

# Theoretical Developments in Dark Matter-Nucleon Scattering

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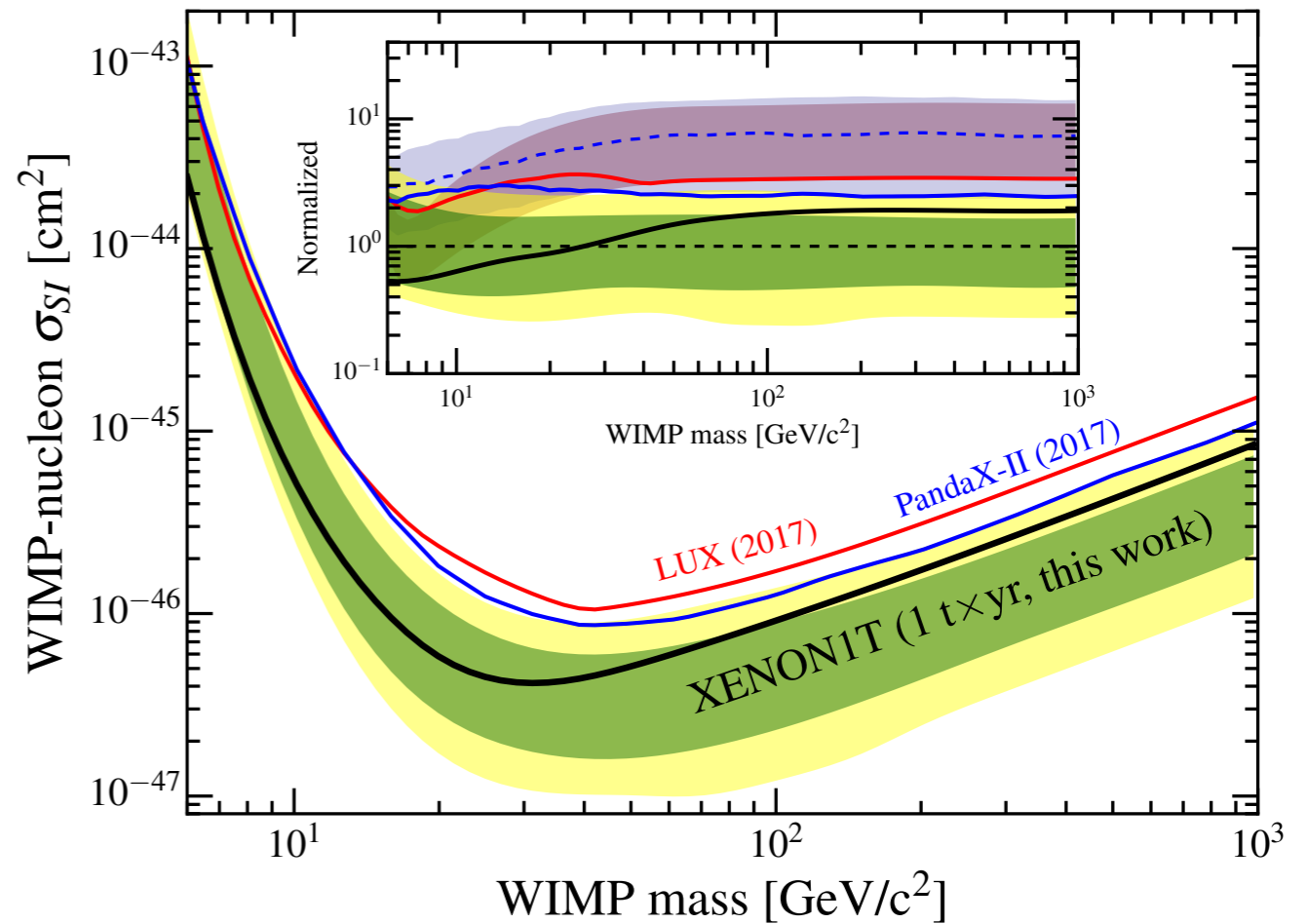
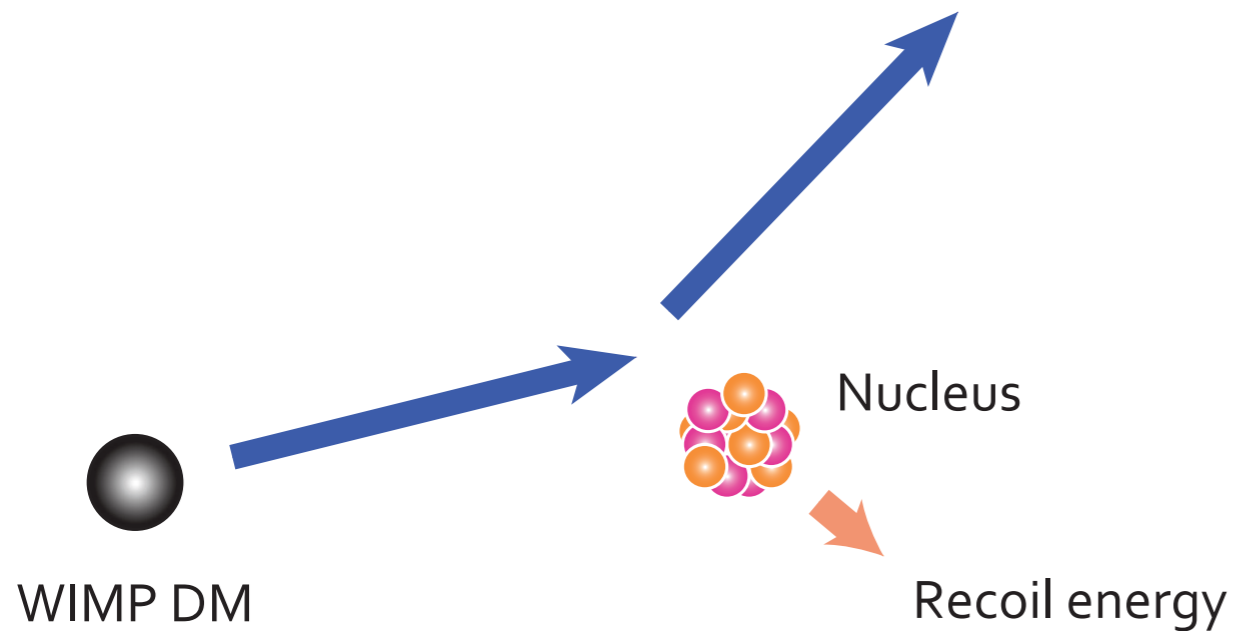
東京大学  
THE UNIVERSITY OF TOKYO

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LAPTh, Annecy, France

# DM Direct Detection experiments



XENON Collaboration, arXiv: 1805.12562.

Ranny Budnik's talk.

Detect recoil energy of DM-nucleus scattering.

Experimental sensitivity has been significantly improving.

# Theory side

On the theoretical side, we calculate

DM-nucleon scattering cross section

## Challenges

- ▶ Nucleon matrix elements.
- ▶ Appropriate treatment for perturbation.
  - Effective theoretical approach.
  - Separation of short/long-distance effects.
  - Renormalization group equations (RGEs).

# Nucleon matrix elements

Nucleon matrix elements of quark scalar operators are defined by

## Mass fractions

$$\langle N | m_q \bar{q}q | N \rangle / m_N \equiv f_{T_q}$$

( $m_N$  : nucleon mass)

We extract these values from

$$\Sigma_{\pi N} \equiv \frac{1}{2}(m_u + m_d) \langle N | \bar{u}u + \bar{d}d | N \rangle$$

$$\sigma_s \equiv \langle N | m_s \bar{s}s | N \rangle$$

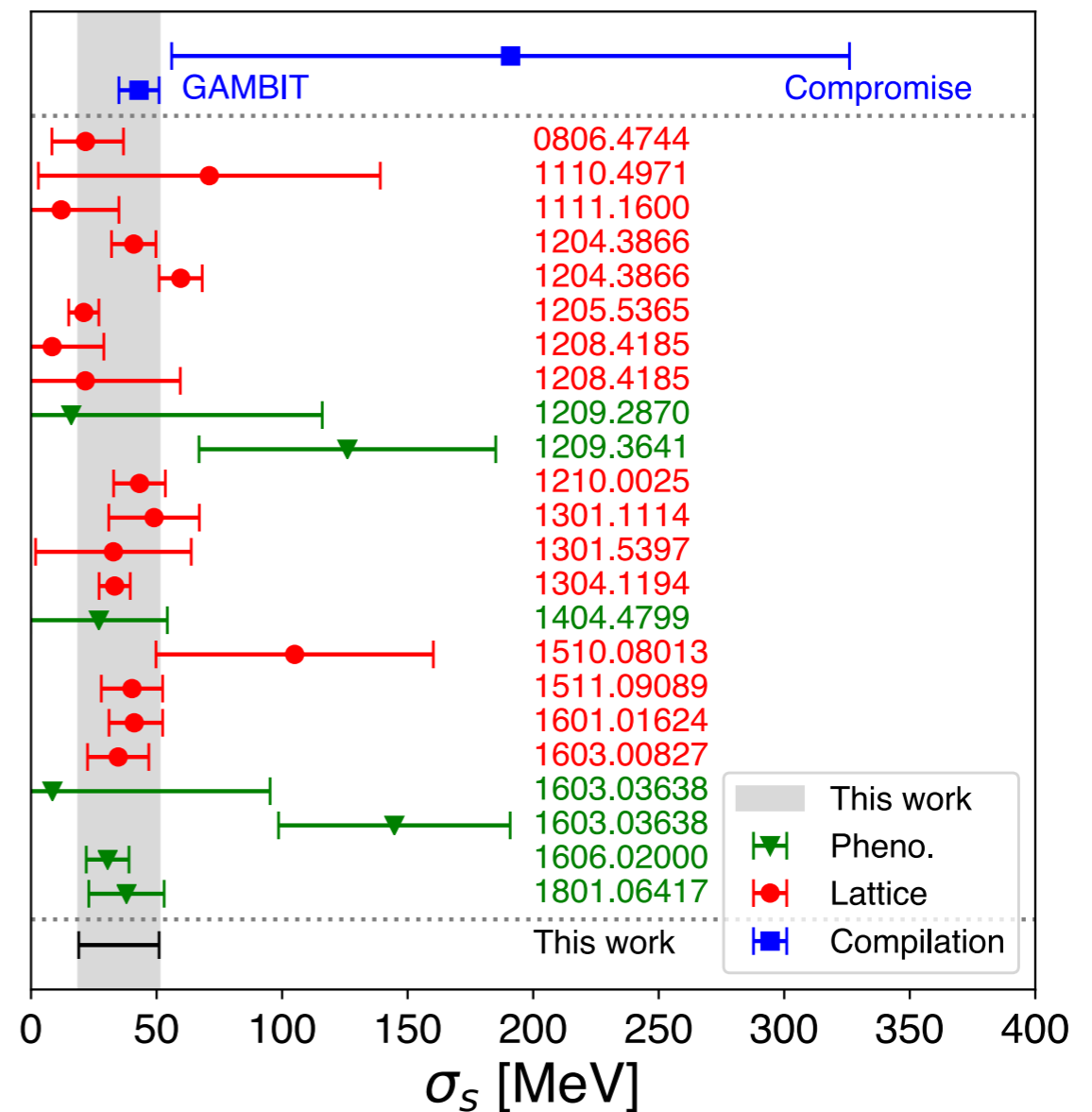
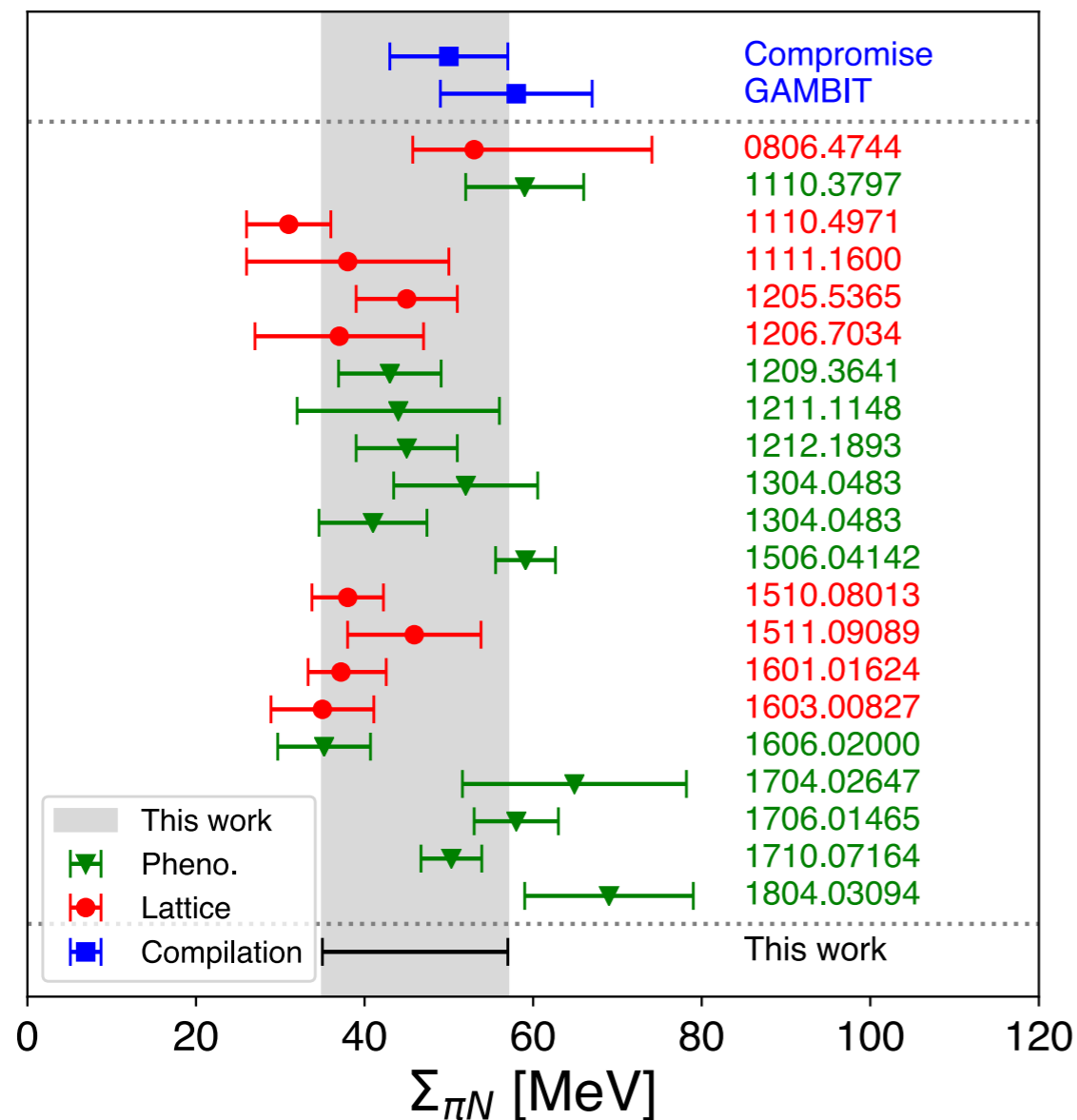
- Lattice simulation
- Chiral perturbation
- Low-energy  $\pi N$  scattering  
*etc.*

Cf.)

$$\sigma_0 = \frac{1}{2}(m_u + m_d) \langle N | \bar{u}u + \bar{d}d - 2\bar{s}s | N \rangle$$

Yields large uncertainty.

