

THE 14TH INTERNATIONAL WORKSHOP ON THE DARK SIDE OF THE UNIVERSE

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Gravitational-wave signals in extended theories of gravity

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Cosmology and Gravity Group - UNIVERSITY OF CAPE TOWN and SKA Cosmology

[1806.05195](#), [1704.08311](#), [1602.03880](#)





Black holes, gravitational waves and fundamental physics: a roadmap

Leor Barack¹, Vitor Cardoso^{2,3}, Samaya Nissanke^{4,5,6}, Thomas P. Sotiriou^{7,8} (editors)

White Paper , [1806.05195](#)

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COST action "Gravitational Waves, Black Holes, and Fundamental Physics"



Outline

a. Motivation?

b. Those interesting fellows: neutron stars

- TOV equation
- A better understanding of Equations of State

c. Full analysis of paradigmatic models in $f(R)$ gravities

- Methodology and mass definition
- Testing R^2 vs. perturbative analyses
- Towards viable cosmological models: the Hu-Sawicki case

d. Gravitational Waves detection?

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Extended theories of gravity: a motivation

- The search for a fully consistent **Quantum Gravity theory** is a very active field of research. GR is consistent *if treated* in the frame of **quantum effective field theories** but it breaks down at Planck scale

J.F. Donoghue and T. Torma,
gr-qc/9405057

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J.F. Donoghue and T. Torma,
gr-qc/9405057

✓ Extended theories of gravity

- must Emulate certain – gravitational – aspects of General Relativity
- must Explain the cosmological evolution in different eras
- are motivated by the cosmological constant (Λ) problem, dark energy, dark matter, singularities...

Extended theories of gravity: a motivation

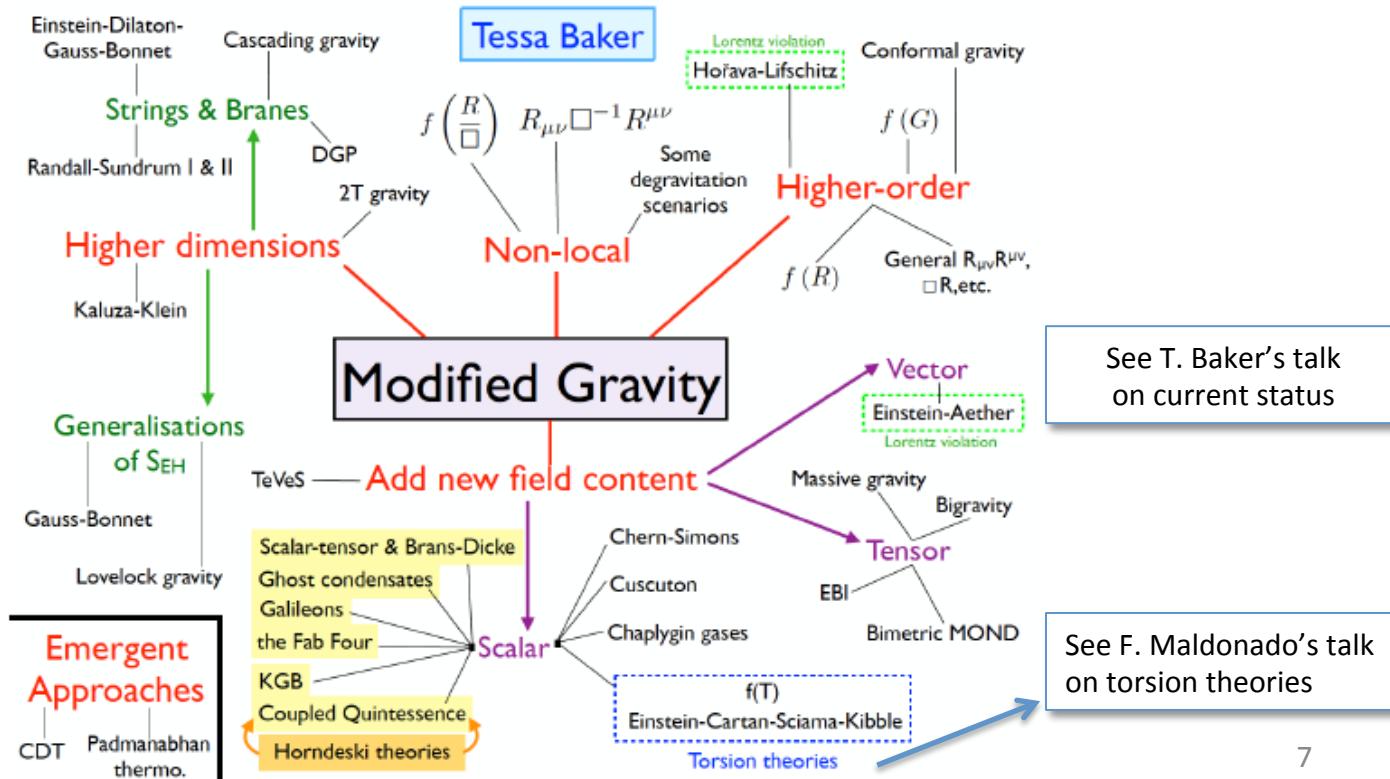
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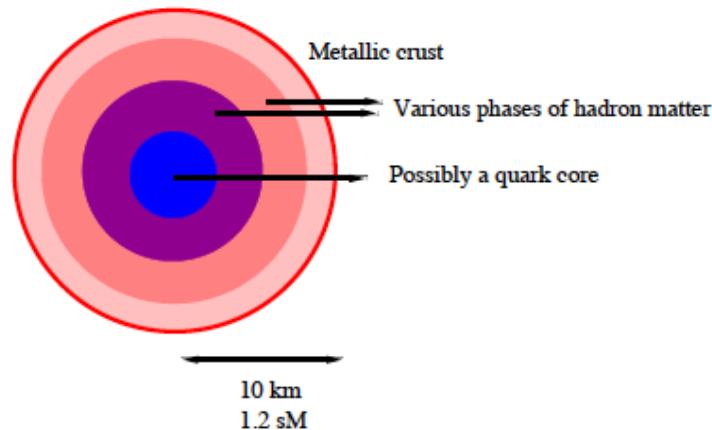
- must Emulate certain – gravitational – aspects of General Relativity
- must Explain the cosmological evolution in different eras
- are motivated by the cosmological constant (Λ) problem, dark energy, dark matter, singularities...

✓ Proposals



✓ Motivation

Often emphasized: Neutron stars test structure of matter

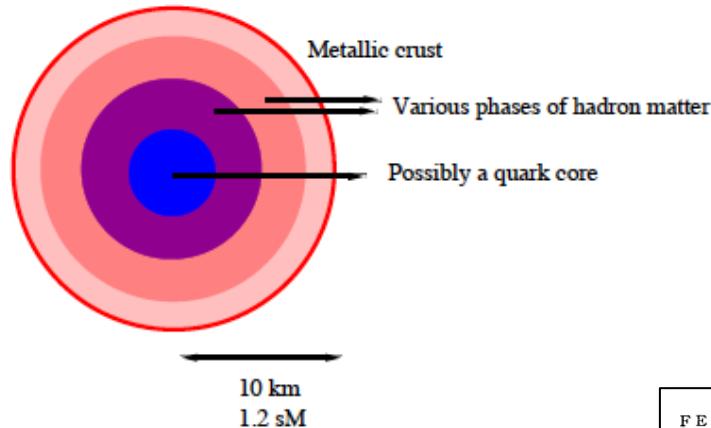


- ▶ $g = O(10^{12})\text{m/s}^2$ vs. $O(300)\text{m/s}^2$ outside
(where GR tests with pulsars are performed)
 $O(10^6)\text{m/s}^2$ at white dwarves.

i.e., General Relativity is extrapolated 6 orders of magnitude to learn about Nuclear Physics only a factor 3-4 away :S

