MODIFIED DARK MATTER:

Relating Dark Energy, Dark Matter, and Baryonic Matter

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References:

Ho, Minic and Ng, Phys. Lett. B693, 567 (2010); Phys. Rev. D85, 104033 (2012).

Edmonds, Farrah, Ho, Minic, Ng, and Takeuchi, ApJ 793, 41 (2014); IJMP A32, 1750108 (2017).

Edmonds, Farrah, Minic, Ng, and Takeuchi, arXiv:1709.04388.

CONTENTS

From cold dark matter (CDM) and modified Newtonian dynamics (MOND) to modified dark matter (MDM)

Constructing MDM via gravitational thermodynamics and entropic gravity

Observational (astronomical) tests of MDM

(Extended) Quanta of MDM obey infinite statistics?

Summary and future work

Units (most of the time): $c = 1, \hbar = 1, k_B = 1$

Why MDM?

- Critique of CDM
- Critique of MOND
- Search for an approach (inspired by quantum gravity) that combines their salient successful features into a unified scheme $\Rightarrow$ MDM

“Missing mass” problem

Zwicky and Rubin and Ford ...: Observational evidence for substantial mass discrepancies between dynamical studies and observations of visible (baryonic) matter on scales spanning galactic to cosmological

Two routes to alleviate the missing mass problem; either change the source side and add missing pieces to the energy momentum tensor T_{ab} or modify the geometric/gravitational side of Einstein's equations

$$G_{\alpha\beta} \ = \ 8\pi G\, T_{\alpha\beta} + \Lambda\, g_{\alpha\beta}$$

Route 1. Change source side: Cold Dark Matter paradigm (as an example)

Route 2. change geometric side: Modified Newtonian Dynamics (as an example)

Cold Dark Matter (CDM) paradigm

At larger scales, dark matter is apparently required to yield:
the correct gravitational lensing; the correct elemental abundances from big bang nucleosynthesis; the correct cosmic microwave background spectrum shapes (including the alternating peaks); the correct large-scale structures ...

And there is a plethora of dark matter candidates: (sterile) neutrinos, supersymmetric particles, axions, and WIMPs etc. ...

Problems with CDM include: The core/cusp problem; the too big to fail problem; the satellite planes problem; at the galactic scale, dark matter can explain the observed asymptotic independence of orbital velocities on the size of the orbit only by fitting data (usually with two parameters for individual galaxies); it can do no better in explaining the observed baryonic Tully-Fisher relation, i.e. the asymptotic-velocity-mass ($v^4 \propto M$) relation; it seems to possess too much power on small scales ($1 \sim 1000$ kpc). In short, CDM works spectacularly well at the cluster and cosmological scales, but not quite so at the galactic scale.

And so far, dark matter detection experiments have failed to detect dark matter particles ...

Modified Newtonian Dynamics (MOND)

Milgrom: the mass discrepancy appears only at accelerations below a certain critical acceleration $a_c \sim 10^{-10}\mathrm{m/s}^2 \approx \dfrac{cH_0}{2\pi}$ (with the Hubble parameter $H_0 = (67.74 \pm 0.46)\,\mathrm{km/s/Mpc.})$

Milgrom, who proposed MOND, postulated that the force on a test mass m, $F = ma$, valid for acceleration $a \gg a_c$, is modified, in the small acceleration limit $a \ll a_c$, to $F = \frac{ma^2}{a_c}$.

The two regions of acceleration are thus connected by an interpolating function $F \;=\; ma\,\mu(a/a_c)$, where $\mu(x) = 1$ for $x \gg 1$ and $\mu(x) = x$ for $x \ll 1$.

For a given (baryonic) source mass M, its gravitational attraction on a test mass m is $F = m(GM/r^2) \equiv ma_N$, where $a_N = GM/r^2$ is the usual Newtonian acceleration without dark matter.

MOND yields:

$$\frac{1}{\mu(a/a_c)}\frac{GM}{r^2} \;=\; a\ .$$

leading to the observed flat rotation curves as well as the Baryonic Tully-Fisher relation .

But there are problems with MOND at the cluster and cosmological scales. It fails to address the dynamics of galactic clusters and other cosmological measurements, in particular, it cannot explain the third and higher CMB peaks, and the shape of matter power spectrum...

$\Rightarrow$ Need an approach (inspired by quantum gravity) that combines the salient successful features of CDM and MOND into a unified scheme
$\Rightarrow$ MDM.

Constructing Modified Dark Matter (MDM)

• Generalizing Jacobson's treatment of gravitational thermodynamics

• Generalizing Verlinde's treatment of entropic gravity (inspired by Jacobson's work)

MDM and Gravitational Thermodynamics

Recall the work of Jacobson:

Start with the thermodynamic relation $dE = TdS$ (for energy E, temperature T and entropy S) in Rindler spacetime.

E denotes the integral of the energy momentum tensor ($T_{\alpha\beta}$) of matter.

For T, use the Unruh temperature associated with the local accelerating (Rindler) observer $T = \frac{\hbar a}{2\pi c k_B}$.

For S, the holographic principle gives $S = \frac{c^3 A}{4G\hbar}$, where A is the area of the Rindler horizon.

Jacobson shows: LHS (of thermodynamic relation) $E \to T_{\alpha\beta}$; and $S \to R_{\alpha\beta}$ (the Ricci tensor) such that RHS $\to G_{\alpha\beta}$, yielding Einstein's equations.

We generalize Jacobson's treatment with a consistent modification of the energy momentum tensor so that the fundamental acceleration ($\sim a_c$ introduced by hand in MOND) emerges naturally.

We assume (1) the validity of Einstein's theory of gravity; (2) a standard energy-momentum tensor.

(1) requires that we preserve the holographic scaling of the area. Then (2), in conjunction with the form of the thermodynamic relation, demands that we change the temperature while preserving the entropy.

$\Rightarrow$ Our model is given by the thermodynamic relation $d\widetilde{E} = \widetilde{T} dS$.

Note: Since the Unruh temperature knows the inertial properties and is fixed by the background, the additional part of the energy-momentum tensor (coming from a modified temperature) will also know the inertial properties and the background.

