

# Primordial Black holes and gravitational waves

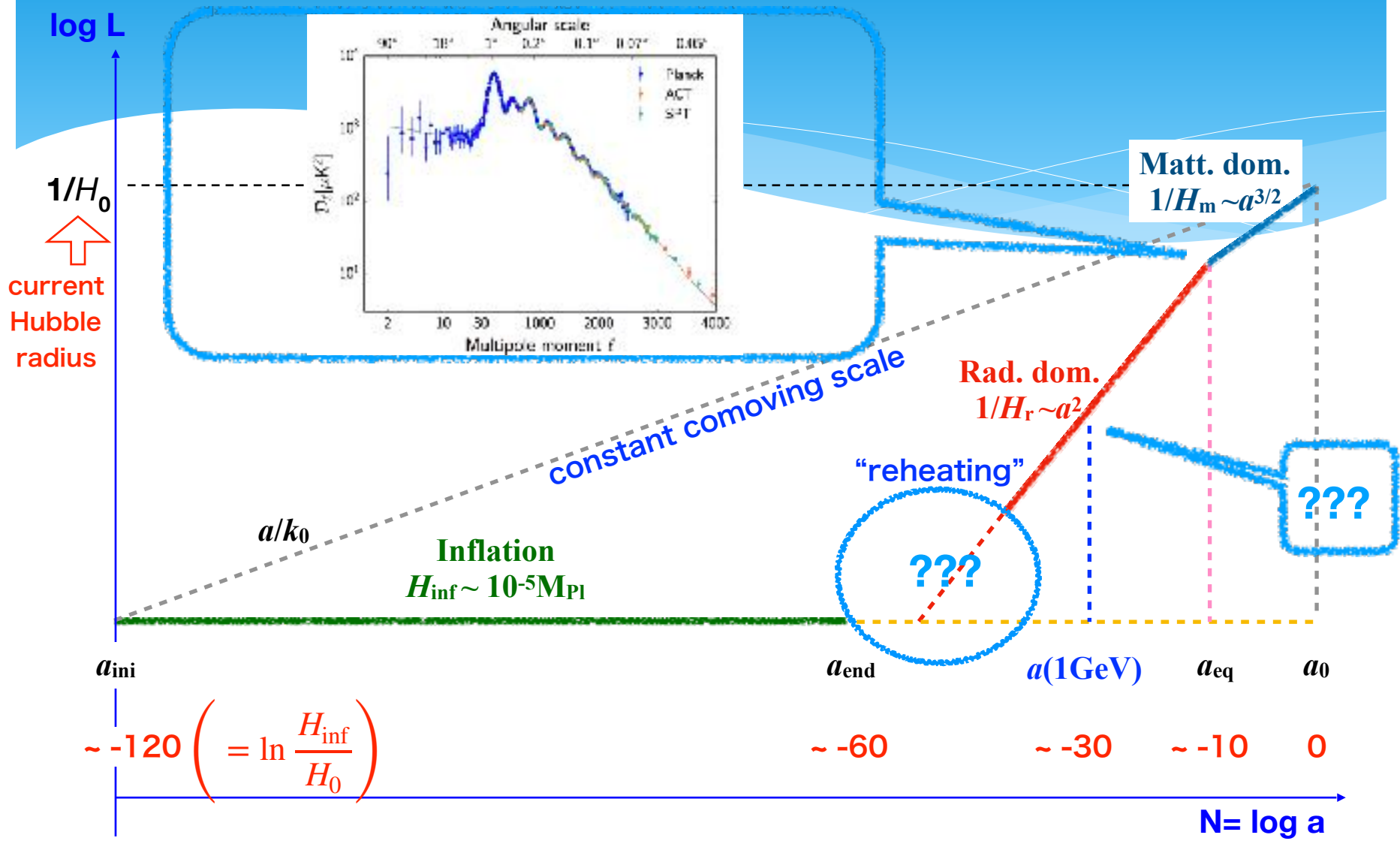
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# Inflation & PBH formation

# cosmic spacetime diagram



# curvature perturbation from inflation

- inflaton ( $\sim$ massless) vacuum fluctuations (=Gaussian)

$$\left| \langle \phi | \vec{k} \rangle \right|^2 = |\varphi_k|^2, \quad \varphi_k \sim \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k t}; \quad \omega_k = \frac{k}{a} \gg H$$

rapid expansion renders oscillations frozen at  $k/a < H$   
 (fluctuations become “classical” on superhorizon scales)

$$\varphi_k \sim \frac{H}{\sqrt{2k^3}}; \quad \frac{k}{a} \ll H \Rightarrow \langle \delta\phi_k^2 \rangle = \left( \frac{H}{2\pi} \right)_{k/a \sim H}^2 \dots \text{almost scale-invariant}$$

for  $\epsilon = -\frac{\dot{H}}{H^2} \ll 1$

- curvature perturbation on comoving slices

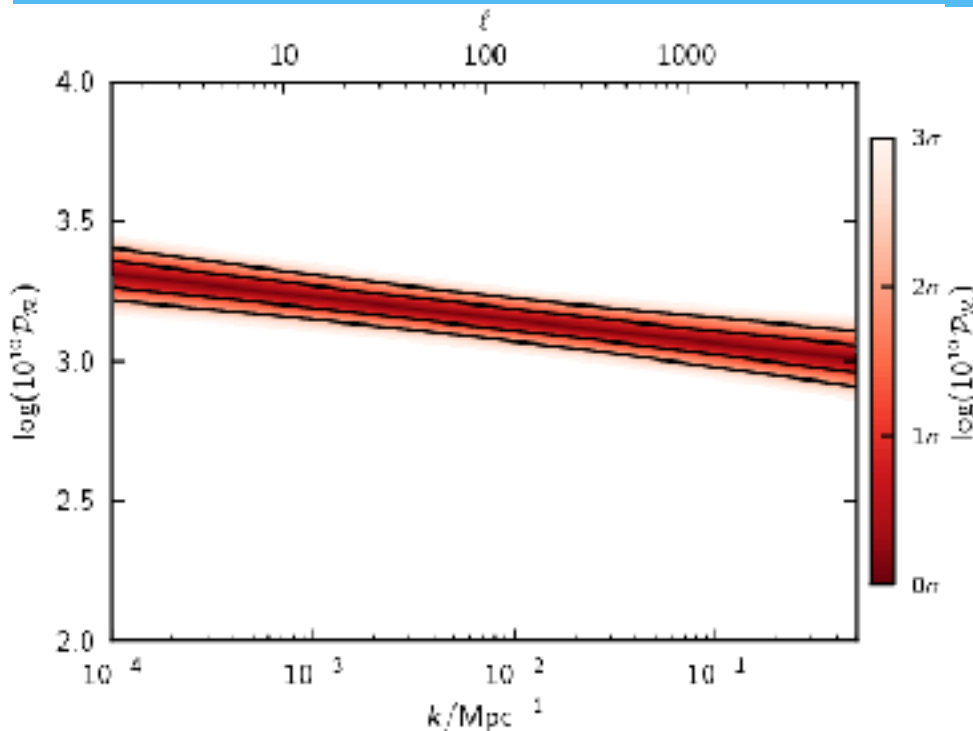
$$\mathcal{R}_c = -\frac{H}{\dot{\phi}} \delta\phi \dots \text{conserved on superhorizon scale}$$

for single-field slow-roll models

(almost scale-invariant if  $\dot{\phi}$  is also slowly varying)

