

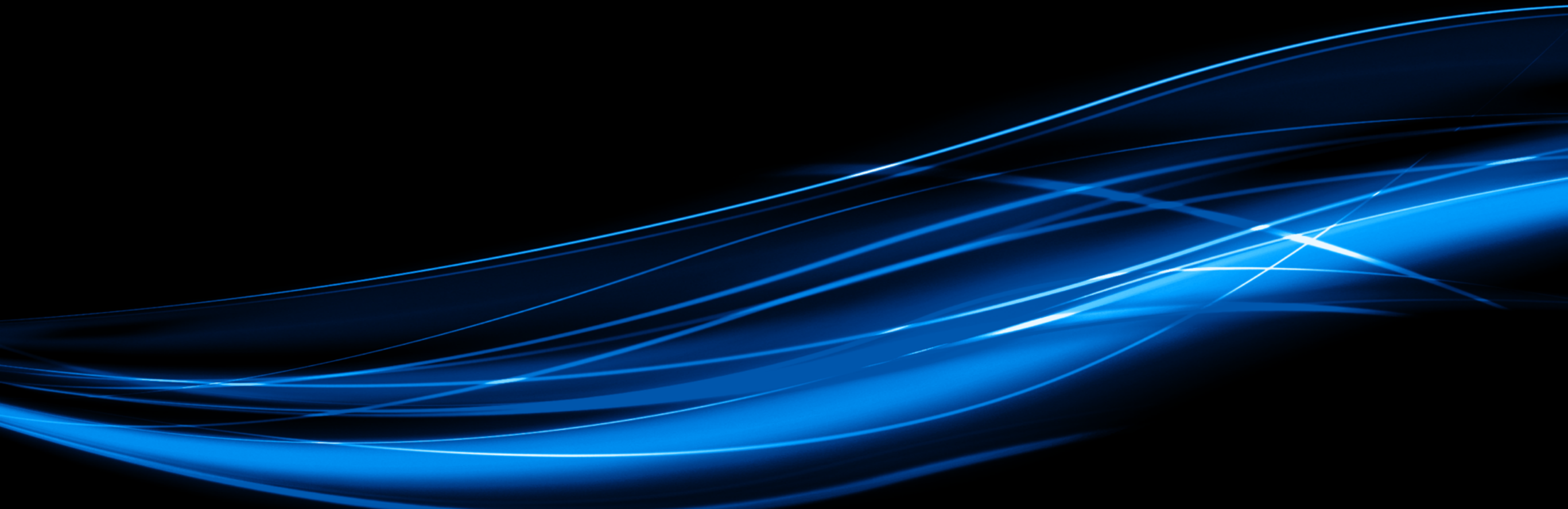
The background features a dark blue field with a complex, abstract pattern of glowing blue lines that form a series of overlapping, wavy bands. These bands create a sense of depth and movement, resembling a topographical map or a data visualization. The overall aesthetic is scientific and futuristic.

# **Modified Gravity:** the Landscape post-GW170817

**Tessa Baker**

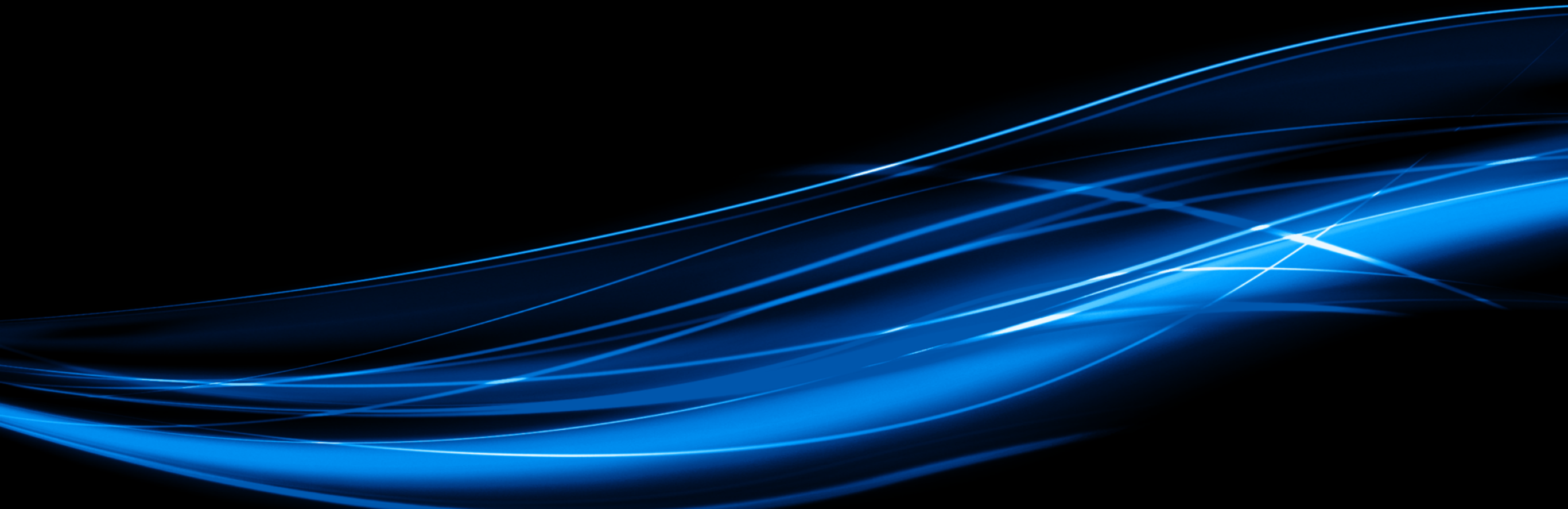
**University of Oxford**

# Outline

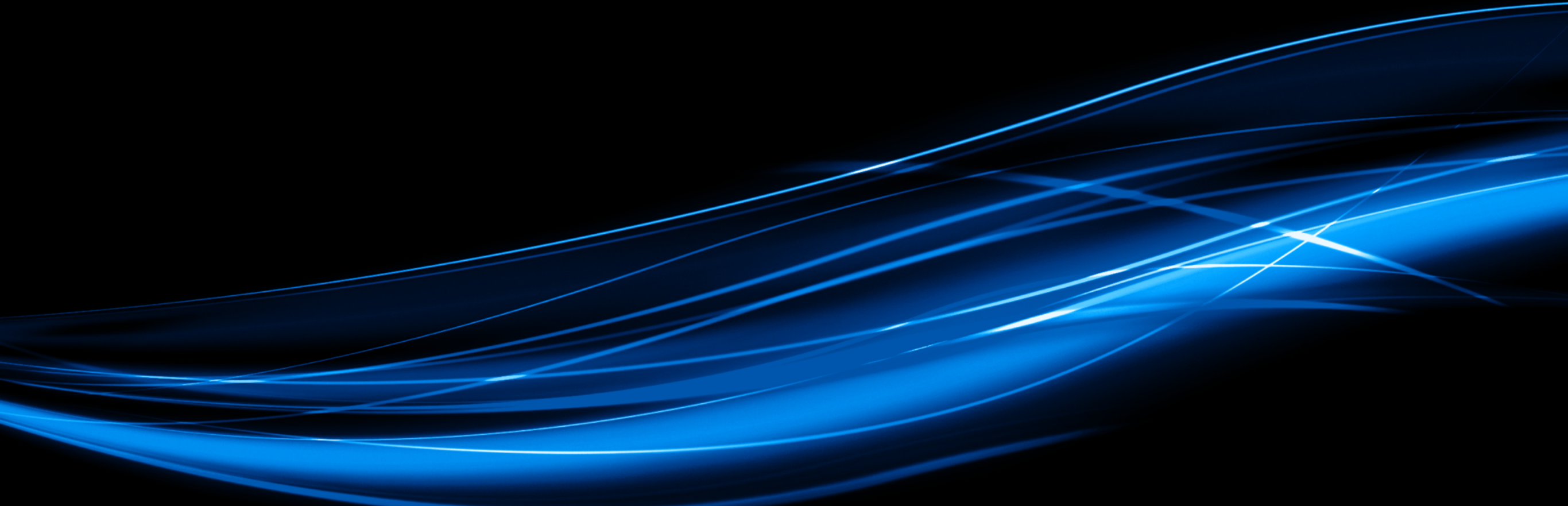


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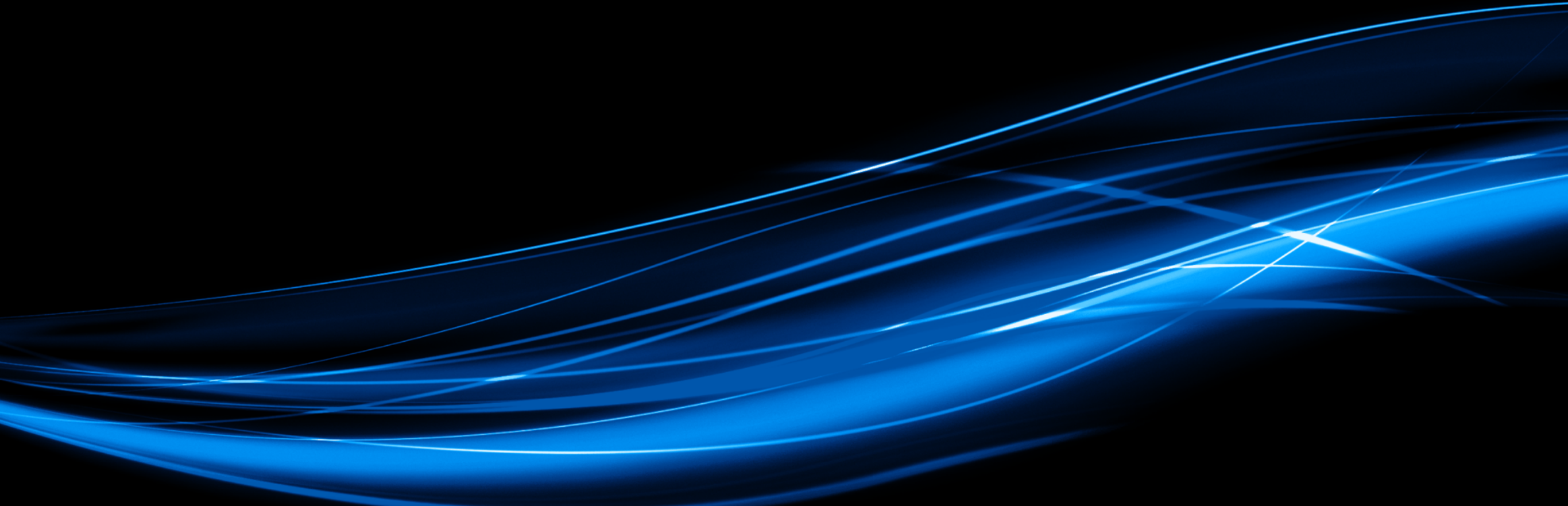
- ◆ Reasons to look beyond GR.