Consider a local observer with local acceleration a in de Sitter space where $a_0 = c^2\sqrt{\Lambda/3} = cH_0$ like our expanding Universe. The Unruh temperature experienced by this observer (Ref.: Deser & Levin) is $T_{a_0+a} = \dfrac{\hbar}{2\pi c k_B}\sqrt{a^2 + a_0^2}$. Define the *effective* (normalized) temperature

$$\widetilde{T} \equiv T_{a_0+a} - T_{a_0} = \frac{\hbar}{2\pi c k_B}\left(\sqrt{a^2 + a_0^2} - a_0\right) \equiv \frac{\hbar \tilde{a}}{2\pi c k_B} .$$

Our proposal is the generalization (from $\Lambda = 0 = a_0$ to $\Lambda \neq 0 \neq a_0$ case): $T \to \widetilde{T}$, hence $a \to \tilde{a}$; $E \to \widetilde{E}$, (hence later: $M \to \widetilde{M}$).

MDM and Entropic Gravity

Recall **Verlinde's "recipe"**:

Verlinde derives

(I) Newton's 2nd law $\vec{F} = m\vec{a}$, by using

(1) First law of thermodynamics $\Rightarrow$ entropic force $F_{entropic} = T\frac{\Delta S}{\Delta x}$, and invoking Bekenstein's original arguments concerning the entropy S of black holes: $\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x$.

(2) The formula for the Unruh temperature, $k_B T = \frac{\hbar a}{2\pi c}$, associated with a uniformly accelerating (Rindler) observer.

- Will generalize the $T \sim a$ relation to $\widetilde{T} \sim \tilde{a}$.

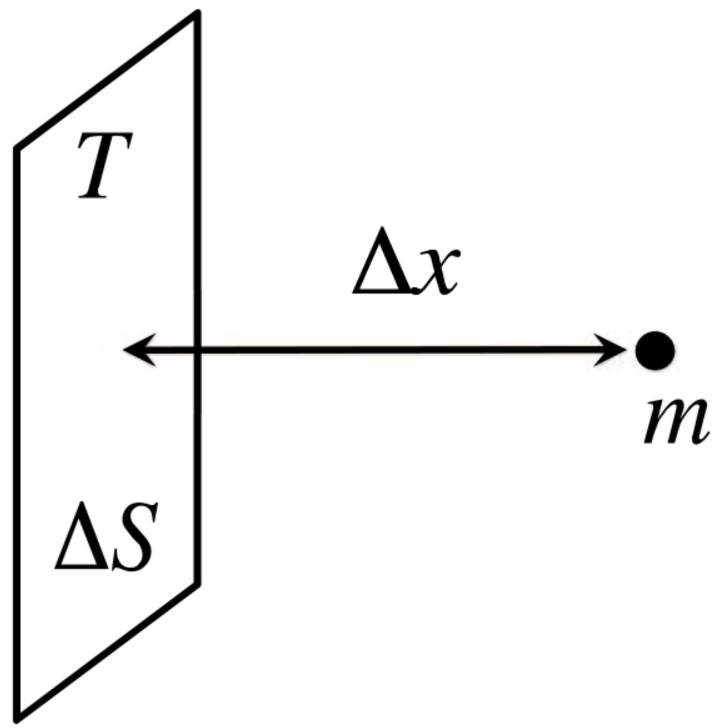
(II) Newton's law of gravity $a = GM/r^2$ by considering an imaginary quasi-local (spherical) holographic screen of area $A = 4\pi r^2$ with temperature T, and using

(1) Equipartition of energy $E = \frac{1}{2}Nk_B T$ with $N = Ac^3/(G\hbar)$ being the total number of degrees of freedom (bits) on the screen;

(2) The Unruh temperature formula and the fact that $E = Mc^2$.

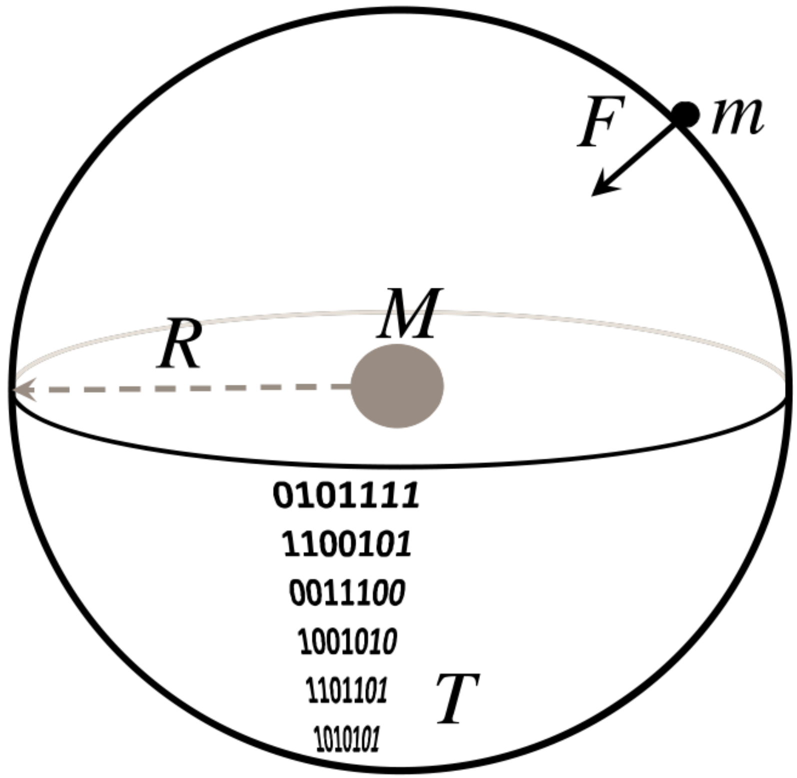
- Will generalize the $T \sim M$ relation to $\widetilde{T} \sim \widetilde{M}$.

A particle with mass approaches a part of the holographic screen



A particle with mass approaches a part of the holographic screen.

A particle with mass m near a spherical holographic screen



A particle with mass m near a spherical holographic screen.

Constructing MDM

(I) Verlinde's approach $\Rightarrow$ the entropic force in de Sitter space is

$$F_{entropic} = \widetilde{T}\,\nabla_x S = m[\sqrt{a^2 + a_0^2} - a_0].$$

For $a \gg a_0$, we have $F_{entropic} \approx ma$.
For $a \ll a_0$: $F_{entropic} \approx m\frac{a^2}{2\,a_0}$, so the terminal velocity v of the test mass m should be determined from $ma^2/(2a_0) = mv^2/r$.