# Recent computations



$$\Sigma_{\pi N} = 46 \pm 11 \text{ MeV}$$


$$\sigma_s = 35 \pm 16 \text{ MeV}$$

# Heavy quark/gluon contribution

Heavy quark/gluon contributions are evaluated with

## Trace anomaly of the energy-momentum tensor

$$\Theta_{\mu}^{\mu} = \frac{\beta(\alpha_s)}{4\alpha_s} G_{\mu\nu}^A G^{A\mu\nu} + (1 - \gamma_m) \sum_q m_q \bar{q}q$$



$m_N$        $\langle N | G_{\mu\nu}^A G^{A\mu\nu} | N \rangle$        $m_N f_{Tq}$

► At leading order

$$\langle N | \alpha_s G_{\mu\nu}^A G^{A\mu\nu} | N \rangle = -\frac{8\pi}{9} m_N f_{TG} \qquad 1 - \sum_{q=u,d,s} f_{Tq} \equiv f_{TG}$$

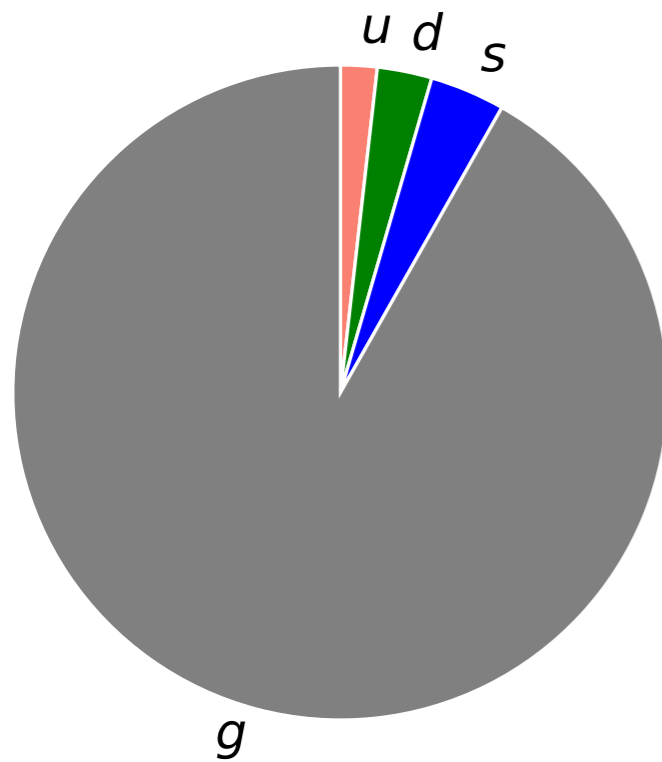
► Perturbative QCD corrections

$\mathcal{O}(\alpha_s^3)$  corrections available.

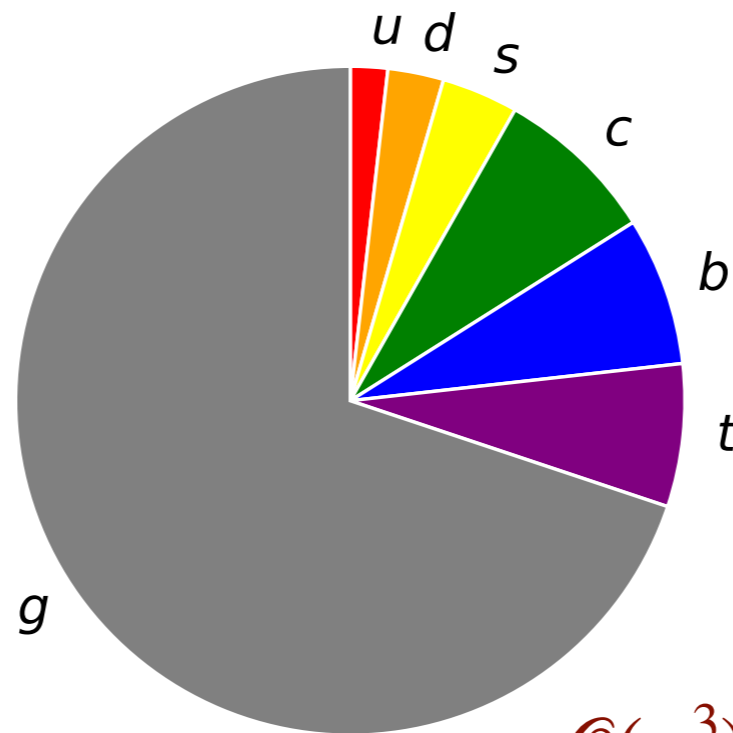
e.g.) L. Vecchi, arXiv: 1312.5695.

# Our compilation for mass fractions

## Three quarks



## Six quarks



$\mathcal{O}(\alpha_s^3)$  pQCD

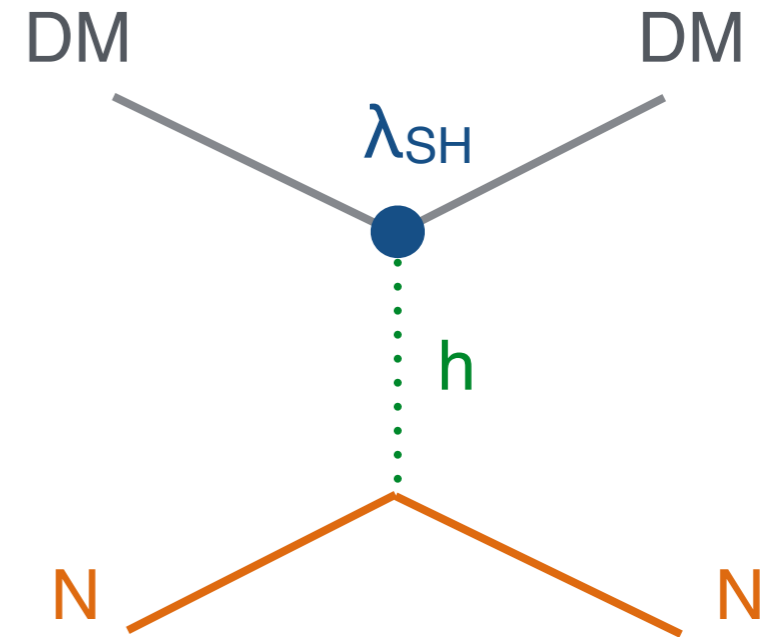
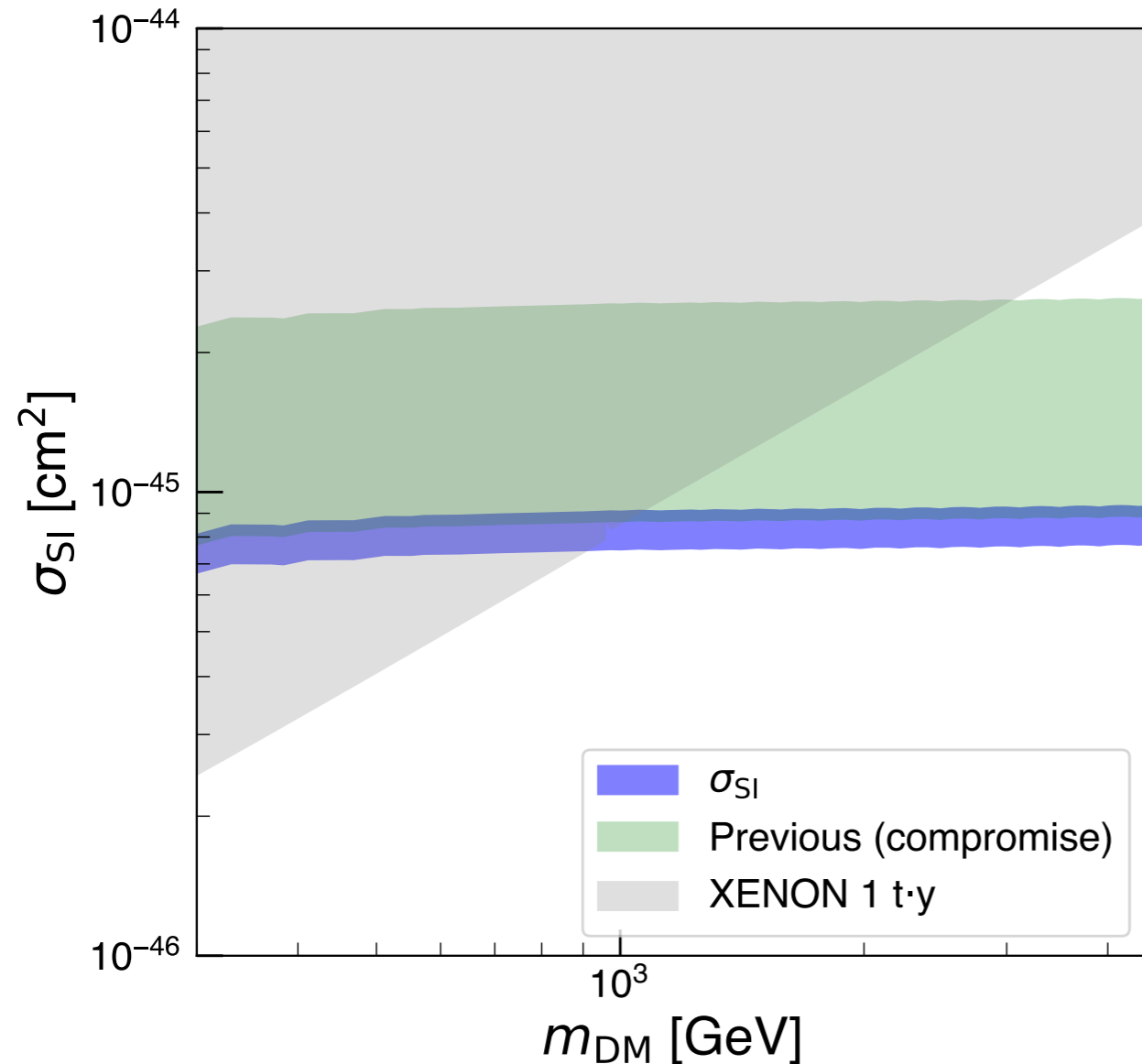
| Proton          |           | Neutron         |           |
|-----------------|-----------|-----------------|-----------|
| $f_{T_u}^{(p)}$ | 0.017(4)  | $f_{T_u}^{(n)}$ | 0.014(3)  |
| $f_{T_d}^{(p)}$ | 0.030(7)  | $f_{T_d}^{(n)}$ | 0.037(9)  |
| $f_{T_s}^{(p)}$ | 0.037(17) | $f_{T_s}^{(n)}$ | 0.037(17) |
| $f_{T_G}^{(p)}$ | 0.916(19) | $f_{T_G}^{(n)}$ | 0.912(20) |

$$f_{T_c}^{(p)} = 0.078(2) \quad f_{T_c}^{(n)} = 0.078(2)$$

$$f_{T_b}^{(p)} = 0.072(2) \quad f_{T_b}^{(n)} = 0.071(2)$$

$$f_{T_t}^{(p)} = 0.069(2) \quad f_{T_t}^{(n)} = 0.068(2)$$

# Singlet real scalar DM

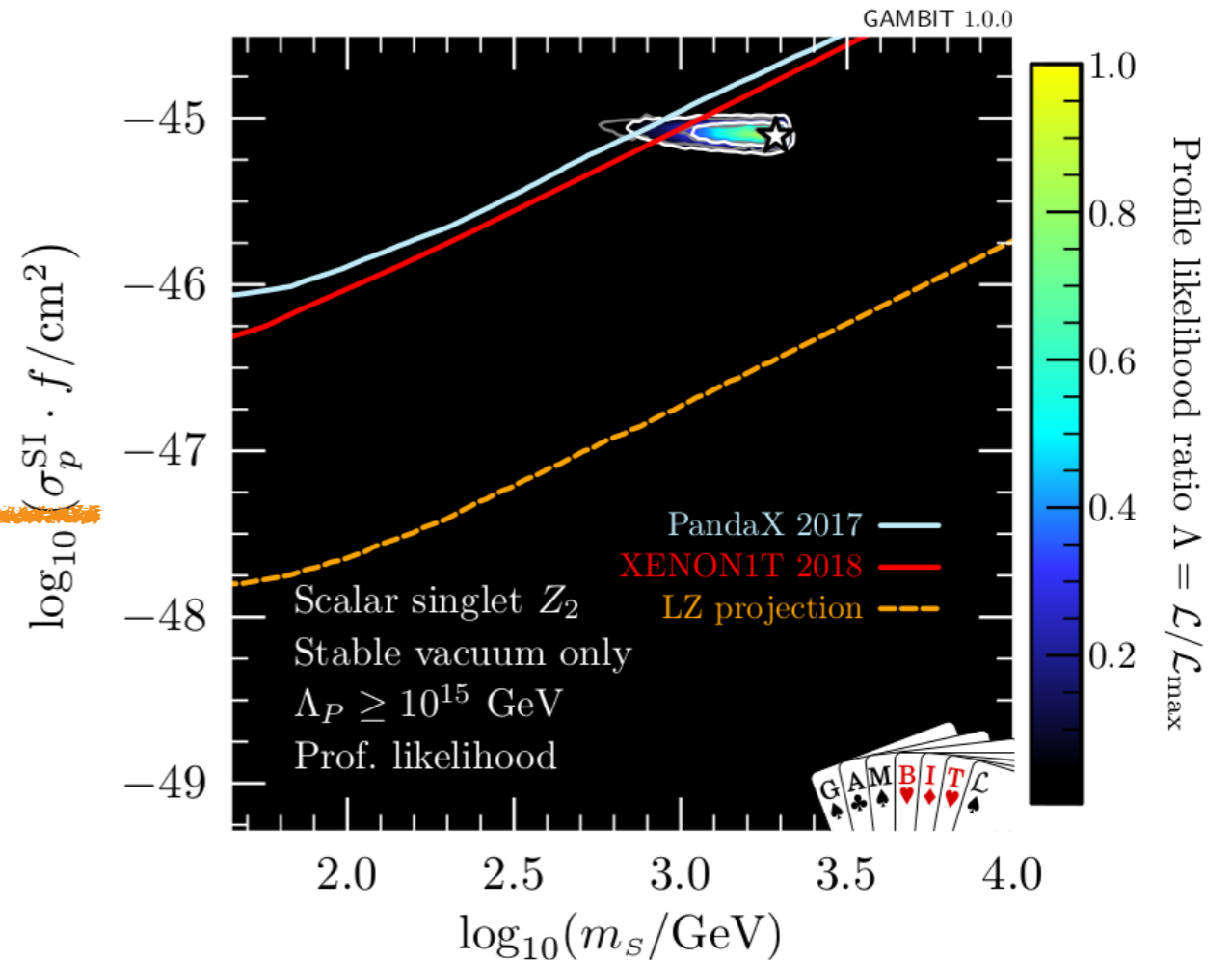
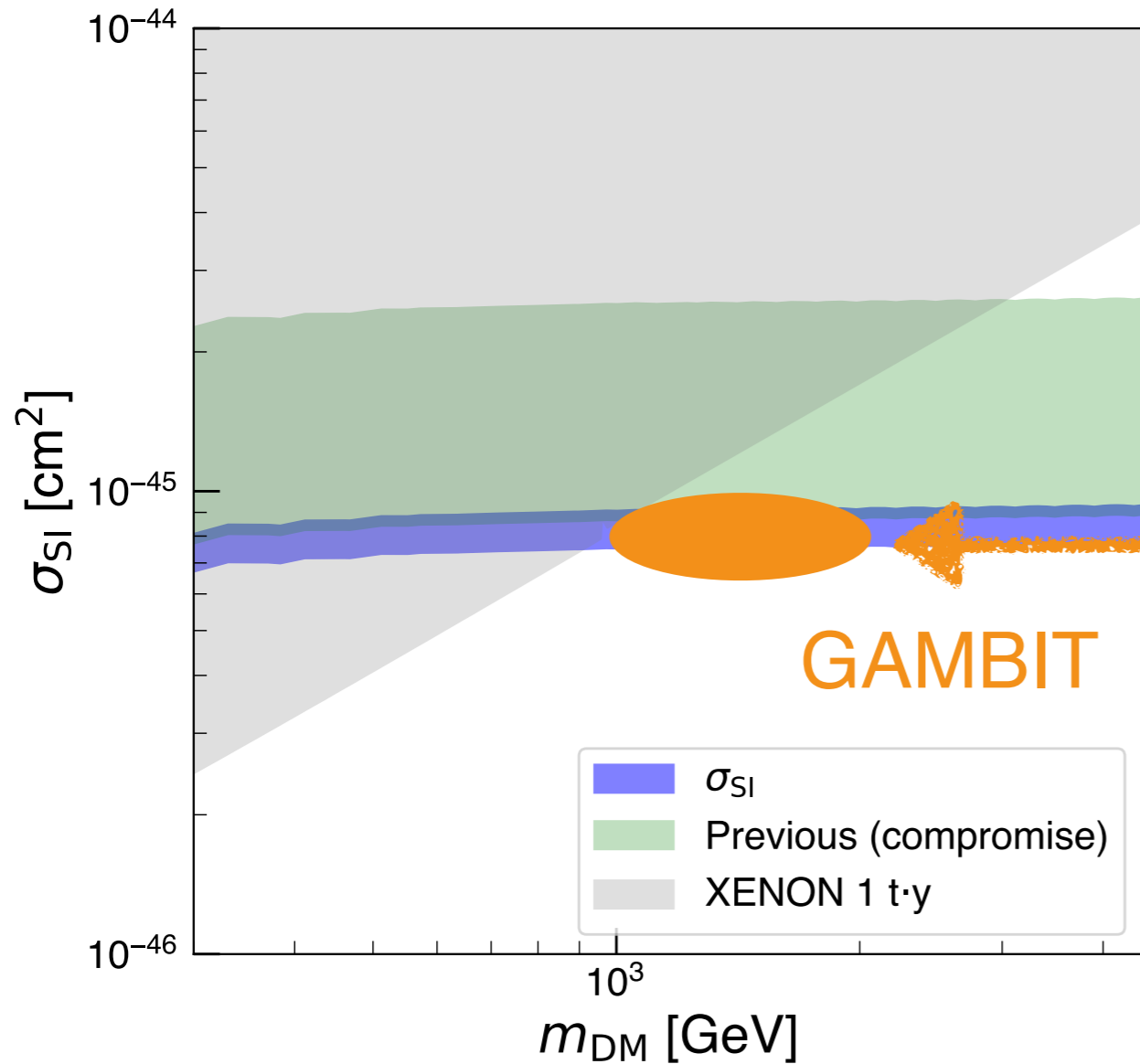