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- ◆ Reasons to look beyond GR.
  - ◆ What happened with GW170817.
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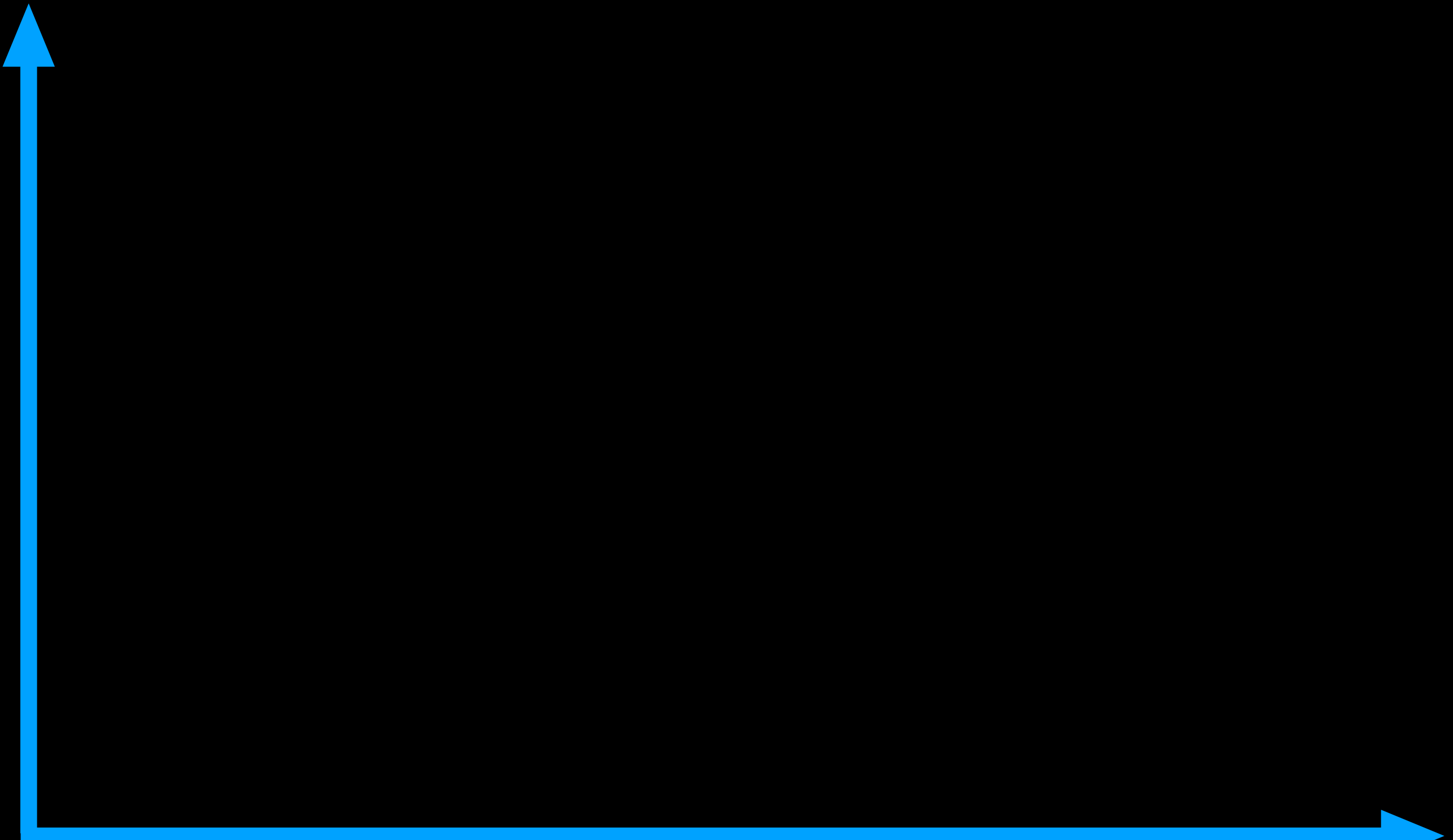
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- ◆ Reasons to look beyond GR.
  - ◆ What happened with GW170817.
  - ◆ What happens next?
- 



# Motivations

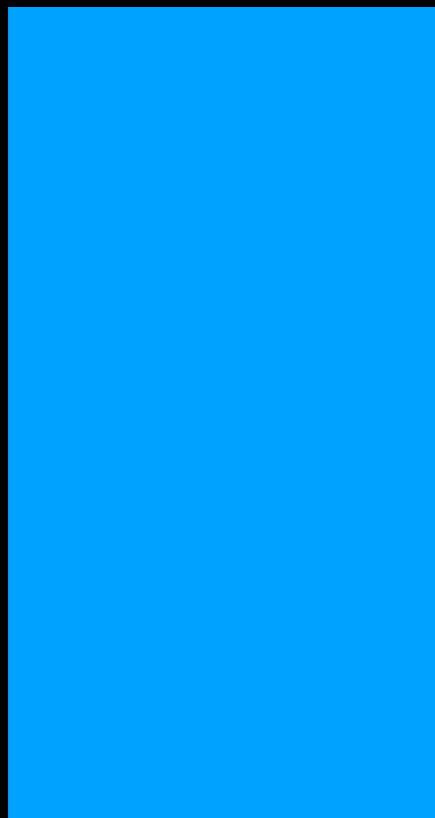
Why are we interested in MG?



# Why are we interested in MG?

**1970s**

**Tests of GR**





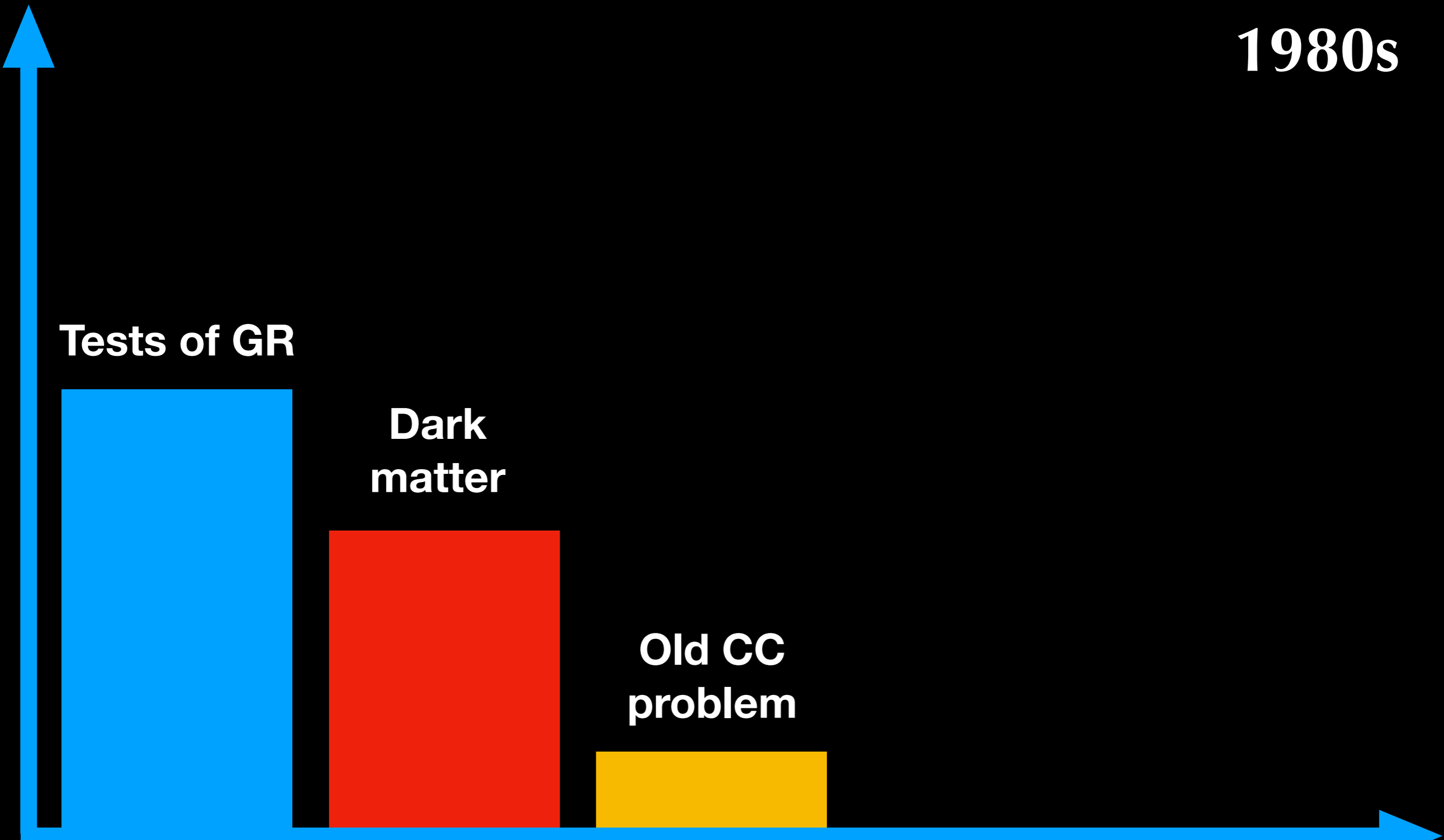
# Why are we interested in MG?