For the small acceleration $a \ll a_0$ regime: The observed flat galactic rotation curves (v is independent of r) and the observed Tully-Fisher relation ($v^4 \propto M$) now require (recall $a_N = GM/r^2$) that $a \approx \left(2\,a_N\,a_0^3\,/\pi\right)^{\frac{1}{4}}$.

But that means $F_{entropic} \approx m\frac{a^2}{2\,a_0} = F_{Milgrom} \approx m\sqrt{a_N a_c}\,$.
We have recovered MoND — provided we identify $a_0 \approx 2\pi a_c$, with the (observed) critical galactic acceleration $a_c \sim \sqrt{\Lambda/3} \sim H \sim 10^{-8} cm/s^2$. Thus from our perspective, MoND is a phenomenological consequence of quantum gravity.

(II) For an imaginary holographic screen of radius r, Verlinde's argument $\Rightarrow$

$$2\pi k_B \widetilde{T} = 2\pi k_B \left(\frac{2\widetilde{E}}{N k_B} \right) = 4\pi \left(\frac{\widetilde{M}}{A/G} \right) = \frac{G\,\widetilde{M}}{r^2},$$

where $\widetilde{M}$ represents the *total* mass enclosed within the volume $V = 4\pi r^3/3$.

$\widetilde{M} = M + M'$ where M' is some unknown mass, i.e., dark matter; consistency $\Rightarrow$

$$M' = \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 M.$$

$$\Rightarrow F_{entropic} = m[\sqrt{a^2 + a_0^2} - a_0] = m\,a_N \left[1 + \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 \right]$$

For $a \gg a_0$, $F_{entropic} \approx ma \approx ma_N$, and hence $a = a_N$. ($M' \approx 0$)

For $a \ll a_0$, $F_{entropic} \approx m\frac{a^2}{2\,a_0} \approx ma_N(1/\pi)(a_0/a)^2$, yielding $a = \left(2\,a_N\,a_0^3/\pi \right)^{\frac{1}{4}}$, as required. ($M' \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r$)

DARK MATTER MASS DENSITY PROFILE ($\rho\prime(r)$)

Consider an ordinary (visible) matter source of radius r_0 with total mass $M(r_0)$:

$$M' \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r \Rightarrow \rho\prime(r) = \frac{M^{1/2}(r_0)(\sqrt{\Lambda}/G)^{1/2}}{r^2}.$$

This result can be compared with the distribution associated with an isothermal Newtonian sphere in hydrostatic equilibrium (used by some dark matter proponents):

$$\rho(r) = \frac{\sigma}{r^2 + r_0^2}.$$

Asymptotically the two expressions agree with σ identified as $M^{1/2}(r_0)(\sqrt{\Lambda}/G)^{1/2}$. A phenomenological check.

Observational tests of MDM

- MDM in galaxies
- MDM in clusters
- MDM & Strong lensing (preliminary)
- MDM & cosmology (preliminary)

Fitting rotation curves with MDM mass profiles

Modified Dark Matter:

$$F_{entropic} = m[\sqrt{a^2 + a_0^2} - a_0] = m\, a_N \left[1 + \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 \right]$$

To determine rotation curves:

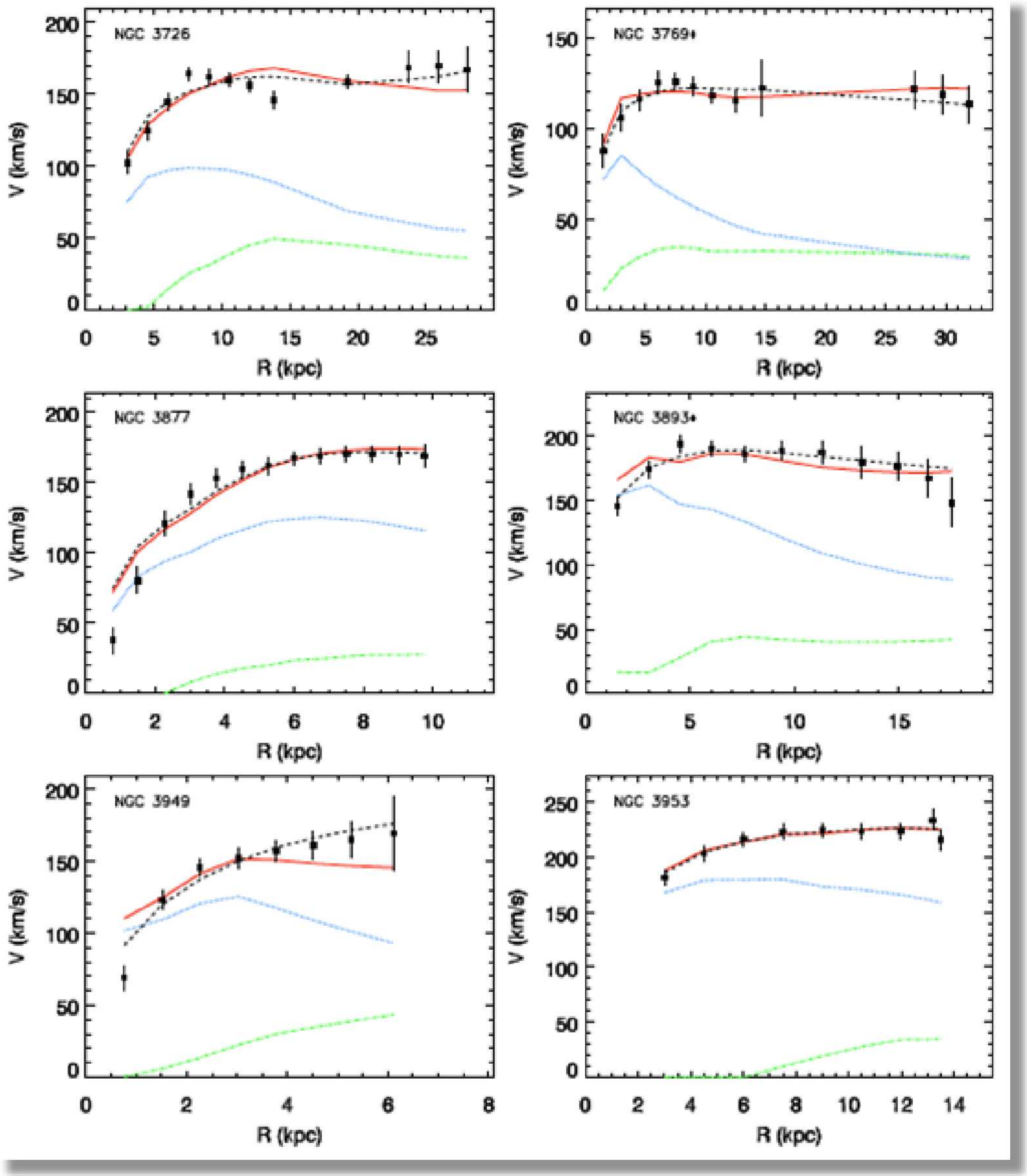
$$F_{entropic} = mv^2/r$$

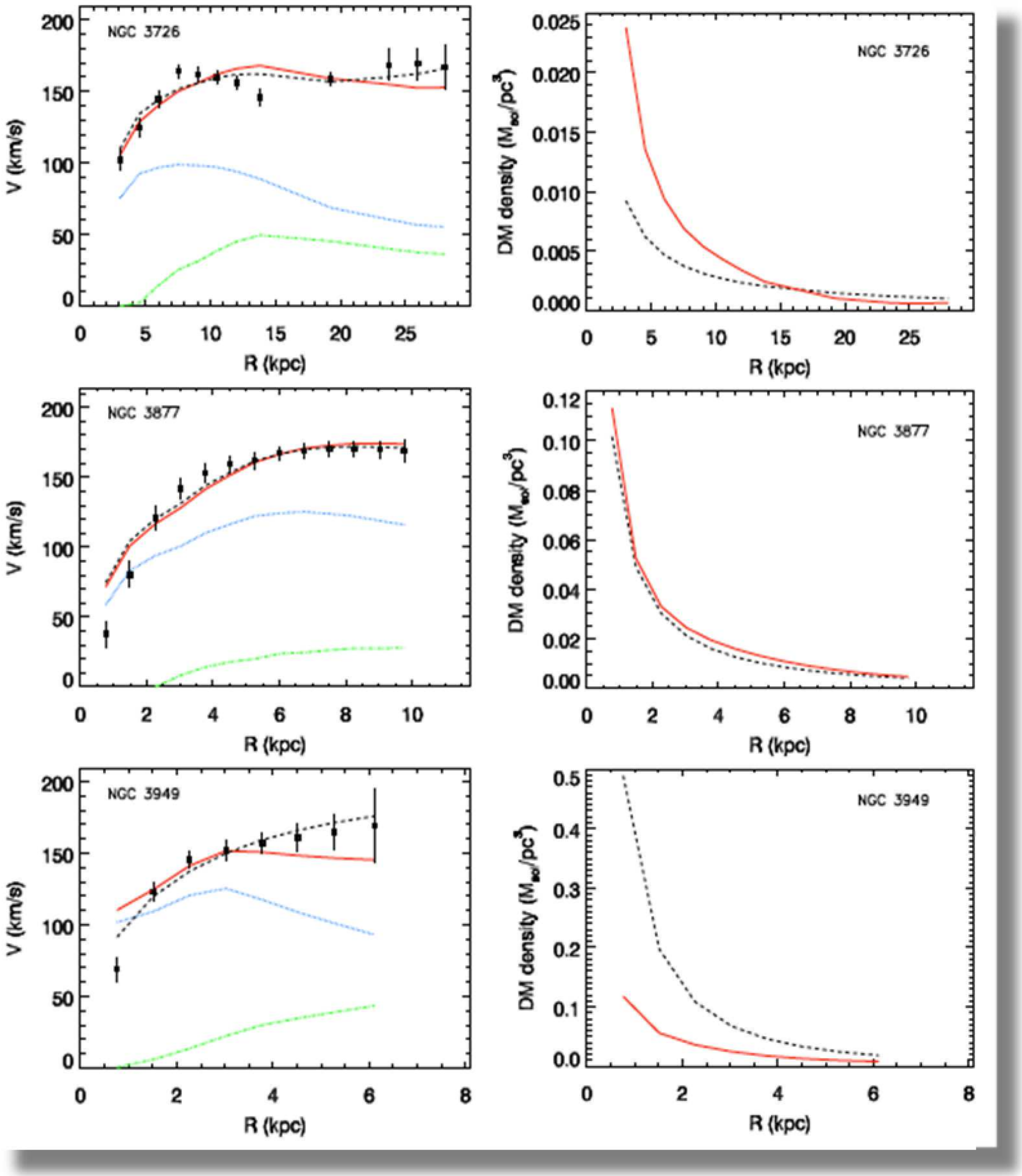
We fit rotation curves for 30 local spiral (HSB as well as LSB) galaxies.

Next 4 slides: Samples of rotation curves and dark matter density profiles.