✓ Motivation

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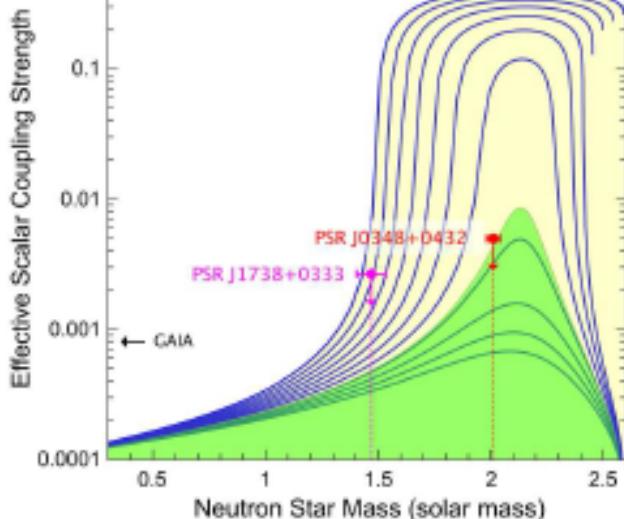
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FEBRUARY 15, 1939 PHYSICAL REVIEW VOLUME 55

On Massive Neutron Cores

J. R. OPPENHEIMER AND G. M. VOLKOFF
Department of Physics, University of California, Berkeley, California
(Received January 3, 1939)

It has been suggested that, when the pressure within stellar matter becomes high enough, a new phase consisting of neutrons will be formed. In this paper we study the gravitational equilibrium of masses of neutrons, using the equation of state for a cold Fermi gas, and general relativity. For masses under $\frac{1}{2}\odot$ only one equilibrium solution exists, which is approximately described by the nonrelativistic Fermi equation of state and Newtonian gravitational theory. For masses $\frac{1}{2}\odot < m < \frac{3}{4}\odot$ two solutions exist, one stable and quasi-Newtonian, one more condensed, and unstable. For masses greater than $\frac{3}{4}\odot$ there are no static equilibrium solutions. These results are qualitatively confirmed by comparison with suitably chosen special cases of the analytic solutions recently discovered by Tolman. A discussion of the probable effect of deviations from the Fermi equation of state suggests that actual stellar matter after the exhaustion of thermonuclear sources of energy will, if massive enough, contract indefinitely, although more and more slowly, never reaching true equilibrium.

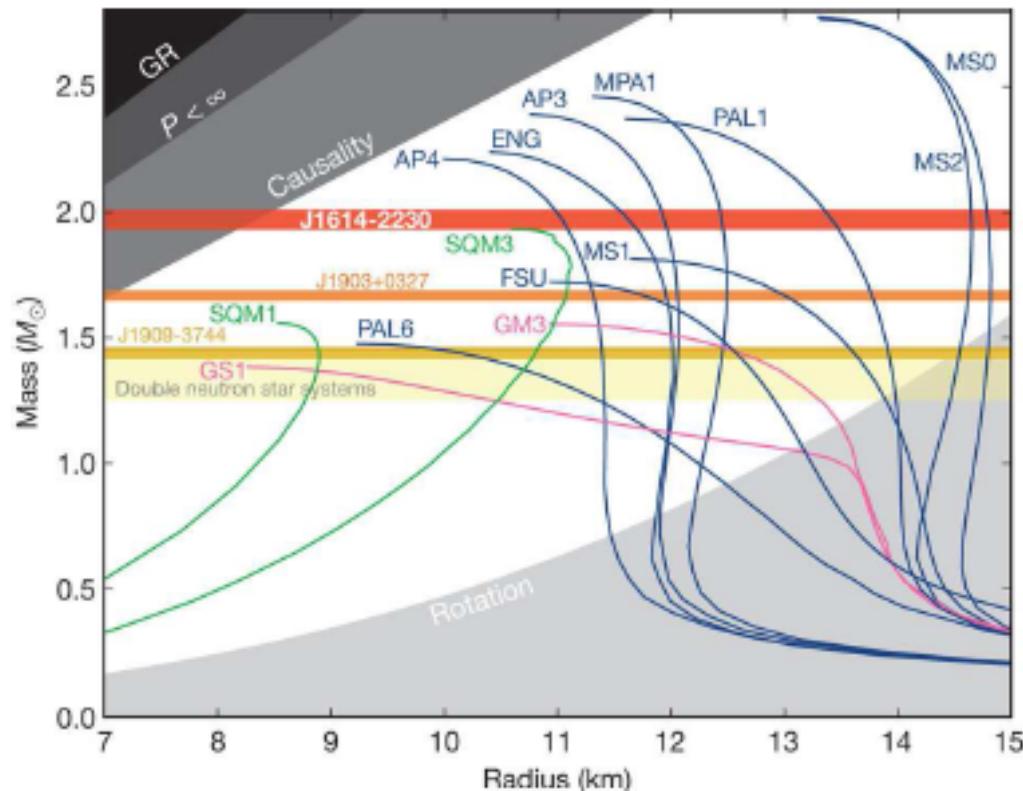
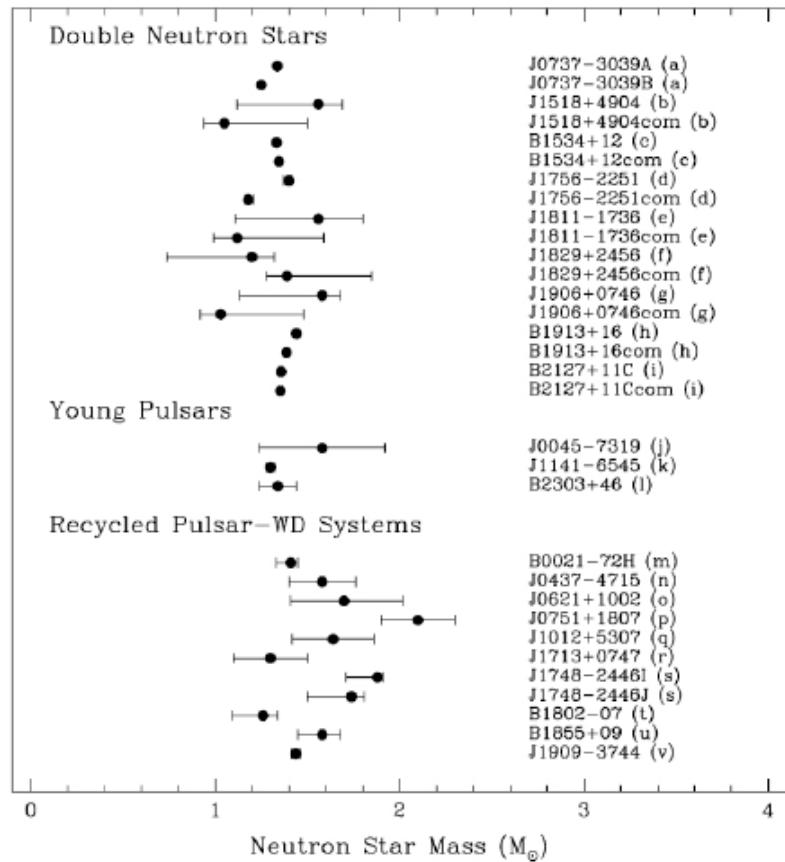


General Relativistic Hydrostatic equilibrium (static, spherical body)

$$\frac{dP}{dr} = -\frac{G_N}{r^2} \frac{(\varepsilon(r) + P(r))(M(r) + 4\pi r^3 P(r))}{1 - \frac{2G_N M(r)}{r}} .$$