$\lambda_{SH}$  is taken so that DM abundance agrees to the observed value.

Uncertainty from nucleon matrix elements reduced to  $O(10)\%$  level.



# Singlet real scalar DM



Felix Kahlhoefer's talk.

In good agreement with the GAMBIT result.

# Perturbative calculation

To perform perturbative calculation at  $O(10)\%$  accuracy

## ▶ RGE/threshold corrections in QCD

- Effective theoretical approach.
- Separation of short/long-distance effects.

## ▶ Electroweak-loop corrections

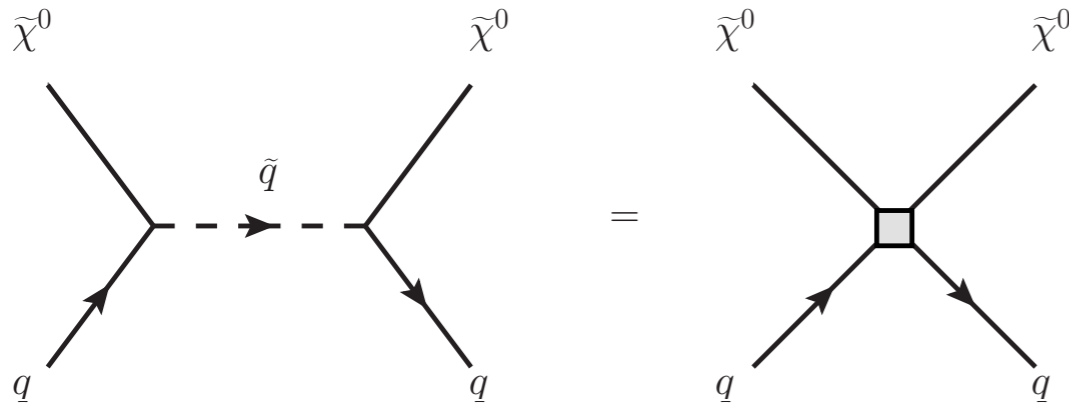
Can be important when

- DM has electroweak charge.
- Tree-level contribution is suppressed.

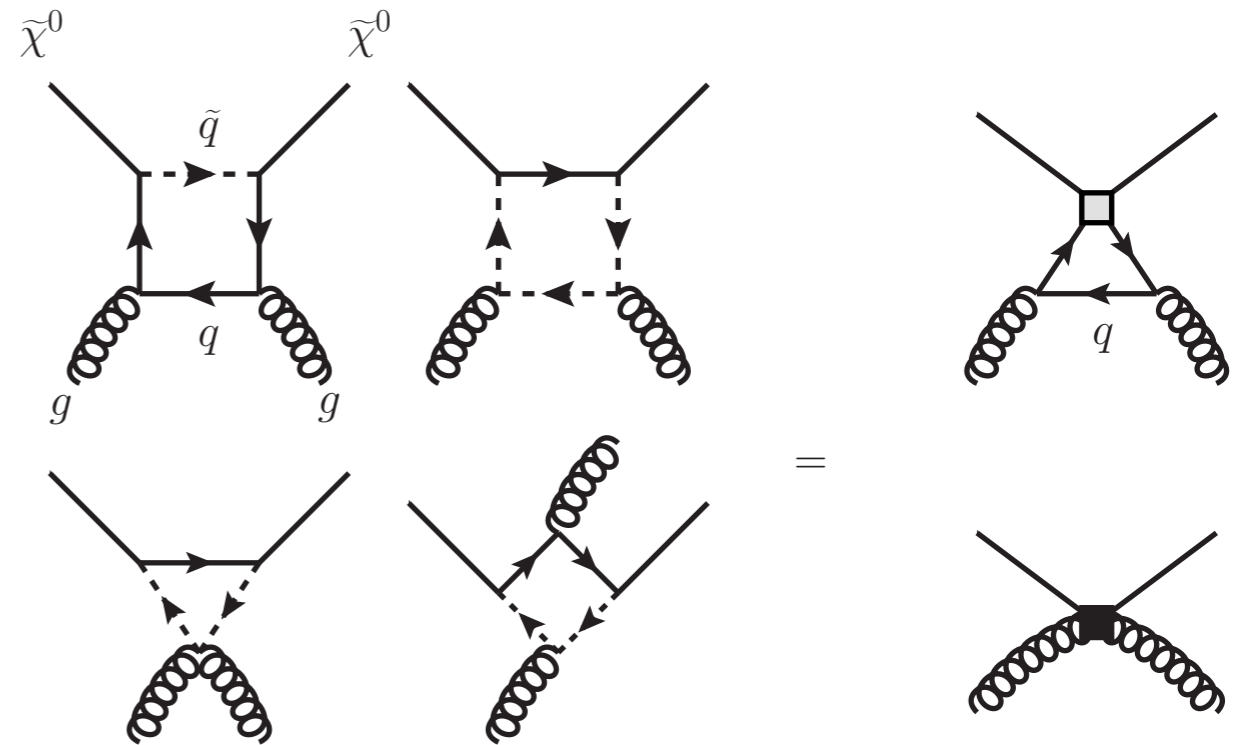
Wino in MSSM, Minimal DM, etc.

# Colored mediators

## Tree-level matching



## One-loop matching



Only the short-distance contribution included into Wilson coefficients.

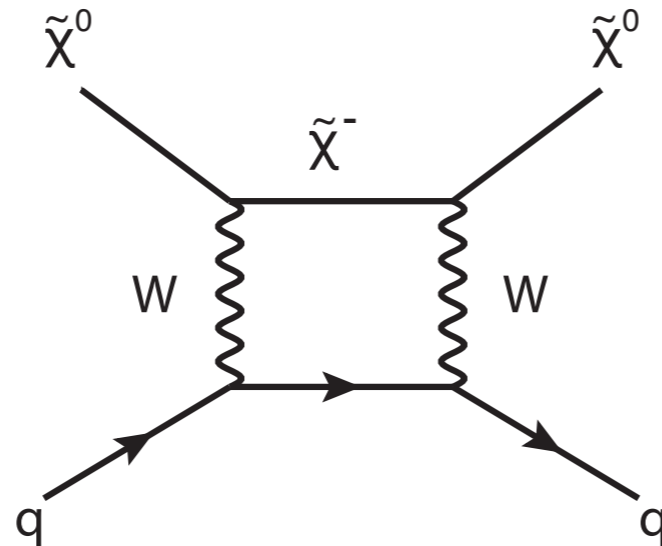
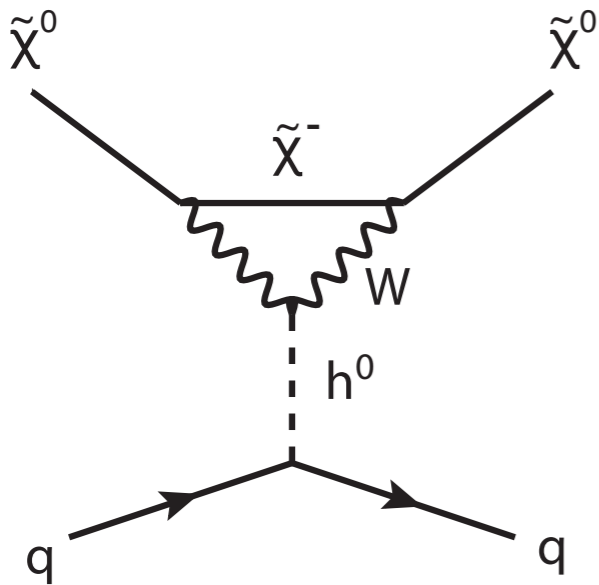
Operators are evolved according to RGEs.

Important especially when mediators are heavy.

➔ **O(10)% improvement.**

# Electroweak loop corrections (n = 3, Y = 0)

## 1-loop (quark)

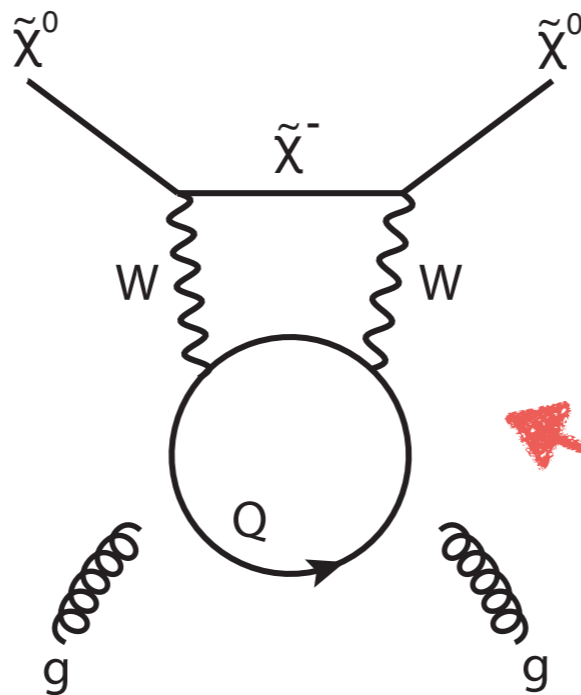
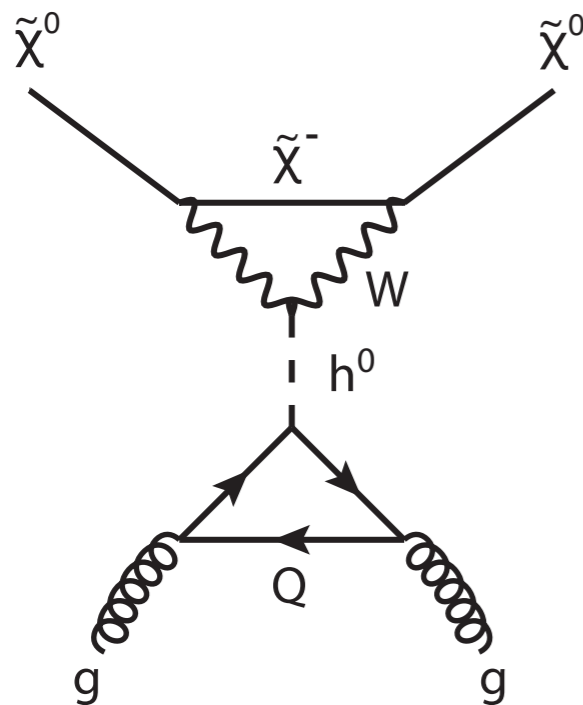


Non-decoupling effects

$$\mathcal{O}\left(\frac{\alpha_2^2}{m_W m_h^2}\right) \quad \mathcal{O}\left(\frac{\alpha_2^2}{m_W^3}\right)$$