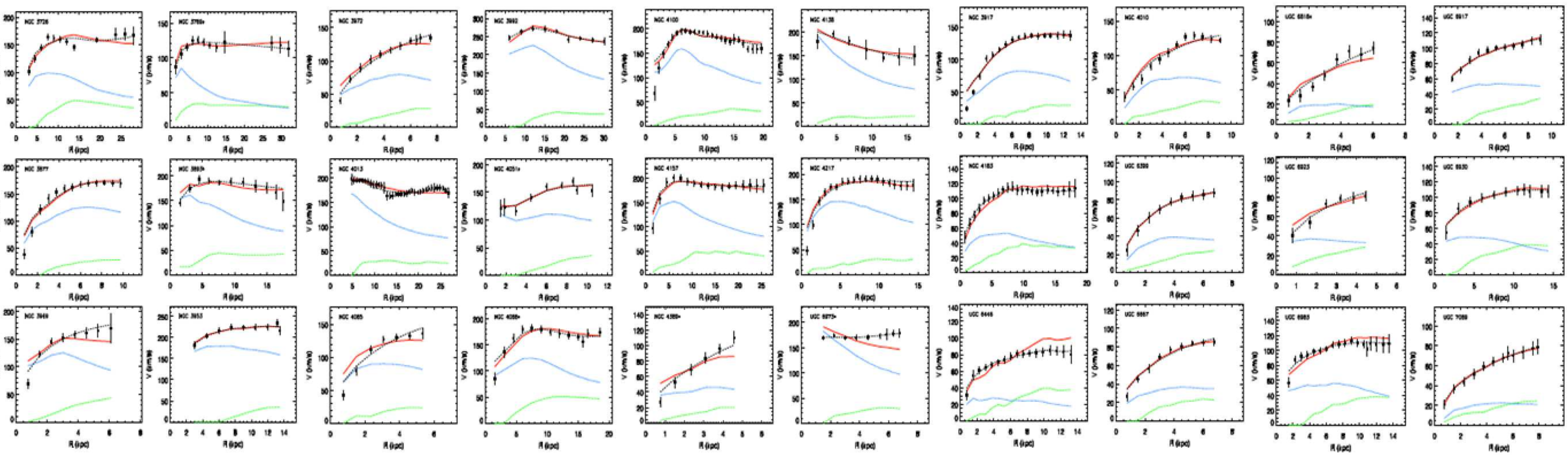
Data - black squares; Stars - blue line; Gas - green line. [Sanders & Verheijen]
MDM - red line; CDM - black dashed line (using NFW profile).

Fitting parameters: MDM (1): mass-to-light ratio M/L; CDM (3): c, v_{200}, M/L.
MDM uses the minimum number of parameters: hence more economical than CDM.


NGC 3726
NGC 3769
NGC 3877
NGC 3893
NGC 3949
NGC 3953
V (km/s)
R (kpc)



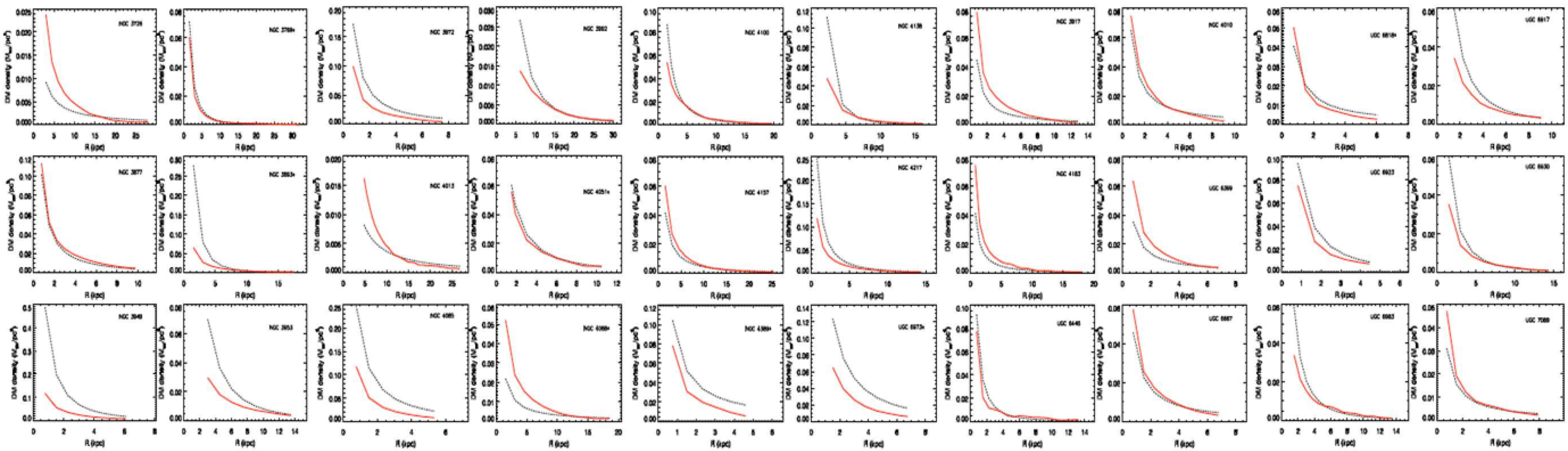
NGC 3726
V (km/s)
R (kpc)
NGC 3726
DM density (M_{sol}/pc^3)
R (kpc)
NGC 3877
V (km/s)
R (kpc)
NGC 3877
DM density (M_{sol}/pc^3)
R (kpc)
NGC 3949
V (km/s)
R (kpc)
NGC 3949
DM density (M_{sol}/pc^3)
R (kpc)



We fit rotation curves for 30 local spiral galaxies, providing the first astrophysical test of MDM

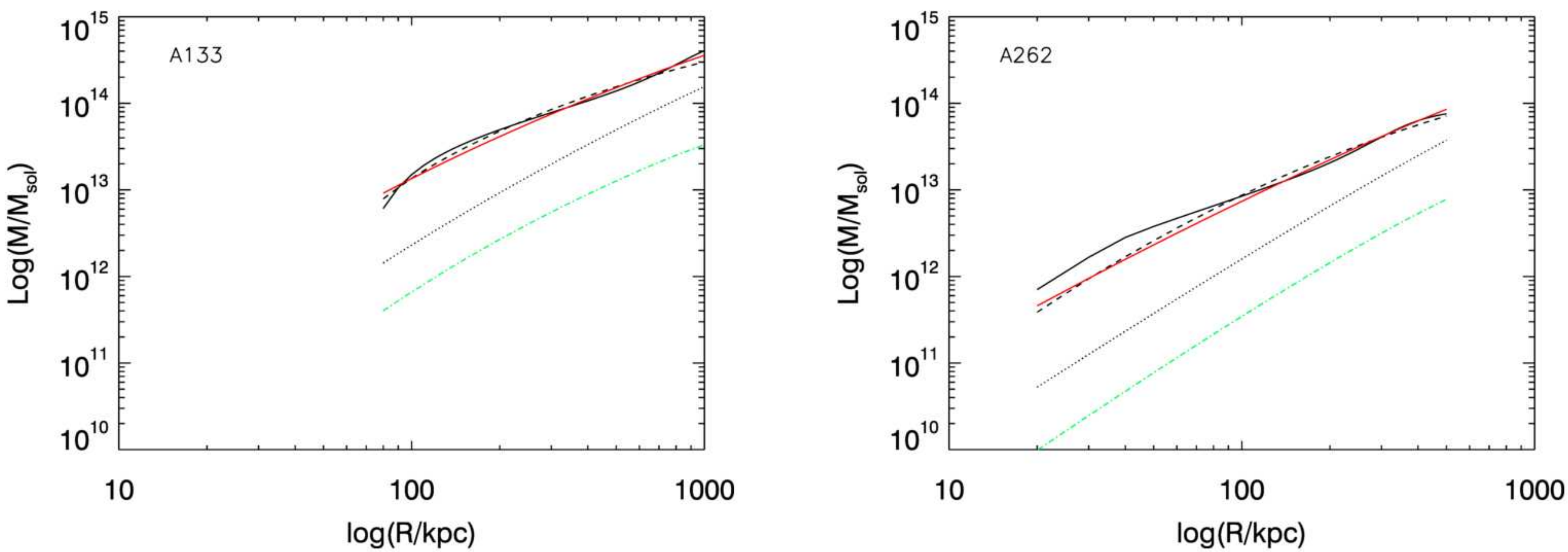
IT PASSED!