# observational constraint on inflation

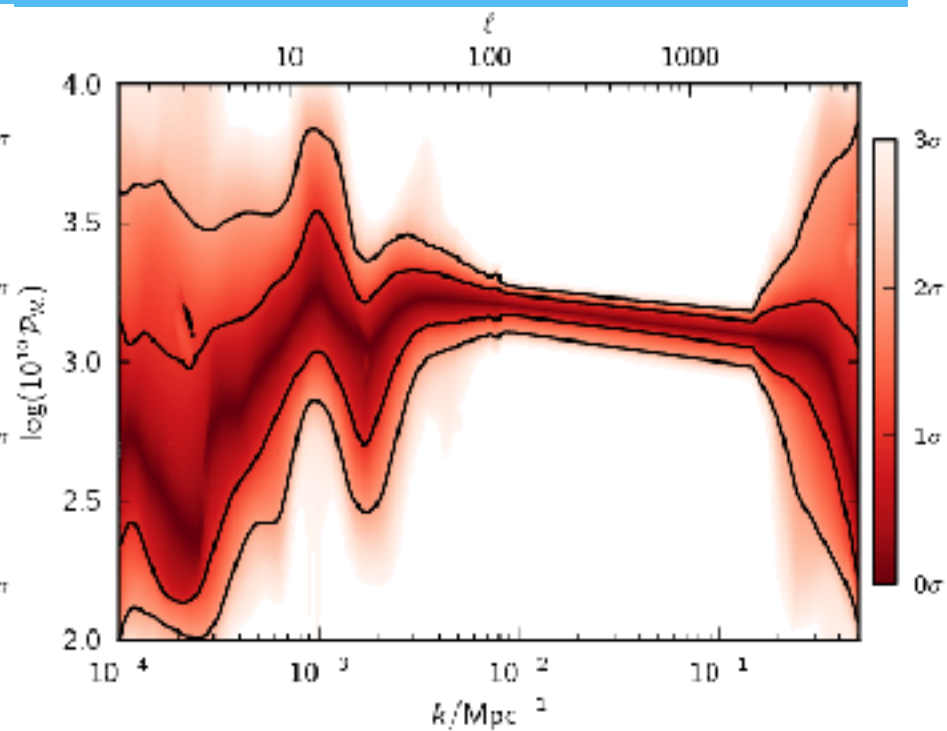
Planck 2015 results XX



power-law

$$\mathcal{P}_{\mathcal{R}} = A(k/k_*)^{n_s-1}$$

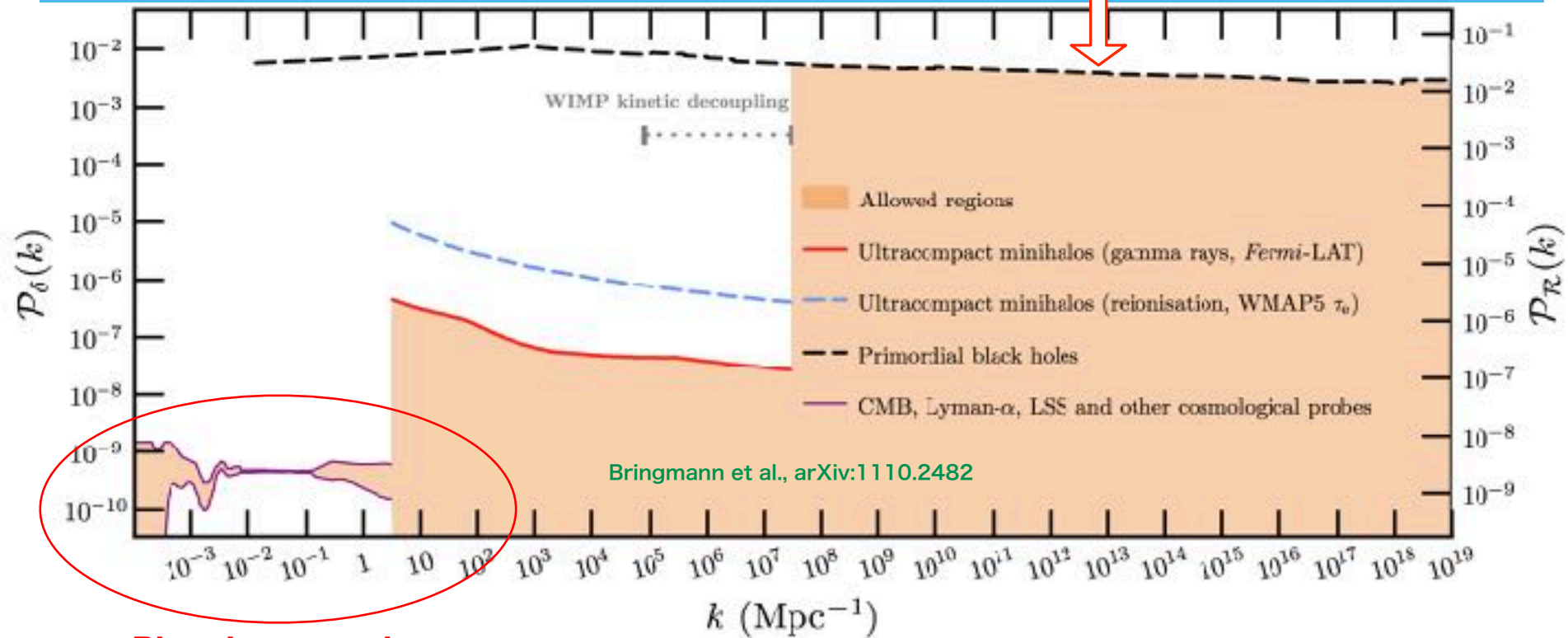
$n_s \approx 0.968 \dots$  almost scale-invariant



piece-wise continuous  
(9 segments)

# observational constraint on inflation

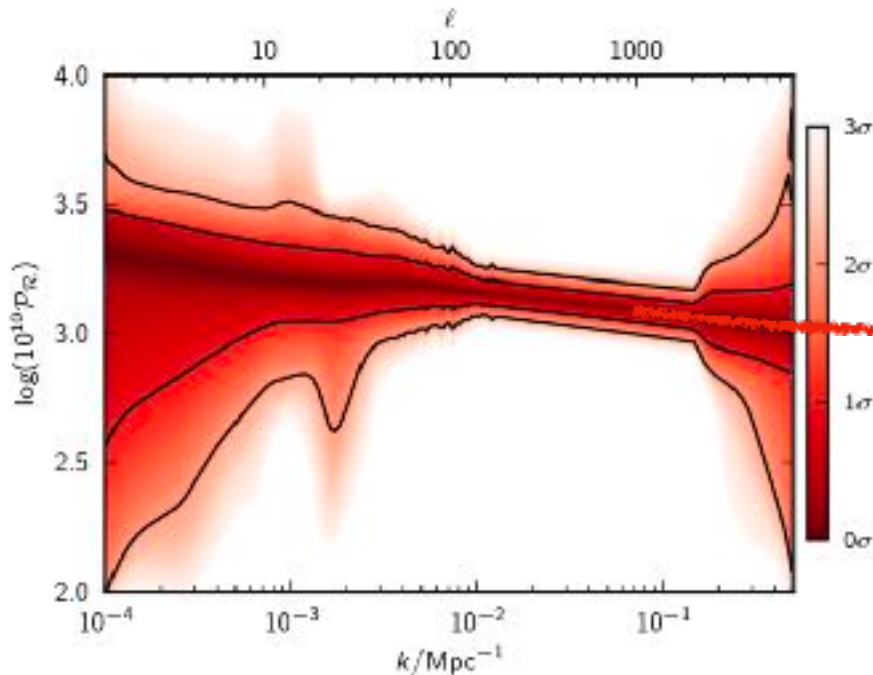
constraints on small scales are from BHs



Planck constraint

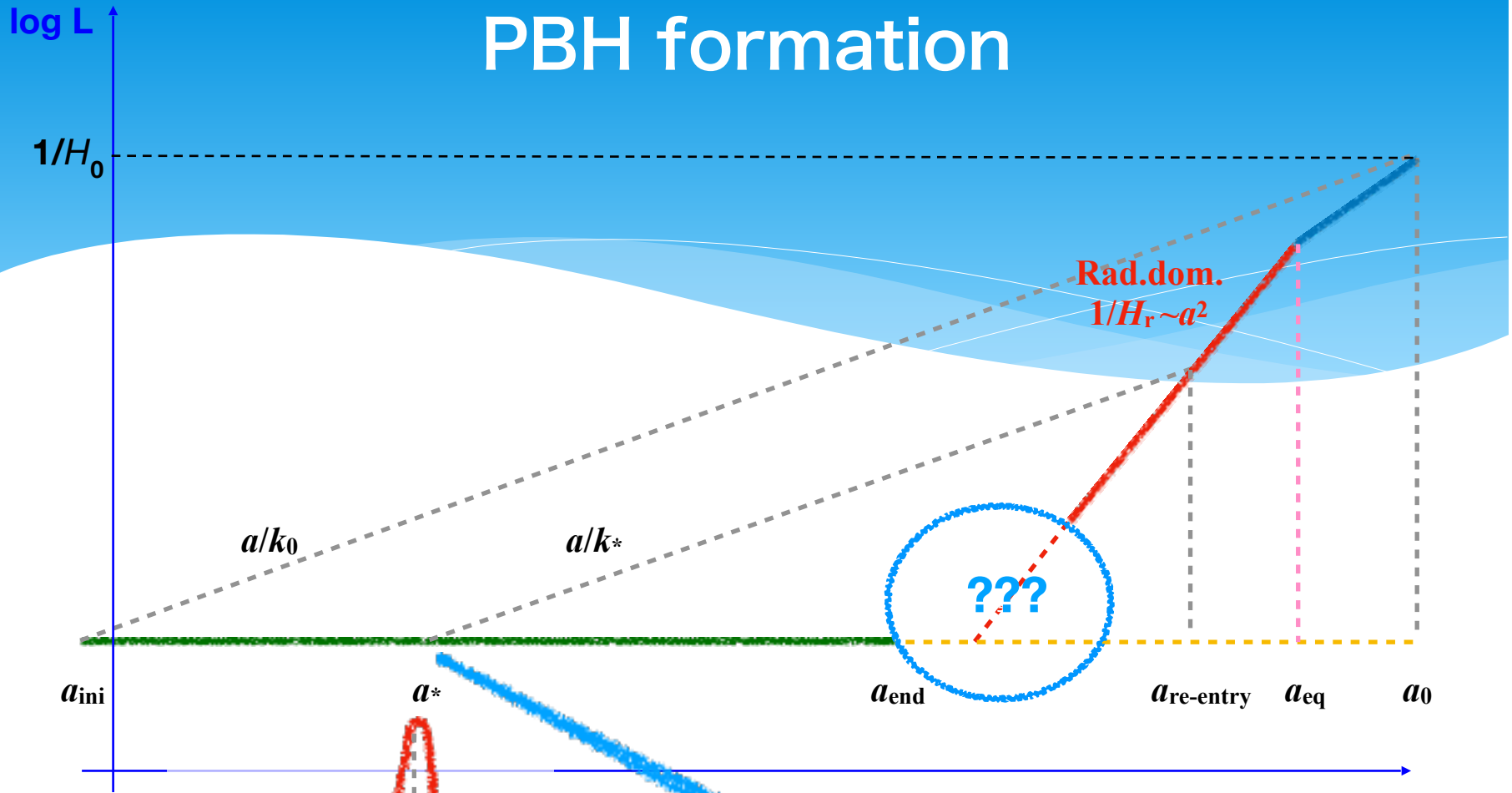
There are some constraints on small scales, but quite weak.