1980s

Tests of GR

Dark  
matter

Old CC  
problem



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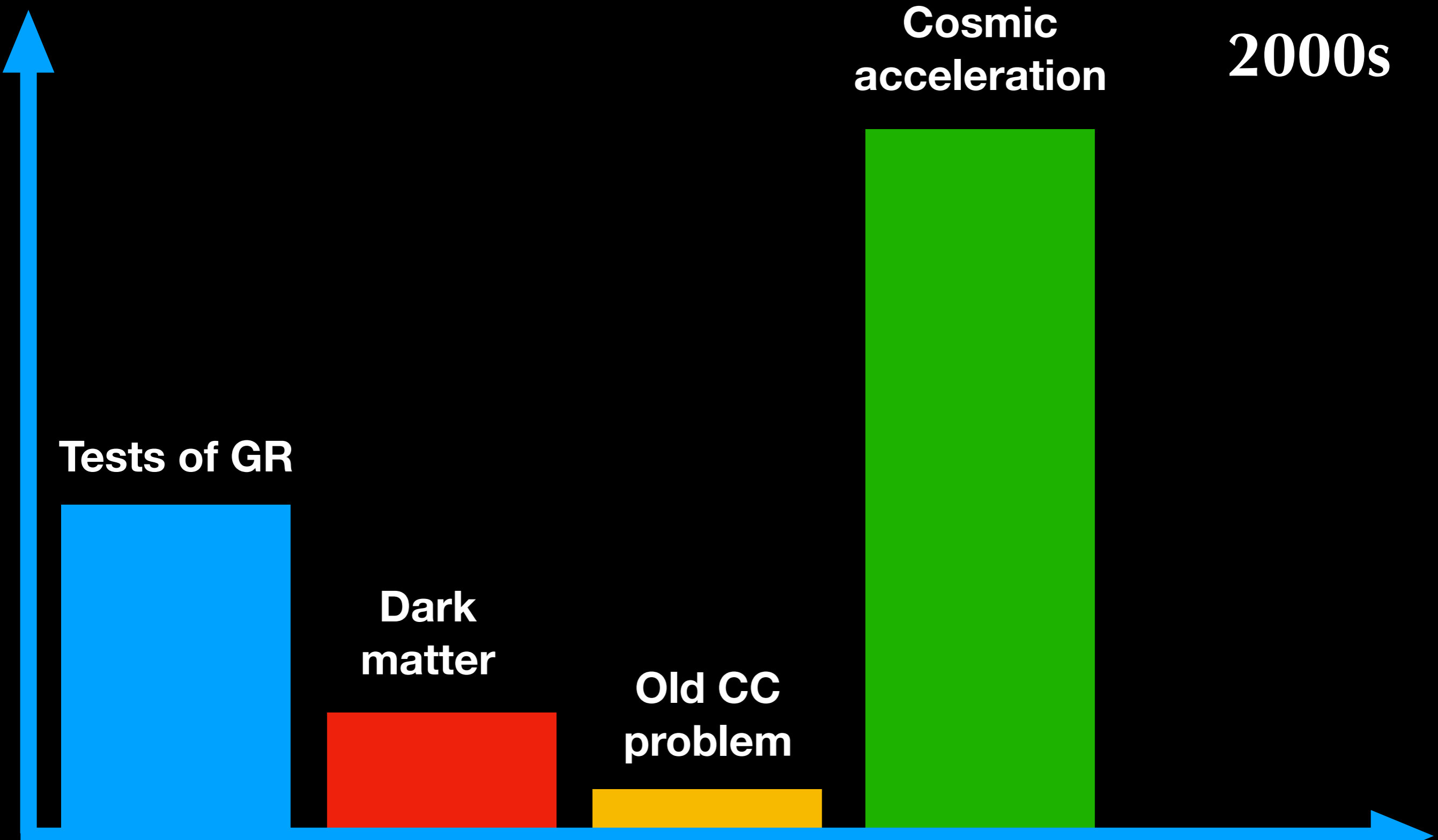
A bar chart with a blue vertical axis and a blue horizontal axis. The chart features three bars of decreasing height from left to right. The first bar is blue and labeled 'Tests of GR'. The second bar is red and labeled 'Dark matter'. The third bar is yellow and labeled 'Old CC problem'. The text '1990s' is located in the upper right area of the chart.

Topic	Relative Interest Level
Tests of GR	High
Dark matter	Medium
Old CC problem	Low

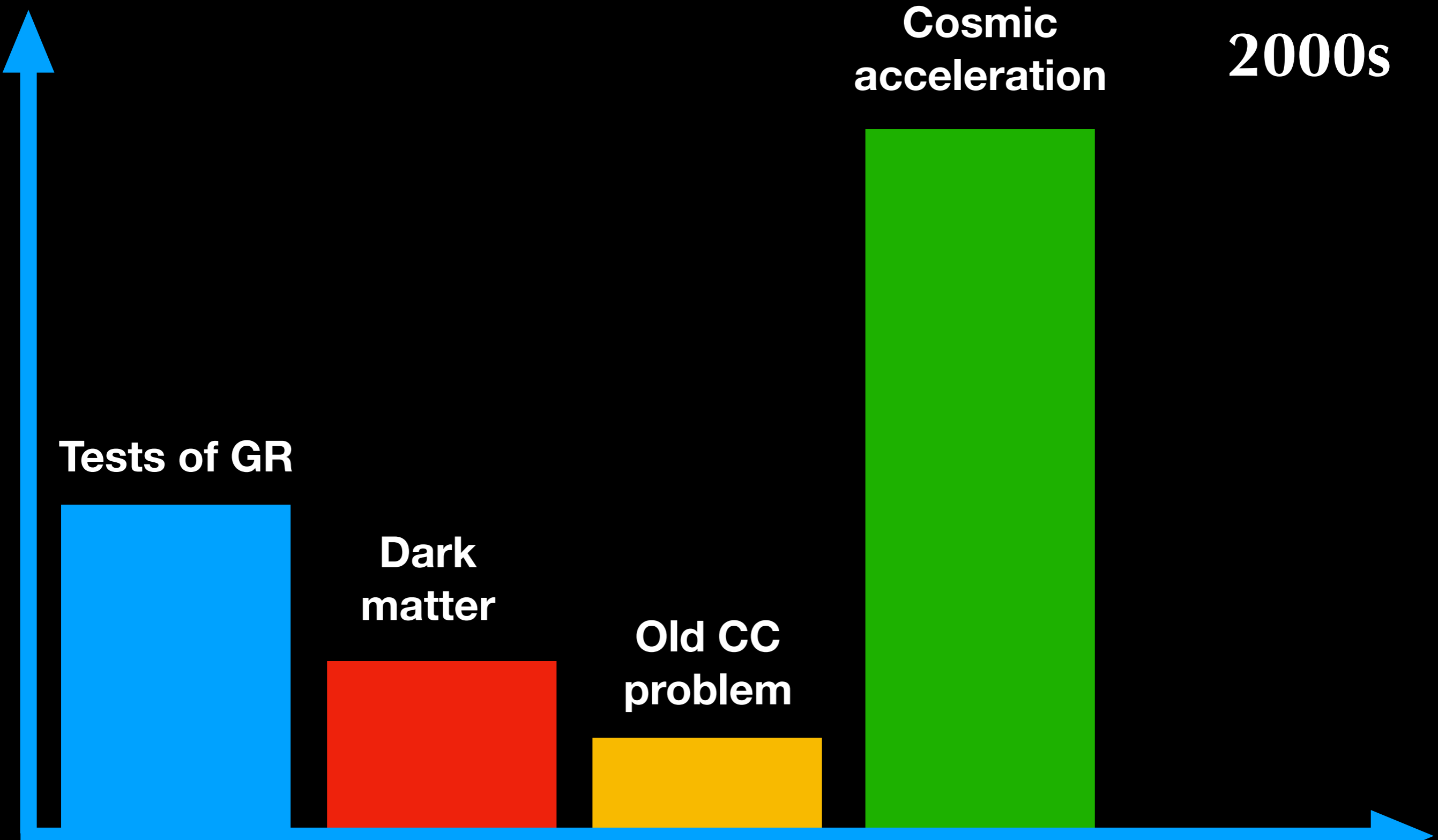
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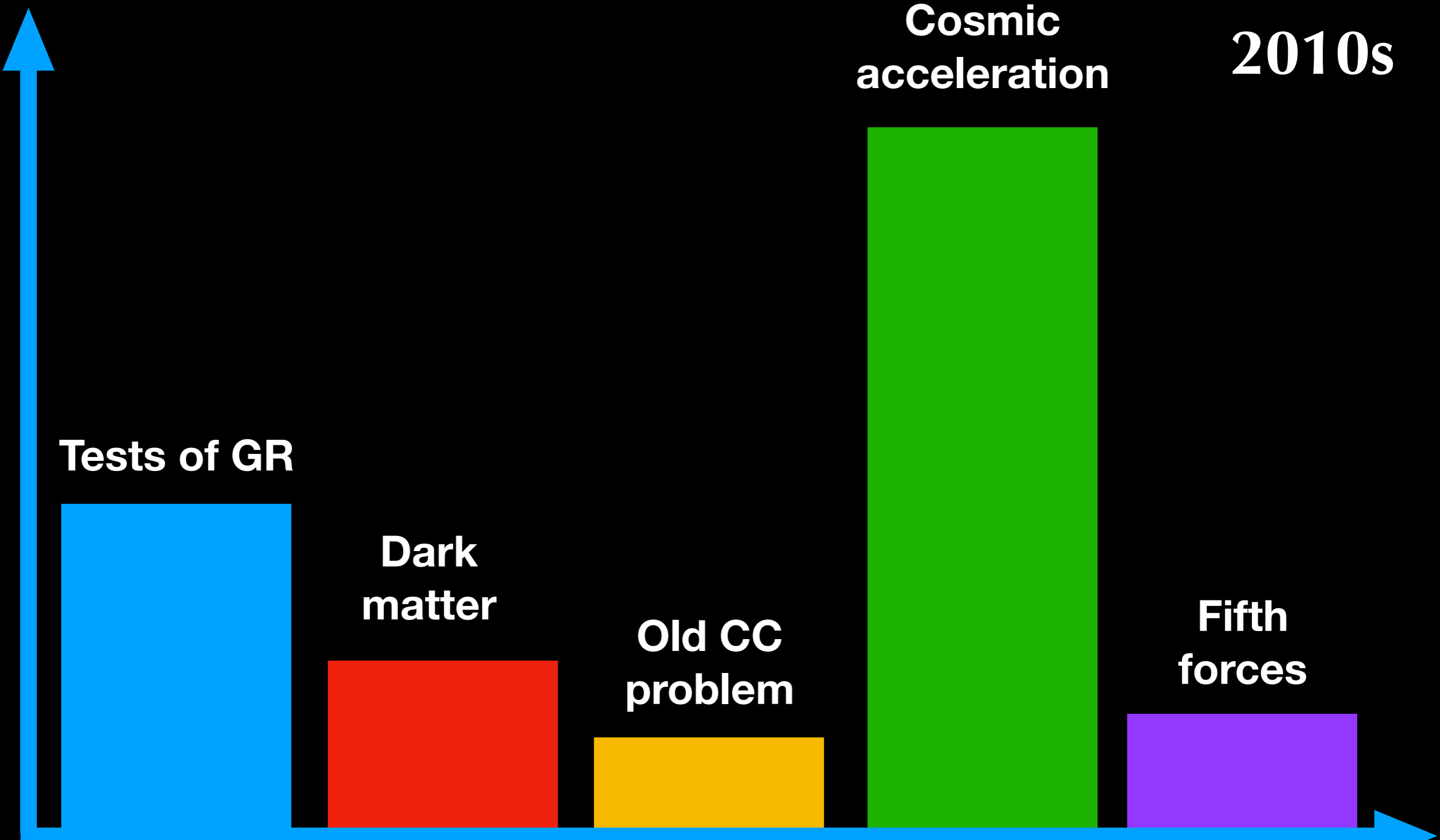
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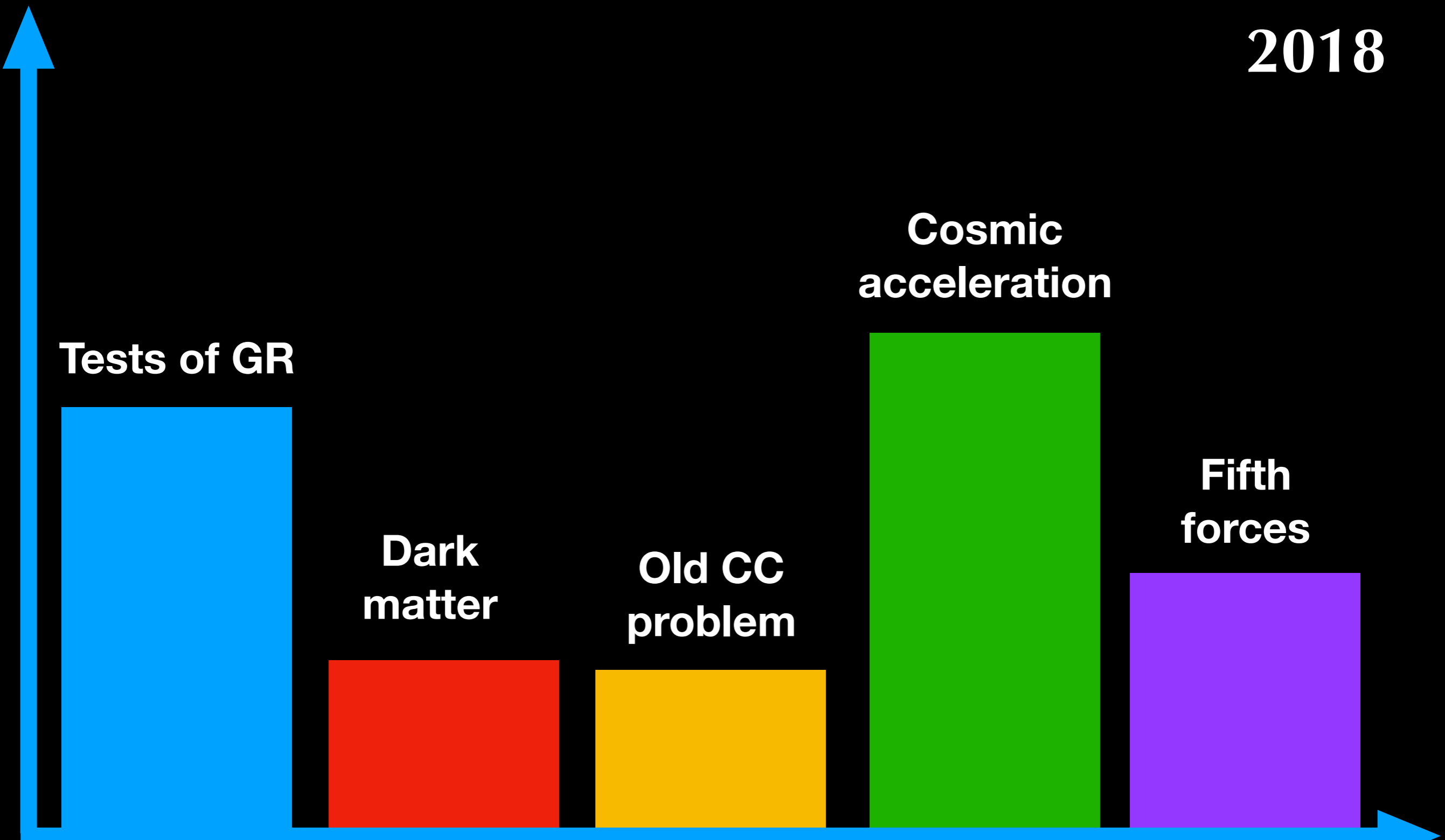
Tests of GR

