Asymptotic Scale Invariance, Vacuum Stability and Higgs Inflation

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arXiv: 1807.***** with Mikhail Shaposhnikov

Introduction

LHC experiment

- Higgs boson $m_h \simeq 125\,\mathrm{GeV}$
- The SM is consistent so far.

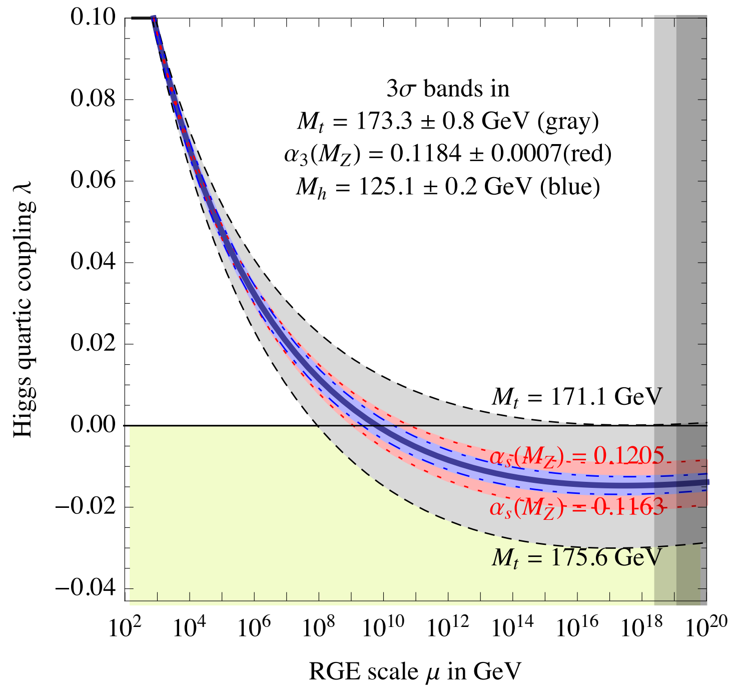
SM is valid up to very high energy scale?

➡ λ crosses zero, or touches zero.

EW vacuum is meta-stable.

Critical

D.Buttazzo, G.Degrassi, P.P.Giardino, G.F.Giudice, F.Sala, A.Salvio, A.Strumia (2014)



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What does this imply?

DM, GW from PT, Strong CP⋯

Asymptotic scale invariance ➡ Higgs inflation

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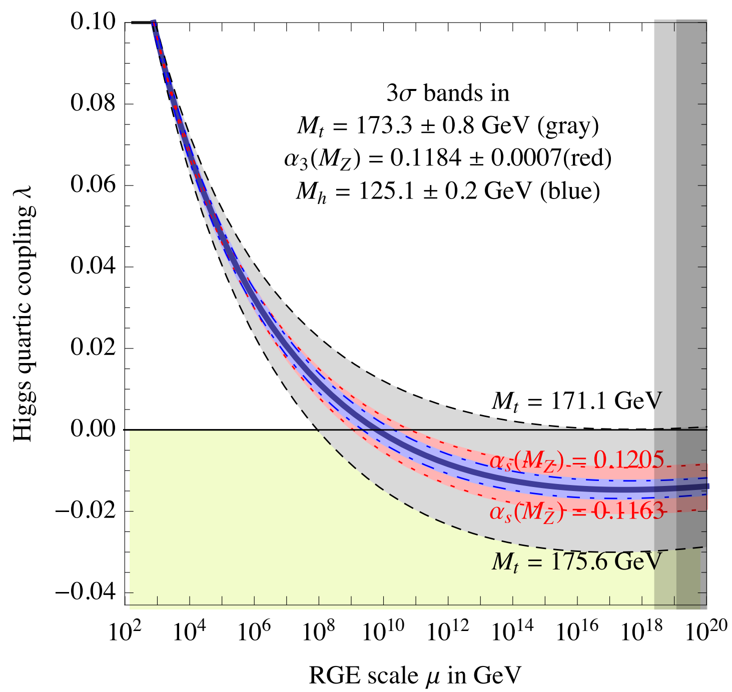

Introduction

Scale Anomaly and Effective Potential

Asymptotic Scale Invariance and Stability

Asymptotic Scale Invariance and Higgs Inflation

Future Directions (DM, GW…??)

Summary

Scale Anomaly and Effective Potential

Invariance under $\begin{cases} x^\mu \to \sigma^{-1} x^\mu \\ \Phi(x) \to \sigma^{d_\Phi} \Phi(x) \end{cases}$

d_Φ : Mass dimension of dynamical fields Φ

Explicit mass scale breaks SI :

$$V = -\frac{\mu_{\rm EW}^2}{2} h^2 + \frac{\lambda}{4} h^4$$

In the SM of particle physics sector, SI is broken by the negative Higgs mass term.

$\boxed{\text{Scale invariant for } h \gg \mu_{\rm EW} \text{ . (approximately)}}$

If you want, $\mu_{\rm EW} \propto \phi$: dynamical but is NOT crucial here.

Scale Anomaly and Effective Potential

SI is anomalous with regularization/renormalization NOT respecting the symmetry.

Dimensional regularization $n = 4 - 2\varepsilon$

$$\frac{\lambda h^4}{4} \quad \Longrightarrow \quad \mu^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{\lambda h^4}{4}$$

An explicit mass scale is introduced for the divergence and defines coupling "constants".

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One-loop (CW) correction to Higgs potential

$$\Delta V \sim (-1)^F m^4 \ln \frac{m^2}{\mu^2}$$

Breaks the scale invariance

Scale Anomaly and Effective Potential

Effective potential

$$V_{\rm eff} = \frac{h^4}{4}\left[\lambda + \frac{B}{2}\ln\frac{h^2}{\mu^2} + \cdots\right]$$