J. Hisano, S. Matsumoto, M. Nojiri, O. Saito (2005)

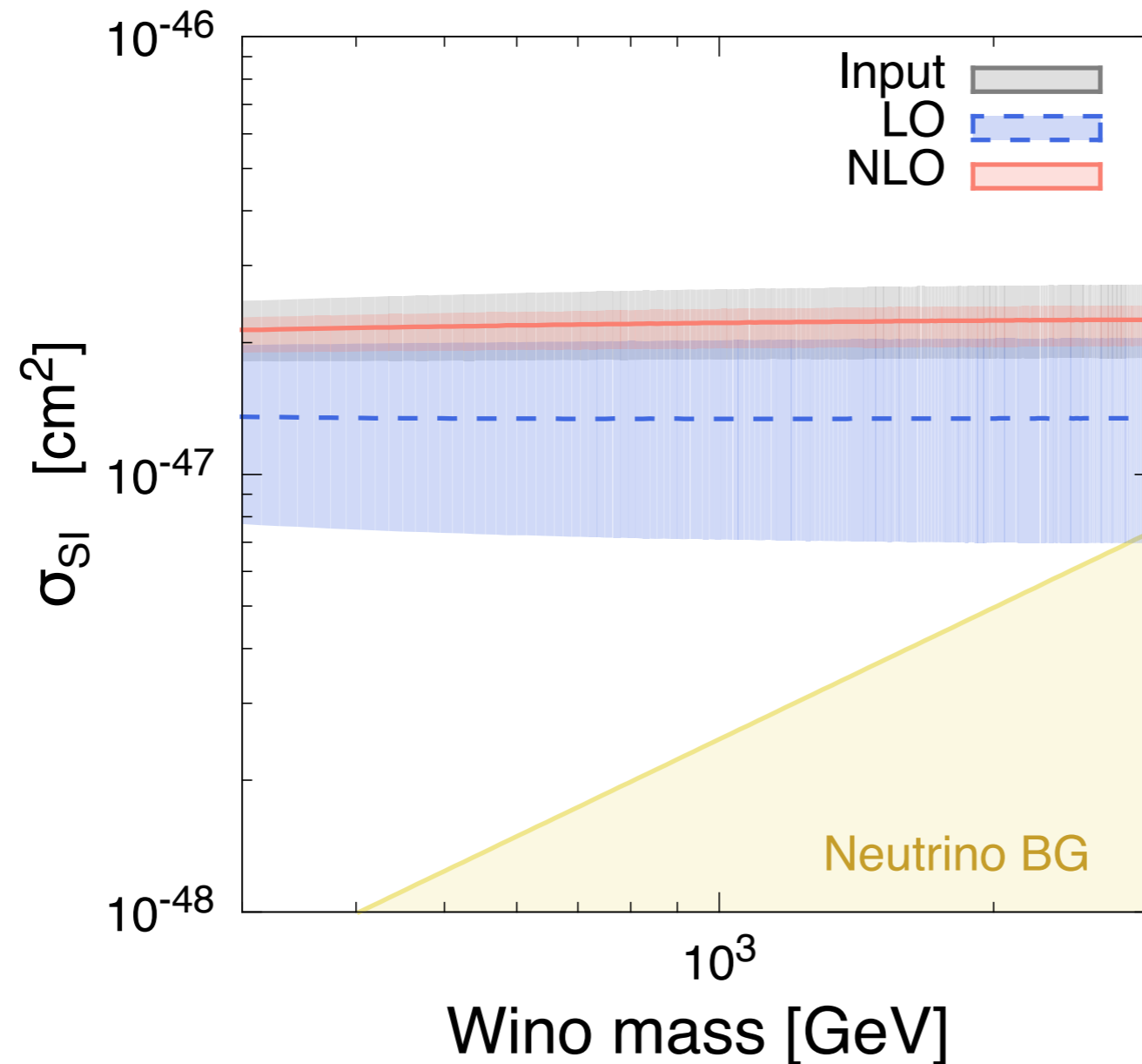
## 2-loop (gluon)



Comparable to the 1-loop contributions

J. Hisano, K. Ishiwata, N. Nagata, Phys. Lett. B690, 311 (2010)

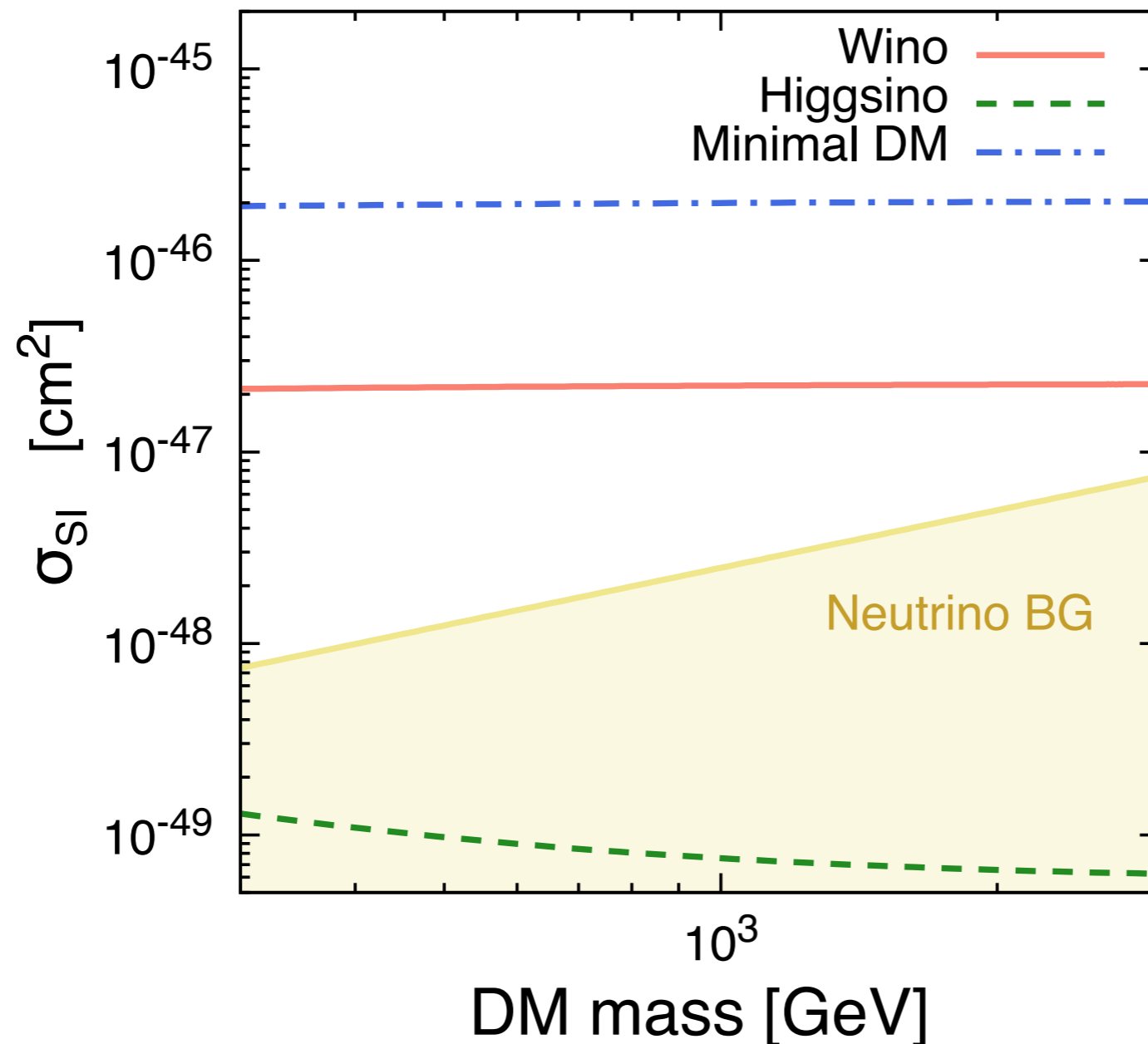
# NLO results



Error was large due to cancellation among various contributions.

With NLO QCD corrections included, uncertainty reduced to O(10)% level.

# NLO results



- ▶ **Triplet** and **quintuplet** cases can be probed.
- ▶ **Doublet** case is below the **neutrino floor**.

# Conclusion

- Direct DM searches are proceeding apace.
- To make the most of these experiments, theoretical uncertainties should be minimized.
- Uncertainties in nucleon matrix elements are controlled within  $O(10)\%$  level.
- Higher-loop corrections can be important.

**Backup**



# Recent computations

| Reference        | $\Sigma_{\pi N}$ | Uncertainties       | $\sigma_s$ | Uncertainties       | Method                          |
|------------------|------------------|---------------------|------------|---------------------|---------------------------------|
| [22]             | 50               | 7                   | 191        | 135                 | Compilation                     |
| [33]             | 58               | 9                   | 43         | 8                   | Compilation                     |
| [36]             | 53               | $2^{+21}_{-7}$      | 21.7       | $^{+15.1}_{-13.4}$  | Lattice                         |
| [37]             | 59               | 7                   |            |                     | B $\chi$ PT, $\pi$ atoms        |
| [38]             | 31               | $3 \pm 4$           | 71         | $34 \pm 59$         | Lattice                         |
| [39]             | 38               | 12                  | 12         | $^{+23}_{-16}$      | Lattice                         |
| [40]             |                  |                     | 40.9       | $7.5 \pm 4.7$       | Lattice                         |
|                  |                  |                     | 59.6       | $5.1 \pm 6.9$       | Lattice                         |
| [41]             | 45               | 6                   | 21         | 6                   | Lattice                         |
| [42]             | 37               | $8 \pm 6$           |            |                     | Lattice                         |
| [43]             |                  |                     | 8.4        | $14.1 \pm 15.0$     | Lattice                         |
|                  |                  |                     | 21.6       | $27.2 \pm 26.3$     | Lattice                         |
| [11]             |                  |                     | 16         | $80 \pm 60$         | B $\chi$ PT                     |
| [44]             | 43               | $1 \pm 6$           | 126        | $24 \pm 54$         | Lattice/B $\chi$ PT             |
| [45]             |                  |                     | 43.2       | 10.3                | Lattice                         |
| [46]             | 44               | 12                  |            |                     | $\pi$ N scattering              |
| [47]             | 45               | 6                   |            |                     | $\pi$ N scattering              |
| [48]             |                  |                     | 49         | $10 \pm 15$         | Lattice                         |
| [49]             |                  |                     | 32.8       | 31.0                | Lattice                         |
| [50]             | 52               | $3 \pm 8$           |            |                     | Lattice/B $\chi$ PT             |
|                  | 41               | $5 \pm 4$           |            |                     | Lattice/B $\chi$ PT             |
| [51]             |                  |                     | 33.3       | 6.2                 | Lattice/B $\chi$ PT             |
| [52]             |                  |                     | 27         | $27 \pm 4$          | Lattice/B $\chi$ PT             |
| [53]             | 59.1             | $1.9 \pm 3$         |            |                     | $\pi$ atoms                     |
| [54]             | 38               | $3 \pm 3$           | 105        | $41 \pm 37$         | Lattice                         |
| [55]             | 45.9             | $7.4 \pm 2.8$       | 40.2       | $11.7 \pm 3.5$      | Lattice                         |
| [56]             | 37.2             | $2.6^{+4.7}_{-2.9}$ | 41.1       | $8.2^{+7.8}_{-5.8}$ | Lattice                         |
| [26]             | 35               | 6.1                 | 34.7       | 12.2                | Lattice                         |
| [57]             |                  |                     | 8.5        | $4.4 \pm 86.6$      | $\pi$ atoms, $\pi$ N scattering |
|                  |                  |                     | 144.7      | $4.6 \pm 45.9$      | $\pi$ atoms, $\pi$ N scattering |
| [58]             | 35.2             | 5.5                 | 30.5       | 8.5                 | B $\chi$ PT                     |
| [59]             | 64.9             | $1.5 \pm 13.2$      |            |                     | Lattice/B $\chi$ PT             |
| [60]             | 58               | 5                   |            |                     | $\pi$ N scattering              |
| [61]             | 50.3             | $1.2 \pm 3.4$       |            |                     | Lattice/B $\chi$ PT             |
| [62]             | 48               |                     | 38         | 15                  | Lattice/B $\chi$ PT             |
| [63]             | 69               | 10                  |            |                     | B $\chi$ PT                     |
| <b>This work</b> | <b>46</b>        | <b>11</b>           | <b>35</b>  | <b>16</b>           | <b>New compilation</b>          |

# PDG method

We use the method adopted by PDG to obtain average and error.

## Wighted average and error

$$\bar{x} + \delta\bar{x} = \frac{\sum_i w_i x_i}{\sum_i w_i} \pm \left( \frac{\chi^2}{(N-1) \sum_i w_i} \right)^{1/2}$$

with **weights** defined by  $w_i = 1/(\delta x_i)^2$  and  $\chi^2$

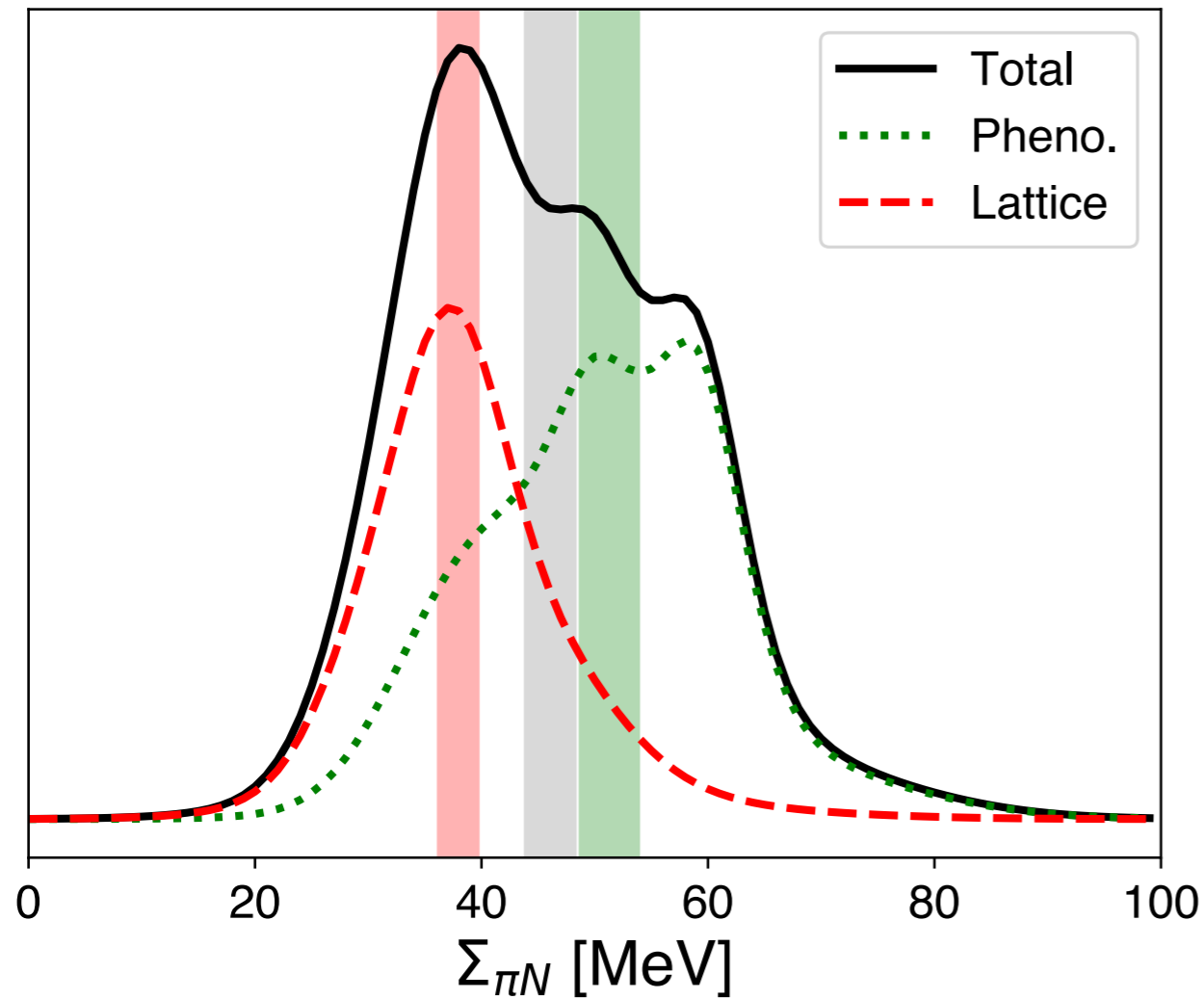
$$\chi^2 = \sum_i w_i (x_i - \bar{x})^2$$