Dark matter density profiles for 30 local spiral galaxies (HSB/LSB)



CDM IN CLUSTERS [D Edmonds et al.]

Plots of total mass of galaxy clusters A133 and A262 (two in a Chandra sample of 13 relaxed galaxy clusters given in A. Vikhlinin et al.) within radius R (assuming spherical symmetry). The solid black line is the virial mass; The dot-dashed green line is gas mass; The dotted black line is MOND (effective mass); The dashed black line is CDM; The solid red line is MDM.



*Have to take into account well-known physical effects (Tolman-Ehrenfest effect) associated with a change of scale (from galactic scale to cluster scale).

(more) MDM IN CLUSTERS

The total gravitating mass in Newtonian, MOND, and MDM dynamics vs observed mass (Unpublished work by D Edmonds et al.):

Galactic Clusters: the sample

White, Jones & Forman (1997, MNRAS 292) tabulated observed temperatures and mass estimates of the hot gas for 207 clusters from X-ray data collected by the *Einstein satellite*.

Mass of stars is estimated using the rough correlation found by David et al. (1990, ApJ, 356). $M_{gas}/M_{stars} \approx 0.5 T_{keV} h_{50}^{-1.5}$.

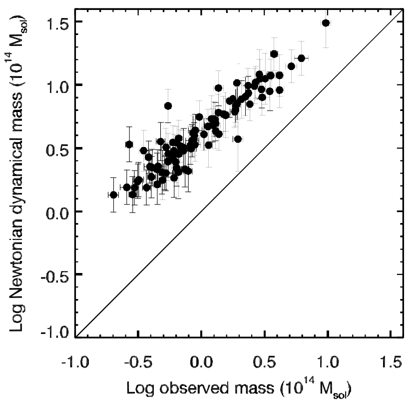
David's correlation and beta-models are imprecise for clusters with small outer radius. We therefore consider only clusters with outer radius ≥ 0.75 Mpc.

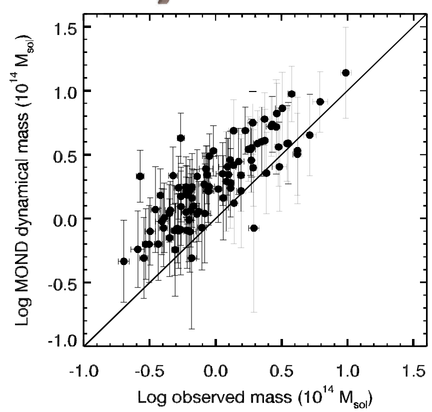
We are left with 93 clusters.

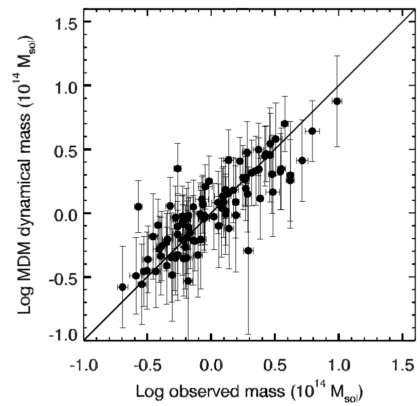
We have adapted Sanders' approach (for MOND) to the case of MDM (to compare MOND with MDM, formerly known as MONDian dark matter).

Galactic Clusters: data fits

Newtonian dynamics	MoNDian dynamics	MoNDian dark matter







$$\frac{M_{Newton}}{M_{obs}} \approx 4.4 \qquad \frac{M_{MoND}}{M_{obs}} \approx 2.1 \qquad \frac{M_{MDM}}{M_{obs}} \approx 1.0$$

DE, Farrah, Ho, Minic, Ng & Takeuchi, 2015 [in preparation];
Sanders 1999, ApJ 512

A comment on strong lensing

Strong lensing: the formation of multiple images of background sources by the central regions of some clusters.

Critical surface density required for strong lensing is $\Sigma_c = \frac{1}{4\pi} \frac{cH_0}{G} F(z_l, z_s)$, with $F \approx 10$, typical observations.

Deep MOND limit: $\Sigma_{MOND} \approx a_c/G$

Numerical values: $a_c \approx cH_0/6$

So, as noted by Sanders, MOND cannot produce strong lensing on its own: $\Sigma_c \approx 5\Sigma_{MOND}$

But MDM mass distribution appears to be sufficient for strong lensing:
$a_0 = cH_0 = 2\pi a_c \approx 6a_c$

- Cosmology: Friedmann's Equations

In a fully relativistic situation, we should use the active gravitational (Tolman-Komar) mass, i.e., replace a non-relativistic source of gravity by a fully relativistic source

$\Rightarrow$ Friedmann's Equations:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3},$$

and

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}.$$

I.e., One can in principle have Einstein's gravity together with a(n additional) Modified Dark Matter source.

If we naively use MoND at the cluster scale, we would be missing the pressure and cosmological constant terms which could be significant. This may explain why MoND doesn't work well at the cluster scale, despite the CDM-MoND duality realized at the galactic scale.

Quanta of MDM obey infinite statistics?