Within GR paradigm, NS can be used to exclude models

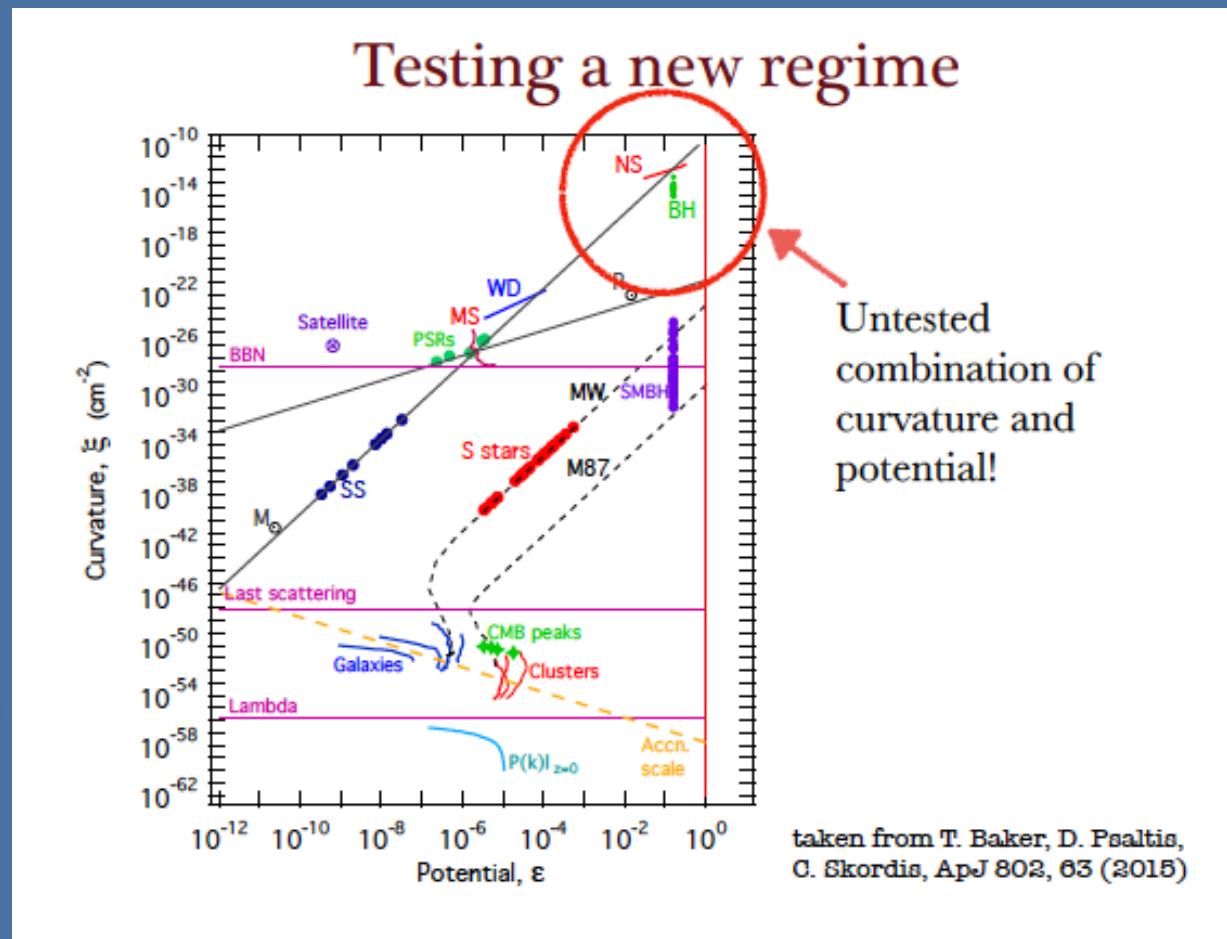
Measured pulsar stars (before 2010)
Phys. Rev. C77 065803 (2008)



- Stars made of free-neutron gas can only host $0.6M_{\text{Sun}}$
- Traditional nuclear-force potential model $1.5\text{-}1.6M_{\text{Sun}}$
- Stiffer chiral interaction $2.2M_{\text{Sun}}$

Observational Evidence that GR Correctly Describes Strong Field Gravity

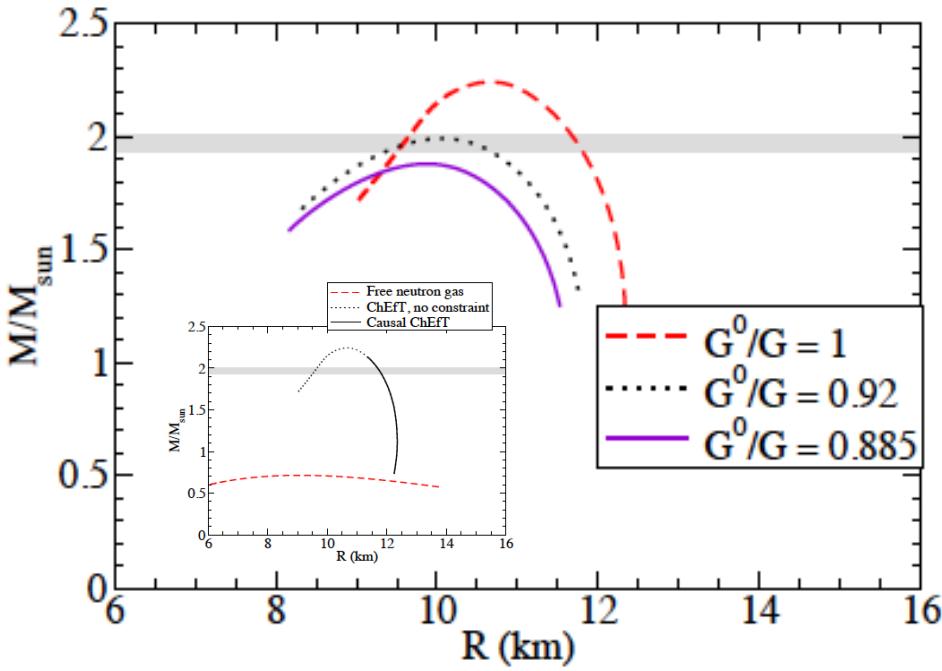
- GW150914 is the first direct confirmation
 - Prior observations
 - neutron stars
 - cosmic evolution *assuming* DE & DM are not gravitational anomalies
 - stellar and supermassive black hole candidates
- are certainly **consistent** with GR, but do not provide over-constrained measurements of strong-field predictions



✓ What do those QCD-inspired EoS say about gravity?

Extensions of General Relativity predict $G_N(g)$ arXiv:0410117

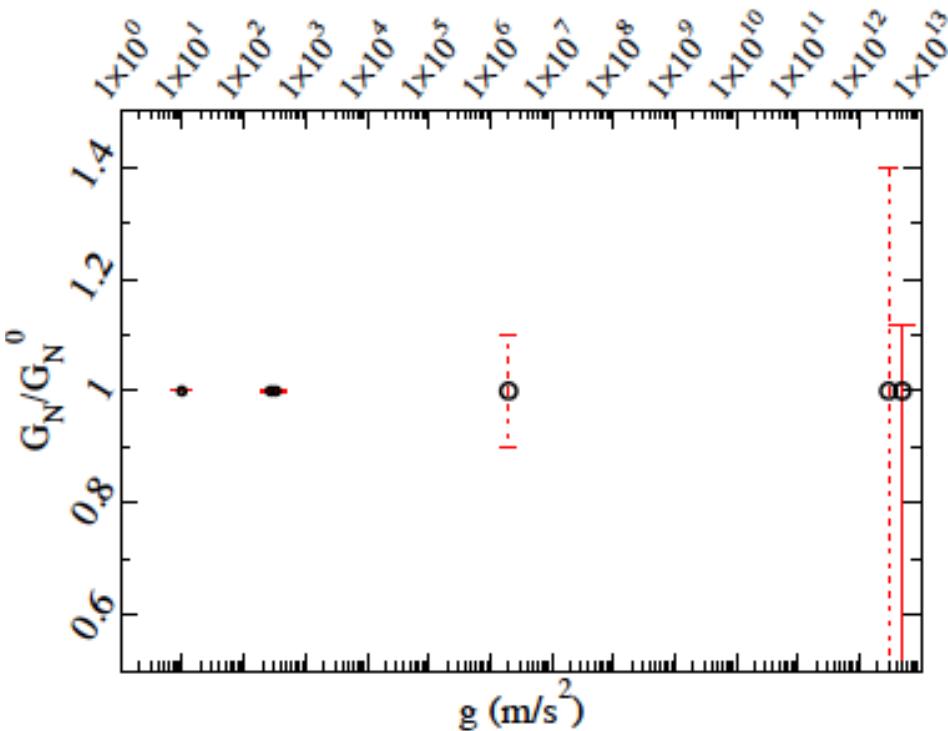
$$\frac{dP}{dr} = -\frac{G_N}{r^2} \frac{(\varepsilon(r) + P(r))(M(r) + 4\pi r^3 P(r))}{1 - \frac{2G_N M(r)}{r}}$$