## Ideogram

$$I(x) = \sum_i \frac{1}{\sqrt{2\pi\delta x_i^2}} \exp \left[ -\frac{(x - x_i)^2}{2\delta x_i^2} \right]$$

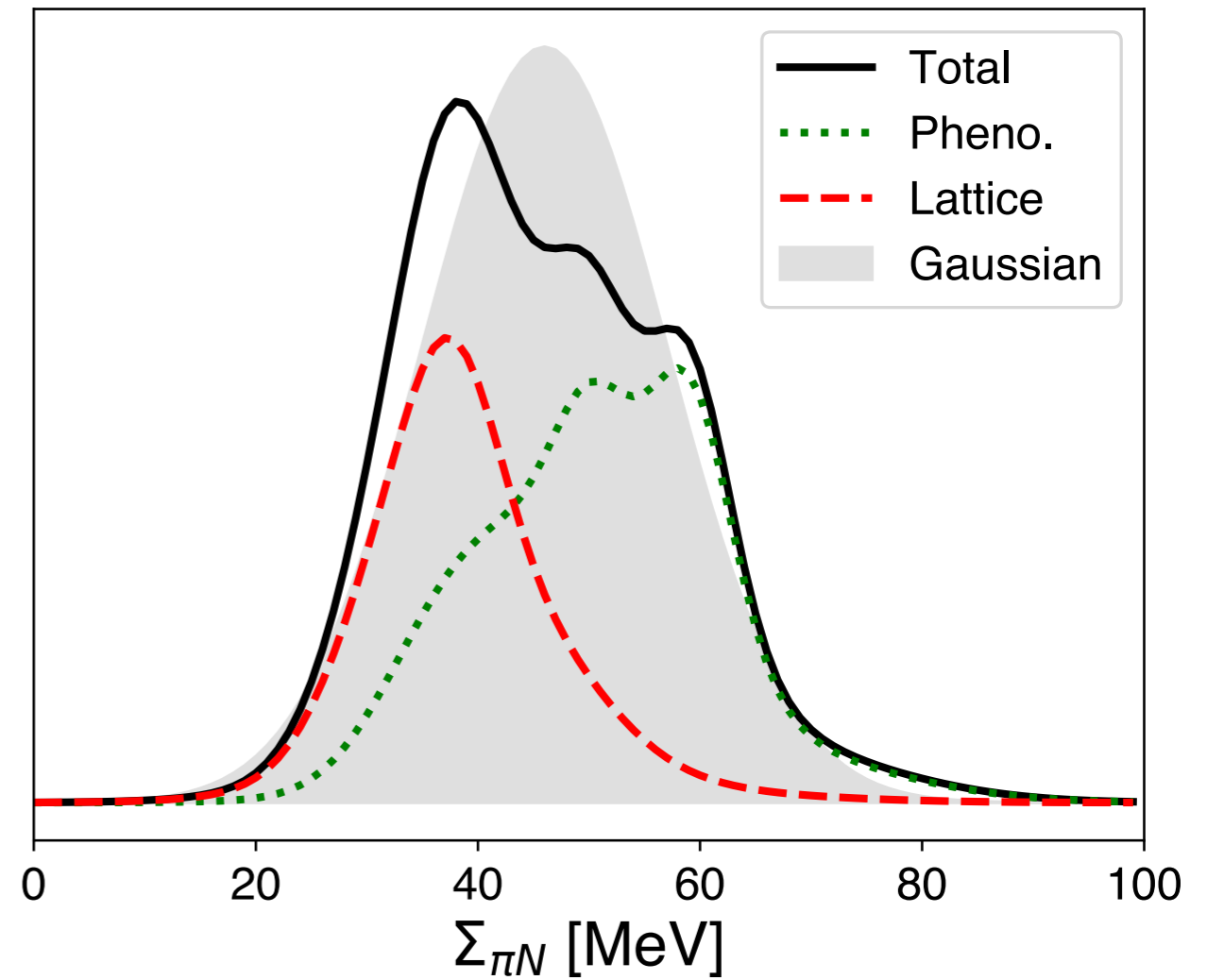
Sum of Gaussians with a central value  $x_i$ , error  $\delta x_i$ , and area proportional to  $1/\delta x_i$ .

# Ideogram: $\Sigma_{\pi N}$



PDG method

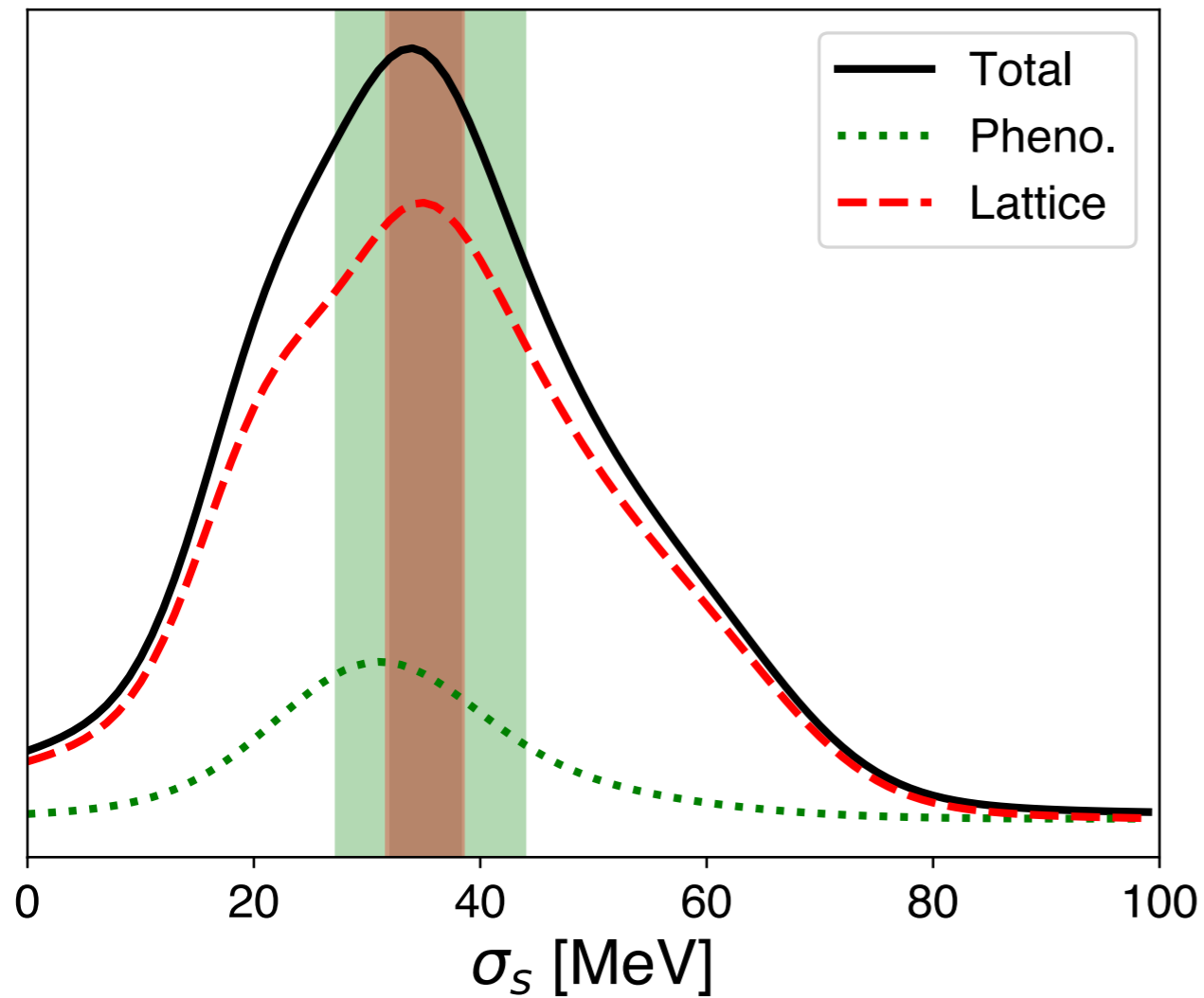
$$\Sigma_{\pi N} = 46.1 \pm 2.2 \text{ MeV}$$



Ours

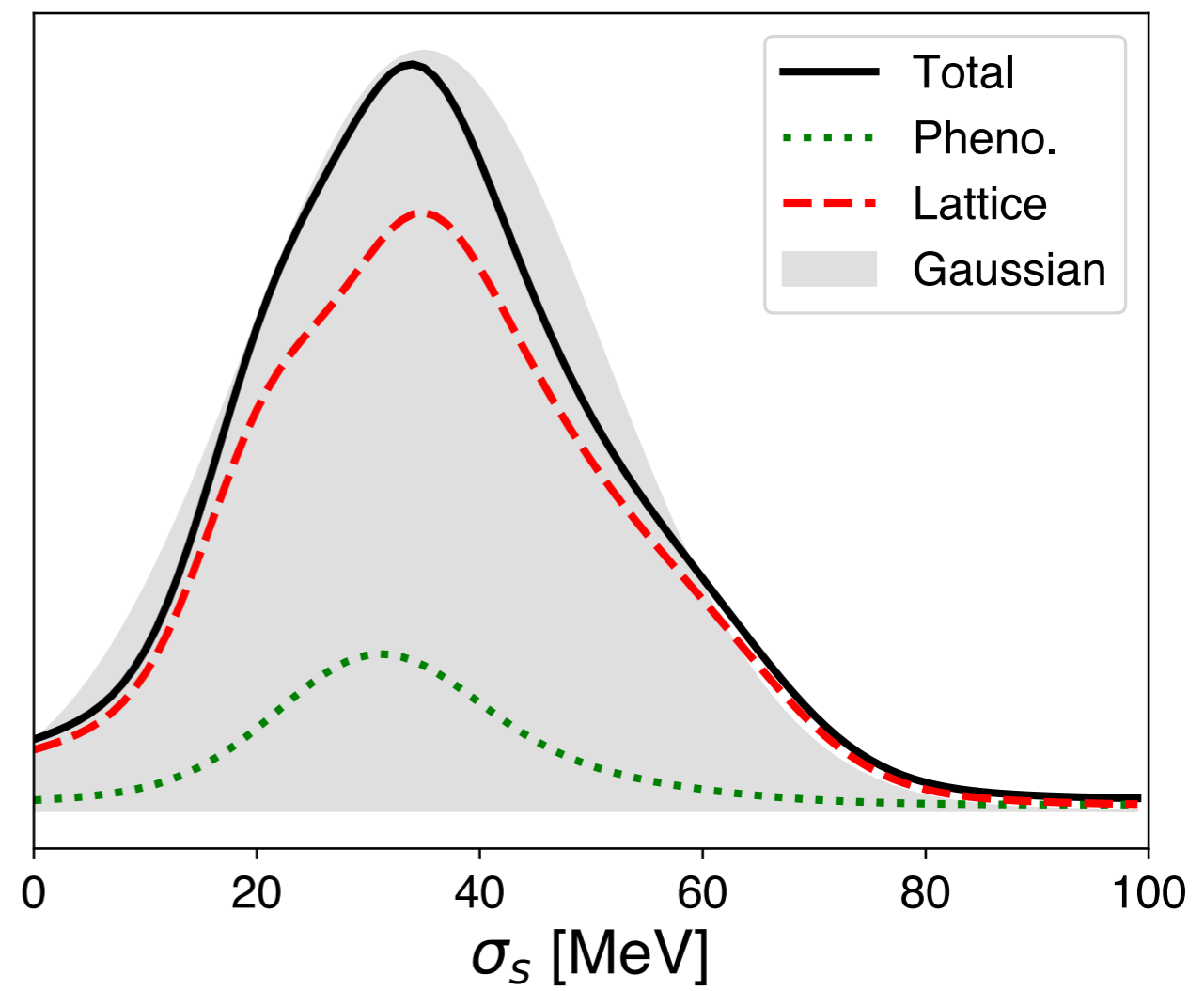
$$\Sigma_{\pi N} = 46 \pm 11 \text{ MeV}$$