# Bayesian reconstruction of the primordial power spectrum with $l < 2300$ . (Planck 2015)



No resolution to say anything precise about higher  $k$ .

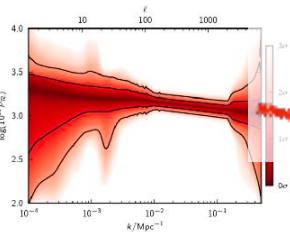
# PBH formation



$N = \log a$

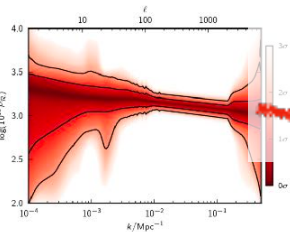
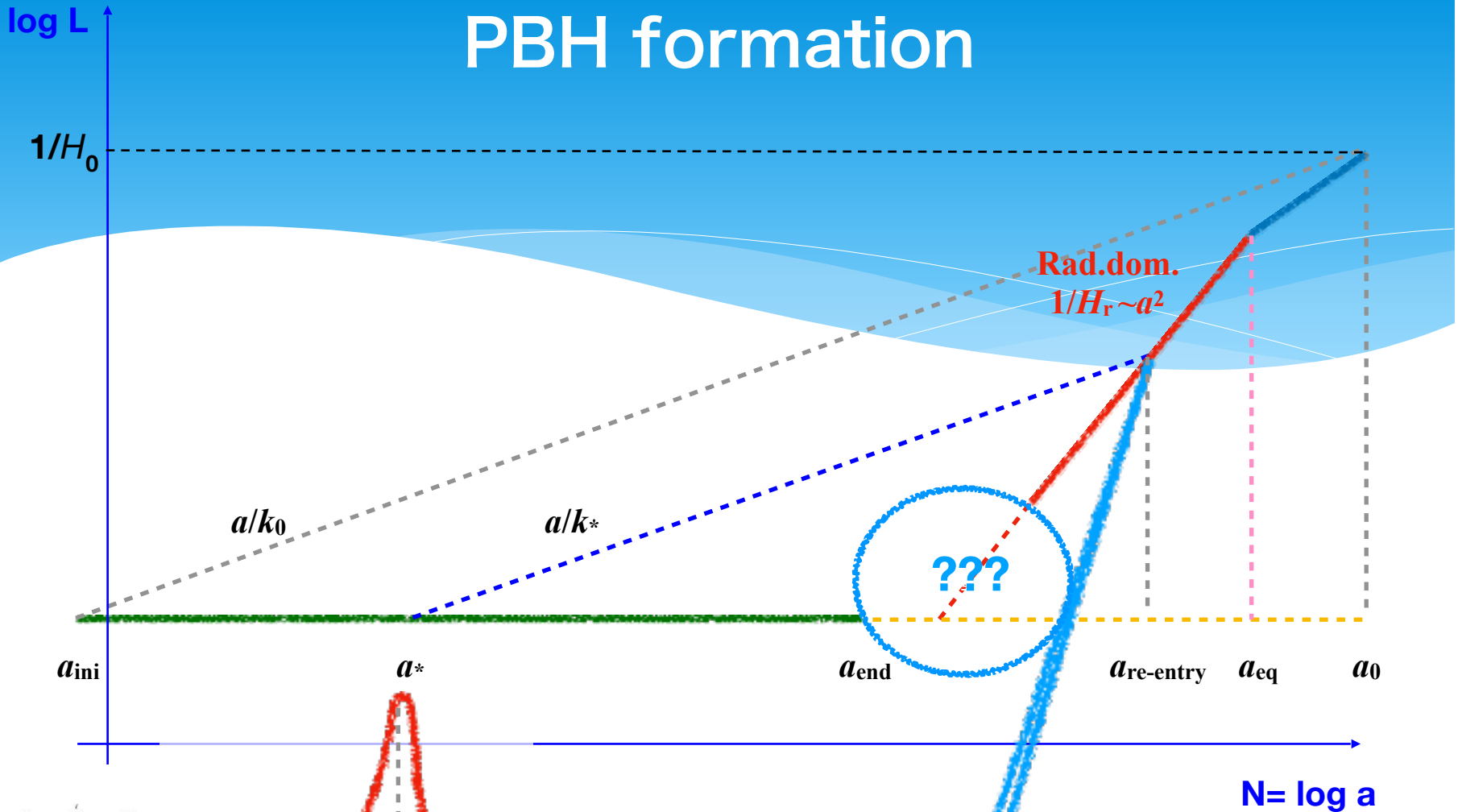
A peak in the primordial curvature perturbation, which leaves horizon and gets frozen at  $a^*$ .

$$k_* = Ha_*$$





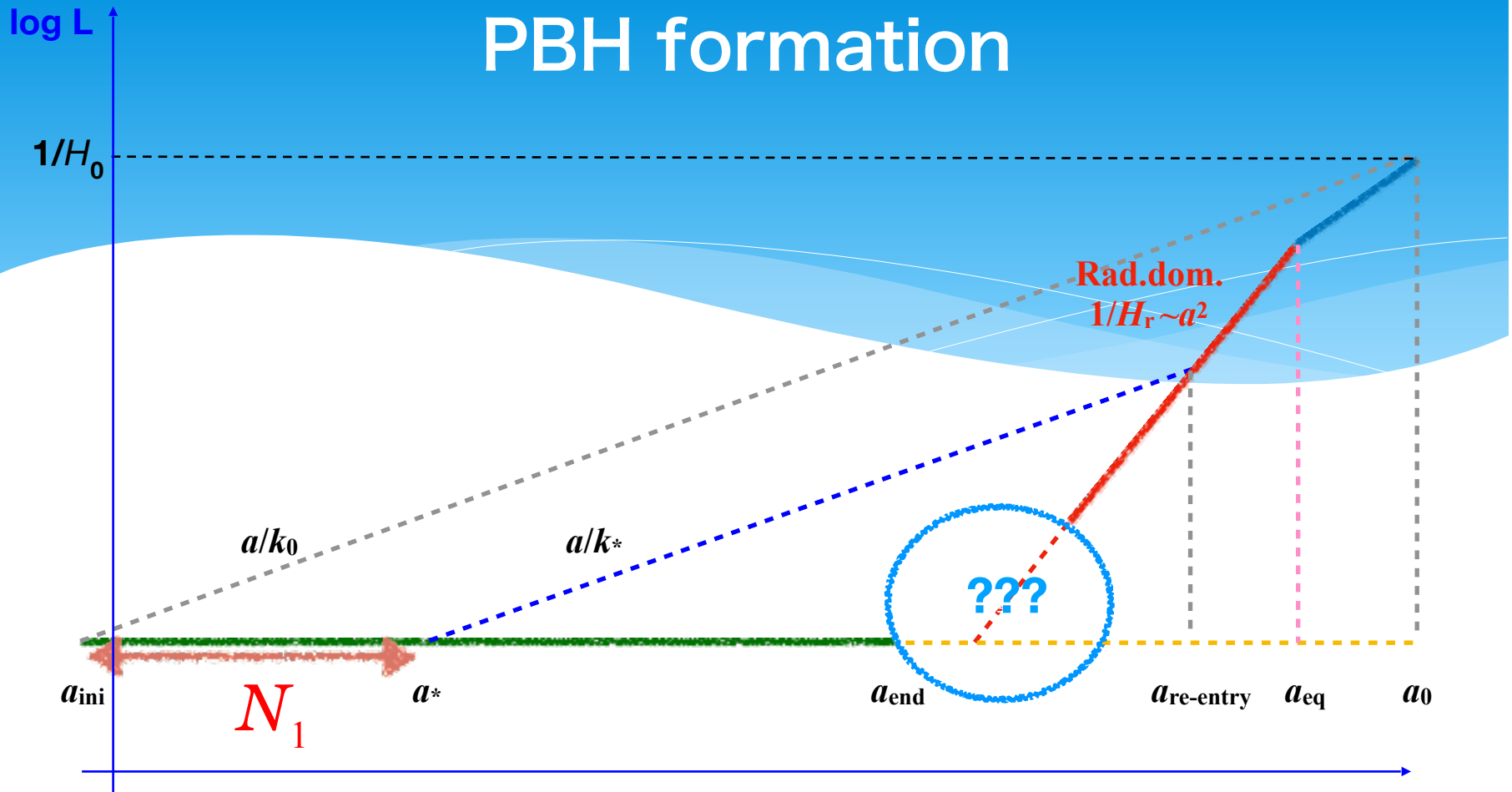
# PBH formation



$$k_* = Ha_*$$

The peak re-enters horizon during radiation era.  
If the amplitude  $> O(0.1)$ , PBH will form.