$G_N \simeq \text{constant}$ $r \rightarrow 0$

$$G_N \propto \frac{1}{k^q} \propto r^q \quad r \rightarrow \infty$$

$$q \simeq 10^{-6}$$



A. Dobado, F. Llanes and V. Oller
Phys. Rev. C 85 (2012) 012801

Paolo Creminelli¹ and Filippo Vernizzi²

$$L_2 \equiv G_2(\phi, X), \quad L_3 \equiv G_3(\phi, X) \square \phi,$$

$$L_4 \equiv G_4(\phi, X) {}^{(4)}R - 2G_{4,X}(\phi, X)(\square \phi^2 - \phi^{\mu\nu}\phi_{\mu\nu}) + F_4(\phi, X)\varepsilon^{\mu\nu\rho}_{\sigma}\varepsilon^{\mu'\nu'\rho'\sigma'}\phi_{\mu}\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}$$

$$\begin{aligned} L_5 \equiv & G_5(\phi, X) {}^{(4)}G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)(\square \phi^3 - 3\square \phi \phi_{\mu\nu}\phi^{\mu\nu} + 2\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}_{\sigma}) \\ & + F_5(\phi, X)\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\mu'\nu'\rho'\sigma'}\phi_{\mu}\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}, \end{aligned}$$

$$L_{c_T=1} = G_2 + G_3 \square \phi + B_4(R + K_{\mu}^{\nu}K_{\nu}^{\mu} - K^2)$$

Jose María Ezquiaga^{1, 2, *} and Miguel Zumalacárregui^{2, 3, †} $c_g = c$ $c_g \neq c$

Horndeski

General Relativity

quintessence/k-essence [42]

Brans-Dicke/ $f(R)$ [43, 44]

Kinetic Gravity Braiding [46]

beyond H.

Derivative Conformal (20) [18]

Disformal Tuning (22)

DHOST with $A_1 = 0$

Viable after GW170817

quartic/quintic Galileons [13, 14]

Fab Four [15, 16]

de Sitter Horndeski [45]

 $G_{\mu\nu}\phi^{\mu}\phi^{\nu}$ [47], Gauss-Bonnet

quartic/quintic GLPV [19]

DHOST [20, 48] with $A_1 \neq 0$

Non-viable after GW170817

(Some) Results in NR and GW in the context of Extended Theories of Gravity

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- ✓ Establishing well-posedness: Existence, uniqueness and continuous dependence on initial data
- ✓ Interpreting well/ill-posedness in the context of effective field theory (EFT). IVP. Hyperbolicity?
 - * Already in GR, hyperbolicity depends on the formalism [ADM weak vs. BSSD strong]
 - $f(R)$: [B. Mongwane, 1610.07224](#) ; Spherical symmetry scalar-tensor theories: [Alcubierre et al., 1207.6142](#)
- ✓ Numerical challenges associated with the above and with having extra fields
- ✓ Parameterisations vs. Non-parameterisation - Theory-agnostic or Specific Tests?

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Theory-agnostic Tests

- PN Tests of GR Double Binary Pulsar [Arun et al. (2006), Mishra et al. (2010)]
- Parameterised post-Einsteinian Formalism
 - PPE-modified Inspiral Waveform Phase [Yunes & Pretorius PRD80 122003 (2009)]
 - PPE Mapping to Specific theories [Yunes, Pretorius & Spergel PRD80 122003 (2010)]
- Generalized IMRPhenom Waveform
 - Inspiral-merger-ringdown Phenomenological D (IMRPhenomD) waveform in GR [Khan et al. (2015)]
 - Possible to model non-GR merger & ringdown (**unknown mapping to specific theories**)

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IV. Gravitational Waves. Prospects and Conclusions

Maximum mass for a given eq. of state

- ▶ Hydrostatic equilibrium:
pressure compensates weight of upper layers
- ▶ Causality limits the achievable pressure, $c^2 \geq c_s^2 = \frac{dP}{d\rho}$
- ▶ But nothing limits the amount of matter falling on the star
- ▶ Only solution: increase density
- ▶ When $R_* < R_{\text{Schwarzschild}} = 2M_*$, Gen. Rel. predicts collapse
- ▶ Finding heavier N-stars tests General Relativity

Differences between EOS

1. What particle content beyond neutrons (protons+leptons, hyperons, pions, kaons, quarks...)
2. What interactions are used (Argonne, Skyrme, chiral...)
3. What computational method (variational, Brückner-Hartree-Fock, rel. mean field, Monte Carlo...)

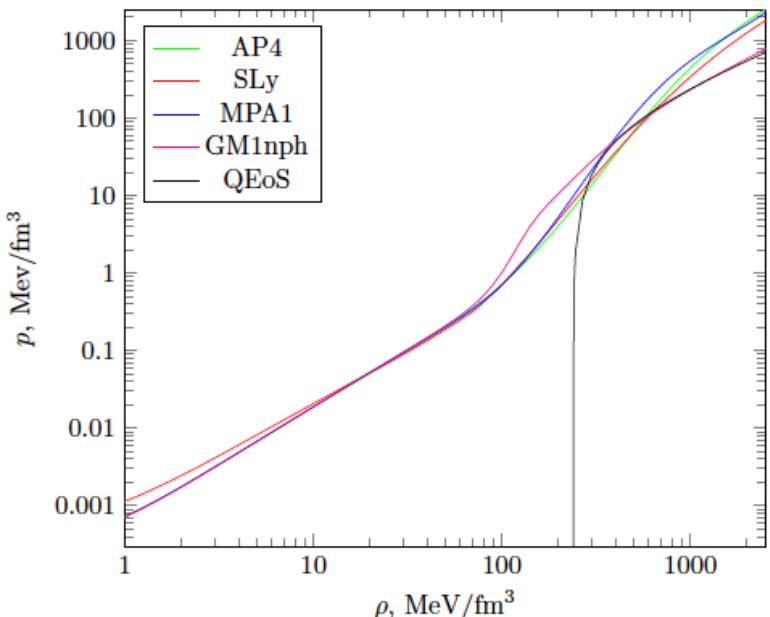
The numerical relativity community does not have the education to distinguish...

Equations of state

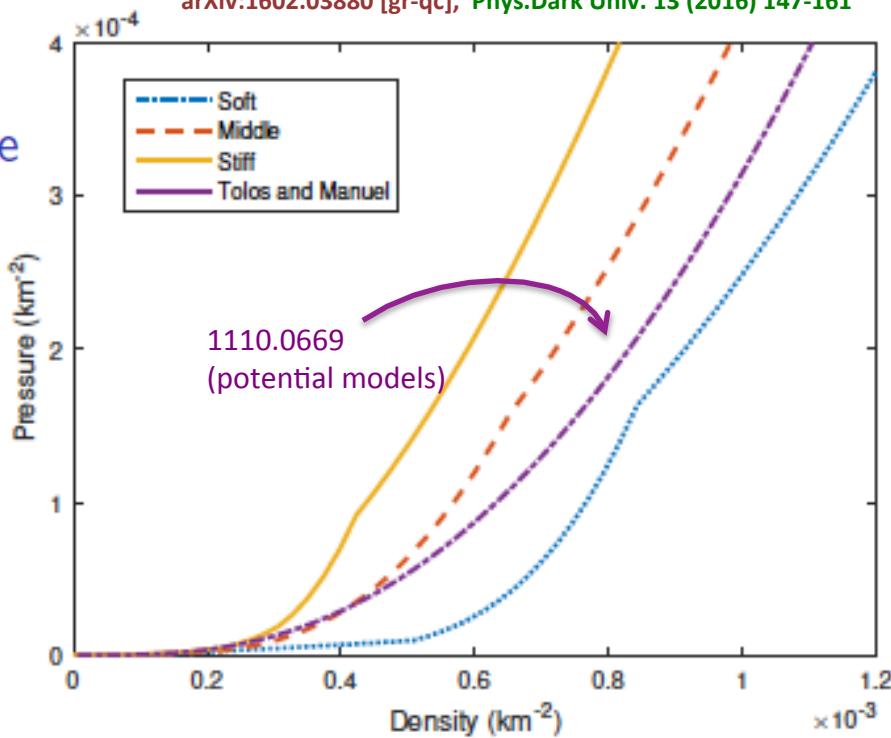
New degrees of freedom → softer eq. of state