# Ideogram: $\sigma_s$



PDG method

$$\sigma_s = 35.2 \pm 3.1 \text{ MeV}$$

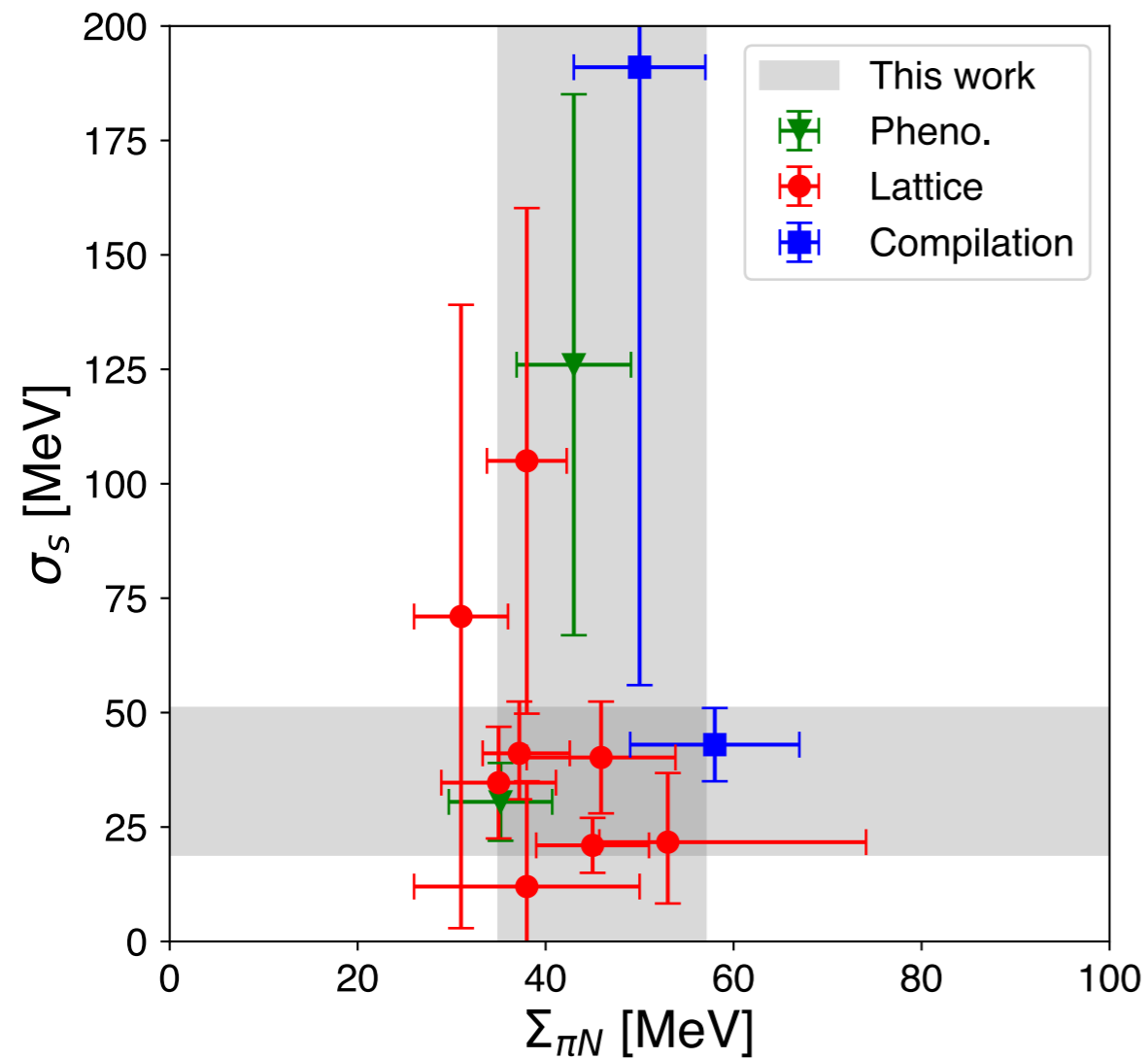


Ours

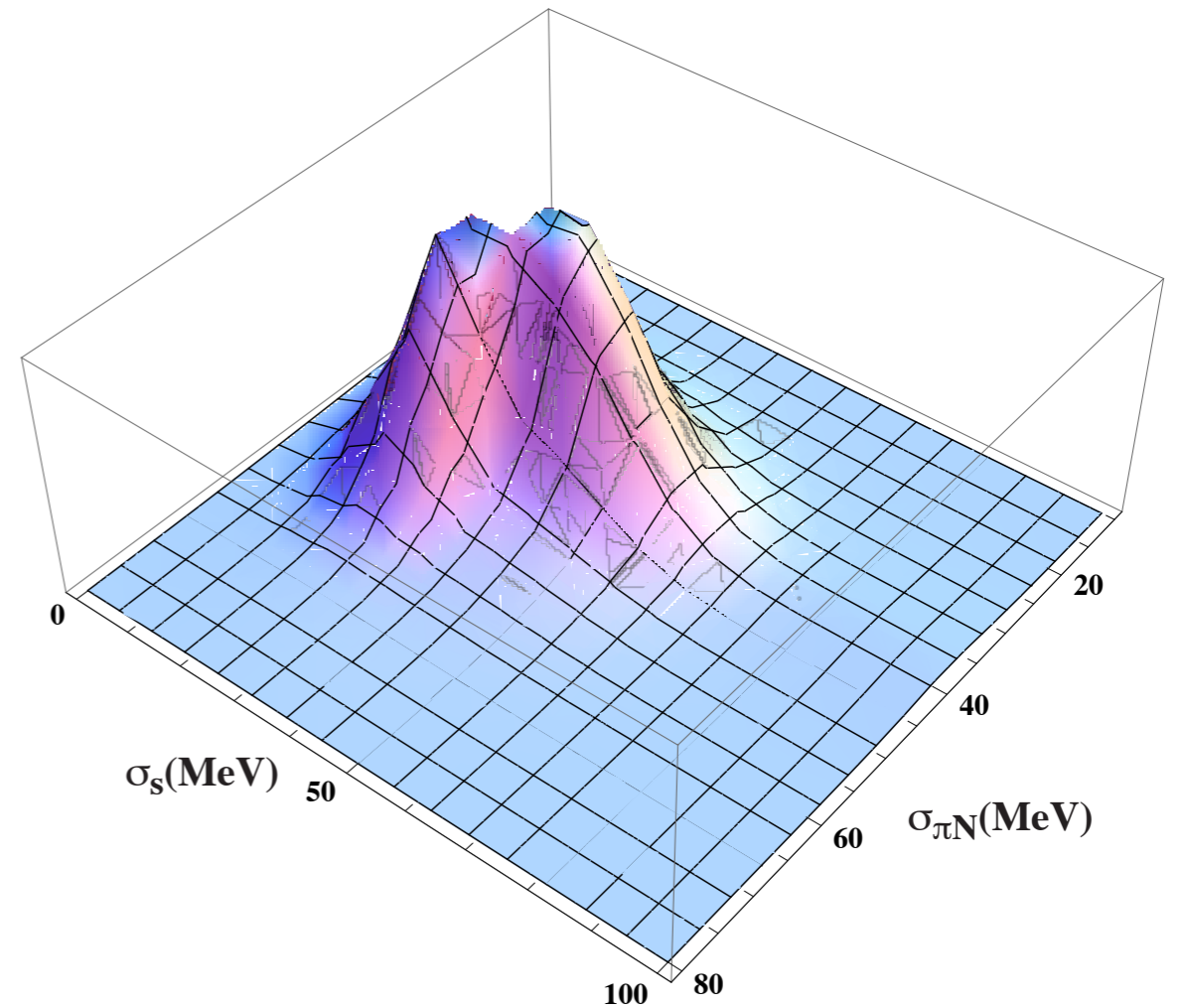
$$\sigma_s = 35 \pm 16 \text{ MeV}$$

# Correlation between $\Sigma_{\pi N}$ and $\sigma_s$

## Scatter plot



## 2-D ideogram



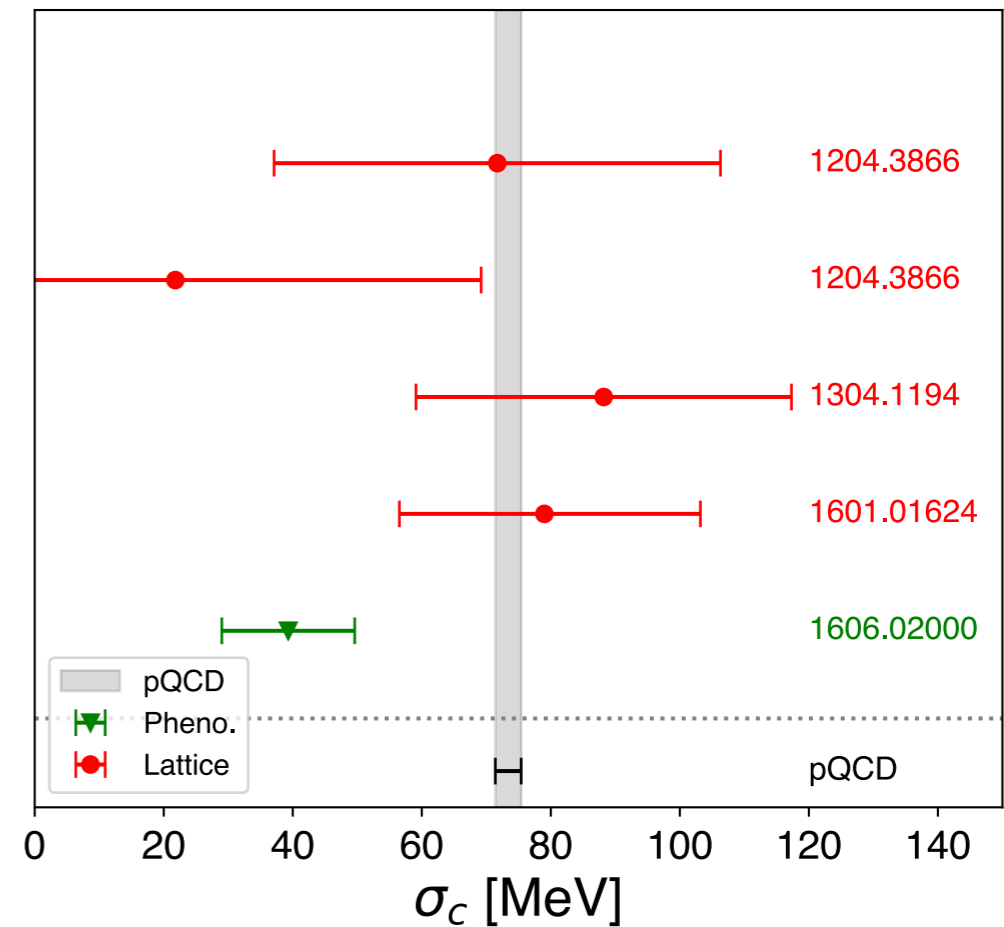
No significant correlation.

# Direct evaluation of $\sigma_c$

## Results

| Reference | $\sigma_c$ | Uncertainties   | Method        |
|-----------|------------|-----------------|---------------|
| [44]      | 71.7       | 34.6            | Lattice       |
|           | 21.8       | 47.4            | Lattice       |
| [55]      | 88.2       | 29.1            | Lattice       |
| [60]      | 79         | $21_{-8}^{+12}$ | Lattice       |
| [62]      | 39.3       | 10.3            | Phenomenology |
| [68]      | 4.3        | 4.4             | Phenomenology |
|           | 12.5       | 13              | Phenomenology |
|           | 32.3       | 33.6            | Phenomenology |

## Scatter plot



Errors are much larger than that obtained from pQCD.

# Singlet real scalar DM

## Lagrangian

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}m^2 S^2 - \frac{1}{2}\lambda_{SH} S^2 |H|^2 - \frac{1}{4!}\lambda_S S^4$$
$$m_{\text{DM}} = m^2 + \frac{\lambda_{SH} v^2}{2}$$

The DM phenomenology is governed by two parameters:

$$m_{\text{DM}} \quad \lambda_{SH}$$

If you require its thermal relic abundance to agree with the observed DM density, only one free parameter remains.

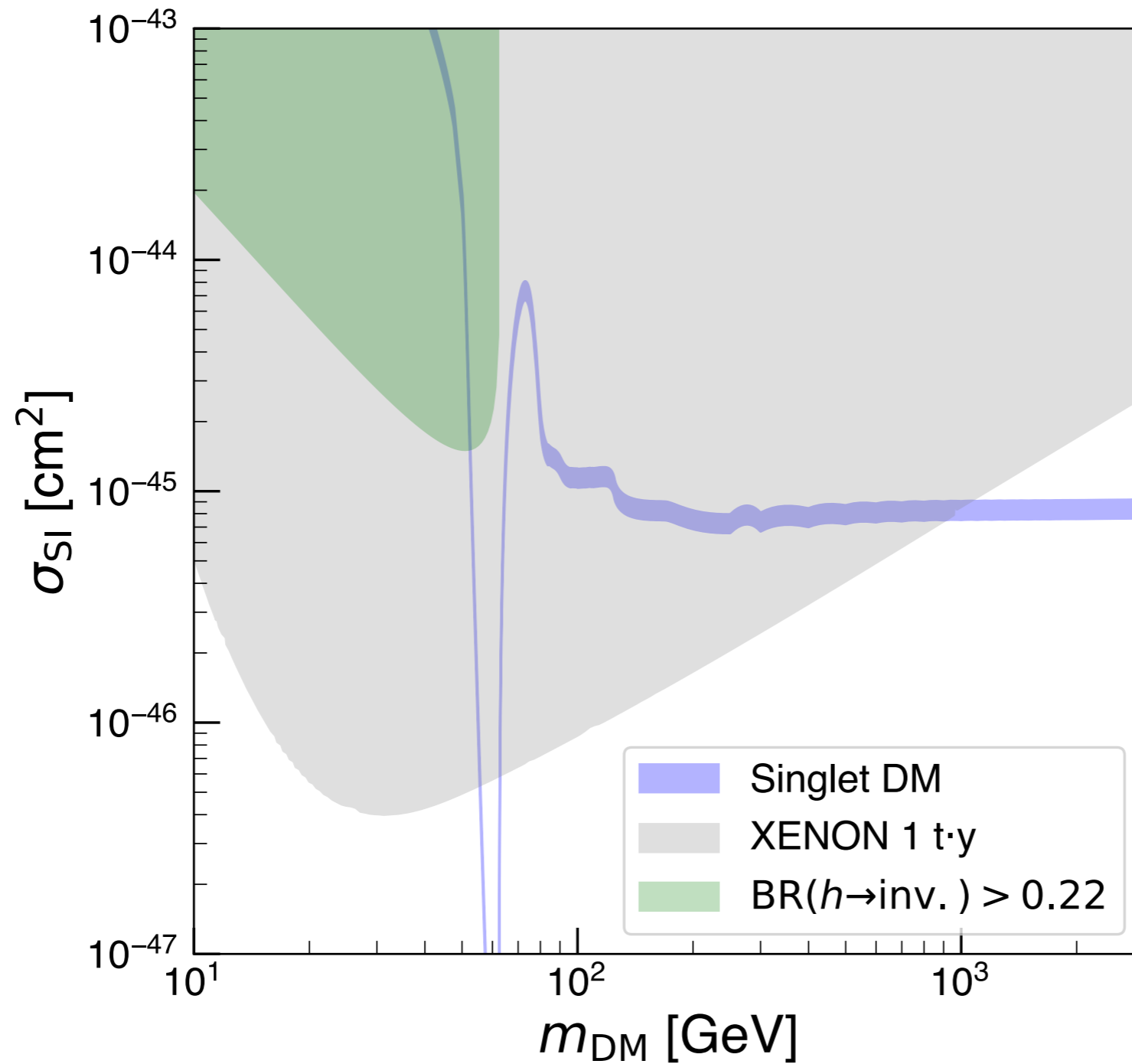
$$\sigma_{\text{ann}} v_{\text{rel}} \simeq \frac{\lambda_{SH}^2}{16\pi m_{\text{DM}}^2}$$

$$\Omega_{\text{DM}} h^2 \simeq \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle}$$

## Observation

$$\Omega_{\text{DM}} h^2 \simeq 0.12$$

# Singlet real scalar DM



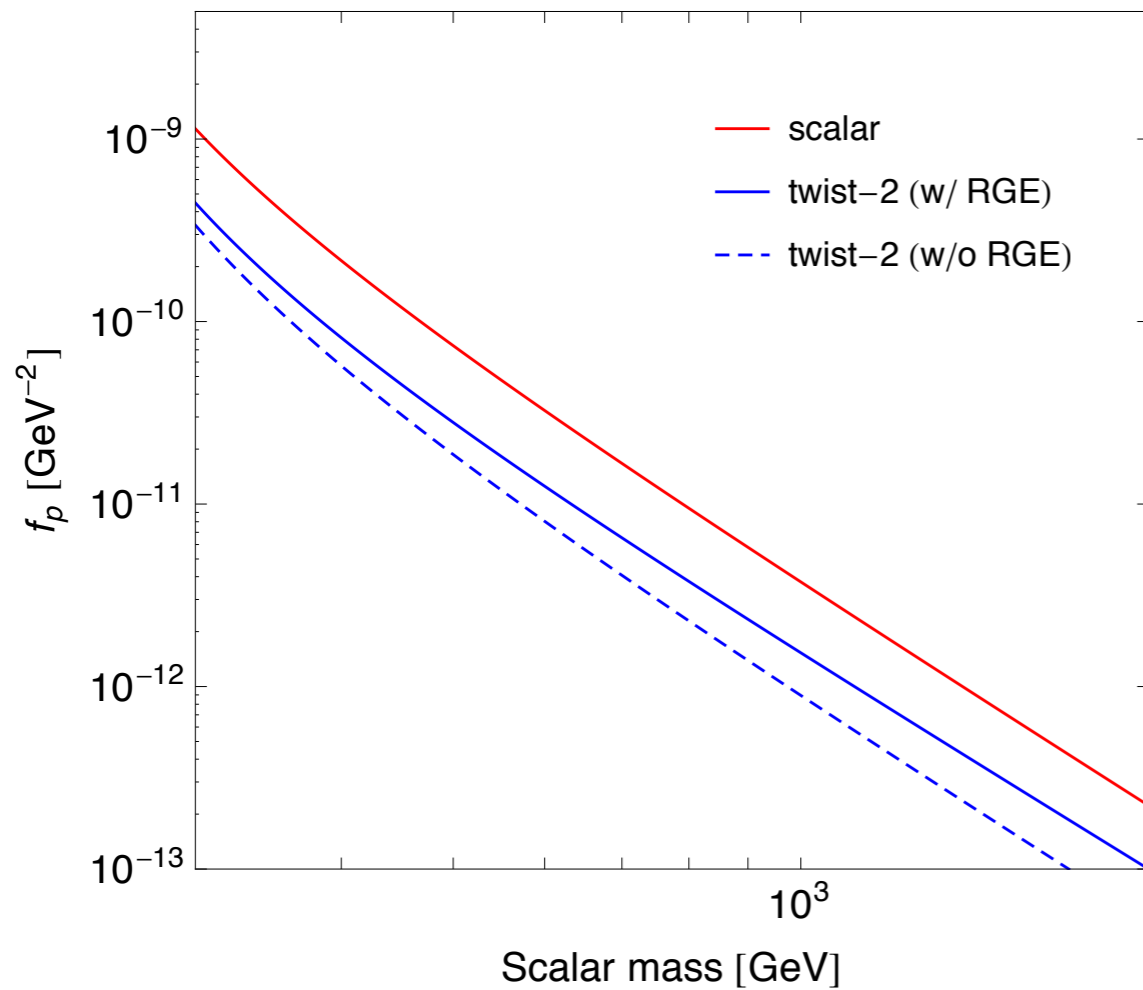
The simple DM model has been constrained stringently.