$$\equiv \frac{\lambda_{\rm eff}(h)}{4}\,h^4$$

Scale Anomaly and Effective Potential

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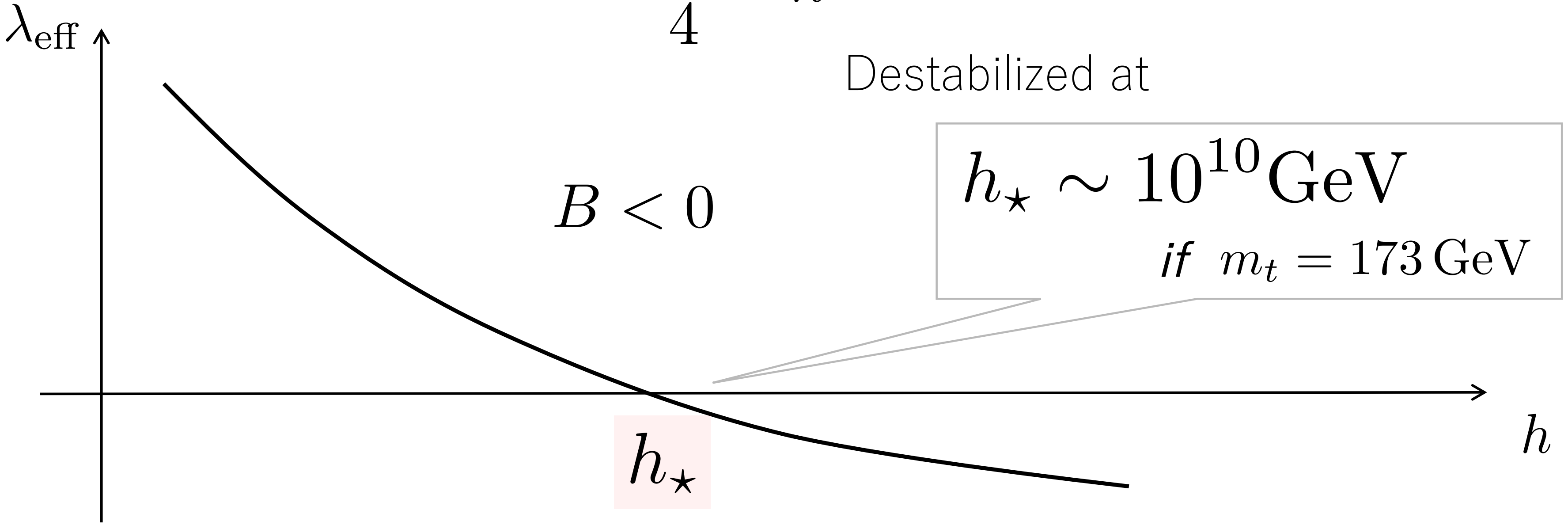

Scale Anomaly and Effective Potential

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$$V_{\text{eff}} = \frac{h^4}{4}\left[\lambda + \frac{B}{2}\ln\frac{h^2}{\mu^2} + \frac{B'}{8}\left(\ln\frac{h^2}{\mu^2}\right)^2 + \cdots\right]$$
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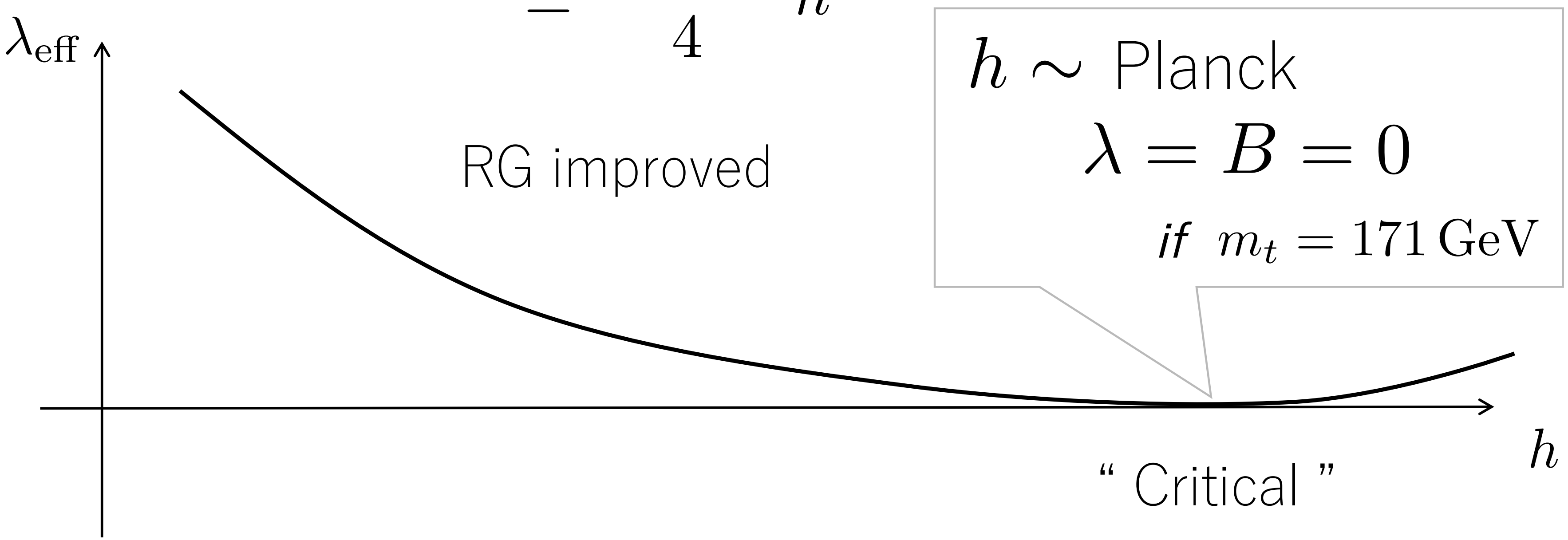

Asymptotic Scale Invariance and Stability

$$\frac{\lambda h^4}{4} \Longrightarrow \omega^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{\lambda h^4}{4}$$

$$\omega^2 = \mu^2 + \alpha h^2$$

Field-dependent

Extended to n-dim. differently from the above.

$\Rightarrow$ A different theory

Asymptotic Scale Invariance and Stability

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Extended to n-dim.
differently from the above.

➡ A different theory

$$\omega^2 = \mu^2 + \alpha h^2 = \mu^2 \left(1 + \frac{h^2}{\mu_\star^2}\right)$$