The neutron is not pointlike

- ▶ Nucleon radius (measured in $e^- p$ scattering) $r_N \simeq 0.88 \text{ fm}$
- ▶ Nuclear radius $r_A \simeq 1.2 \text{ fm}A^{1/3}$
- ▶ When $\rho \simeq 133 \text{ MeV/fm}^3 \simeq 2.8 \text{ } m_\pi^4$
the volume evacuated by the neutrons is important



A. Astashenok, S. Odintsov, AdLCD,
arXiv:1704.08311 [gr-qc], Class.Quant.Grav. 34 (2017) no.20, 205008



- Traditionally, **potential-inspired Nuclear forces**
- **Effective Field Theories (EFT) in Chromodynamics**

K. Hebeler et al., Astrophys. J. 773 (2013) 11
[arXiv:1303.4662 [astro-ph.SR]]

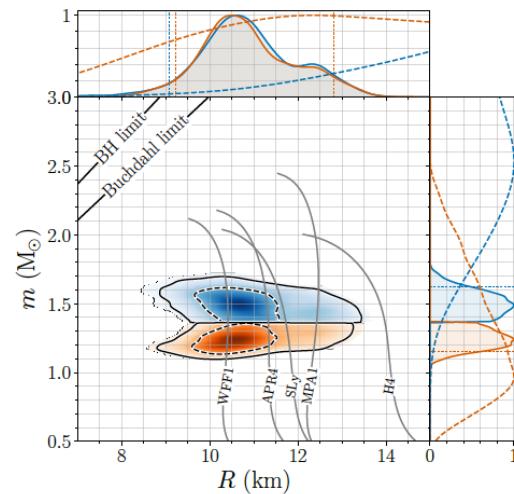
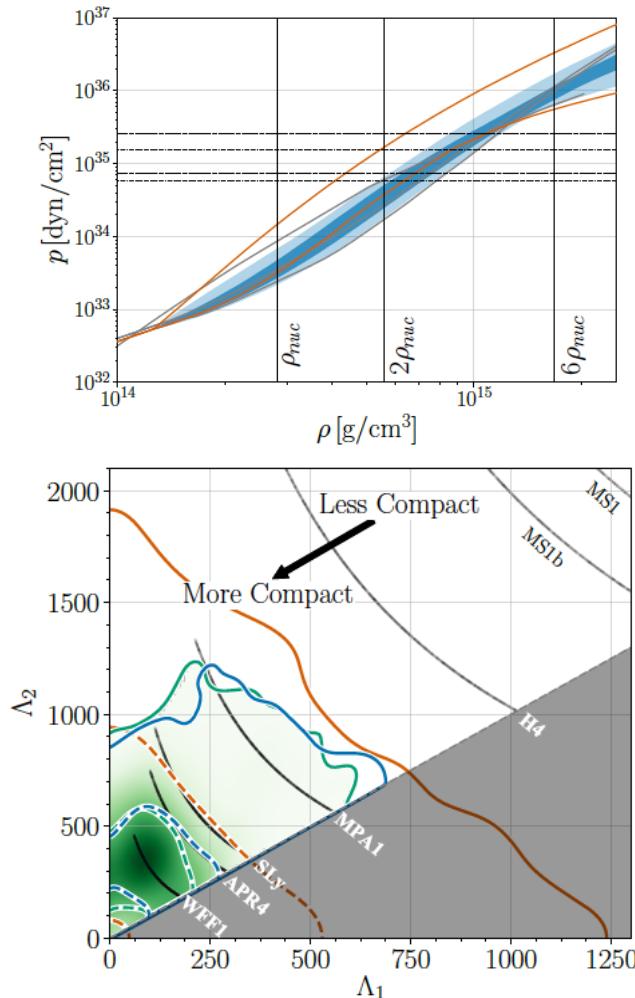
with **stiff**, **middle** and **soft** bands depending on c_s

GW170817: Measurements of neutron star radii and equation of state

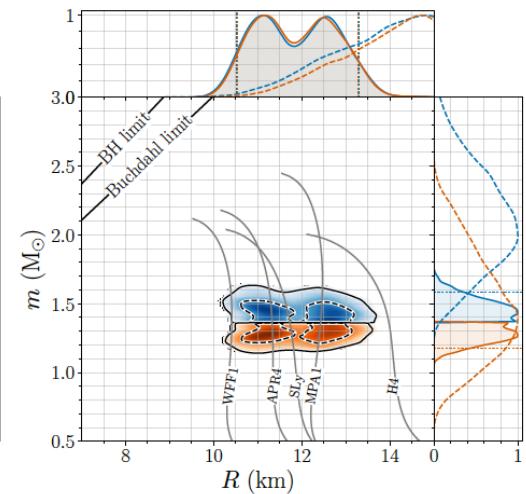
[1806.11581](#) LIGO –Virgo

Use of constraints for NS components of GW170817 tidal deformabilities Λ_1, Λ_2

EoS $p(\rho)$ constraints become possible having assumed a EoS parameterisation and a $1.97M_{\text{Sun}}$ lower mass limit



Insensitive EoS



Parameterised EoS

- ✓ Not large radii and that **soft** EoS such as APR4 are favoured over **stiff** ones such as H4 or MS1
- ✓ Imposing a common EoS for the BNS components leads to a reduction of the 90% credible interval width for the radius measurement of almost a factor of two.

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$f(R)$ procedure: solving the field equations (I of II)

Unlike earlier studies, e.g., Astashenok et al., 1309.1978 we do not rely on perturbation theory

- ▶ Solar system tests: $|f(R_{ss})| < 10^{-6}$ but what is R_{ss} ?
- ▶ $R_{FRW} \sim 3 \times 10^{-46} \text{ km}^{-2}$
- ▶ $R_{Schwz} = 0$; dimensionally, Kretschmann's scalar
 $R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = \frac{12r_s^2}{r^6} \sim 3 \times 10^{-17} \text{ km}^{-2}$ (at $r = R_\odot$)
- ▶ $R + aR^2 + O(R^4)$ a can currently be huge

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 - ▶ $R + aR^2 + O(R^4)$ *a can currently be huge*
- Other approaches include: **Scalar-tensor picture**, Capozziello *et al.*, PLB 742 (2015) 160
Yazadjiev *et al.*, JCAP 1406 003 (2014)
- Full resolution, Ganguly *et al.*, PRD 89 064019 (2014)
Capozziello *et al.*, arXiv:1509.04163 [gr-qc].
- ...but a) assuming Schwarzschild as pure exterior solution
 b) Labelling the star mass as in GR

$f(R)$ procedure: solving the field equations (I of II)

Unlike earlier studies, e.g., Astashenok et al., 1309.1978 we do not rely on perturbation theory

- ✓ Method here (non-perturbative)
 - Non-linear coupled ODE system solved by Runge-Kutta
 - Initial conditions to obtain regularity, finite pressure, matching at the radius
 - No assumption on Schwarzschild at the star's radius

$f(R)$ procedure: solving the field equations (II of II)

- ✓ **Stability conditions for $f(R)$:** oscillations around GR remain finite

$$\delta = \frac{A'}{2A} - \frac{B'}{2B} - \frac{2}{r} - \frac{2R_0 f_{3R}(R_0)}{f_{2R}(R_0)} < 0 \quad \gamma = \frac{A}{3f_{2R}(0)} (1 + f_R(0)) < 0$$

- ✓ **Boundary conditions** $p(r_\odot) = 0$ (for larger radii, $\rho = p = 0$)