# RGE effects

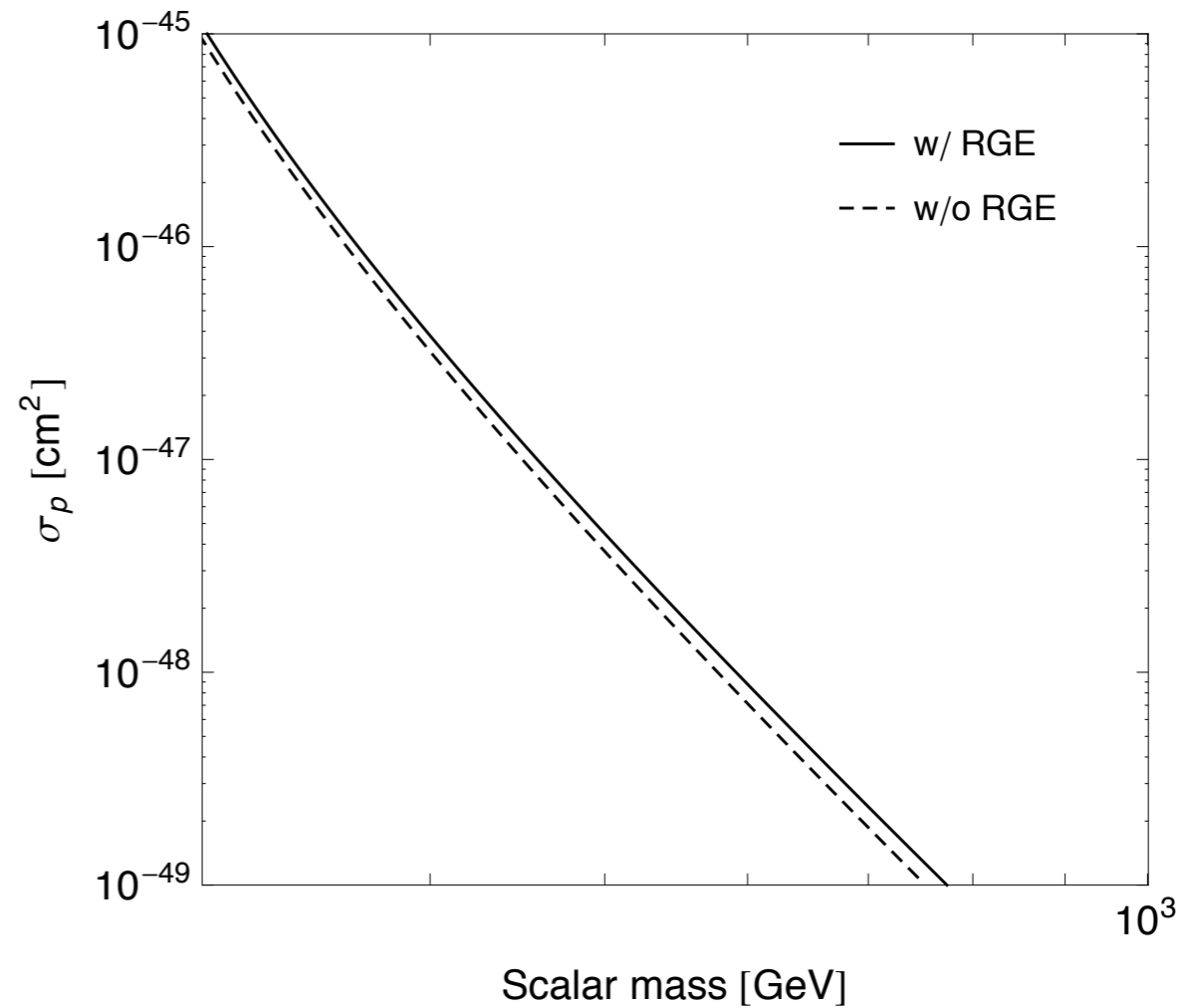
## Effective coupling

M=200 [GeV]



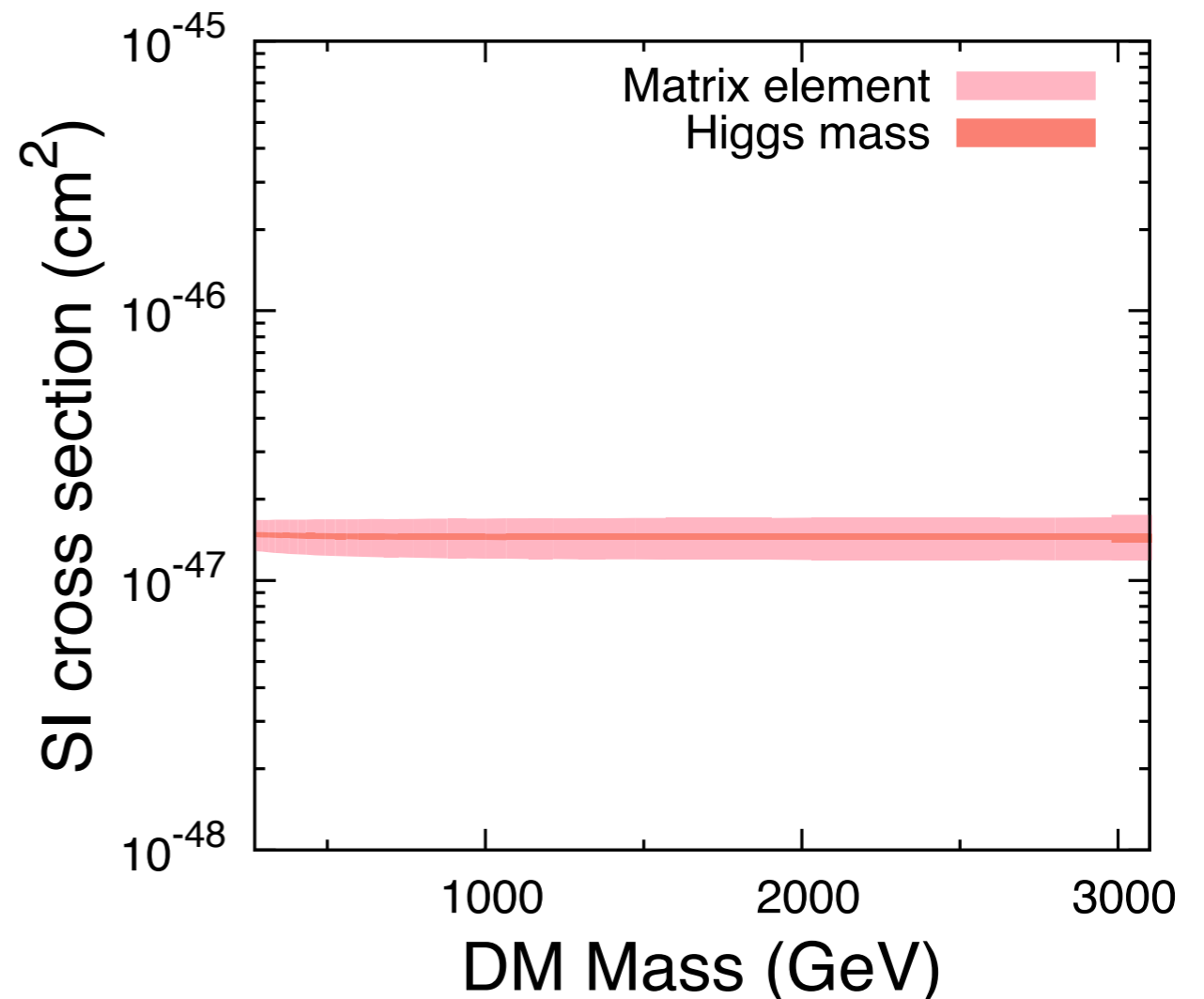
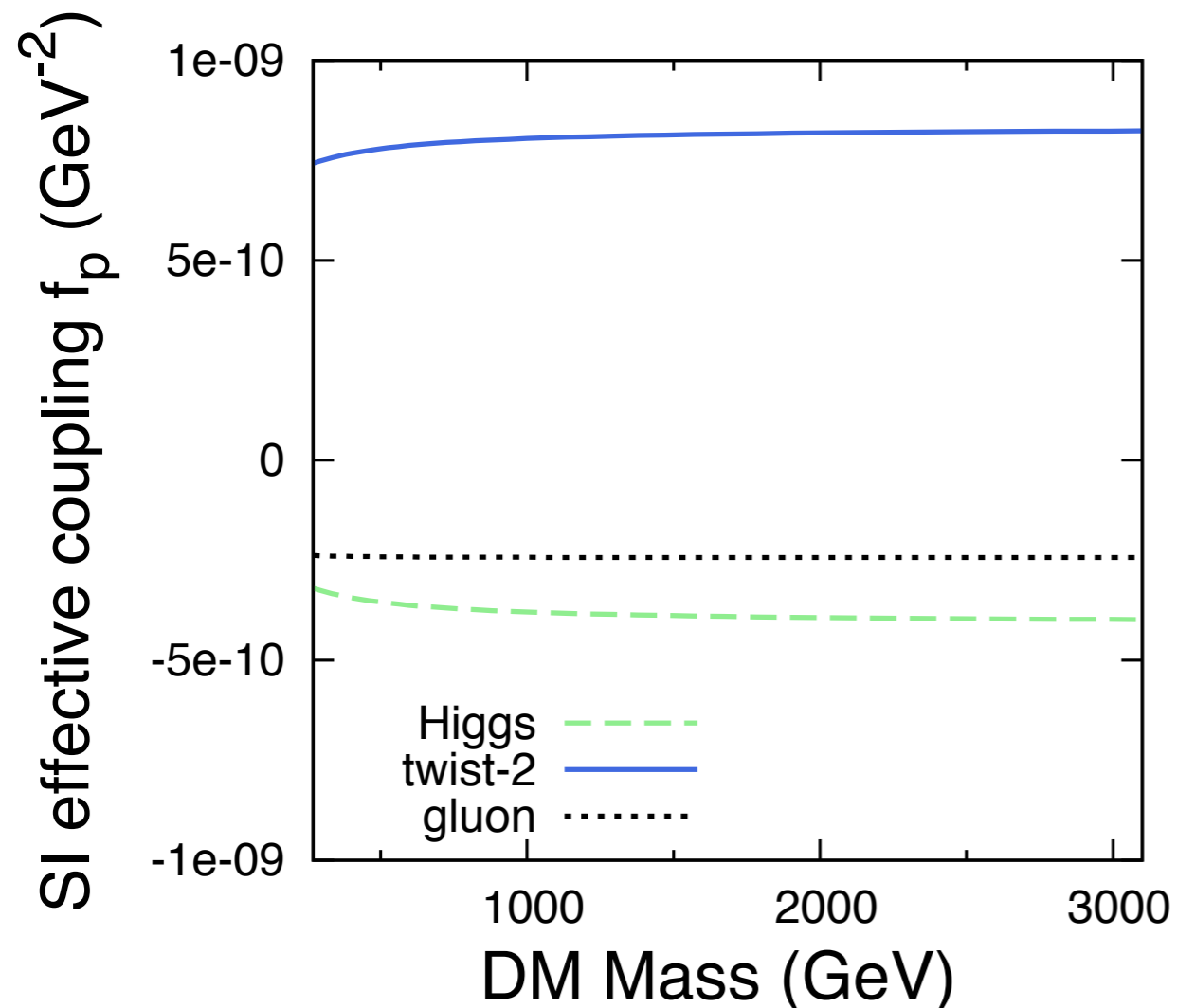
## SI cross section

M=200 [GeV]



O(10)% effects.