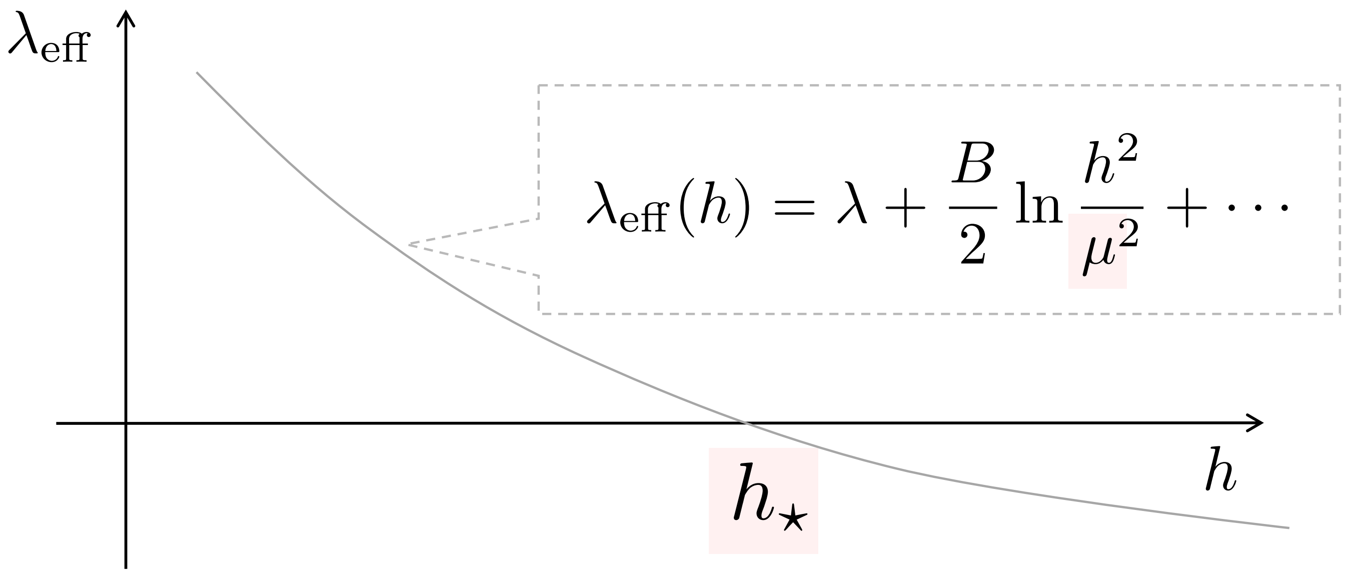
Explicit mass scale is negligible for

$$h \gg \mu_\star$$

Asymptotic Scale Invariance

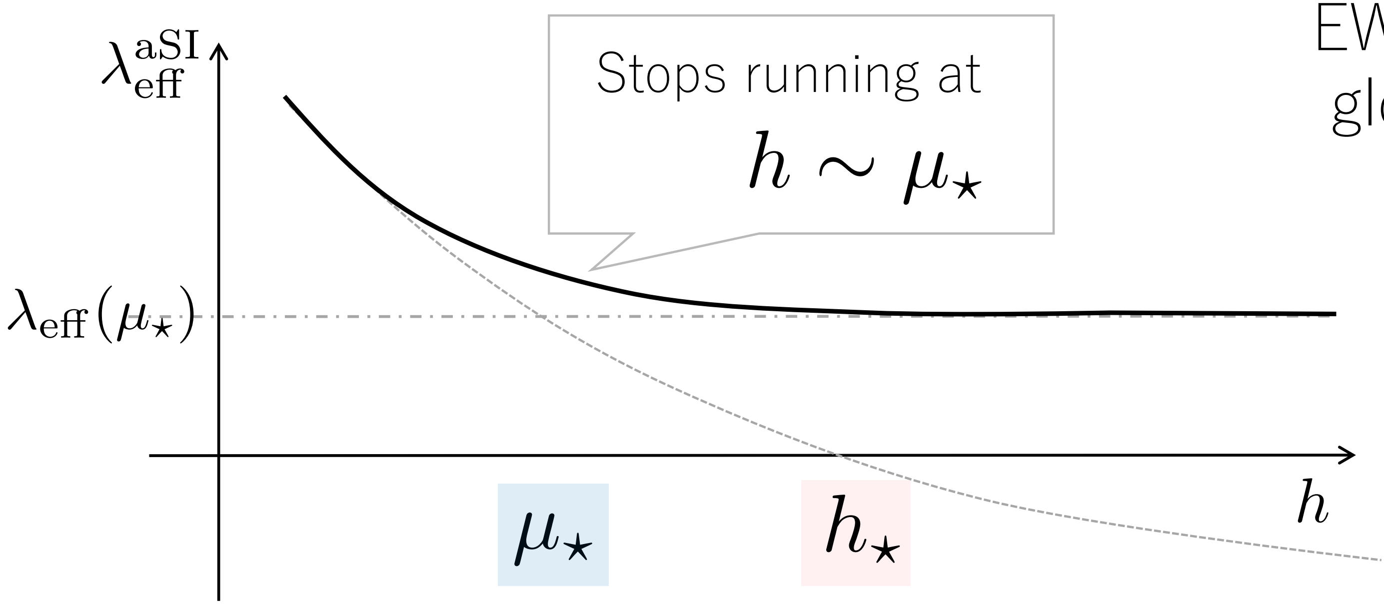
Asymptotic Scale Invariance and Stability

$$\lambda_{\text{eff}}^{\text{aSI}}(h) = \lambda + \frac{B}{2}\ln\frac{h^2}{\omega^2} + \cdots$$



Asymptotic Scale Invariance and Stability

$$\lambda_{\rm eff}^{\rm aSI}(h) = \lambda + \frac{B}{2}\ln\frac{h^2}{\mu^2(1+h^2/\mu_\star^2)} + \cdots \Rightarrow \lambda_{\rm eff}(\mu_\star) > 0$$

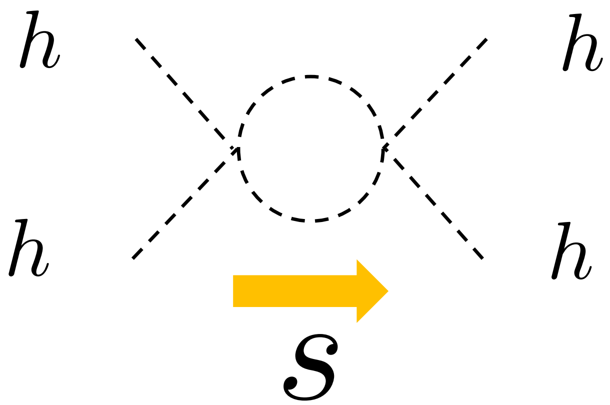


EW vacuum is global minimum

If $\mu_\star < h_\star$

Asymptotic Scale Invariance and Stability

Couplings run as energy scale of scattering increases.



$$\lambda_{\mathrm{eff}} \sim \lambda + \frac{\lambda^2}{16\pi^2} \ln \frac{s}{\omega^2}$$

Fixed

$$\omega^2 = \mu^2 \left(1 + \frac{h^2}{\mu_\star^2}\right)$$

Asymptotic Scale Invariance and Stability

$$\omega^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{\lambda h^4}{4}$$: Non-renormalizable

$$\omega^2 \propto \mu_\star^2 + h^2$$

Non-polynomial operators are needed for renormalization

$$\frac{h^{4+2k}}{(\mu_\star^2 + h^2)^k} \qquad (k \geq 1)$$

$\sim h^4$ for $h \gg \mu_\star$

Asymptotically SI

Asymptotic Scale Invariance and Stability

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Asymptotically SI

Up to which energy scale is this effective theory valid?