Minkowski spacetime asymptotically required

$$\lim_{r \rightarrow \infty} B = \lim_{r \rightarrow \infty} A = 1 \quad \lim_{r \rightarrow \infty} R = \lim_{r \rightarrow \infty} R' = 0$$

- ✓ **Free initial conditions** $\{A, B, B', R, R', p\}$

$$A(0) = 1, B'(0) = 0, \boxed{p(0) = p_0} \text{ and } R'(0) = 0 \rightarrow \text{Family of stars}$$

It remains two initial conditions $B(0)$ and $R(0)$ \longrightarrow Shooting method

Intermission: a correct label for the mass

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✓ Newtonian gravity

$$\frac{1}{2} m \dot{r}^2 + \frac{m L^2}{2r^2} - \frac{M_{\text{Newton}} m}{r} = E$$

$$M_{\text{Newton}} = \int_0^{r_\odot} 4\pi r^2 \rho_{\text{matter}}(r) dr$$

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✓ Einsteinian gravity

$$\frac{1}{2} m \dot{r}^2 + \frac{m L^2}{2r^2} \left(1 - \frac{2M_{\text{GR}}}{r} \right) - \frac{M_{\text{GR}} m}{r} = E \quad \begin{bmatrix} M_{\text{GR}} = \int_0^{r_\odot} 4\pi r^2 \rho_{\text{Rel}}(r) dr \\ \rho_{\text{Rel}}(r) = \rho_{\text{matter}}(r) + \rho_{\text{energy}}(r) \end{bmatrix}$$

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✓ Beyond Einsteinian gravity

$$B(r) \equiv 1 - \frac{2M(r)}{r}, \quad A(r) \equiv \frac{1 + U(r)}{B(r)}$$

$$\frac{m \dot{r}^2}{2} (1 + U(r)) + \frac{m L^2}{2r^2} \left(1 - \frac{2M(r)}{r}\right) - \frac{M(r)m}{r} = E$$

$$M_{f(R)}(r) \equiv \frac{M(r)}{1 + U(r)} \longrightarrow M_{f(R)}^\infty = \lim_{r \rightarrow \infty} M_{f(R)}(r)$$

Gravitational mass

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$$\frac{m \dot{r}^2}{2} (1 + U(r)) + \frac{m L^2}{2r^2} \left(1 - \frac{2M(r)}{r}\right) - \frac{M(r)m}{r} = E$$

$$\begin{aligned} \lim_{r \rightarrow \infty} U(r) &= 0 \\ \lim_{r \rightarrow \infty} \frac{M(r)}{r} &= 0 \end{aligned}$$

Boundary conditions

$$M_{f(R)}(r) \equiv \frac{M(r)}{1 + U(r)}$$



$$M_{f(R)}^\infty = \lim_{r \rightarrow \infty} M_{f(R)}(r)$$

Gravitational mass

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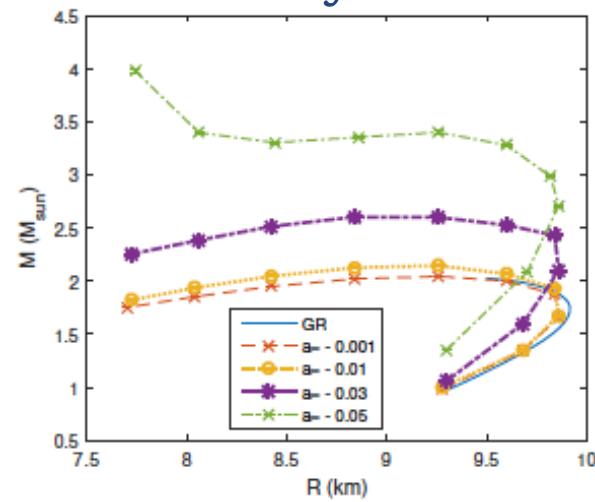
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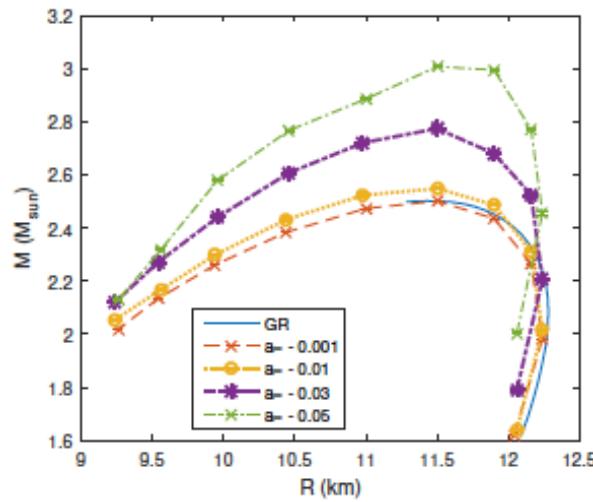
IV. Gravitational Waves. Prospects and Conclusions

R^2 model: Mass-Radius diagrams

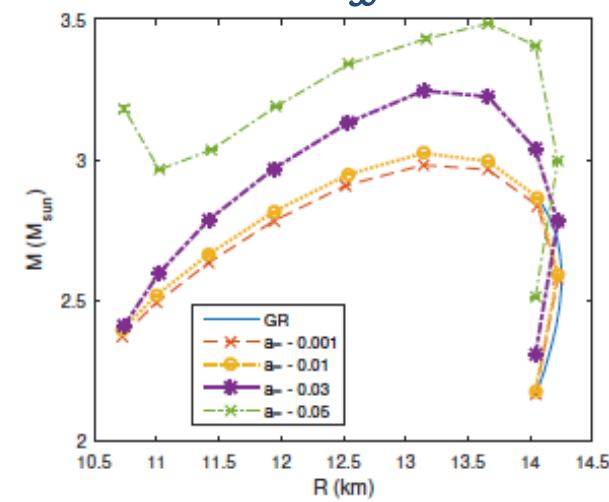
Soft



Middle



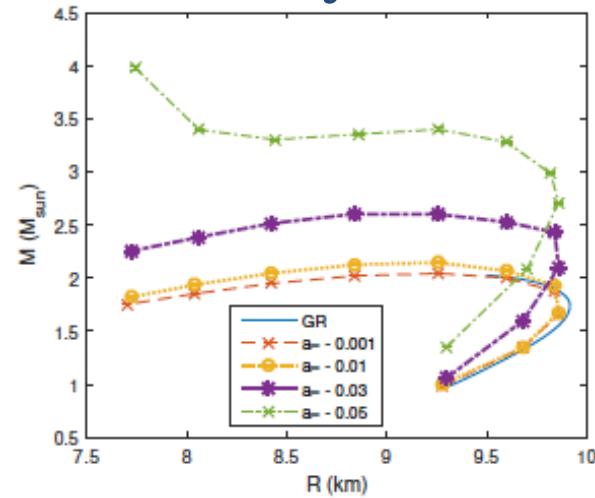
Stiff



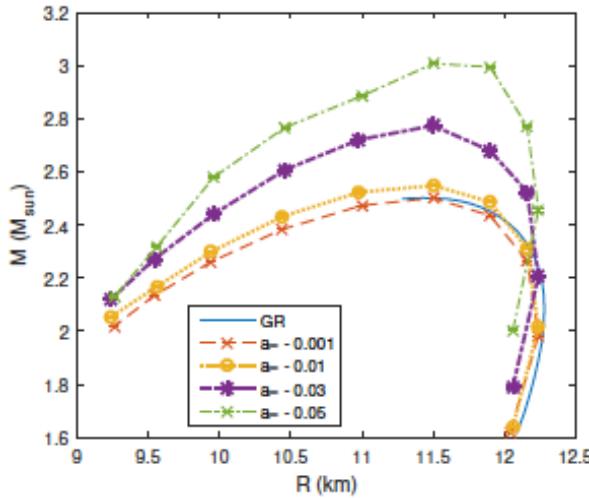
M. Aparicio, AdLCD, F. Llanes-Estrada, V. Zapatero
arXiv:1602.03880 [gr-qc], Phys.Dark Univ. 13 (2016) 147-161