- MDM via gravitational Born-Infeld theory
- Infinite statistics

- **Modified Dark Matter via Gravitational Born-Infeld Theory**

A particularly useful reformulation of MDM is via an effective gravitational dielectric medium, motivated by the analogy between Coulomb's law in a dielectric medium and Milgrom's law for MoND. [E.g., write Milgrom's μ as $1+\chi$ with χ being interpretted as "gravitational susceptibility".]

$\Rightarrow$ MoNDian force law is recovered if the quanta of MDM obey the so-called infinite statistics (as described by the Cuntz algebra (a curious average of the bosonic and fermionic algebras) $a_i\, a_j^\dagger = \delta_{ij}$. See next 2 slides.)

Note: Theories of particles obeying ∞ statistics are non-local [Fredenhagen; Greenberg]

(The fields associated with infinite statistics are not local, neither in the sense that their observables commute at spacelike separation nor in the sense that their observables are pointlike functionals of the fields.)

Can expect unusual dynamics and interactions with ordinary matter (?)

Perhaps this explains the difficulty in detecting dark matter.

INFINITE STATISTICS

[Doplicher, Haag, & Roberts; Govorkov; Greenberg; ...]

• q-deformation of the Heisenberg algebra $(-1 \leq q \leq 1)$

$$a_k a_l^\dagger - q a_l^\dagger a_k = \delta_{kl}$$

($q = \pm 1$ corresponds to bosons/fermions)

• Take $q = 0 \Rightarrow a_k a_l^\dagger = \delta_{kl}$

• Any 2 states obtained by acting on $|0>$ with creation operators in different order are orthogonal to each other

$$< 0|a_{i1}...a_{iN} a_{jN}^\dagger ... a_{j1}^\dagger|0> = \delta_{i1,j1}...\delta_{iN,jN}$$

implying that particles obeying inf. stat. are virtually distinguishable

• The partition function is $Z = \Sigma e^{-\beta H}$, NO Gibbs factor

The only known consistent statistics in greater than 2 space dimensions without the Gibbs factor is the quantum Boltzmann statistics, aka infinite statistics

In infinite statistics, all representations of the particle permutation group can occur.

Theories of particles obeying ∞ statistics are non-local

[Fredenhagen; Greenberg]

Number operator

$$n_i = a_i^\dagger a_i + \sum_k a_k^\dagger a_i^\dagger a_i a_k + \sum_l \sum_k a_l^\dagger a_k^\dagger a_i^\dagger a_i a_k a_l + \ldots,$$

and Hamiltonian, etc., are both nonlocal and nonpolynomial in the field operators

• TCP theorem and cluster decomposition still hold

QFTs with ∞ statistics remain unitary

Nonlocality in ∞ statistics can be a virtue

• may be related to nonlocality in holographic principle

SUMMARY

- By generalizing entropic gravity to de Sitter space, and accounting for Milgrom's scaling, we are led to a new form of dark matter.
- Modified dark matter (MDM) behaves like MOND at galactic scales but like CDM at cluster and cosmic scales.
- We fit rotation curves for 30 local spiral galaxies, it PASSES!
- We also test MDM at cluster scales, and again it fares well.
- Preliminary work on strong gravitational lensing and MDM-cosmology is promising.
- Speculation: "particles" constituting DM obey ∞ statistics. If correct, this may explain the difficulty in detecting dark matter.

FUTURE WORK: **(STAY TUNED)**

1. Gravitational lensing; Can it distinguish MDM from CDM?
2. Study interactions of MDM (quanta obeying infinite statistics) with ordinary (baryonic) matter $\Rightarrow$ particle phenomenology. The Bullet Cluster; How strongly coupled is MDM to baryonic matter? How does MDM self-interact?
3. Tests at cosmological scales (acoustic oscillations measured in the CMB...); Simulations of structure formation?
4. NGC1052-DF2 (A galaxy lacking dark matter): surrounding dark matter (to be detected by, e.g., gravitational lensing)?
5. EDGES (21-cm anomaly)? If confirmed ...?
6. Stars made of quanta obeying infinite statistics?
7. Can quantum gravity be actually the origin of particle statistics and the underlying statistics is infinite statistics in that ordinary particles obeying Bose or Fermi statistics are actually some sort of collective degrees of freedom? (What are the effects on the idea of grand unification?)

Back-up slides:

NFW density profile

more detailed discussion of cosmology

more detailed discussion of gravitational Born-Infeld theory and infinite statistics for quanta of MDM

For the CDM fits, we use the NFW density profile:

$$\rho'(r) = \frac{\rho_0}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}.$$

Here $r_s = \frac{r_{200}}{c}$ which designates the 'edge' of the halo, within which objects are assumed to be virialized, usually taken to be the boundary at which the halo density exceeds 200 times that of the background. The parameter c is a dimensionless number that indicates how centrally concentrated the halo is.

The velocity curves are then determined by

$$v(r) = v_{200}\sqrt{\frac{\ln(1+cx) - cx/(1+cx)}{x\left[\ln(1+c) - c/(1+c)\right]}},$$

where v_{200} is the Newtonian velocity at r_{200}. This equation is fit to the data with c, v_{200}, and M/L as free parameters.

Cosmology: Friedmann's Equations.

The FRW metric: $ds^2 = -dt^2 + R(t)(dr^2 + r^2\, d\Omega^2)$, where $R(t)$ is the scale factor.

Assume that the matter sources in the universe form a perfect fluid, with four velocity $u_\mu (= (1, \vec{0})$ in rest frame of fluid).

Consider Verlinde's imaginary holographic screen of comoving radius r (with physical radius $\tilde{r} = rR(t)$).

In a fully relativistic situation, we replace $\tilde{M}$ by the active gravitational (Tolman-Komar) mass $\mathcal{M} = \frac{1}{4\pi G} \int dV\, R_{\mu\nu} u^\mu u^\nu$, and by Einstein's field equation, $\mathcal{M} = 2 \int dV\, \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} + \frac{\Lambda}{8\pi G} g_{\mu\nu} \right) u^\mu u^\nu = \left(\frac{4}{3} \pi r^3 R^3 \right) \left[(\rho + 3p) - \frac{\Lambda}{4\pi G} \right]$

$\Rightarrow$ Friedmann's Equations:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3},$$

and

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}.$$

• Thus one can in principle have Einstein's gravity together with a(n additional) MoNDian dark matter source.