Asymptotic Scale Invariance and Stability

$$\frac{h^{4+2k}}{(\mu_\star^2 + h^2)^k}$$ Non-polynomial operators required

J.M.Cornwall, D.N.Levin, G.Tiktopoulos (1974)

Unitarity bound

N-particle amplitude

$$\mathcal{M}_N \sim E^{4-N}$$

at most

Tree unitarity violation

at $\Lambda \sim \sqrt{\mu_\star^2 + h^2}$

Strong coupling

or

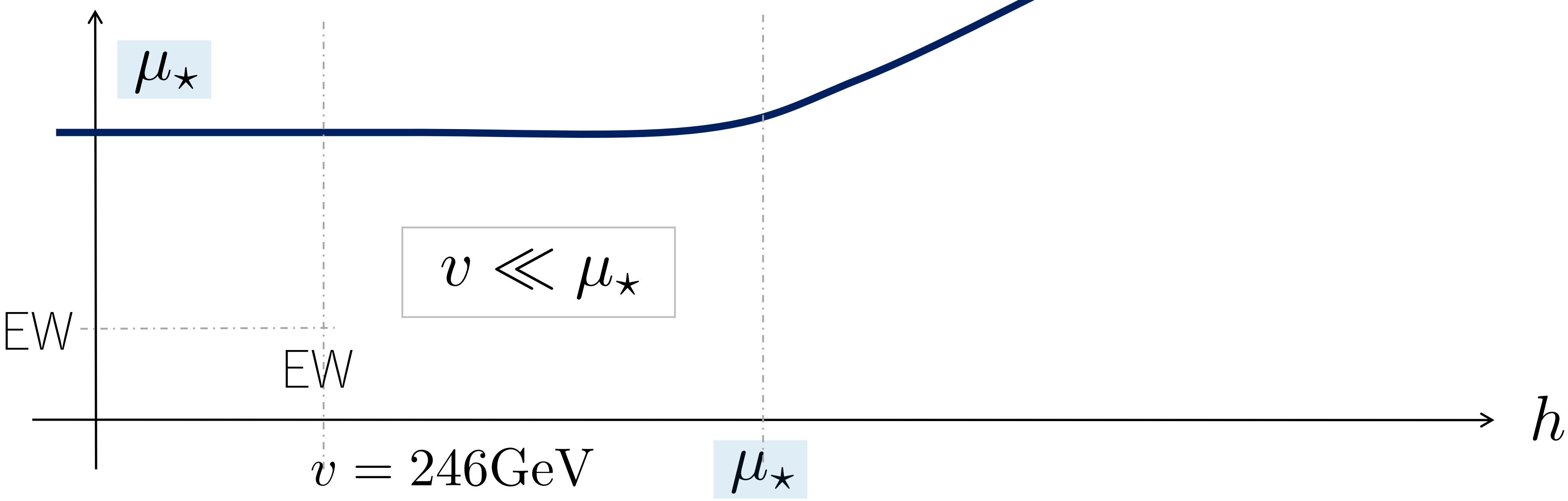
New physics

Asymptotic Scale Invariance and Stability

$$\frac{h^{4+2k}}{(\mu_\star^2 + h^2)^k}$$

Non-polynomial operators required

$\Lambda \sim \sqrt{\mu_\star^2 + h^2}$: Field-dependent



Asymptotic Scale Invariance and Stability

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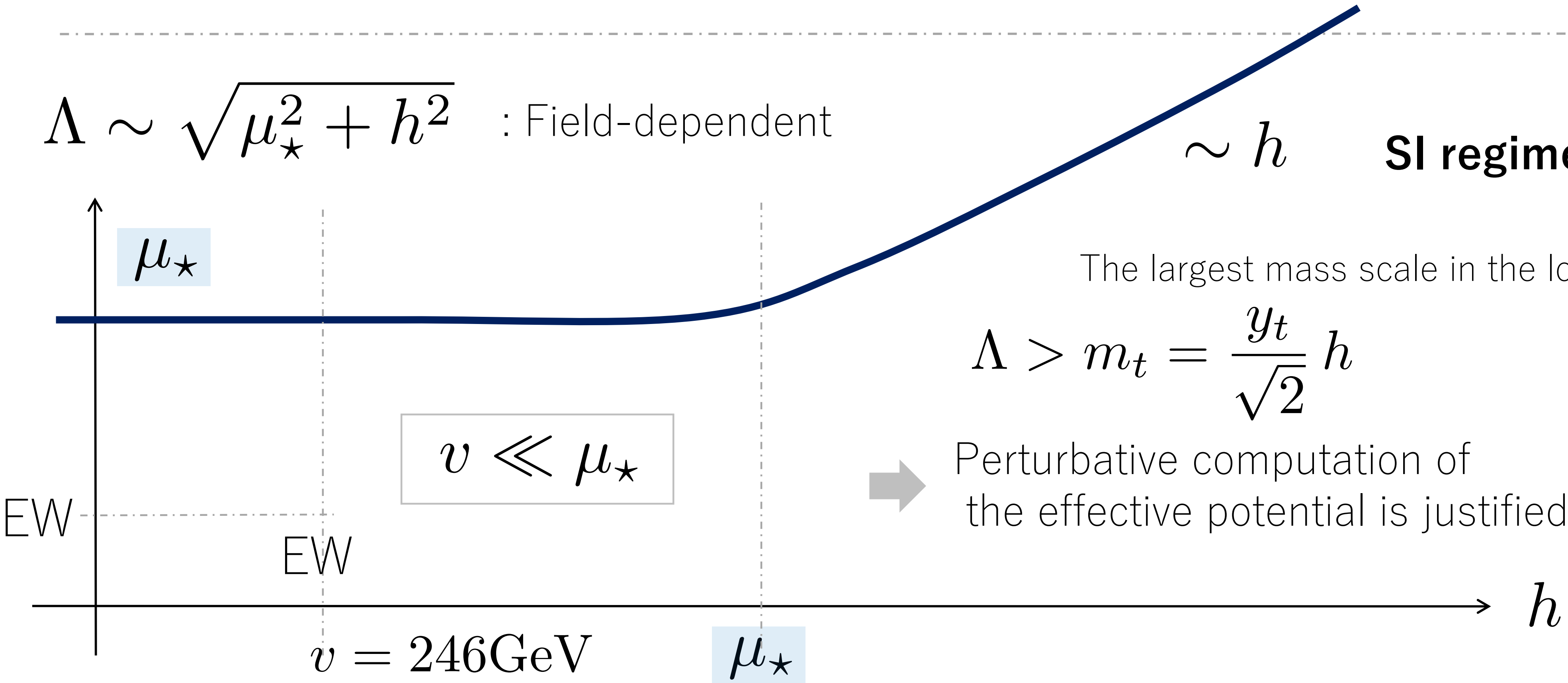