R^2 model: Mass-Radius diagrams

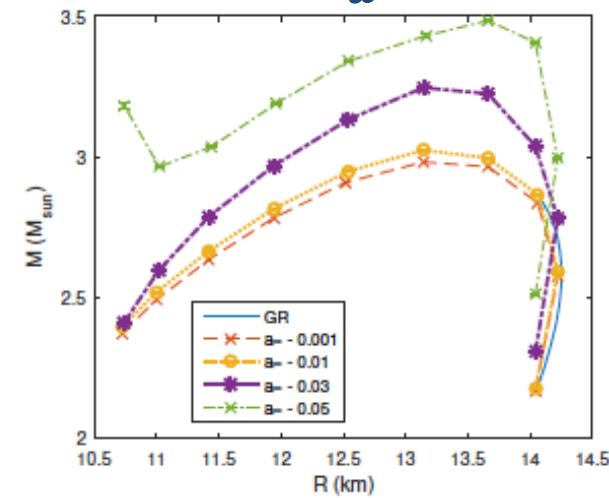
Soft



Middle



Stiff



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- ✓ For each radius masses larger than the GR counterparts
- ✓ $M(r)$ increases with the radial coordinate till reaching the limiting value $M_{f(R)}$
- ✓ For a given mass, the radii are significantly smaller than in GR. Much of the mass seen from the observer is external field, as given by the Ricci scalar damped oscillations outside the star
- ✓ Finding two-solar (or heavier) masses stars does not constrain this class of models!!

Outline

I. Motivation and state of the art

II. Those interesting fellows: static neutron stars

- Tolman-Oppenheimer-Volkov equation
- A better understanding of Equations of state

III. Full analysis of paradigmatic models in $f(R)$ gravities

- Methodology and mass definition
- Testing R^2 model
- Towards viable cosmological models: the Hu-Sawicki case

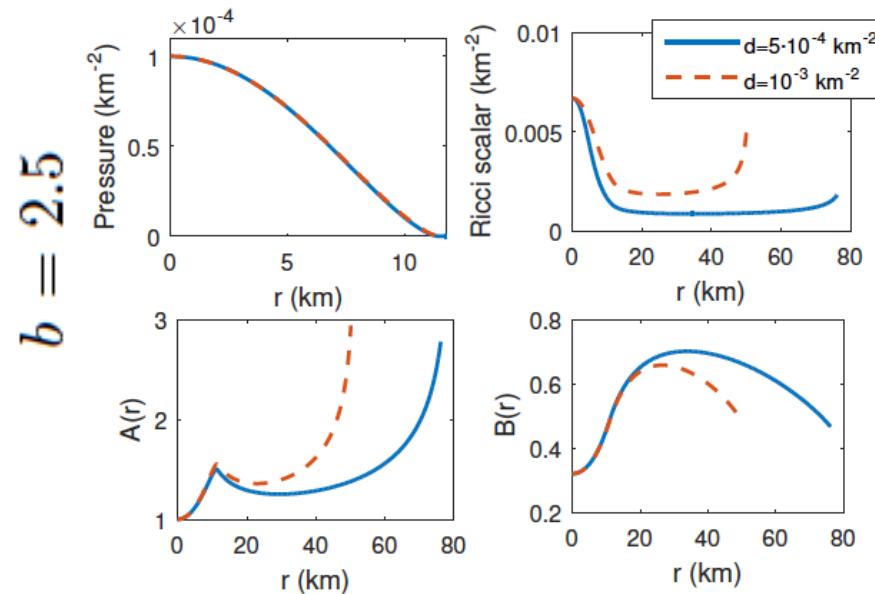
IV. Gravitational Waves. Prospects and Conclusions

Hu-Sawicki model

$$f(R) = -cH_0^2 \frac{b(R/cH_0^2)^n}{e(R/cH_0^2)^n + 1} \xrightarrow{n=1} f(R) = -\frac{bR}{1 + b \frac{R}{d}}$$

W. Hu and I. Sawicki, PRD 76, 064004 (2007)
 [arXiv:0705.1158 [astro-ph]].

$$|R| > |d| \longrightarrow f(R) \simeq -d + \frac{d^2}{bR}$$



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$$|R| > |d| \longrightarrow f(R) \simeq -d + \frac{d^2}{bR} \quad \text{Schwarzschild de-Sitter solutions}$$

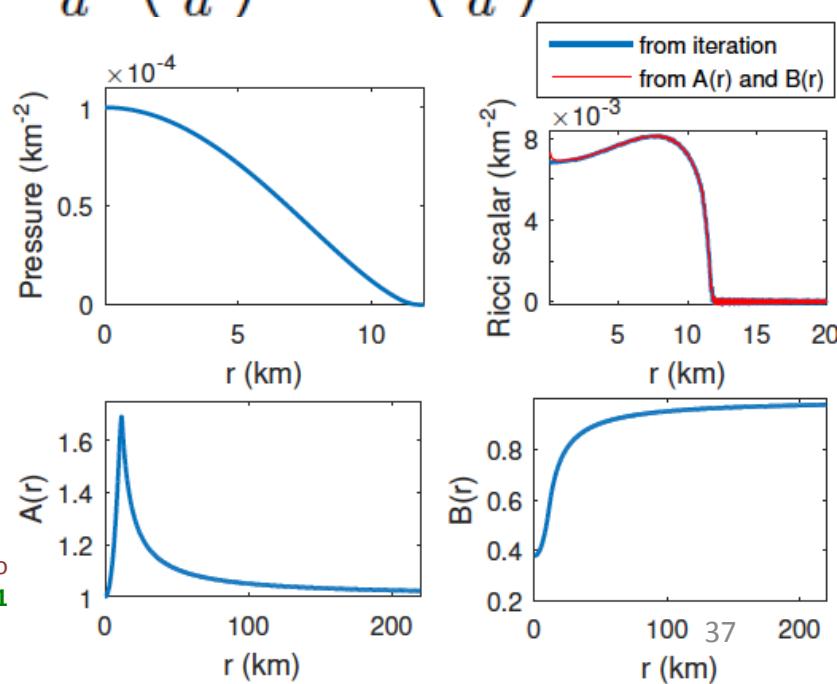
$$|R| < |d| \longrightarrow f(R) \simeq -b\frac{R}{d} + \frac{b^2}{d} \left(\frac{R}{d}\right)^2 - \frac{b^3}{d^2} \left(\frac{R}{d}\right)^3 + \mathcal{O}\left(\frac{R}{d}\right)^4$$

$$d < 0, \quad b < 1$$

Matches perturbatively to GR solutions

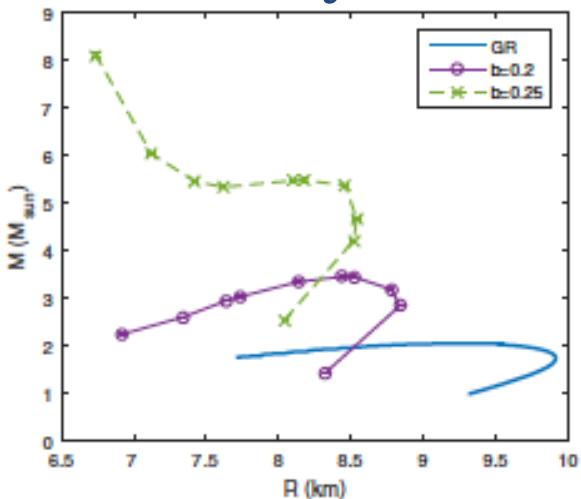
$$d = -1 \text{ km}^{-2} \quad b = 10^{-2}$$