# LO results ( $n = 3, Y = 0$ )

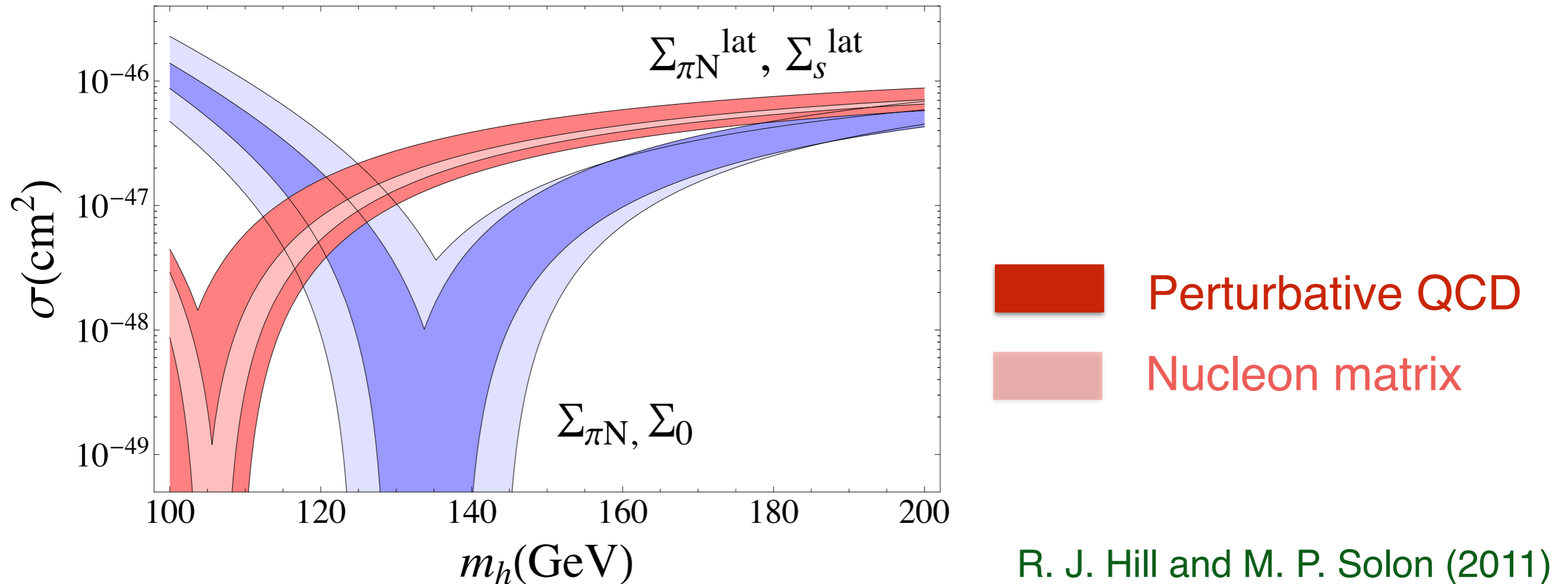


- **Cancellation** among the contributions.
- Resultant scattering cross sections are found to be quite suppressed.

$$\mathcal{O}(10^{-47}) \text{ cm}^2$$

# Uncertainty

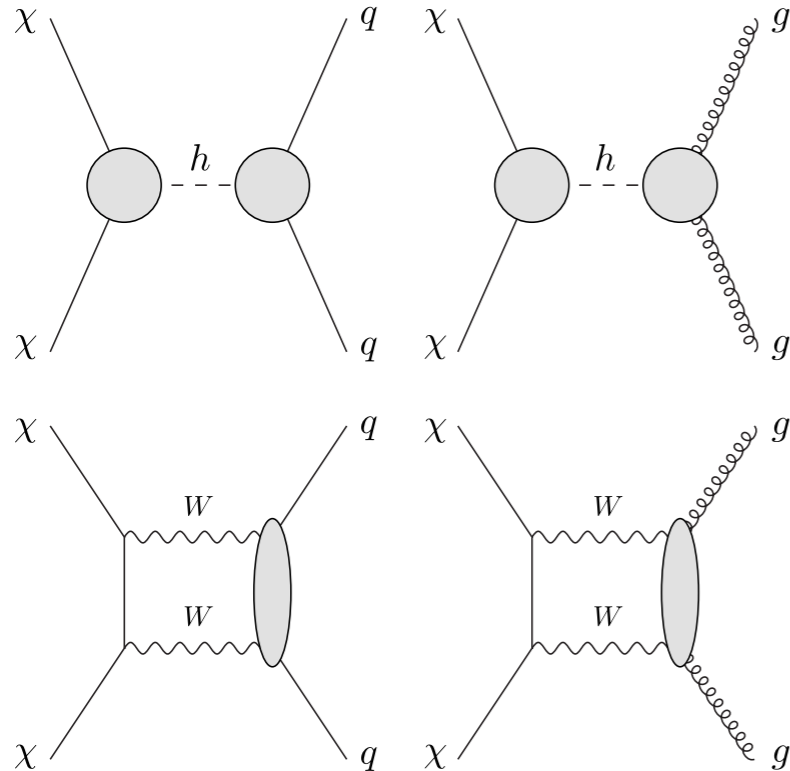
Cancellation results in a large uncertainty.



- Uncertainty mainly comes from perturbative QCD
- Need for **NLO computation** !

# NLO calculation

## Diagrams



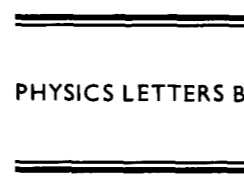
## # of loops

| Operators   |                         | Higgs  |        | Box    |                             |
|-------------|-------------------------|--------|--------|--------|-----------------------------|
| Parton      | Type                    | LO     | NLO    | LO     | NLO                         |
| Quark       | Scalar $C_S^q$          | 1-loop | 2-loop | -      | 2-loop                      |
| (1st&2nd)   | Twist-2 $C_{T_{1,2}}^q$ | -      | -      | 1-loop | 2-loop                      |
| Quark       | Scalar $C_S^b$          | 1-loop | 2-loop | 1-loop | 2-loop (neglected)          |
| (b-quark)   | Twist-2 $C_{T_{1,2}}^b$ | -      | -      | 1-loop | 2-loop (neglected)          |
| Gluon       | Scalar $C_S^G$          | 2-loop | 3-loop | 2-loop | 3-loop                      |
| (1st & 2nd) | Twist-2 $C_{T_{1,2}}^G$ | -      | -      | -      | 2-loop                      |
| Gluon       | Scalar $C_S^G$          | 2-loop | 3-loop | 2-loop | 3-loop (3rd gen. neglected) |
| (3rd)       | Twist-2 $C_{T_{1,2}}^G$ | -      | -      | -      | 2-loop (3rd gen. neglected) |

## OPEs of current-current correlators



9 June 1994

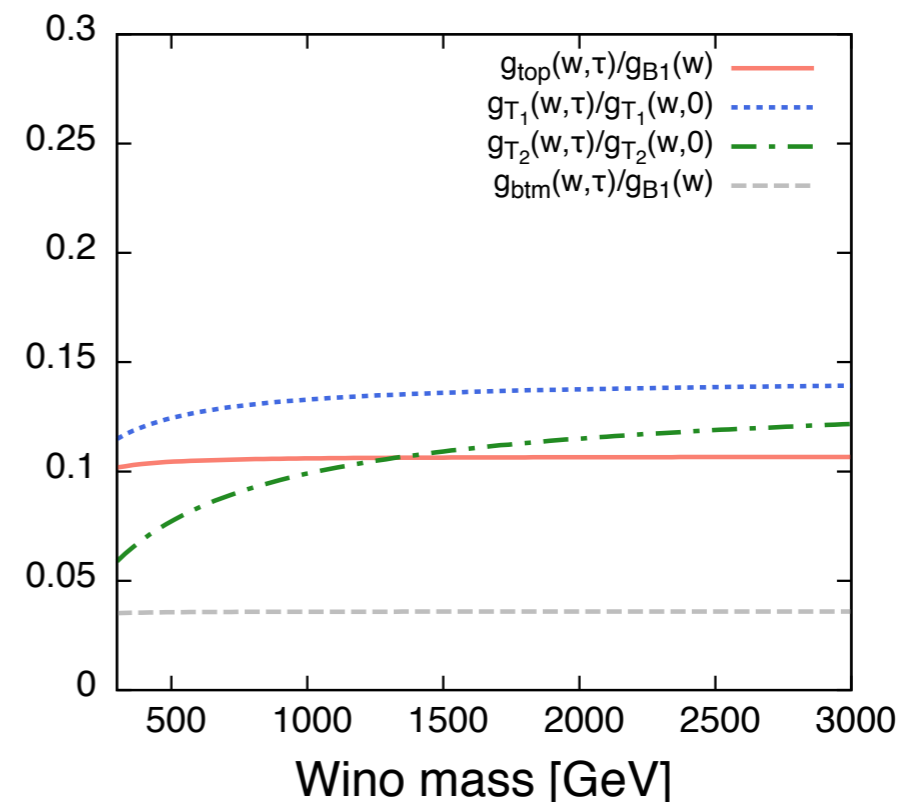


Physics Letters B 329 (1994) 103–110

Two-loop gluon-condensate contributions to heavy-quark current correlators: exact results and approximations\*

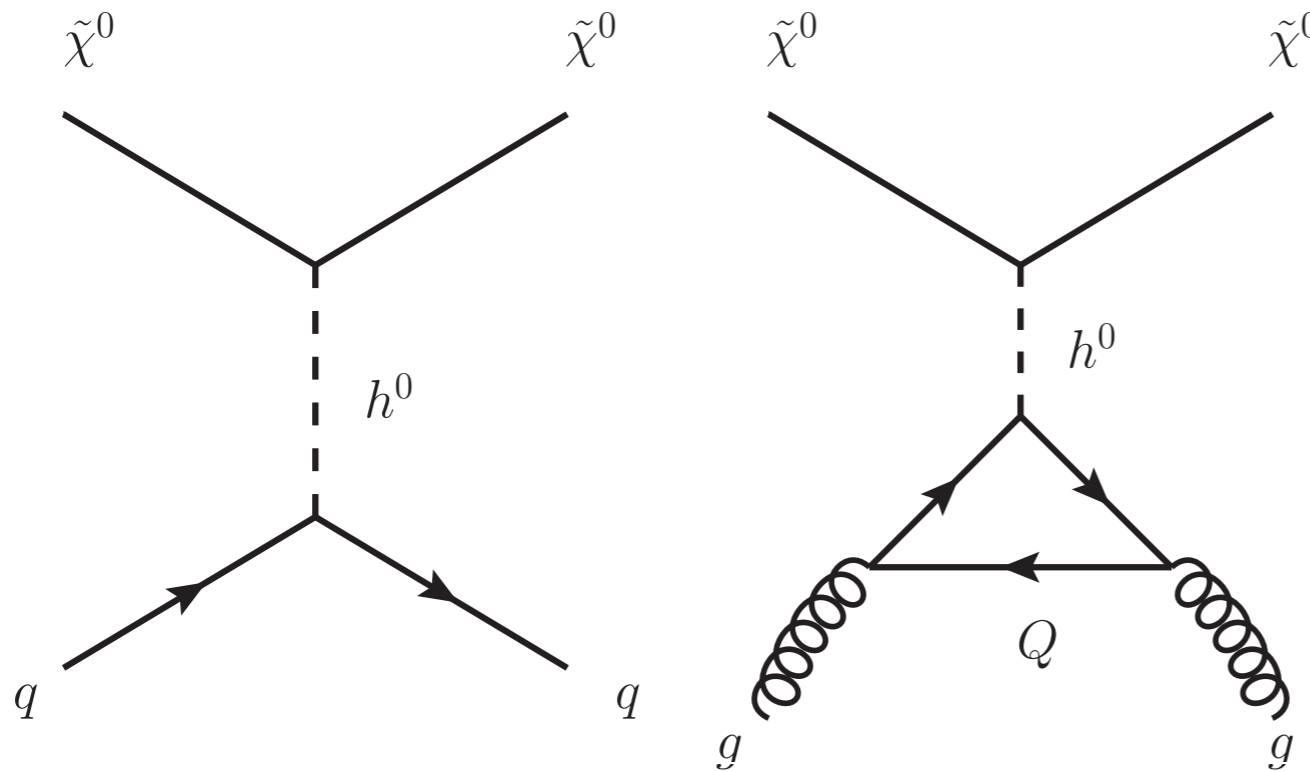
D.J. Broadhurst<sup>a,1</sup>, P.A. Baikov<sup>b,2</sup>, V.A. Ilyin<sup>b,3</sup>, J. Fleischer<sup>c,4</sup>, O.V. Tarasov<sup>c,5</sup>,  
V.A. Smirnov<sup>d,6</sup>

## 3rd gen. contribution



# Wino-like DM in high-scale SUSY

## Tree-level contribution



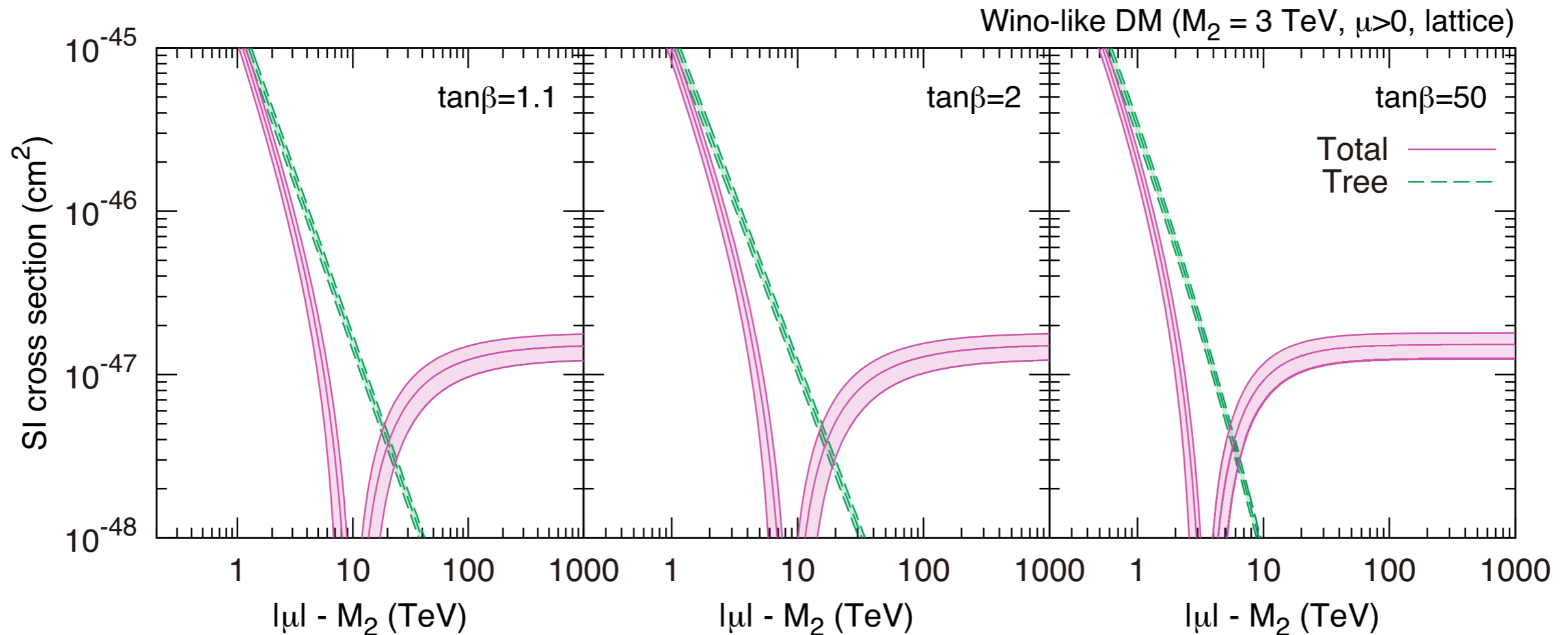
## Effective coupling

$$C_q^H = \frac{g_2^2}{2m_W m_h^2} (Z_{12} - Z_{11} \tan \theta_W) (Z_{13} \cos \beta - Z_{14} \sin \beta)$$

( $Z_{ij}$ : Neutralino mixing matrix)

➔ 
$$C_q^H \simeq \frac{g_2^2 (M_2 + \mu \sin 2\beta)}{2m_h^2 (M_2^2 - \mu^2)} \quad (|\mu \pm M_2| \gg m_Z)$$

# Wino-like DM in high-scale SUSY



The loop contribution needs to be included when higgsino is much heavier than wino.