The departure from MoND happens when (we replace $\tilde{M}$ with $\mathcal{M}$, i.e. when) a non-relativistic source is replaced by a fully relativistic source. In that case $\sqrt{a^2 + a_0^2} - a_0 = \frac{G\,\mathcal{M}}{\tilde{r}^2}$, where $\tilde{r} = rR(t)$ is the physical radius, i.e.,

$$\sqrt{a^2 + a_0^2} - a_0 = \frac{G\,(\,M(t) + M'(t)\,)}{\tilde{r}^2} + 4\pi G\,p\,\tilde{r} - \frac{\Lambda}{3}\,\tilde{r}.$$

If we naively use MoND at the cluster scale, we would be missing $4\pi G\,p\,\tilde{r} - \frac{\Lambda}{3}\,\tilde{r}$ which could be significant. This may explain why MoND doesn't work well at the cluster scale, despite the CDM-MoND duality realized at the galactic scale.

Modified Dark Matter via Gravitational Born-Infeld Theory

A particularly useful reformulation of MoND is via an effective gravitational dielectric medium, motivated by the analogy between Coulomb's law in a dielectric medium and Milgrom's law for MoND. It has been known to (Blanchet, Milgrom and) others that the MoNDian force law can be formulated as being governed by a nonlinear generalization of Poisson's equation which describes the nonlinear electrostatics embodied in the Born-Infeld theory.

Consider the Born-Infeld (BI) theory defined with the following Lagrangian density (b being a dimensionful parameter; the second form is for the case of $\vec{B} = 0$):

$$L_{BI} = b^2 \left(1 - \sqrt{1 - \frac{E^2 - B^2}{b^2} - \frac{(\vec{E} \cdot \vec{B})^2}{b^4}} \right) \longrightarrow b^2(1 - \sqrt{1 - E^2/b^2}),$$

(a nonlinear electrodynamics motivated by the analogy with relativistic mechanics given by $L_{particle} = mc^2(1 - \sqrt{1 - v^2/c^2})$ with c $\Longleftrightarrow b$).

Let us set $\vec{B} = 0$, concentrate on the Hamiltonian density H, supply an extra overall factor of $\frac{1}{4\pi}$ and use the notation $\vec{D} = \epsilon \vec{E}$.

Then the corresponding gravitational Hamiltonian density reads:

$$H_g = \frac{b^2}{4\,\pi} \left(\sqrt{1 + \frac{D_g^2}{b^2}} - 1 \right).$$

Let $A_0 \equiv b^2$ and $\vec{A} \equiv b\,\vec{D}_g$, then the Hamiltonian density becomes

$$H_g = \frac{1}{4\,\pi} \left(\sqrt{A^2 + A_0^2} - A_0 \right).$$

Assume there exists an energy equipartition, then the effective gravitational Hamiltonian density is equal to

$$H_g = \frac{1}{2}\,k_B\,T_{\text{eff}}\,,$$

where T_{eff} is an effective temperature associated which the energy.

(Note that this energy density is energy per unit volume. But we can regard it as energy per degree of freedom by recalling that volume, which usually scales as entropy S, scales as the number of degrees of freedom N in a holographic setting. Parenthetically $S \sim N$ is one of the features of infinite statistics.)

The Unruh temperature formula $T_{\text{eff}} = \frac{\hbar}{2\,\pi\,k_B}\,a_{\text{eff}}$ implies that the effective acceleration is given by

$$a_{eff} = \sqrt{A^2 + A_0^2} - A_0\,.$$

The equivalence principle suggests that we should identify (at least locally) the local accelerations $\vec{a}$ and $\vec{a}_0$ with the local gravitational fields $\vec{A}$ and $\vec{A}_0$ respectively, viz., $\vec{a} \equiv \vec{A}, \quad \vec{a}_0 \equiv \vec{A}_0\,.$ Then a_{eff} should be identified as:

$$a_{\text{eff}} \equiv \sqrt{a^2 + a_0^2} - a_0\,,$$

which, in turn, implies that the Born-Infeld inspired force law takes the form (for a given test mass m)

$$F_{\text{BI}} = m\left(\sqrt{a^2 + a_0^2} - a_0\right)\,,$$

which is precisely the MoNDian force law derived previously!

(Note: For a typical acceleration of order 10 ms^{-2}, the corresponding effective temperature is of order $k_B\, T_{\rm eff} \sim 10^{-24}$ eV, much smaller than the mass of any viable *cold* dark matter candidate.)

To be a viable cold dark matter candidate, the quanta of our MoNDian dark matter are expected to be much heavier than $k_B\, T_{\rm eff}$.

Recall that the equipartition theorem in general states that the average of the Hamiltonian is given by

$$\langle H \rangle = -\frac{\partial \log Z(\beta)}{\partial \beta}\,,$$

where $\beta^{-1} = k_B T$ and Z denotes the partition function. To obtain $\langle H \rangle = \frac{1}{2}\, k_B\, T$ per degree of freedom, we require Z to be of the Boltzmann form

$$Z = \exp(\,-\beta\, H\,)\,.$$

(The conventional quantum-mechanical Bose-Einstein or Fermi-Dirac statistics would not lead to $\langle H \rangle = \frac{1}{2}\, k_B\, T$ per degree of freedom at low temperataure.)

Thus the validity of $H_g = \frac{1}{2}\, k_B\, T_{\rm eff}$ for very low temperature $T_{\rm eff}$ somehow requires a unique quantum statistics with a Boltzmann partition function.
This is precisely what is called the infinite statistics as described by the Cuntz algebra (a curious average of the bosonic and fermionic algebras)

$$a_i\, a_j^\dagger = \delta_{ij}\,.$$

Thus, by invoking infinite statistics, the assumption of energy equipartition $H_g = \frac{1}{2}\, k_B\, T_{\rm eff}$, even for very low temperature $T_{\rm eff}$, is justified.

Recap: (i) the relation between our force law that leads to MoNDian phenomenology and an effective gravitational Born-Infeld theory; (ii) the need for infinite statistics of some microscopic quanta which underly the thermodynamic description of gravity implying such a MoNDian force law.

(A side remark: From the Matrix theory point of view, we expect infinite statistics and an effective theory of the gravitational Born-Infeld type to be closely related.)