# PBH formation



$N = \log a$

**PBH mass:**  $M_{PBH} = \gamma M_H \sim \frac{M_{Pl}^2}{H} = 10^{58} M_{Pl} e^{-2N_1} = M_{Pl} 10^{58-0.87N_1}$

**Inverse relation:**  $N_1 = 44.4 - \frac{1}{2} \ln \left( \frac{M_{PBH}}{10^{16} \text{ g}} \right)$

**PBH mass scale does NOT depend on the reheating physics**

# Primordial Black Holes

# What are Primordial BHs?

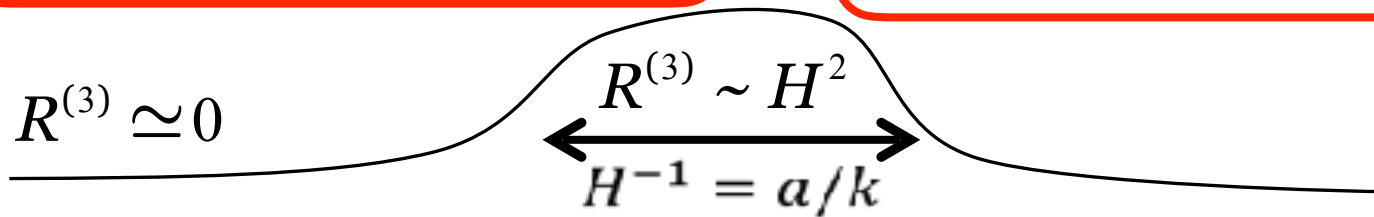
- **PBH = BH formed before recombination epoch** (ie at  $z \gg 1000$ )  
conventionally during radiation-dominated era
- **Hubble size region with  $\delta\rho/\rho=O(1)$**  collapses to form BH  
Carr (1975), ....
- Such a large perturbation may be **produced by inflation**  
Carr & Lidsey (1991), ...
- PBHs may dominate **Dark Matter**.  
Ivanov, Naselsky & Novikov (1994), ...
- **Supermassive BHs** ( $M \gtrsim 10^6 M_{\odot}$ ) may originate from PBHs.
- ....

# Curvature perturbation to PBH

- gradient expansion/separate universe approach

$$6H^2(t, x) + R^{(3)}(t, x) = 16\pi G\rho(t, x) + \dots \quad \text{Hamiltonian constraint (Friedmann eq.)}$$

$$\Rightarrow R^{(3)} \approx -\frac{4}{a^2} \nabla^2 \mathcal{R}_c \approx \frac{8\pi G}{3} \delta\rho_c \quad \Rightarrow \quad \frac{\delta\rho_c}{\rho} \approx \mathcal{R}_c \quad \text{at} \quad \frac{k^2}{a^2} = H^2$$



- If  $R^{(3)} \sim H^2$  ( $\Leftrightarrow \delta\rho_c / \rho \sim 1$ ) collapses to form BH

Young, Byrnes & MS '14

$$M_{\text{PBH}} \sim \rho H^{-3} \sim 10^5 M_{\odot} \left(\frac{t}{1\text{s}}\right) \sim 20 M_{\odot} \left(\frac{k}{1\text{pc}^{-1}}\right)^{-2}$$

- Spins of PBHs are expected to be very small

# examples

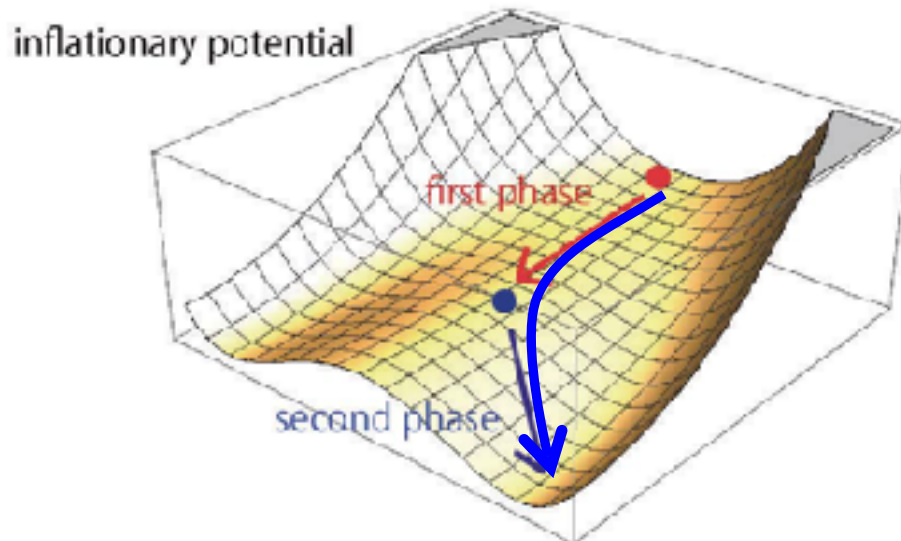
## hybrid-type inflation

Garcia-Bellido, Linde & Wands '96, ...

$\mathcal{R}_C$  grows near the saddle point  
non-Gauss may become large

Abolhasani, Firouzjahi & MS '11, ...

Pattison et al. 1707.00537

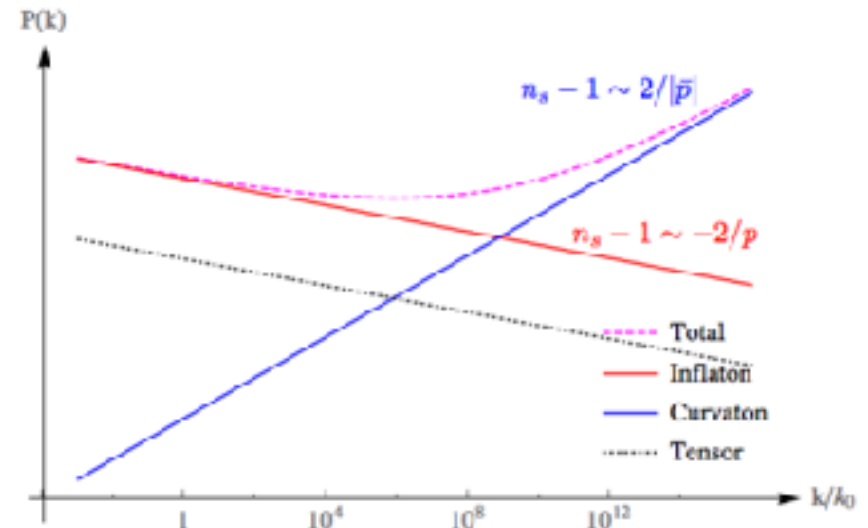


## non-minimal curvaton

Domenech & MS '16

$$L = -\frac{1}{2} f(\phi) g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi$$

$$-\frac{1}{2} h(\phi) m^2 \chi^2$$



# Accretion to PBH?

## ➤ Bondi accretion

$$\dot{M} = \lambda \cdot 4\pi r_B^2 \rho c_s : \quad c_s = \sqrt{P / \rho} \left( = 1 / \sqrt{3} \right), \quad r_B = \frac{GM}{c_s^2}, \quad \lambda \lesssim O(1)$$

### • accretion rate/Hubble time

$$\Rightarrow \frac{\dot{M}}{HM} = \lambda \frac{3}{4} \frac{H}{H_M} : \quad M = \frac{4\pi\rho_M}{3} \left( c_s H_M^{-1} \right)^3 = \frac{c_s^3}{2GH_M}, \quad \frac{H}{H_M} = \left( \frac{a_M}{a} \right)^2$$

↪ horizon size at the time of PBH formation

$$\Rightarrow \int_{a_M}^{\infty} \frac{\dot{M}}{H} \frac{da}{a} \simeq \lambda \frac{3}{8} M \quad \text{PBH mass can increase by a factor of 1.5 at most}$$

Mass increase can be ignored, given other ambiguities

# Effect on CMB?

accretion can lead to radiative emission

- Eddington luminosity: max luminosity from

accretion  $L_{\text{edd}} = \frac{4\pi GMm_p c}{\sigma_T}$ ;  $m_p =$  proton mass  
 $\sigma_T =$  Thomson cross section