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matter

Old CC  
problem

Cosmic  
acceleration

Fifth  
forces



# Classification

# Classification

**Scalar**

**Vector**

**Tensor**



# Classification

## Scalar

k-essence

$f(R)$

Galileons

KGB

Horndeski

DHOST

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Generalised Proca

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Massive gravity

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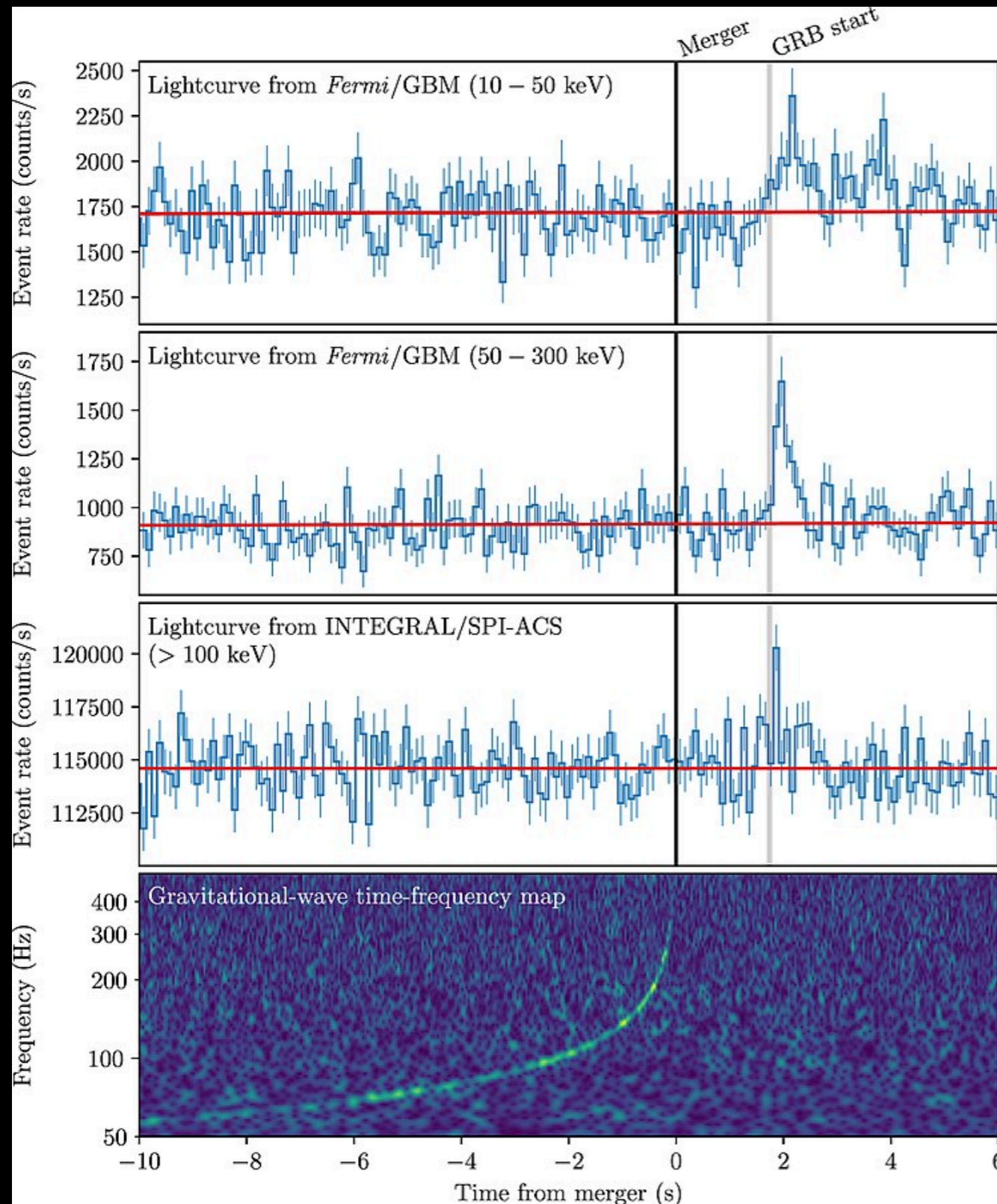
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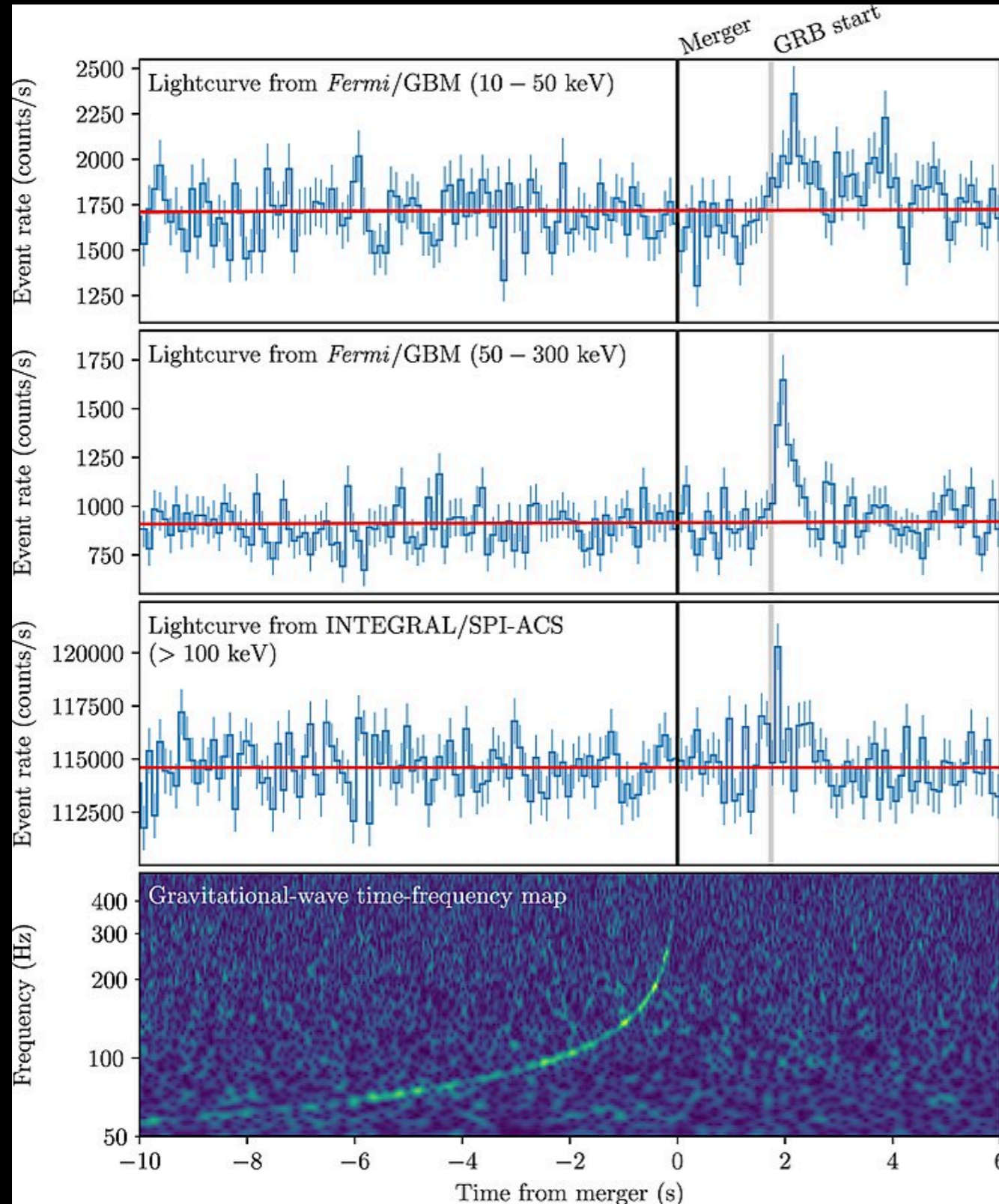
# Gravitational Waves