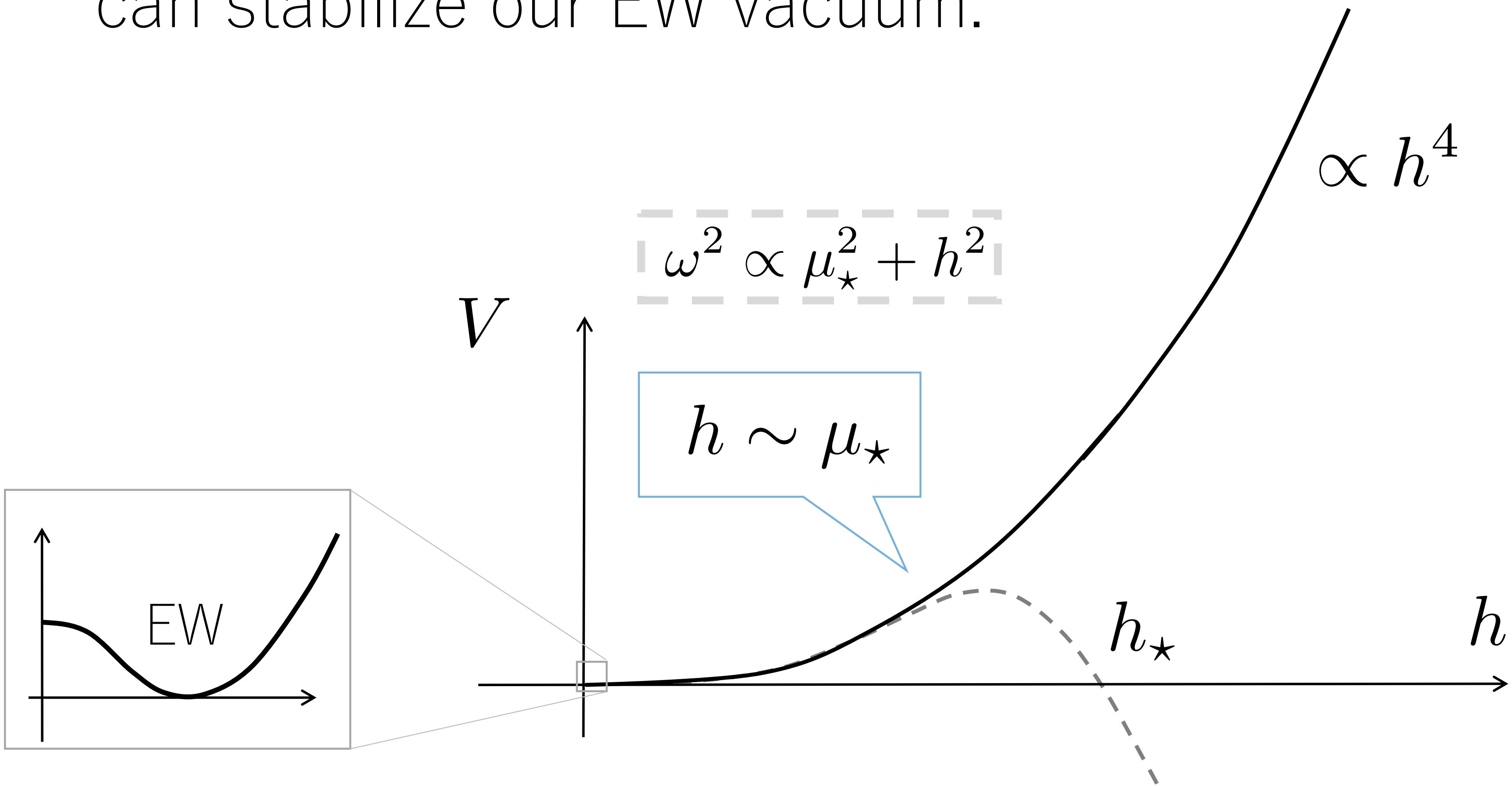
The largest mass scale in the loops

$$\Lambda > m_t = \frac{y_t}{\sqrt{2}} h$$

Perturbative computation of
the effective potential is justified.

Asymptotic Scale Invariance

can stabilize our EW vacuum.



Asymptotic Scale Invariance and Higgs Inflation

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_{\mathrm{P,eff}}^2}{2} R - \frac{g^{\mu\nu}}{2}\partial_\mu h \partial_\nu h - V(h) + \cdots$$

Effective
Planck mass

$$M_{\mathrm{P,eff}}^2 = M_{\mathrm{P}}^2 + \xi h^2$$

F.Bezrukov, M.Shaposhnikov (2007)

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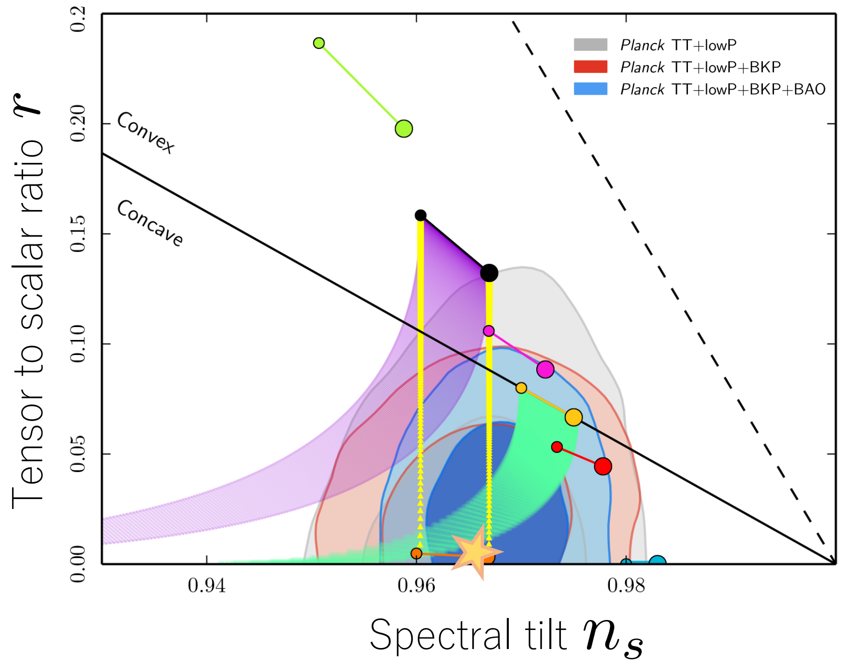
Effective Planck mass

$$M_{\mathrm{P,eff}}^2 = M_{\mathrm{P}}^2 + \xi h^2$$

F.Bezrukov, M.Shaposhnikov (2007)

$\xi \sim 10^4 \sqrt{\lambda}$ Large non-minimal coupling

$$\Rightarrow \quad A_s \simeq 2.2 \times 10^{-9}$$



Asymptotic Scale Invariance and Higgs Inflation

$$\frac{\lambda h^4}{4} \Longrightarrow \omega^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{\lambda h^4}{4}$$

Renormalization prescription	$\omega^2 \propto$	
I	$M_{\rm P}^2 + \xi h^2 = M_{\rm P,eff}^2$	F.Bezrukov, M.Shaposhnikov (2007)
II	$M_{\rm P}^2$ (constant)	A.O.Barvinsky, A.Y.Kamenshchik, A.A.Starobinsky (2008)

Asymptotic Scale Invariance and Higgs Inflation

$$\frac{\lambda h^4}{4} \Rightarrow \omega^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{\lambda h^4}{4}$$