$L = \epsilon L_{\text{edd}}$ ;  $\epsilon \leq 1$  ... luminosity from PBH

- energy output/Hubble time

$$\frac{\dot{\rho}_R}{H\rho_R} = \epsilon \frac{n_{PBH} L_{\text{edd}}}{H\rho_R} = \epsilon \frac{\rho_{PBH}}{\rho_R} \frac{4\pi Gm_p}{\sigma_T H} = \epsilon f_{PBH} \left( \frac{a}{a_{eq}} \right)^3 \frac{4\pi Gm_p}{\sigma_T H_{eq}}$$
$$\simeq 10^{-4} \epsilon f_{PBH} \left( \frac{a}{a_{eq}} \right)^3; \quad f_{PBH} = \frac{\Omega_{PBH}}{\Omega_{CDM}}$$

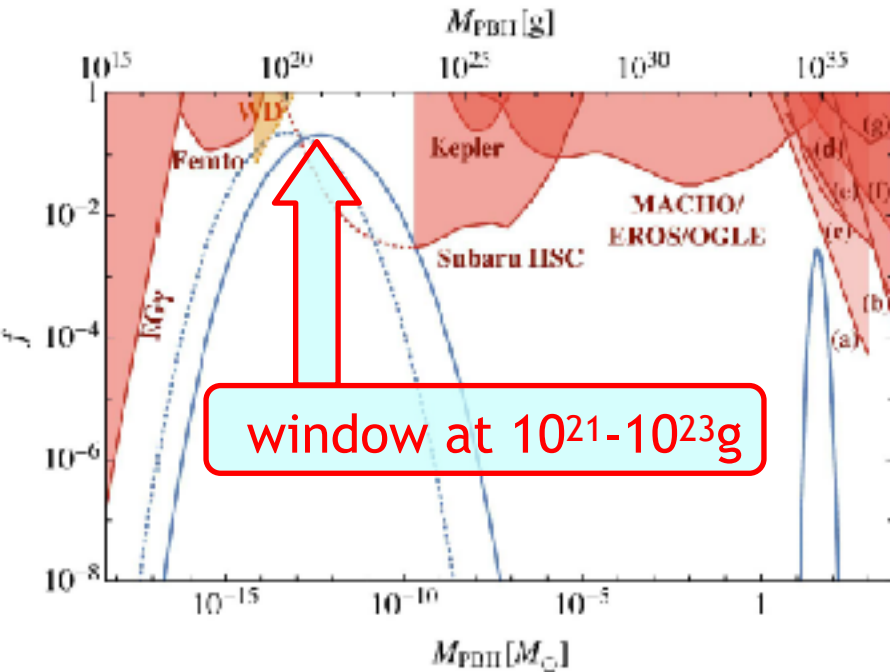
small, but may not be entirely negligible...



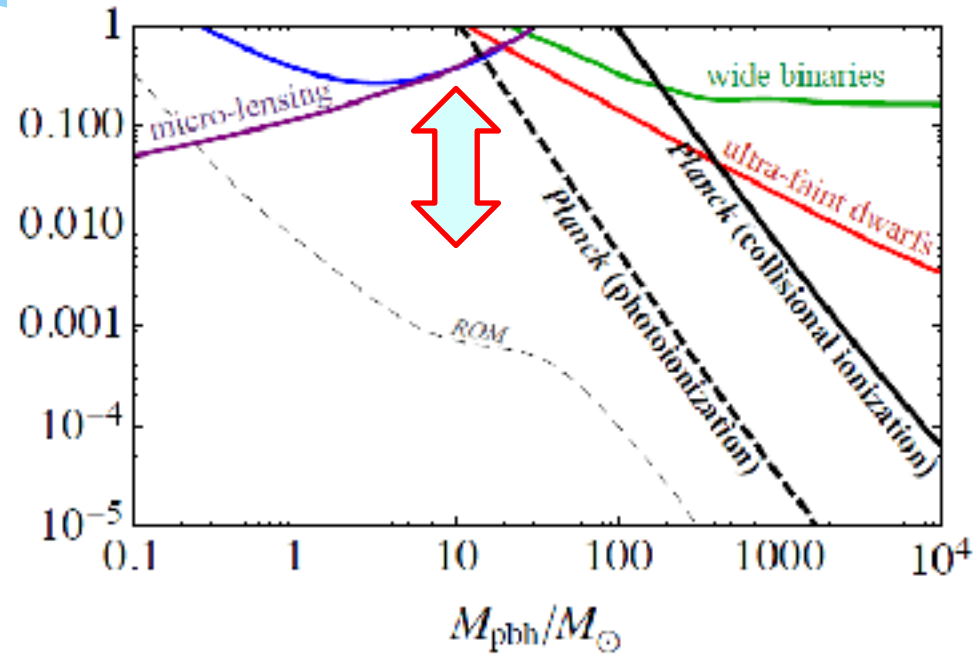
# Constraints on PBHs

LIGO BBHs may occupy ~10% of DM

Can DM be PBHs?



Inomata et al., 1711.06129  
(2-field model)

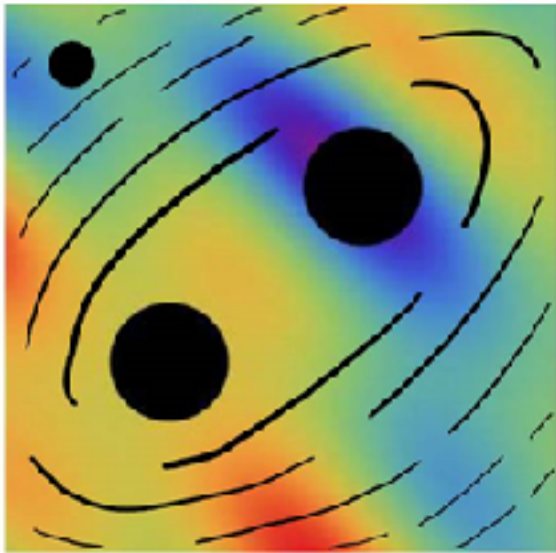


Ali-Haimoud & Kamionkowski, 1612.05644

Ricotti, Ostriker & Mack ('08) overestimated the accretion effect

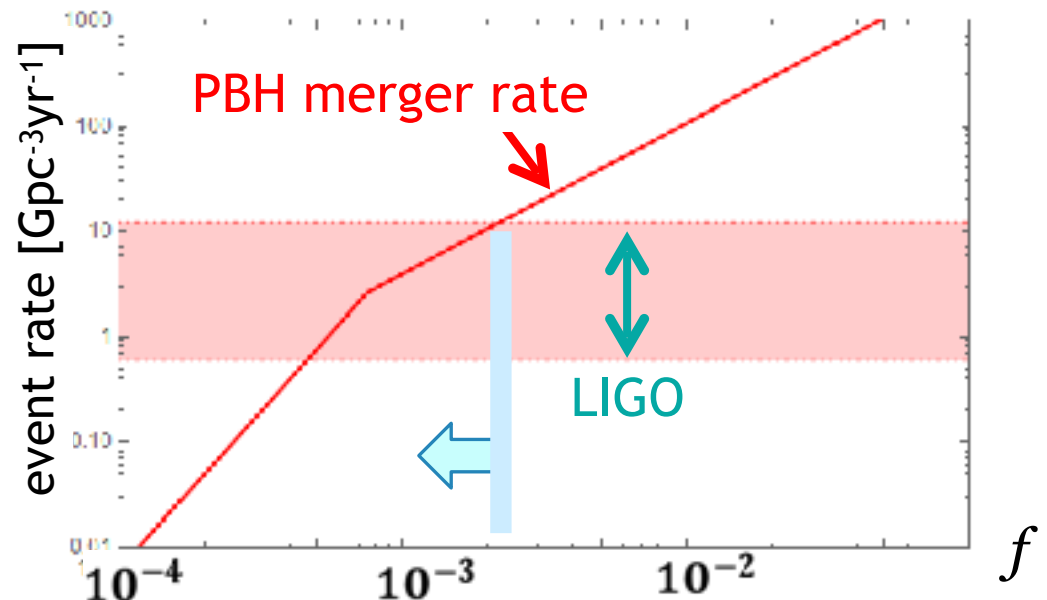
# LIGO BHs = PBHs?

