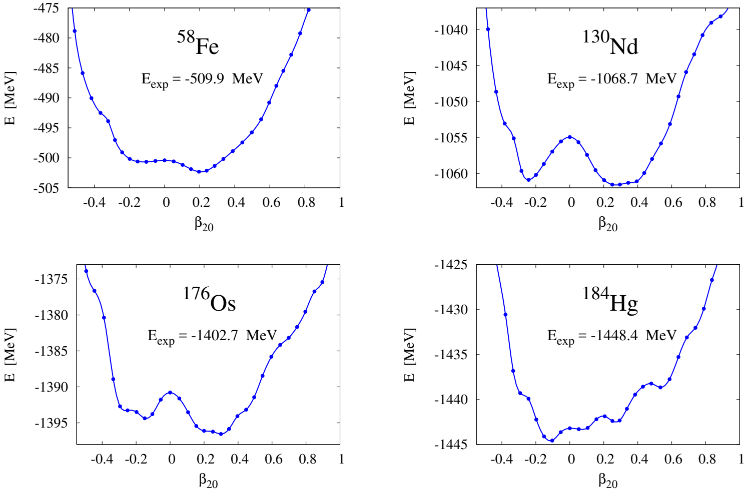
Results are not too bad but it's easy to find examples with less encouraging agreement...

Single particle energies in ^{208}Pb



Density of state scales as expected

Deformed nuclei



Conclusion

- The two-body finite-range pseudopotential complemented with a non-local three-body contact term gives acceptable results but not competitive yet with other existing effective interactions
- It does not contain density dependent terms and is used consistently in all channels: usable with no technical difficulties in beyond mean-field calculations
- The local version of the two-body terms give encouraging results, the non local version will not hurt
- Local version implemented in $\texttt{FINRES}_4$ (spherical solver), under construction in 3D codes `HFBTEMP` (M. Kortelainen) and `HFODD` (J. Dobaczewski *et al.*)

Open questions: Different "flavors" of spherical results

- Calculations of spherical nuclei with a spherical code give nice results
- Calculations for the **same** spherical nuclei with a code allowing deformation give calamitous results
  - Collapse of the local part of the pairing density $\tilde{\rho}(\mathbf{r},\mathbf{r}) = 0$
  - Huge pairing energies
  - Unphysical binding energies
  - Although nuclei are perfectly spherical...
- No bug found so far...
- So what ?
- On a deformed basis, non local densities might not fulfill

$$\rho_q(\mathbf{r}_1,\mathbf{r}_2) = \sum_{\ell j m_\ell} \rho_q(r_1,r_2)\, Y^{(\ell)*}_{m_\ell}(\hat{r}_1)\, Y^{(\ell)}_{m_\ell}(\hat{r}_2)$$

even for a spherical nucleus ?

Open questions: Effective mass, how large should it be ?

	$\rho_{\rm sat}$ ($\rm fm^{-3}$)	B (MeV)	K_∞ (MeV)	m^*/m	J (MeV)	L (MeV)
NLO	0.1599	-16.17	229.8	0.4076	31.96	64.04
N^2LO	0.1601	-16.09	230.0	0.4061	31.95	64.68
$N^2LO + 3B$	0.1600	-16.02	258.6	0.7000	32.00	35.94
$N^2LO + 3B$ ($U_0 = 0$)	0.1600	-16.42	276.3	0.5500	35.00	44.99

- For mean-field calculations ?
- For beyond mean-field calculations ?

Open questions: What drives the parameters to regions with finite-size instabilities ?

- Several Skyrme interactions are plaged with finite-size instabilities
- Isovector instabilities are more likely to occur when the interaction is tuned to give attractive pairing
- Can it be avoided ?
- Does a finite-range help ?