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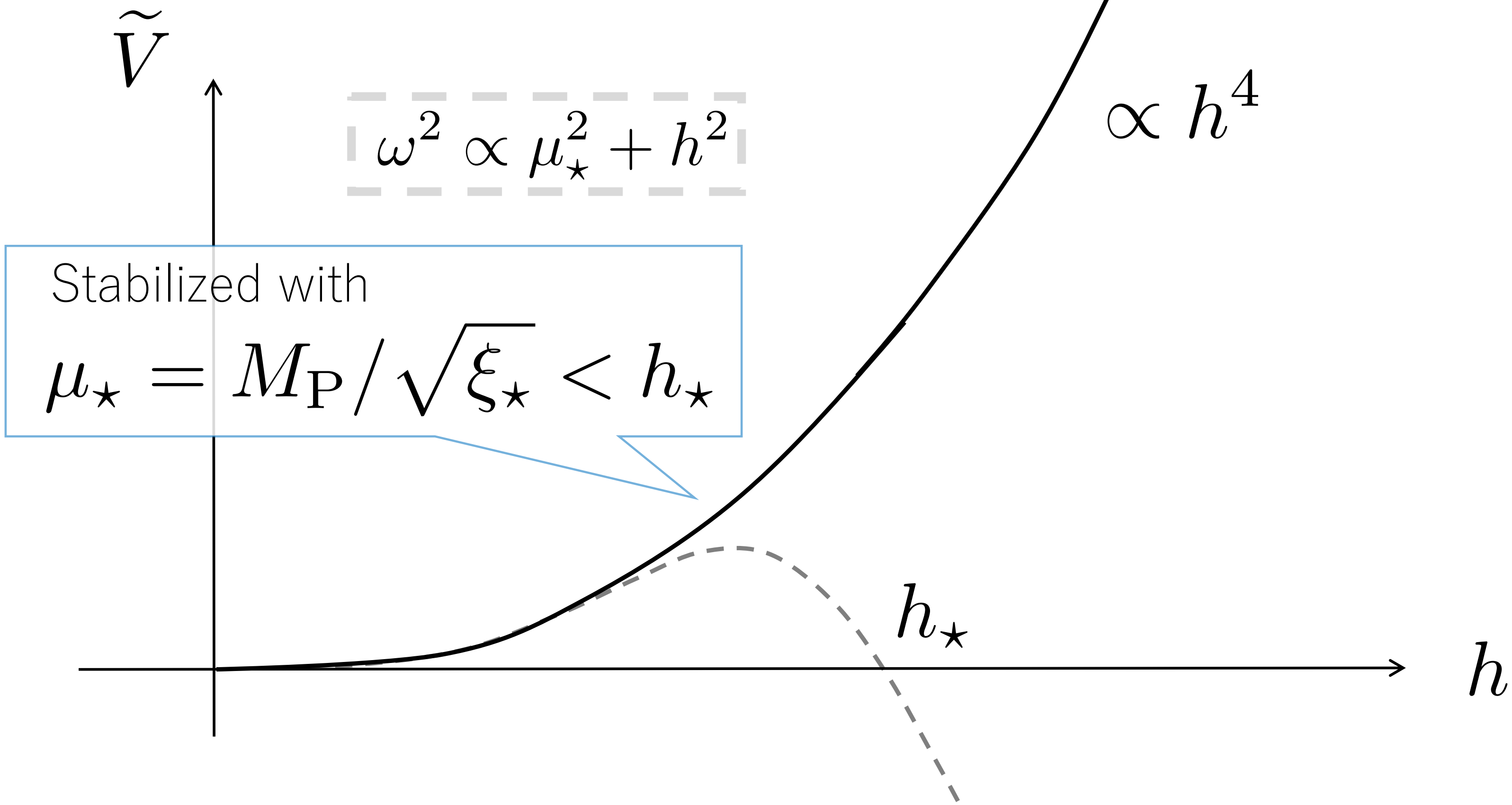
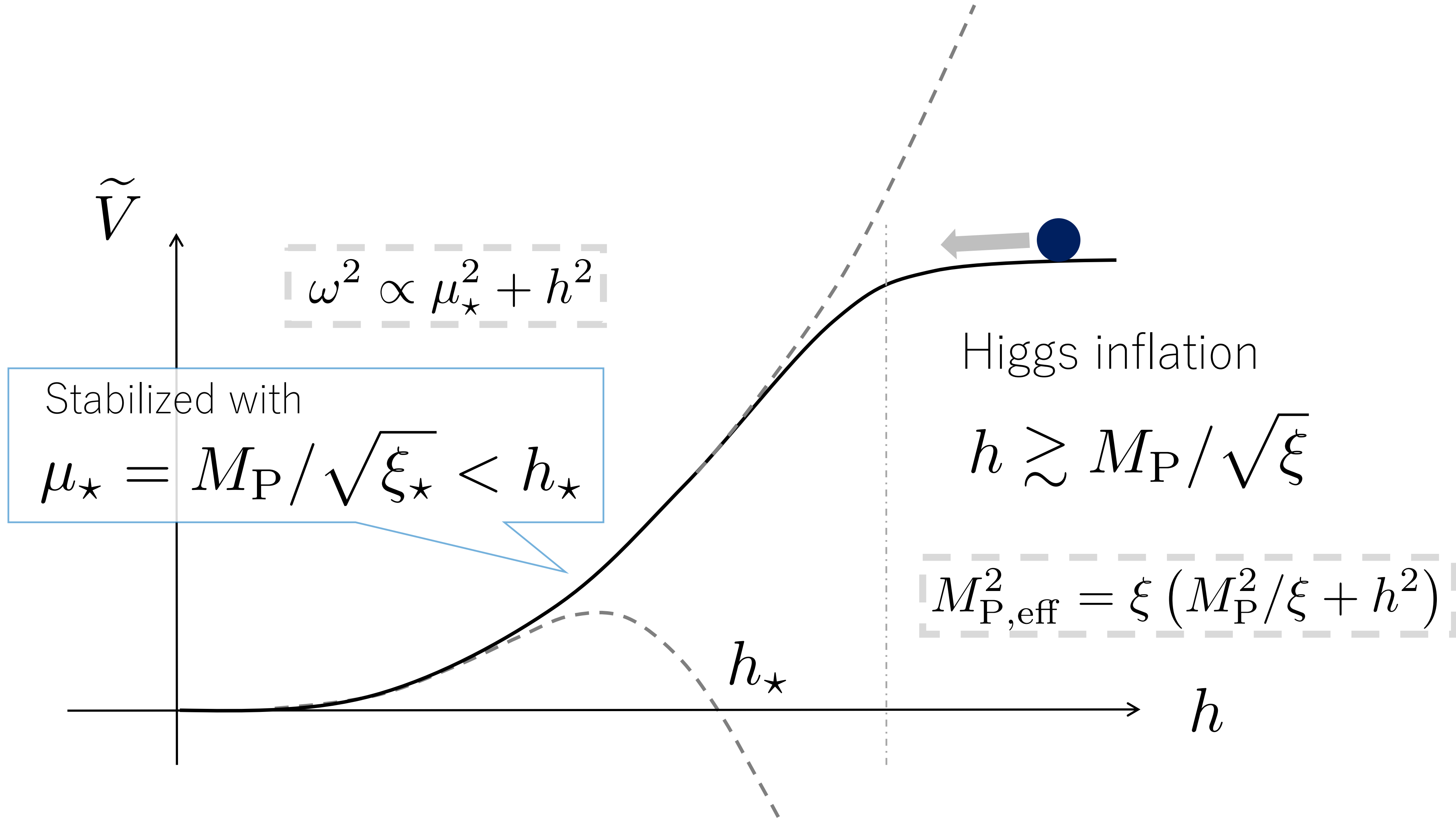
$$\Rightarrow \quad \mu_\star^2 + h^2 \propto M_P^2 + \xi_\star h^2$$

$$\xi_\star = M_{\mathrm{P}}^2/\mu_\star^2 \neq \xi$$

Asymptotic Scale Invariance and Higgs Inflation



Asymptotic Scale Invariance and Higgs Inflation



Asymptotic Scale Invariance and Higgs Inflation

- ✓ Perturbative computation of effective potential is justified.