MS, Suyama, Tanaka & Yokoyama '16



3-body interaction  
leads to formation of  
BH binaries

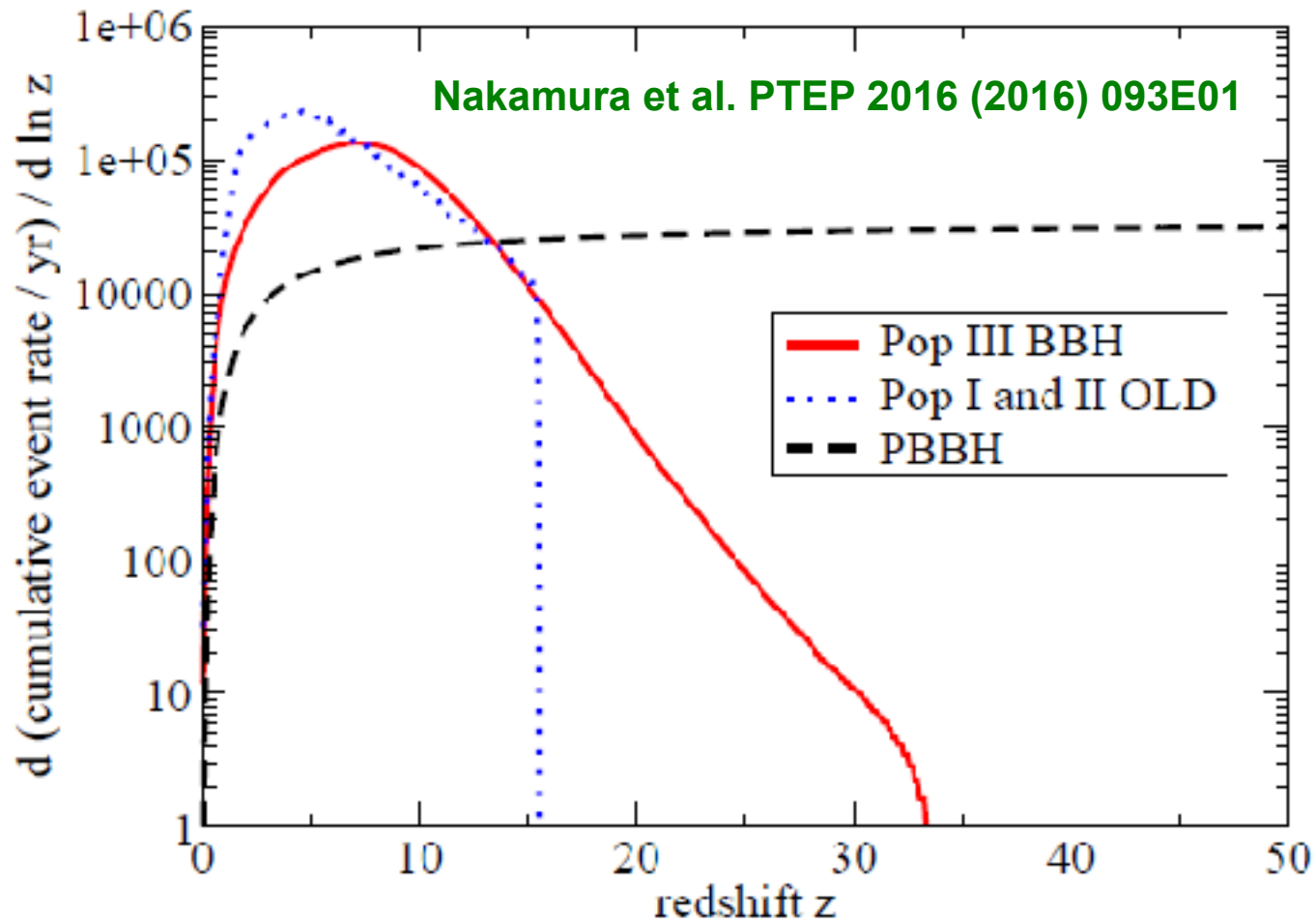
$$M_{PBH} \simeq 20 \left( \frac{k}{\text{kpc}^{-1}} \right)^{-2} M_{\odot} \simeq 20 \left( \frac{100 \text{MeV}}{T} \right)^2 M_{\odot}$$



$f$  = fraction of PBH in dark matter  
tightest constraint at  $M \sim 10M_{\odot}$

(cf. Ali-Haïmoud et al., 1709.06576) 18

# testing PBH hypothesis



# testing PBH hypothesis 2

Kocsis, Suyama, Tanaka, Yokoyama, arXiv:1709.09007

BBH Merger Rate at time  $t$ : mass function

$$\mathcal{R}(m_1, m_2, t) = \frac{n_{\text{BH}}}{2} f(m_1) f(m_2) P_{\text{intr}}(m_1, m_2, t)$$

intrinsic probability

$$P_{\text{intr}}(m_1, m_2, t) \propto g(m_1) g(m_2) m_t^\alpha : m_t = m_1 + m_2$$

$$\Leftrightarrow \alpha(m_1, m_2, t) \equiv -m_t^2 \frac{\partial^2}{\partial m_1 \partial m_2} \ln \mathcal{R}(m_1, m_2, t)$$

- PBH binary scenario

$$\frac{36}{37} < \alpha < \frac{22}{21}$$

- Dynamical formation in dense stellar systems

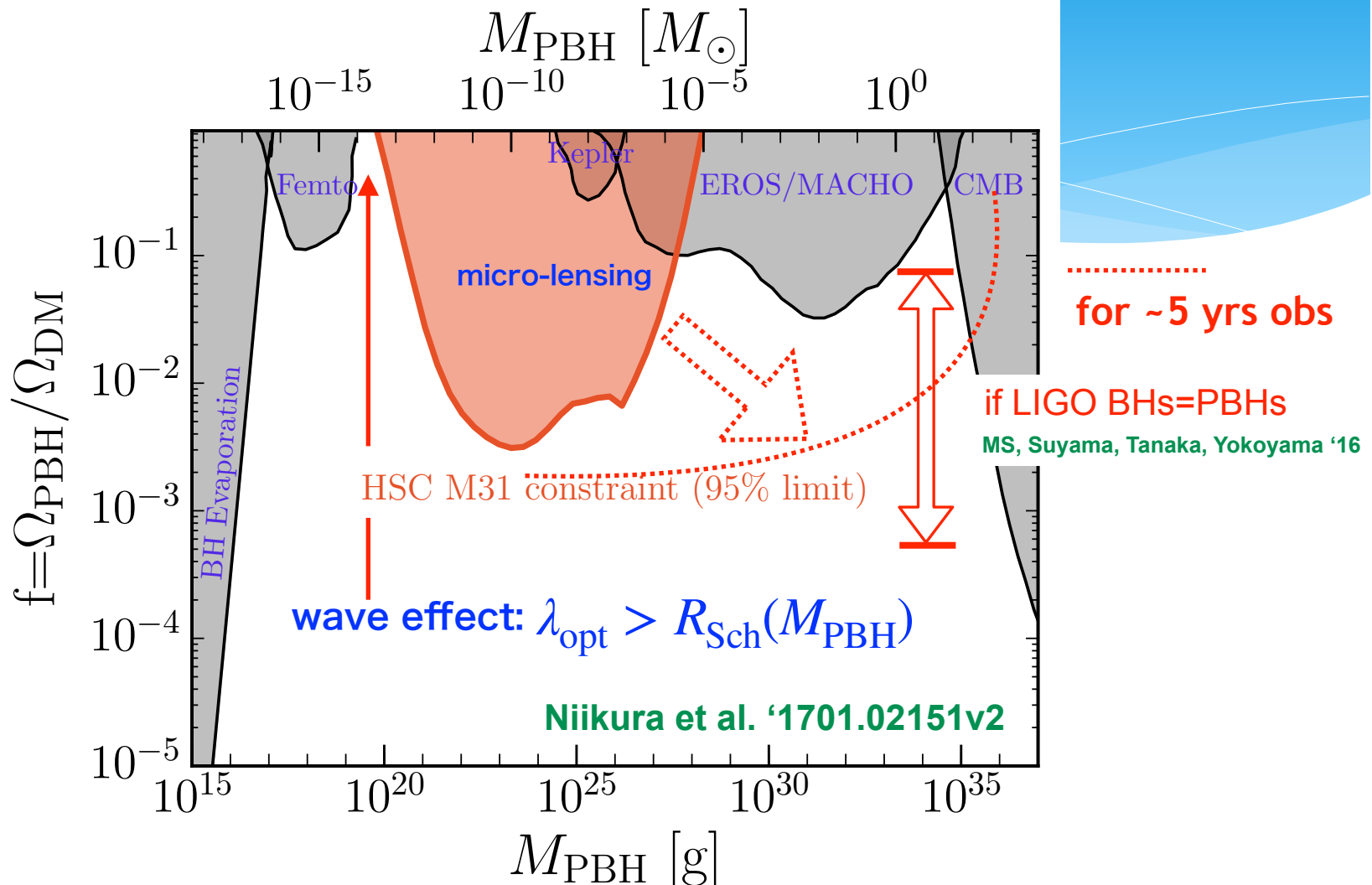
$$\alpha \approx 4$$

clearly distinguishable!

O'Leary et al (2016)

PBHs = CDM?