# GW170817 & EM Friends



$$\Delta t \simeq 1.7 \text{ s}$$

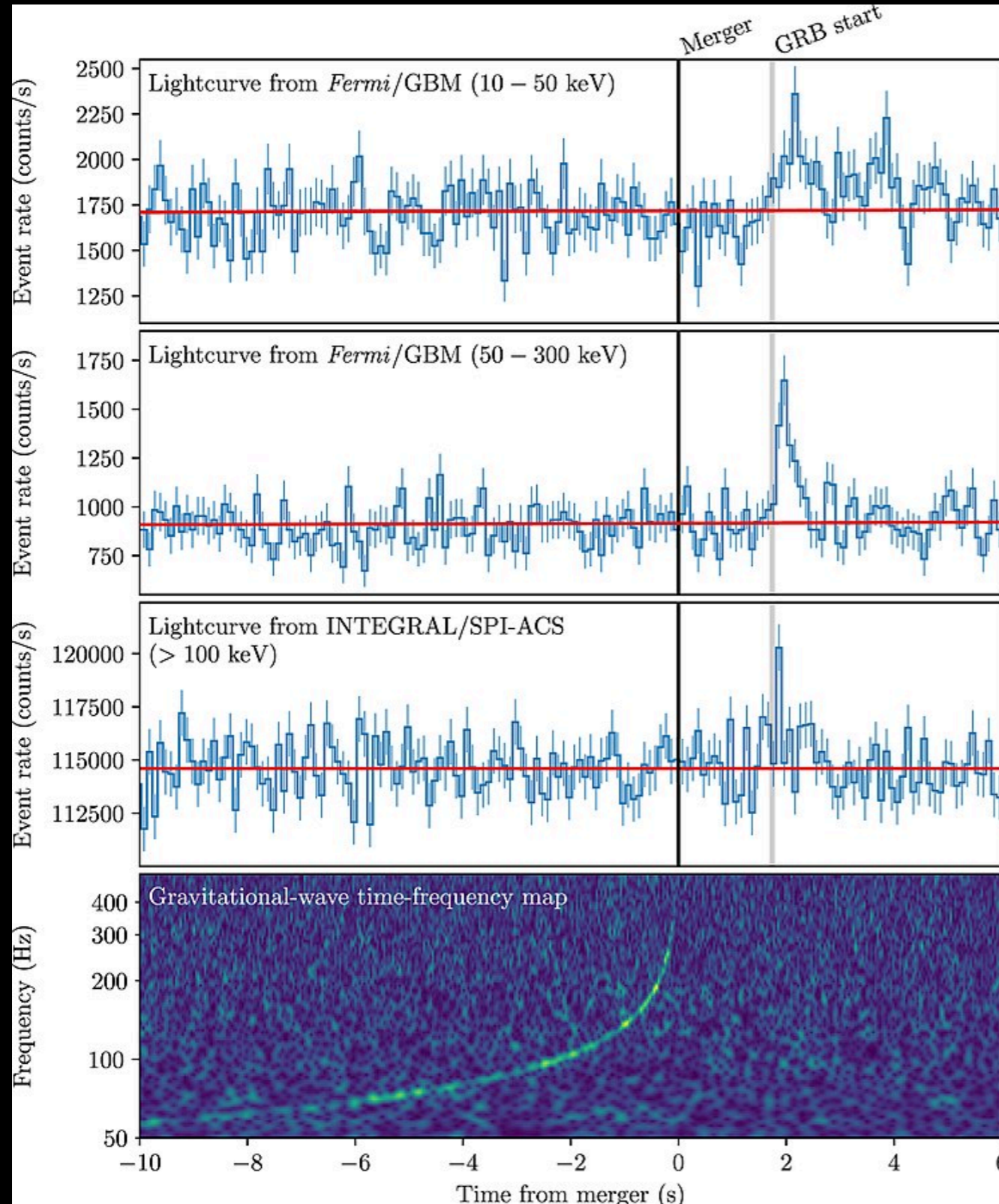
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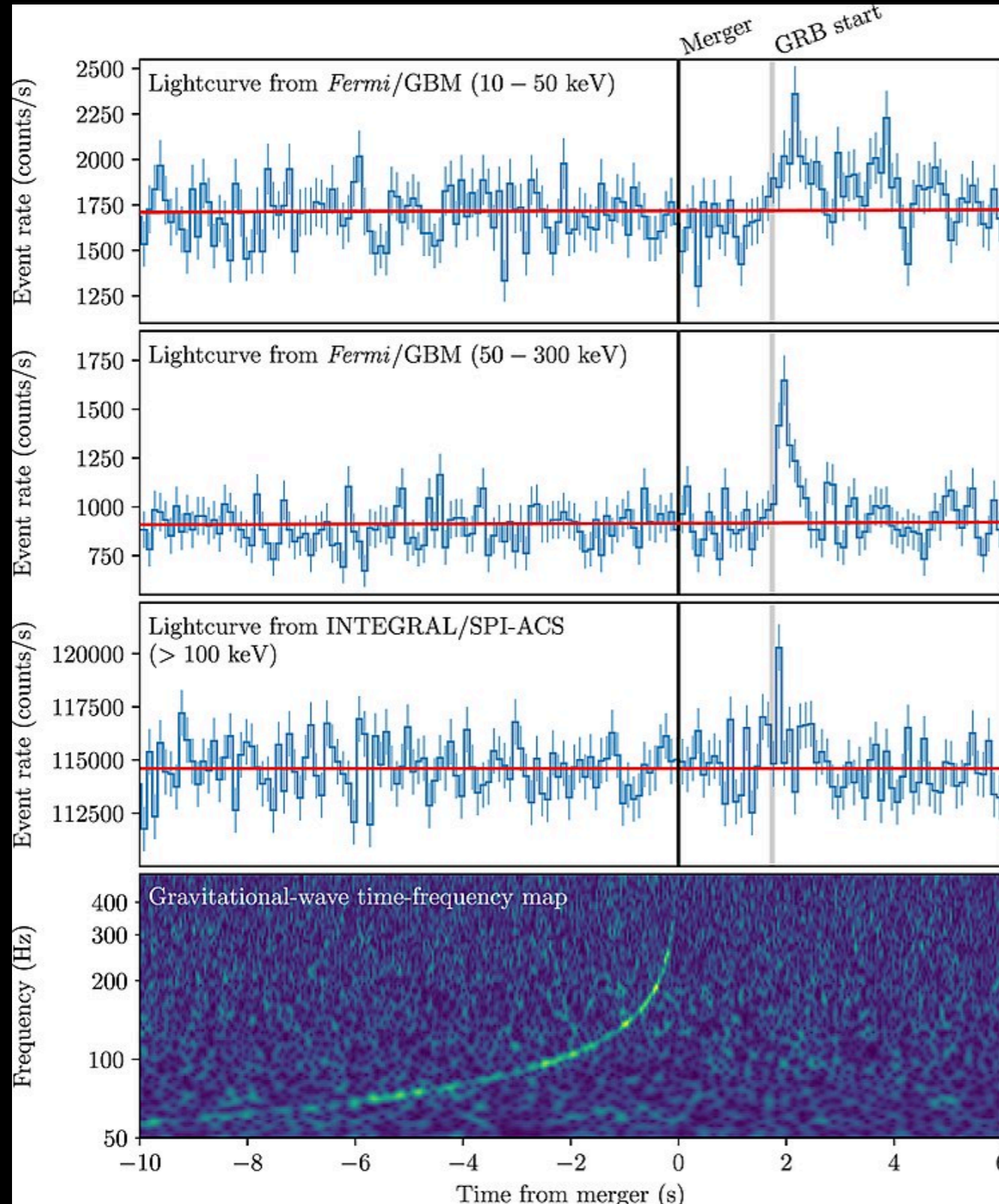
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**More conservative:**

$$|\alpha_T| \lesssim 10^{-12}$$

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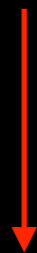
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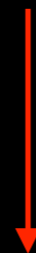
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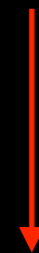
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$\Rightarrow$   $f[R]$  gravity fits the template, so it survives.

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This is not competitive with existing Solar System bounds:

$$m_g \lesssim 10^{-32} \text{ eV}$$

(from Lunar Laser Ranging & Earth-Moon precession)



**Where Next?**



# Survivors: DHOST

Derivative Higher-Order Scalar-Tensor theories.

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Vainshtein screening broken *inside* matter:

$$\frac{d\Phi}{dr} = \frac{\tilde{G}M}{r^2} + F(G_4, A_3) \tilde{G} \frac{d^2 M}{dr^2}$$

Ben Achour 2016  
Langlois & Noui 2016  
Crisostomi et al. 2016

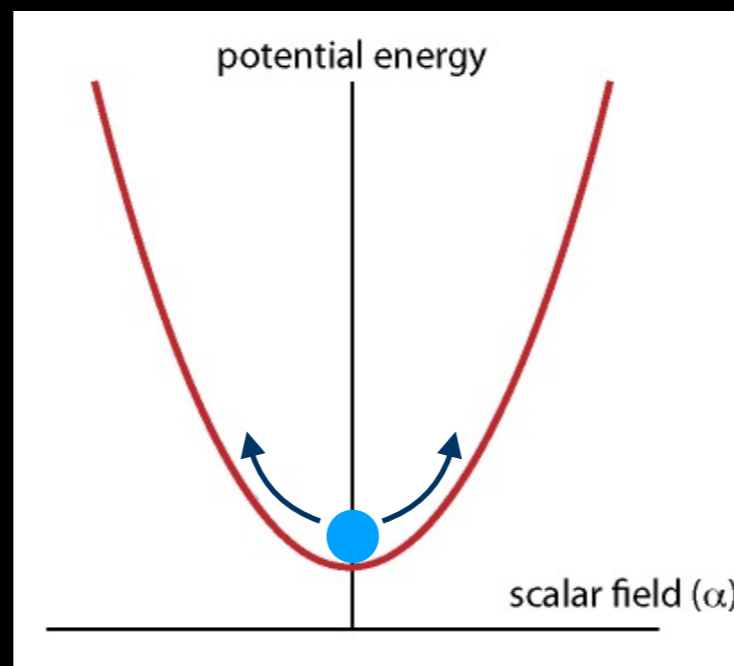
# Survivors: frozen fields

Recall for Horndeski:

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On a cosmological background, make:  $X = \frac{1}{2}\dot{\phi}^2 \ll H_0^2$

$\Rightarrow$  Frozen/slowly-evolving scalar can have  $\alpha_T \simeq 0$ .