$$\Lambda > m_t$$ (the largest mass scale in the loops)

- ✓ Generation of inflaton (Higgs) fluctuation is also computable.

$$\Lambda > H > k_{\mathbf{fluc}}$$ during Higgs inflation

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$$\Lambda > H > k_{\mathrm{fluc}}$$ during Higgs inflation

?? Reheating temperature becomes very high.

$$T_{\mathrm{rh}} > \Lambda|_{h=0} = \mu_\star$$ (zero mode vanishes after thermalization)

➡ Thermal history after inflation
($\Longrightarrow$ Inflationary observable)
cannot be discussed.

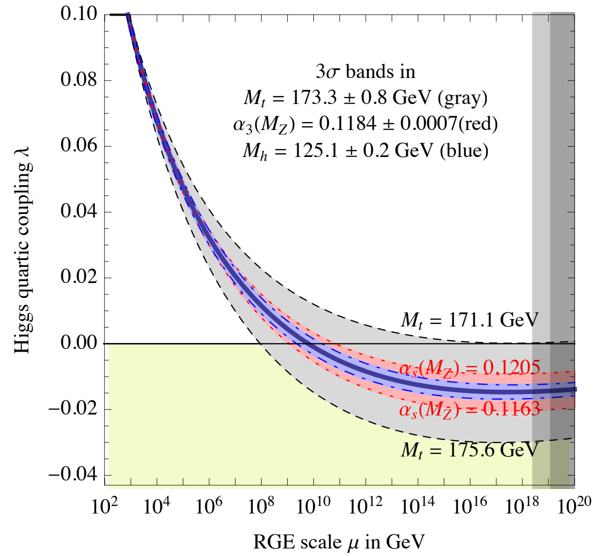
1, Theory above Λ ?

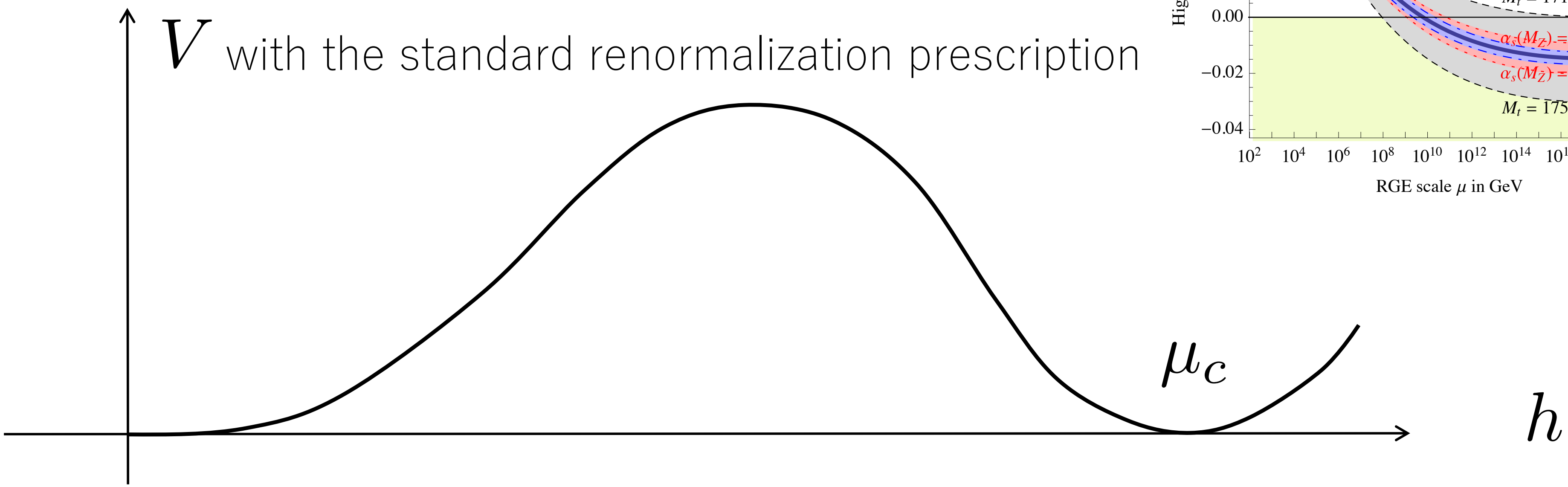
2, When $T_{\mathrm{rh}} < \Lambda$?

Asymptotic Scale Invariance and Higgs Inflation

Critical case

$$V_{\text{eff}} = \frac{h^4}{4}\left[\cancel{\lambda_c} + \cancel{\frac{B_c}{2}\ln\frac{h^2}{\mu_c^2}} + \frac{B_c'}{8}\left(\ln\frac{h^2}{\mu_c^2}\right)^2 + \cdots\right]$$