# PBH constraints: revised



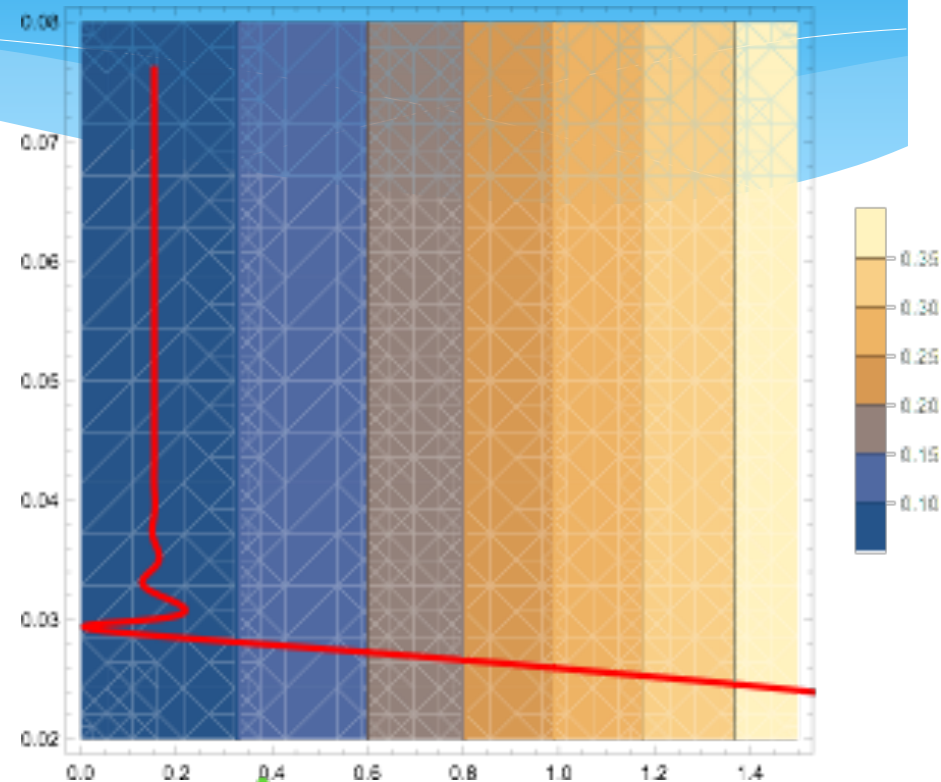
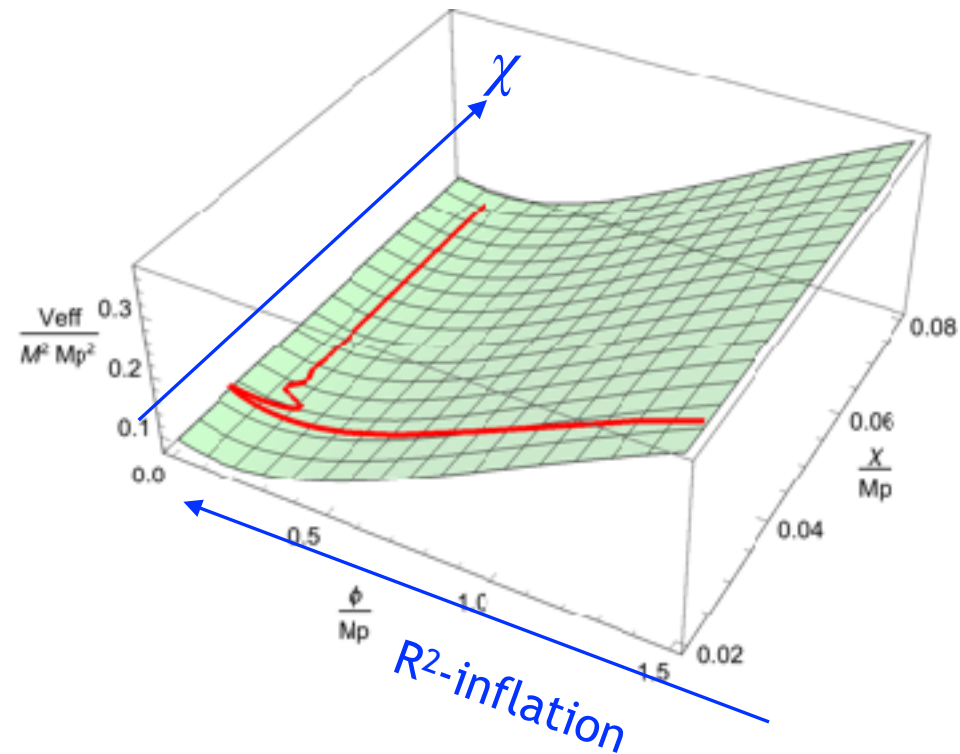
only narrow window at  $M_{\text{PBH}} \approx 10^{20} \text{ g}$

$\longleftrightarrow T_{\text{re-entry}} \sim 10^3 \text{ TeV}$

# monochromatic PBH production

Zhang, Hwang, Pi & MS '18

$$\mathcal{L} = R + \frac{R^2}{6M^2} - \frac{1}{2}(\partial\chi)^2 - V(\chi)$$



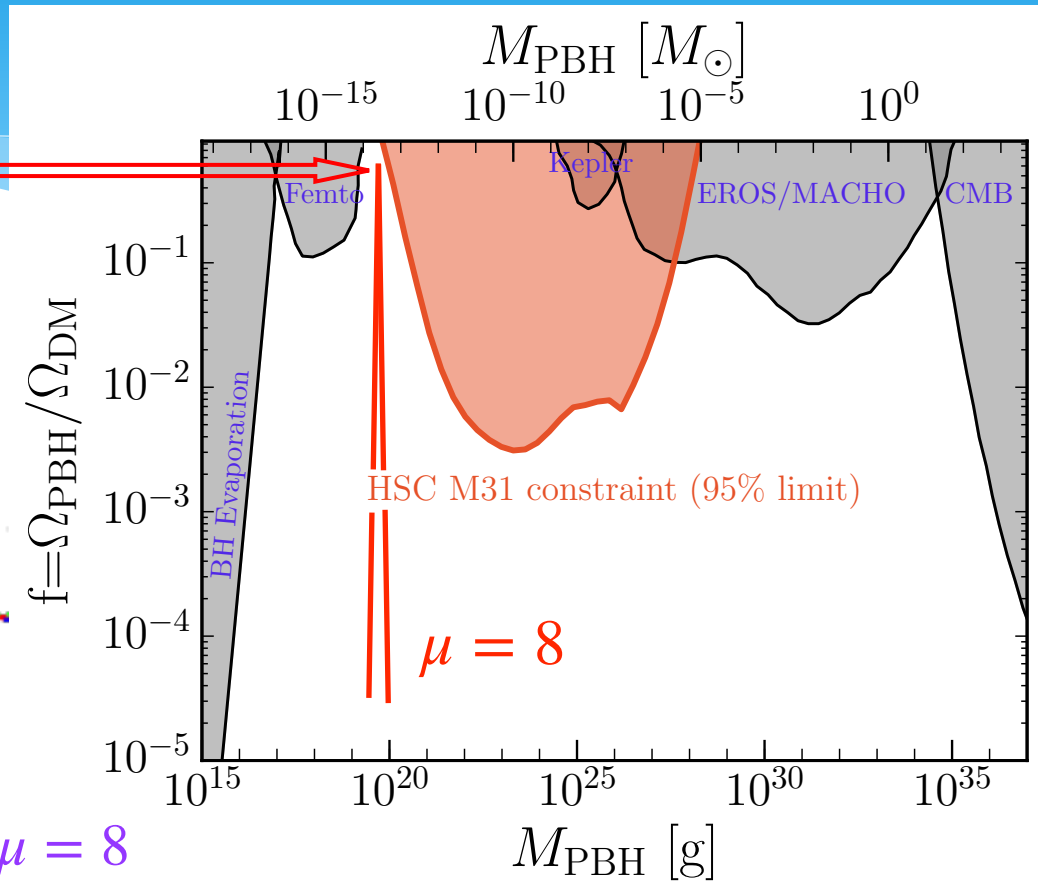
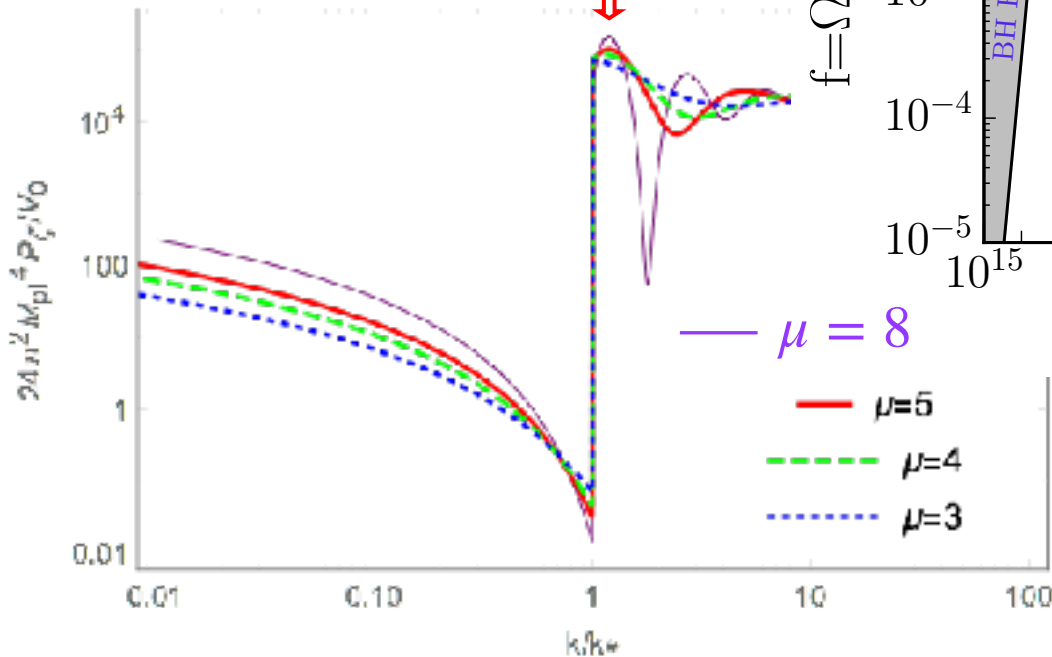
End of the 1st stage of inflation

sharp peak in  $P(k)$   $\rightarrow$  spike in  $f$

$$f \propto \exp \left[ -\frac{1}{\mathcal{P}(k)} \right]$$

spike

sharp peak



$$\mu^2 \approx \frac{H_{2\text{nd}}^2}{H_{1\text{st}}^2}$$



# 2<sup>nd</sup> order GW constraints on PBH

Saito & Yokoyama '09, Alabidi et al. '12, ...