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**Parameterised** tests of gravity.

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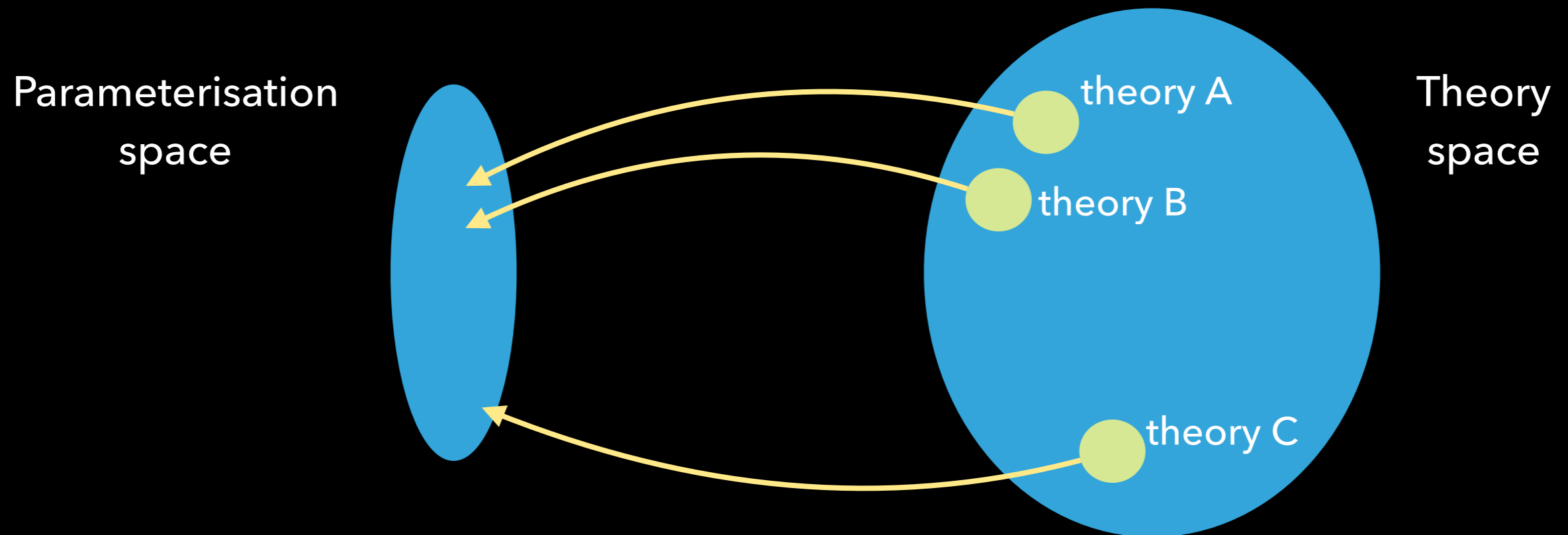
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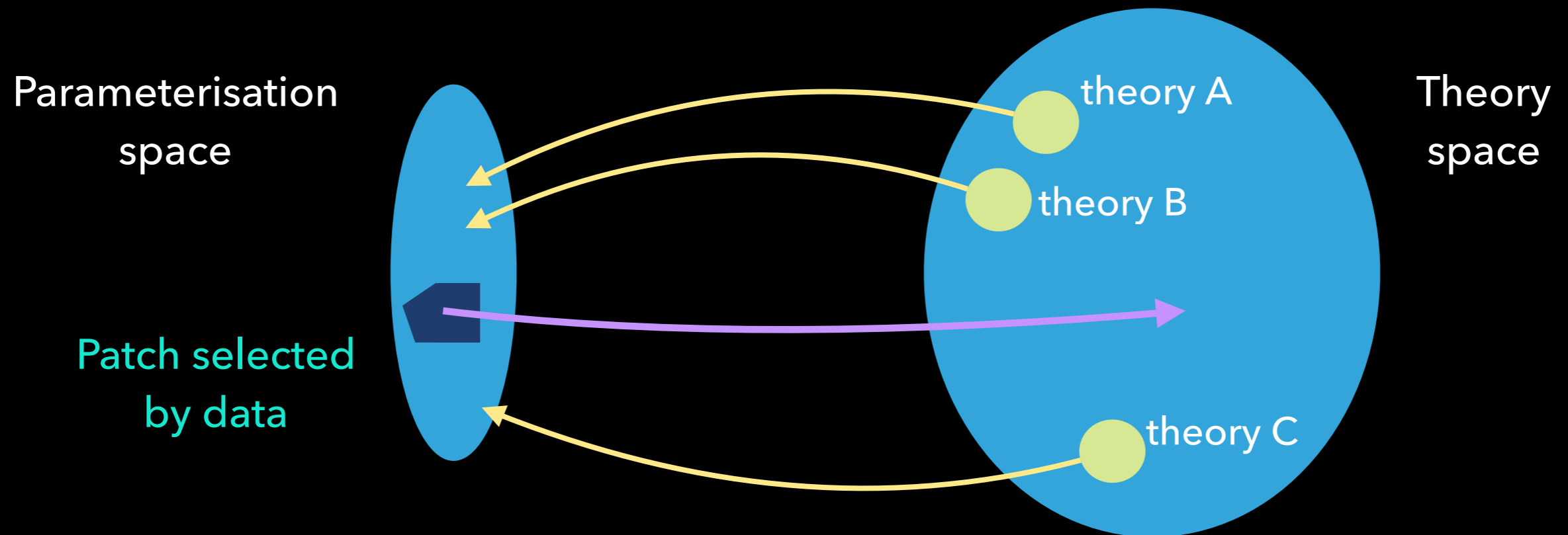
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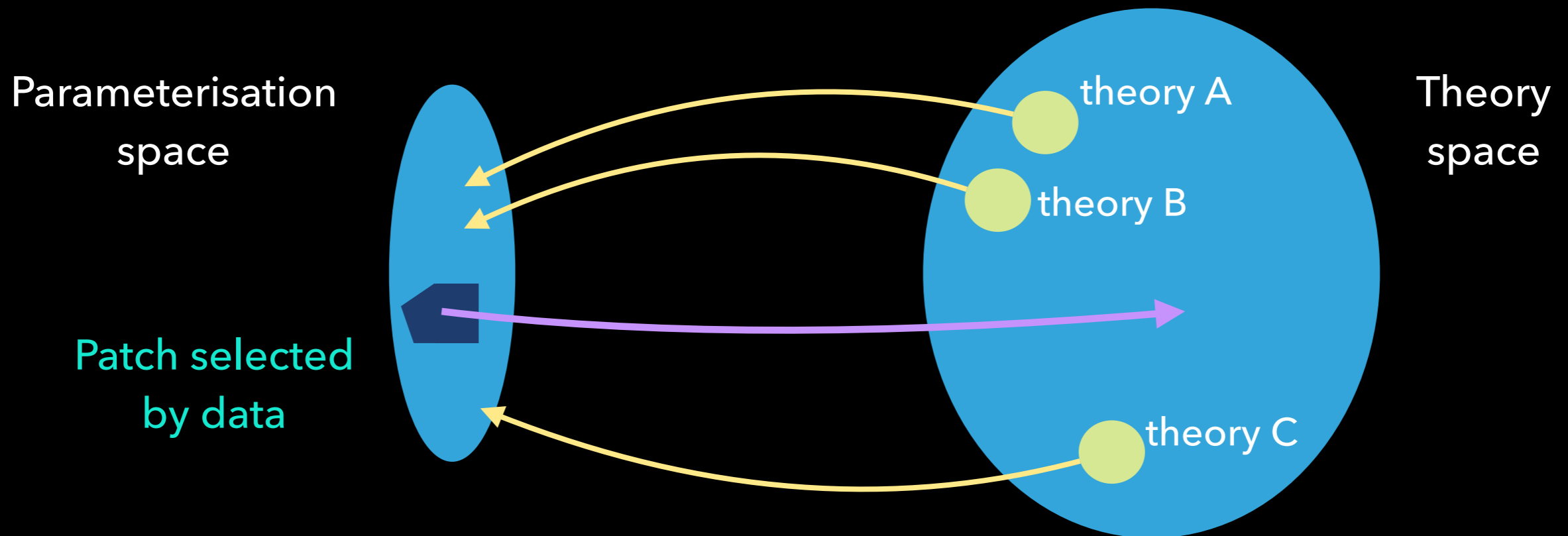
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No 100% perfect solution, but can find parameter set for each family of scalar-, vector-, and tensor-tensor theories.



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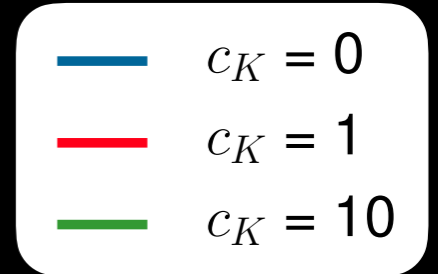
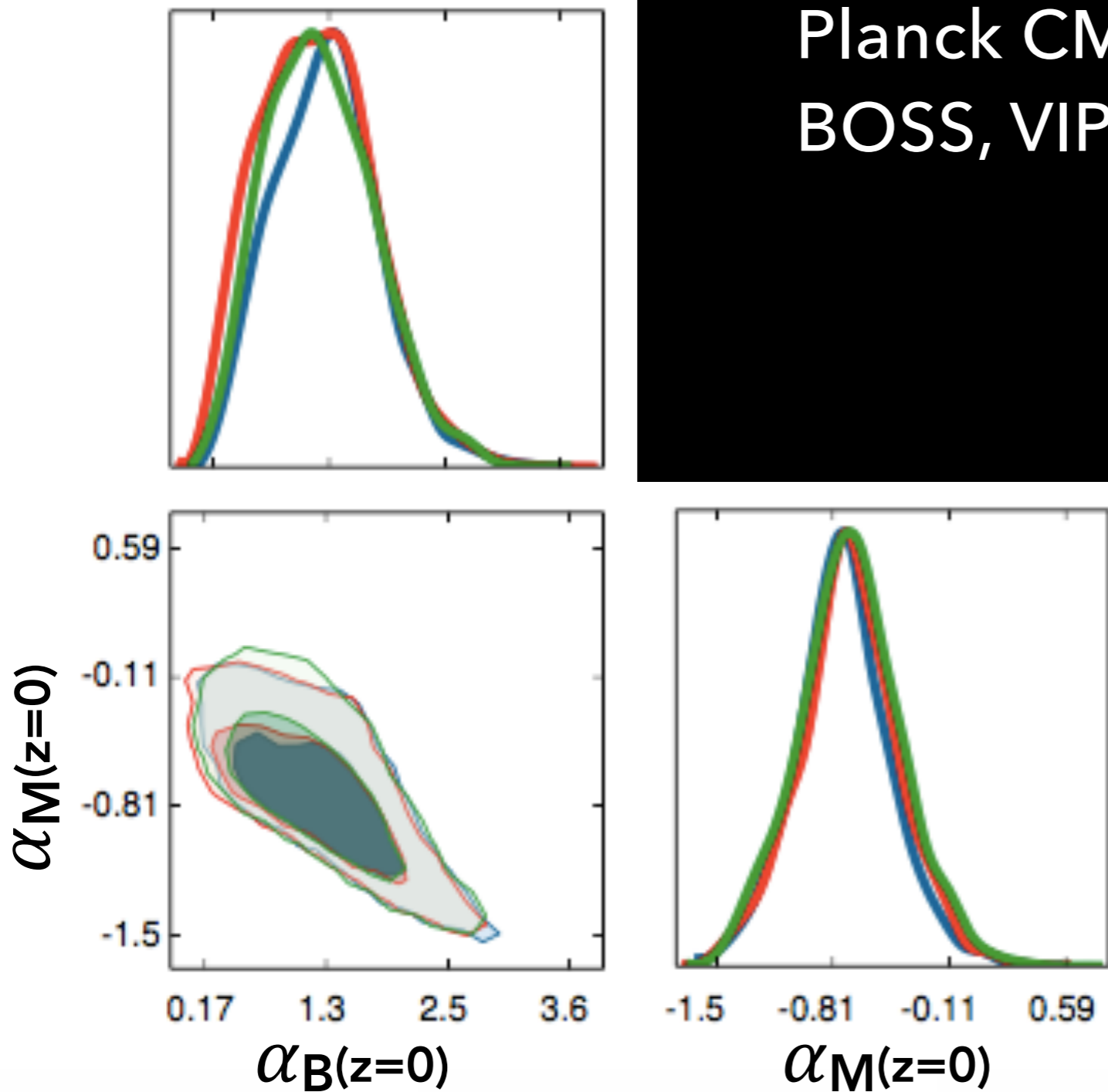
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$\alpha_H(t)$  : disformal symmetries of the metric.