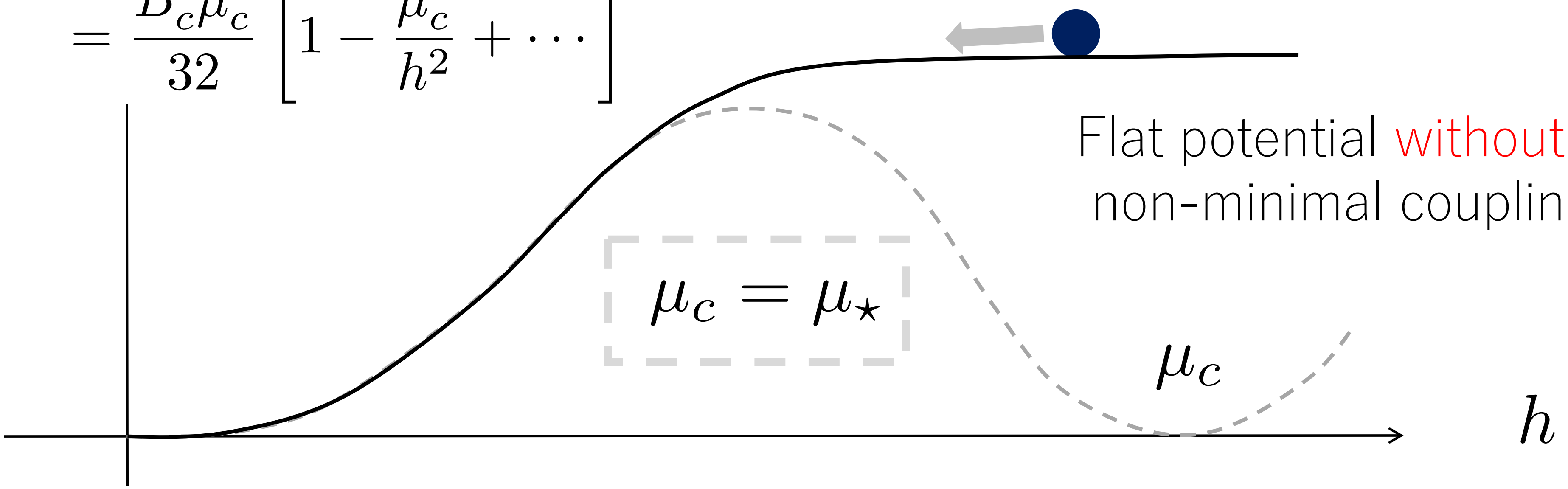


Asymptotic Scale Invariance and Higgs Inflation

Critical case

$$V_{\text{eff}} = \frac{h^4}{4}\left[\cancel{\lambda_c} + \cancel{\frac{B_c}{2}\ln\frac{h^2}{\mu_c^2+h^2}} + \frac{B_c'}{8}\left(\ln\frac{h^2}{\mu_c^2+h^2}\right)^2 + \cdots\right]$$

$$= \frac{B_c'\mu_c^4}{32}\left[1 - \frac{\mu_c^2}{h^2} + \cdots\right]$$



Flat potential without non-minimal coupling ξ

Asymptotic Scale Invariance and Higgs Inflation

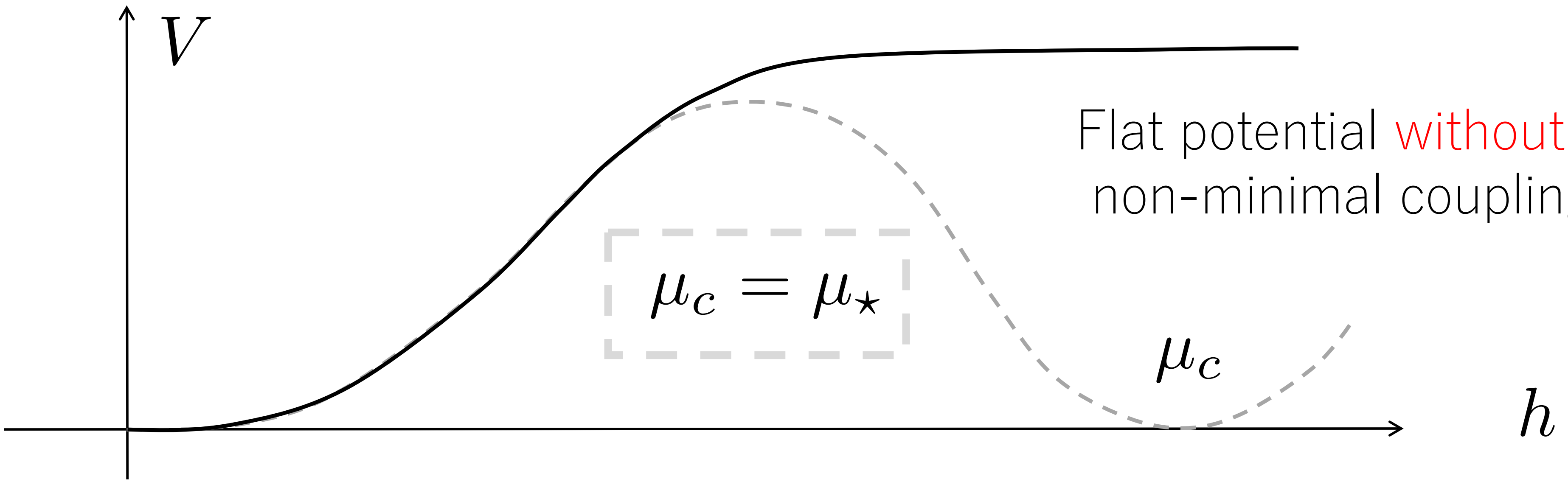
Critical case

$$\mu_c \sim 10^{17}\mathrm{GeV} < M_P$$

$$\Rightarrow \quad A_s \simeq 2.2 \times 10^{-9}$$

$$T_{\rm rh} \lesssim V^{1/4} < \mu_c = \Lambda|_{h=0}$$

Perturbative computation is valid.



Flat potential without non-minimal coupling ξ

Future directions (DM, GW⋯??)

Critical case ($T_{\mathbf{rh}} < \Lambda$, Asymptotic SI itself needs nothing new for cosmology.)

⇓

Any reasonable way to extend the SM??

Future directions (DM, GW···??)

Critical case ($T_{\mathbf{rh}} < \Lambda$, Asymptotic SI itself needs nothing new for cosmology.)

$n_s \approx 0.975$ ➡ 2σ level ➡ Non-standard thermal history is preferred.

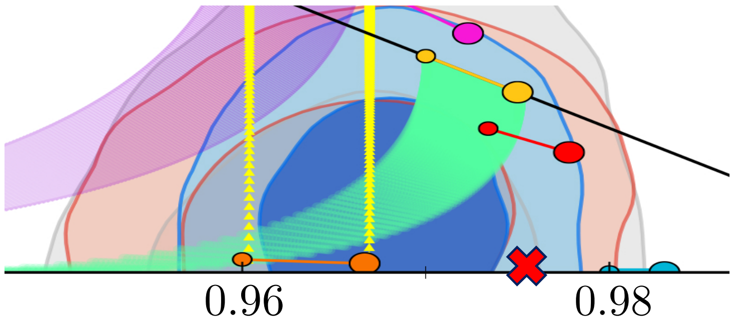
$$n_s \approx 1 - \frac{3}{2N} \quad \text{with} \quad N = 60$$

$$\Delta N \sim 15$$

Super-cooling stage before a phase transition?

Typically, in scale invariant models

Gravitational waves, DM production after PT



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Theory above Λ ?

Needs to guarantee
the absence of $\Lambda^2 h^2$

⇒

DM production with $T > \Lambda$

Phase transition at $T \sim \Lambda$

Summary

Asymptotic Scale Invariance can be responsible for our EW vacuum stability.

Perturbative computation of the effective potential is valid because tree-unitary violation scale Λ is larger than any others.

Higgs inflation is also possible.

However, $T_{\mathrm{rh}} > \Lambda$ requires a theory above Λ .

$T_{\mathrm{rh}} < \Lambda$ for the critical case. The theory below Λ is enough.

Thank you