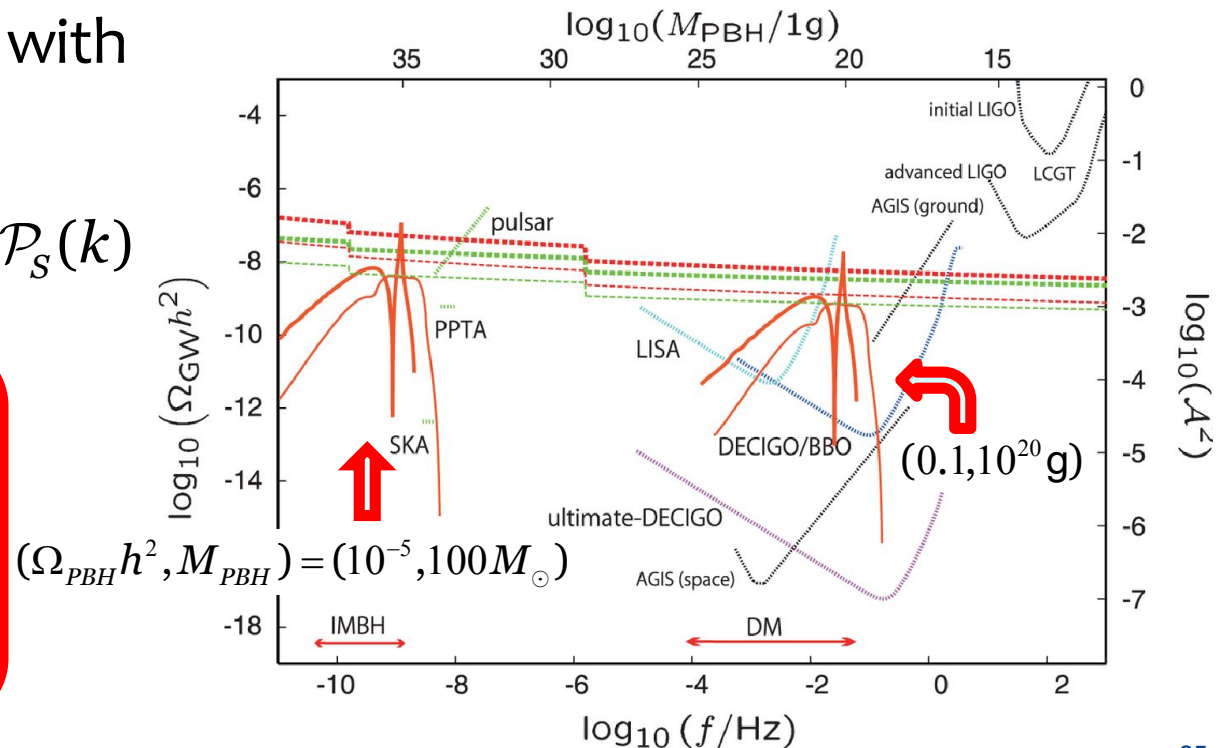
- Non-negligible PBH formation means  $\mathcal{P}_S(k) \sim 10^{-2.5} - 10^{-2}$

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - a^{-2}\Delta h_{ij} = S_{ij} \quad S_{ij} \simeq \frac{1}{a^2} \partial_i \mathcal{R}_c \partial_j \mathcal{R}_c + \dots \sim \frac{k^2}{a^2} \mathcal{P}_S(k)$$

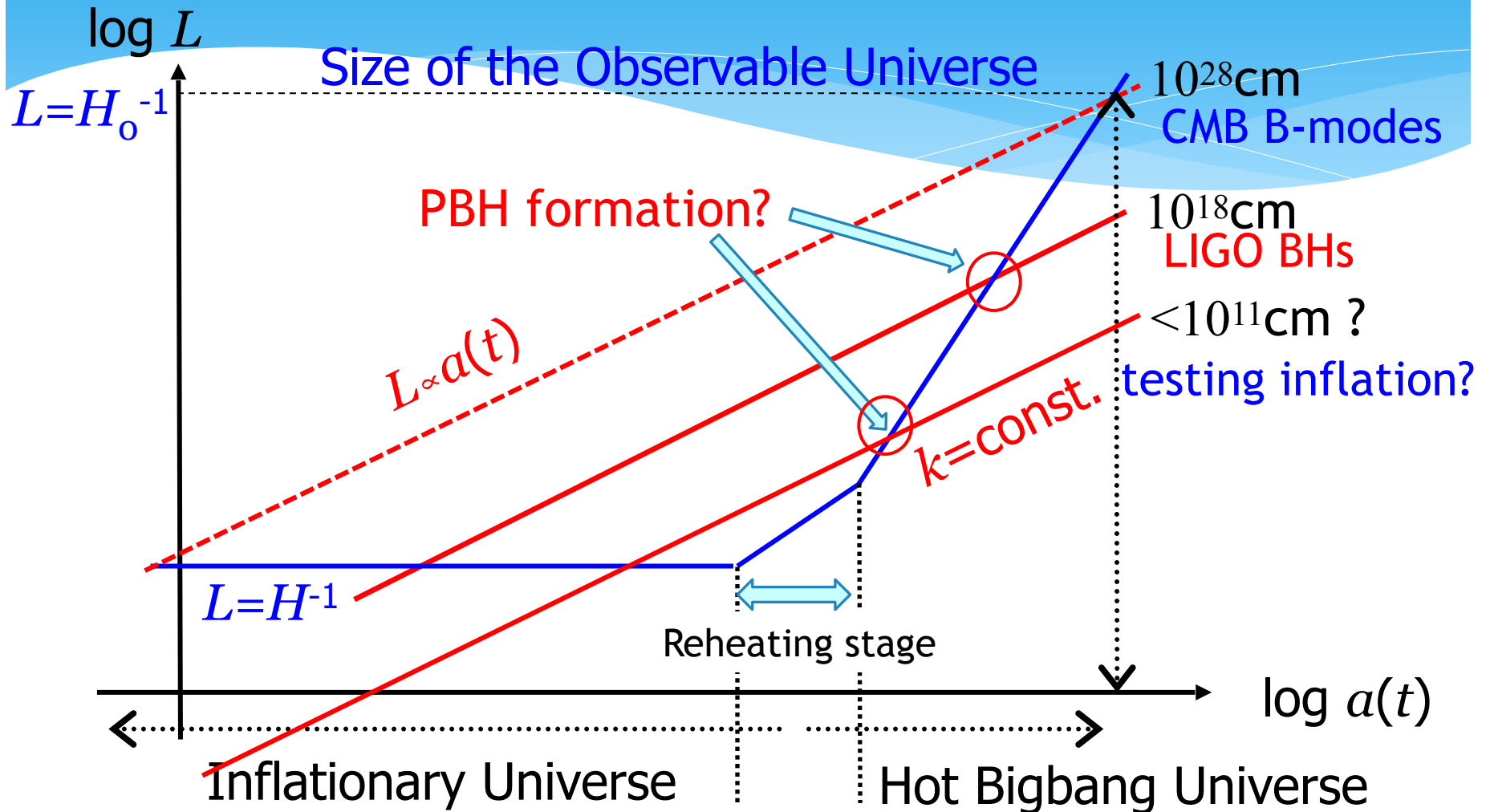
- GWs are produced with amplitude:

$$h_{ij} \sim \frac{k^2}{a^2 H^2} \mathcal{P}_S(k) \sim \mathcal{P}_S(k)$$

2<sup>nd</sup> order GWs would dominate at  $f > 10^{-10}$  Hz ( $k > 10^4$  Mpc<sup>-1</sup>)



# testing inflation by GW astronomy



# Summary

- \* **Inflation** has become the **standard model** of the Universe.  
further tests are needed to confirm inflation.
- \* Inflation can produce **large curvature perturbation on small scales**.  
**PBHs** are virtually the only probe on very small scales.
- \* **LIGO BHs** may be **primordial**.  
advanced GW detectors(+G lensing) will prove/disprove the scenario.
- \* **CDM** can be dominated by PBHs of  $M \sim 10^{20} \text{g}$ .  
**secondary** GWs may be detected by future GW detectors.
- \* **Multi-frequency GW** astronomy/astrophysics is arriving!

GWs will be **an essential tool** for proving/  
falsifying PBH scenarios