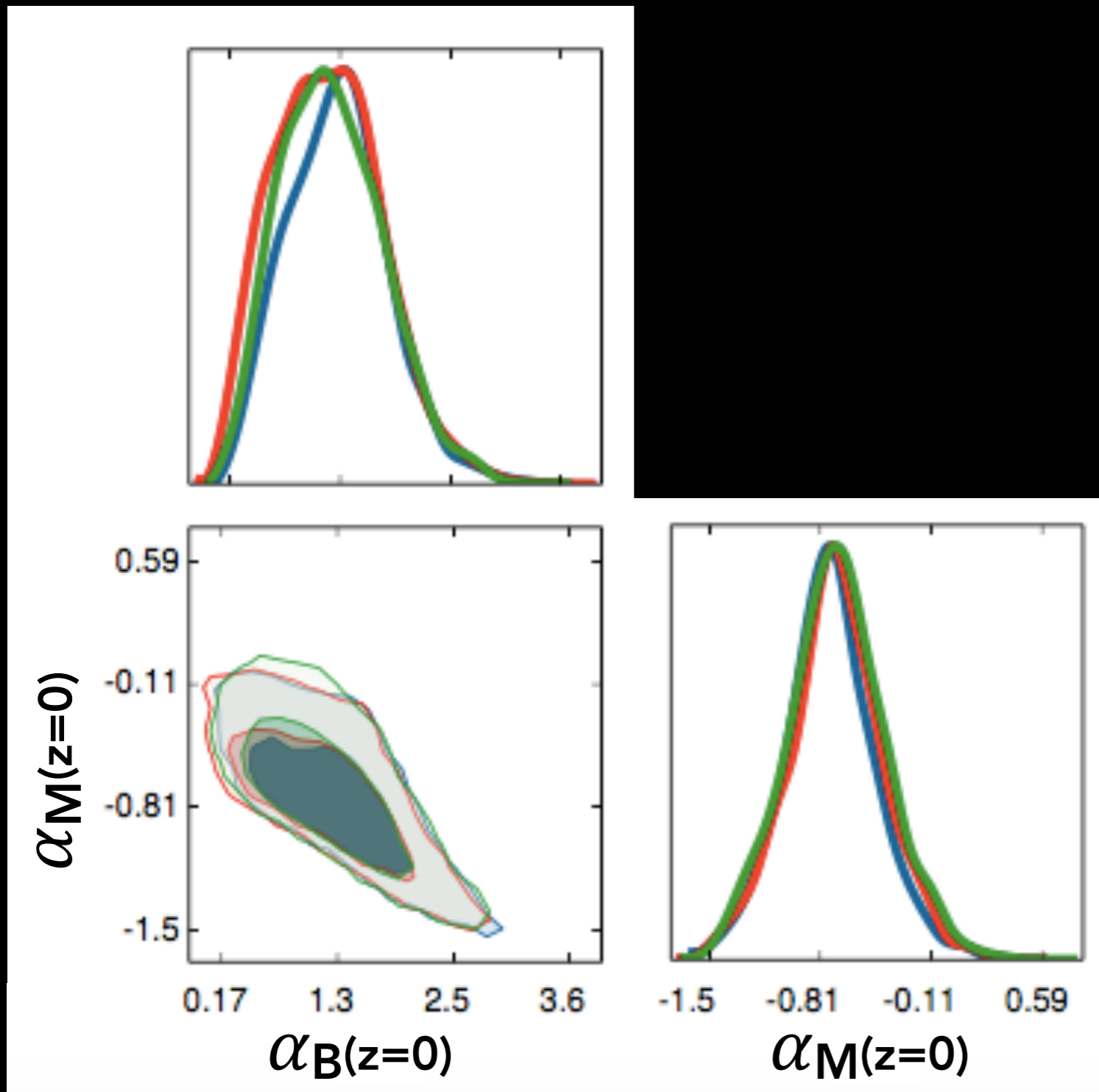
# Current Constraints

Planck CMB data + galaxy surveys:  
BOSS, VIPERS, WiggleZ.





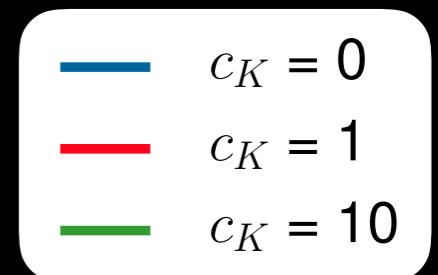
# Current Constraints



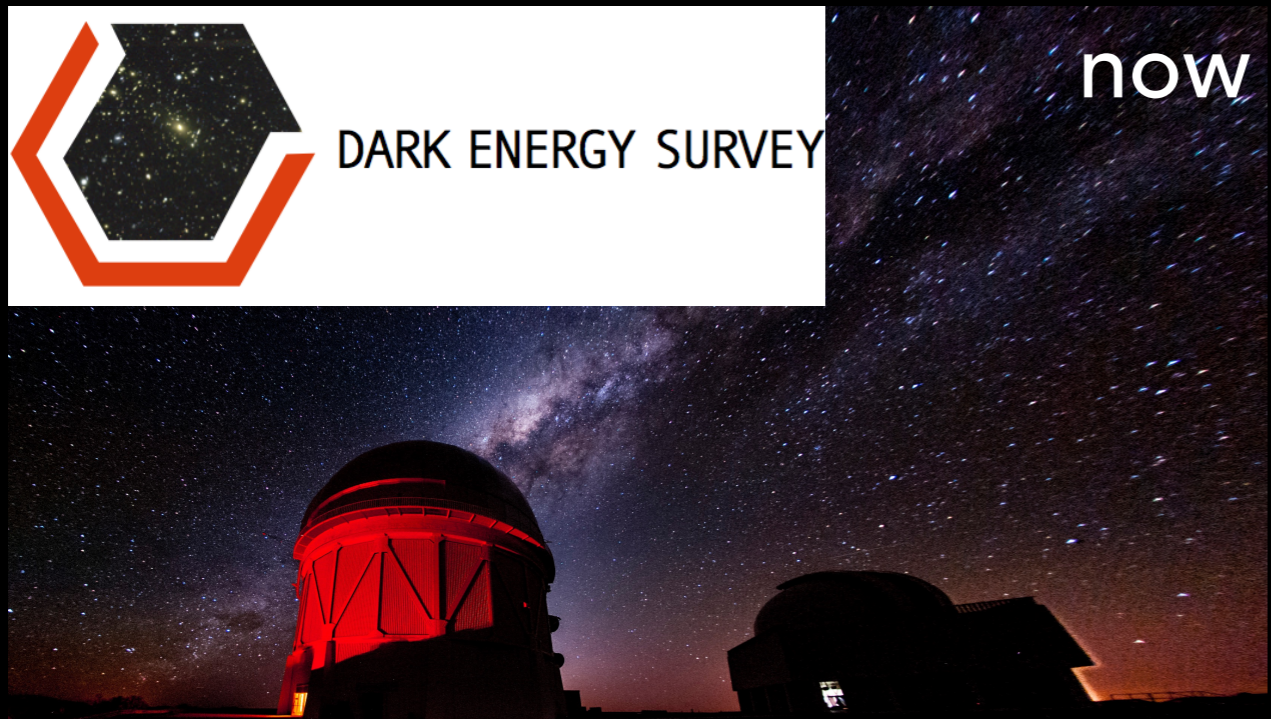
$2\sigma$  constraints:

$$0.24 < \alpha_B < 2.32$$

$$-1.36 < \alpha_M < -0.13$$



# Experiments



**DARK ENERGY SURVEY**

now

The image shows a large astronomical telescope dome at night, illuminated with red light, set against a starry sky with the Milky Way visible.



**euclid**

2021

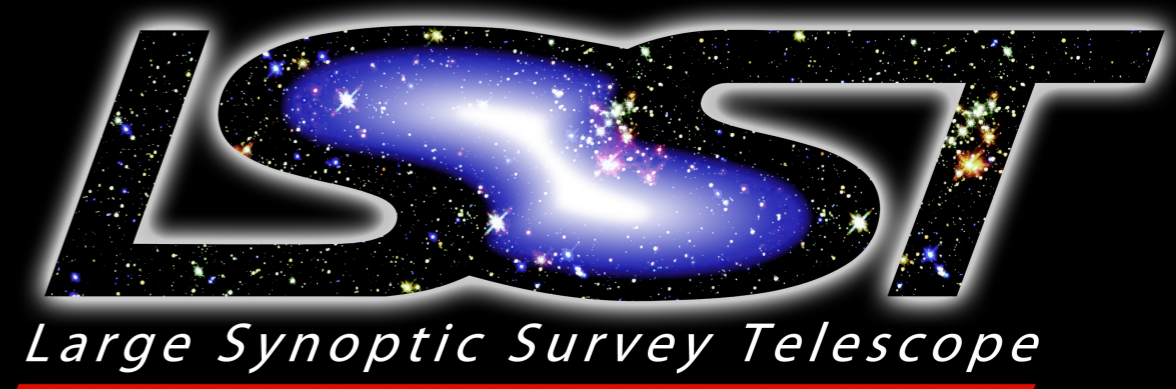
The image shows the Euclid satellite in space, featuring a large solar panel and a complex instrument structure, set against a starry background.

2023



**SKA**  
SQUARE KILOMETRE ARRAY

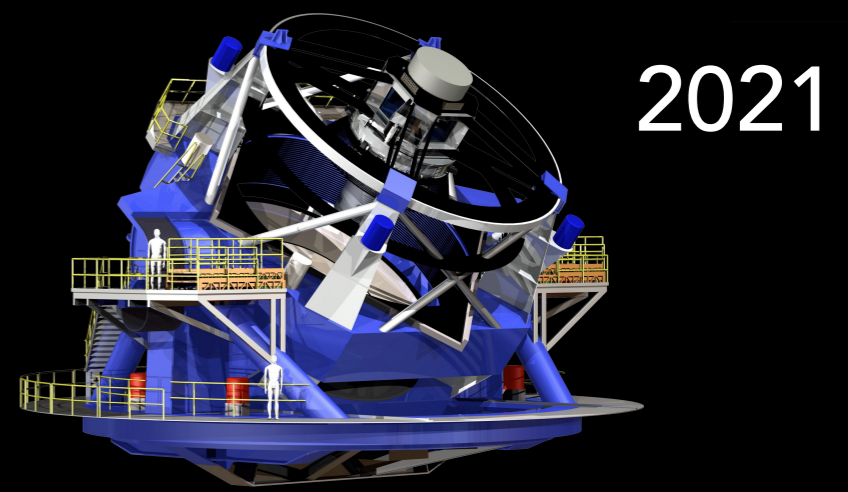
The image shows a vast array of radio telescope dishes in a desert landscape, with the SKA logo overlaid.



**LSST**

*Large Synoptic Survey Telescope*

The logo for the Large Synoptic Survey Telescope (LSST) features the letters 'LSST' in a stylized font, with a glowing galaxy filling the interior of the letters.



2021

The image shows a large radio telescope dish mounted on a complex blue and white structure, with a person standing on a platform for scale.

# Conclusions

# Conclusions

- ◆ MG is alive and well, BUT motivations no longer focus solely on cosmic acceleration.

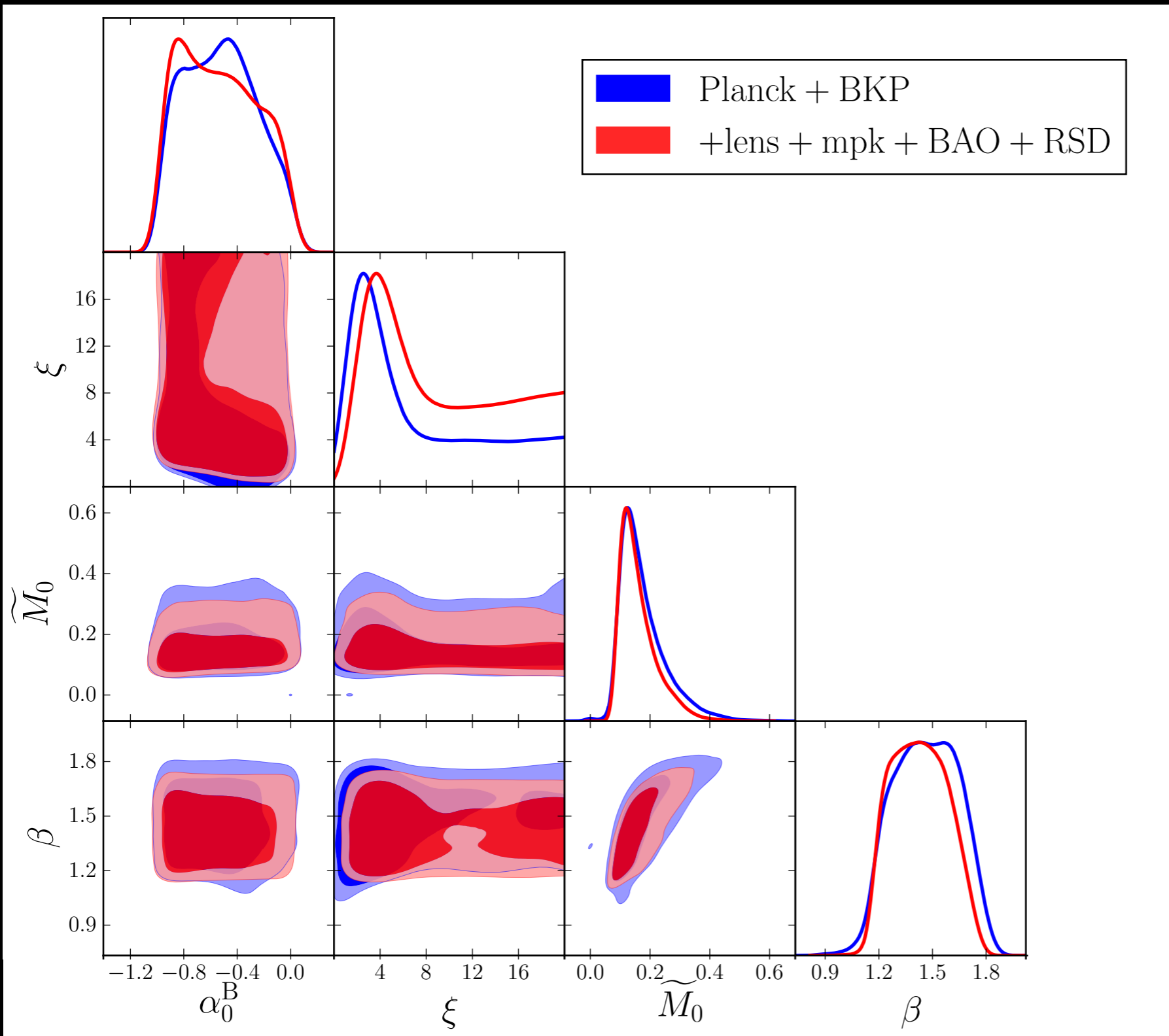
# Conclusions

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- ◆ MG is alive and well, BUT motivations no longer focus solely on cosmic acceleration.
- ◆ GW results have pruned the model space and sparked new growth areas.
- ◆ Parameterised tests are emerging as the smart way to test for new, unknown physics.

# Current Constraints



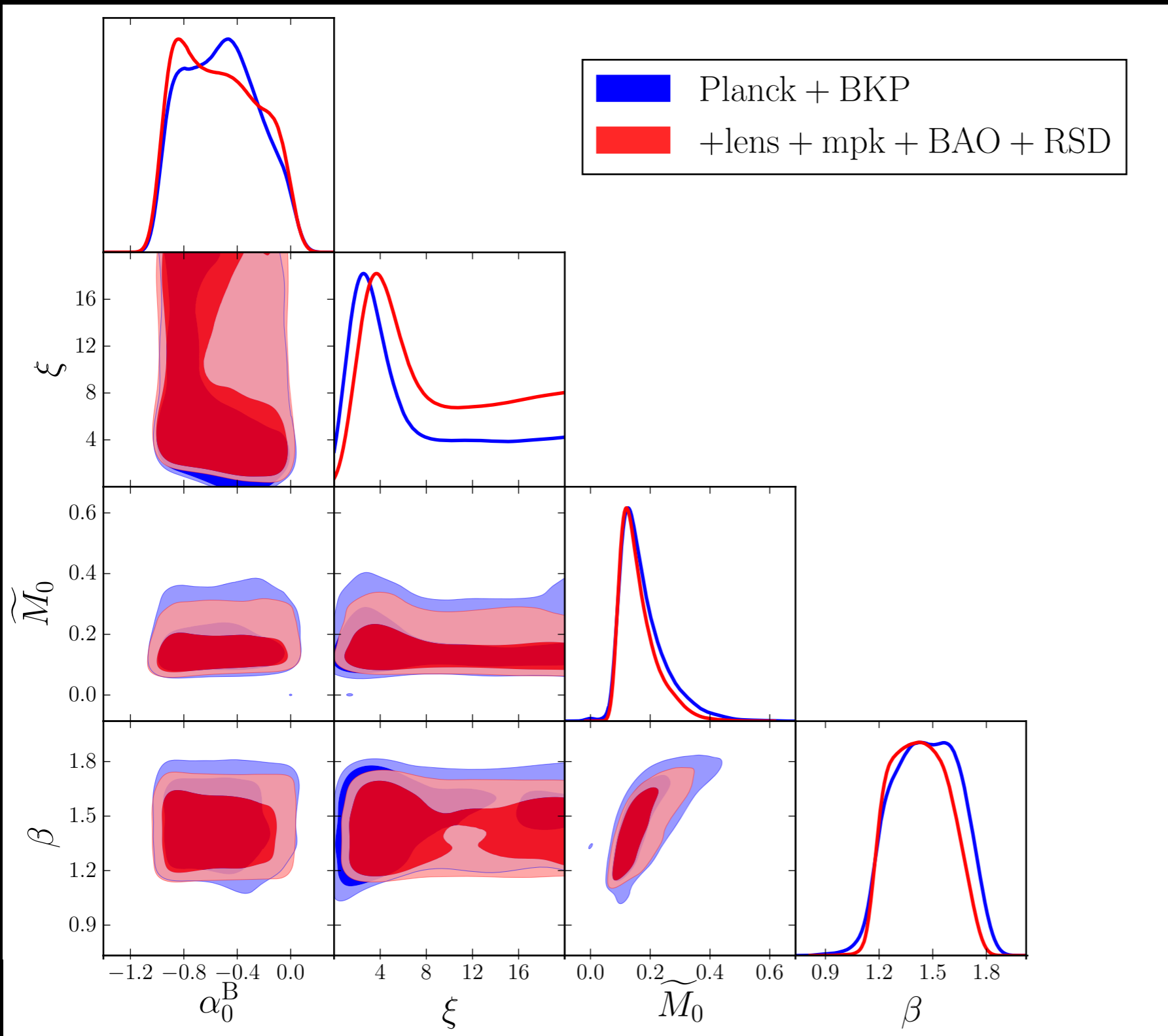
$$\alpha_B(z) = \alpha_0^B a^\xi$$

$$\frac{\delta M(z)}{M_{Pl}} = \tilde{M}_0 a^\beta$$

SDSS (galaxy survey)  
+ Planck CMB +  
BOSS BAOs & RSDs  
+ lensing data.

Kreisch & Komatsu,  
1712.02710

# THE CURRENT STATE OF PLAY



$$\frac{\delta M(z)}{M_{Pl}} = \tilde{M}_0 a^\beta$$

$$\alpha_B(z) = \alpha_0^B a^\xi$$

Caution: stability conditions lead to non-trivial contours.

Kreisch & Komatsu,  
1712.02710