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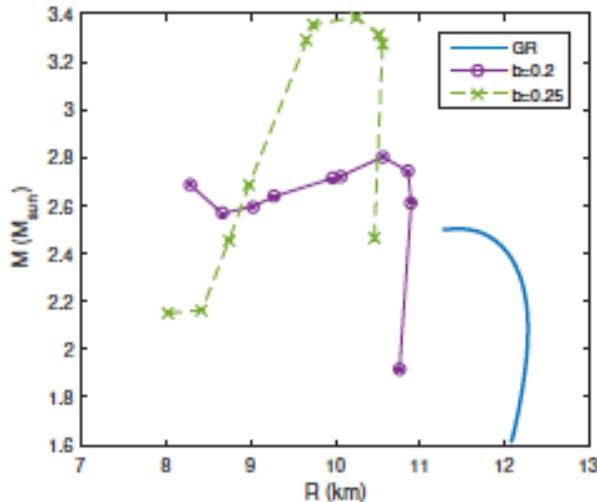
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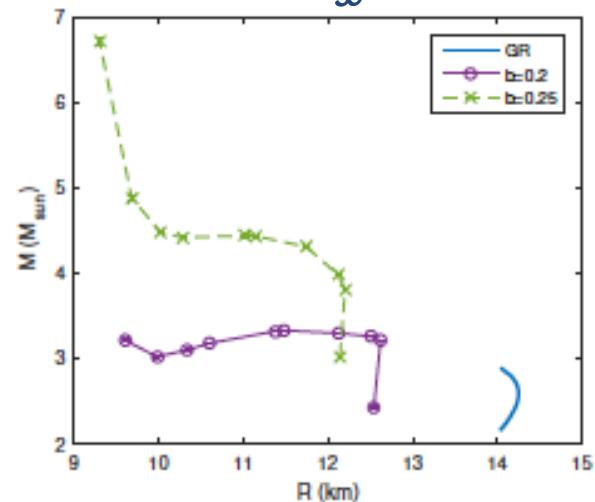


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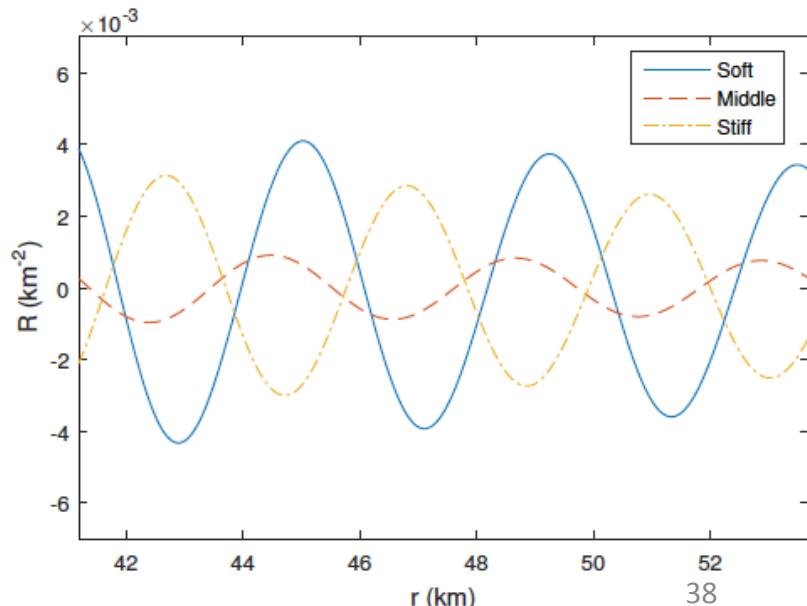
- ✓ As in R^2 model, the maximum achievable mass is not monotonic with the sound speed (EoS stiffness)

Why such differences?



- ✓ $f(R)$ contribution in this case increases the maximum star masses and reduces the radii

- ✓ No Birkhoff theorem in $f(R)$ theories [to complement Clifton 20]



Gravitational waves in $f(R)$ theories?

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Yes, there are (extra longitudinal mode)

Ananda *et al.*, Rev. D 77 (2008) 024033 [arXiv:0708.2258]
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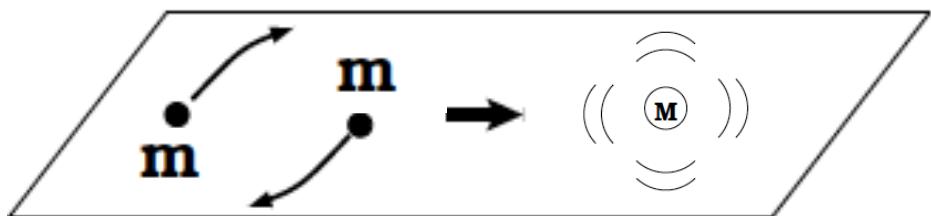
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Large amount of energy stored in the distorted external spacetime during a merger of two NS,



large amount of energy expected to be released

[in contrast with GR mergers of two NS, which can only emit a fraction in form of gravitational waves]



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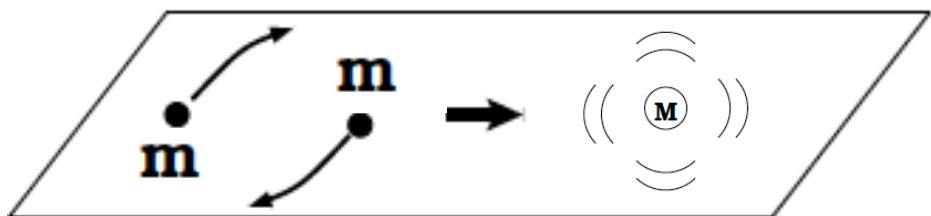
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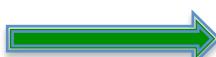
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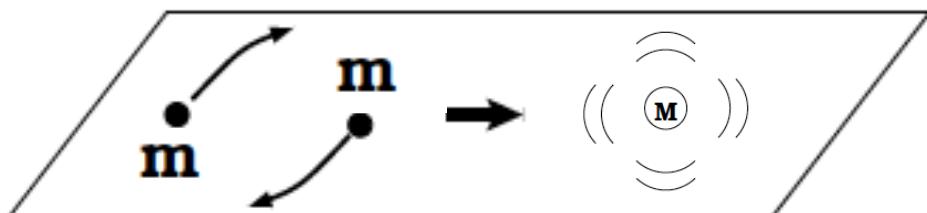
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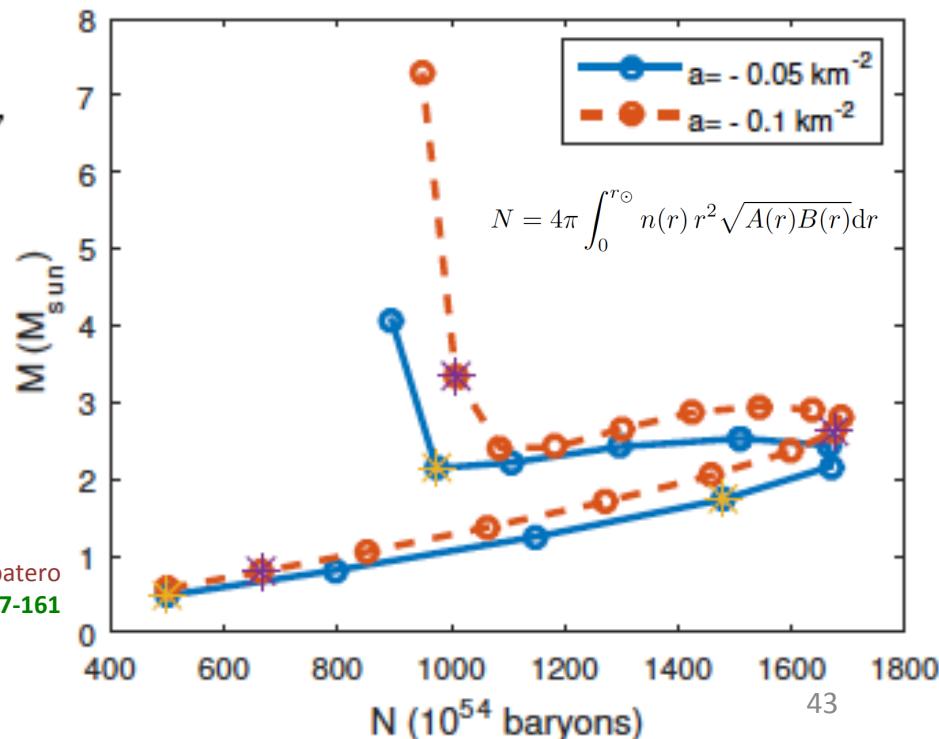
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Outline

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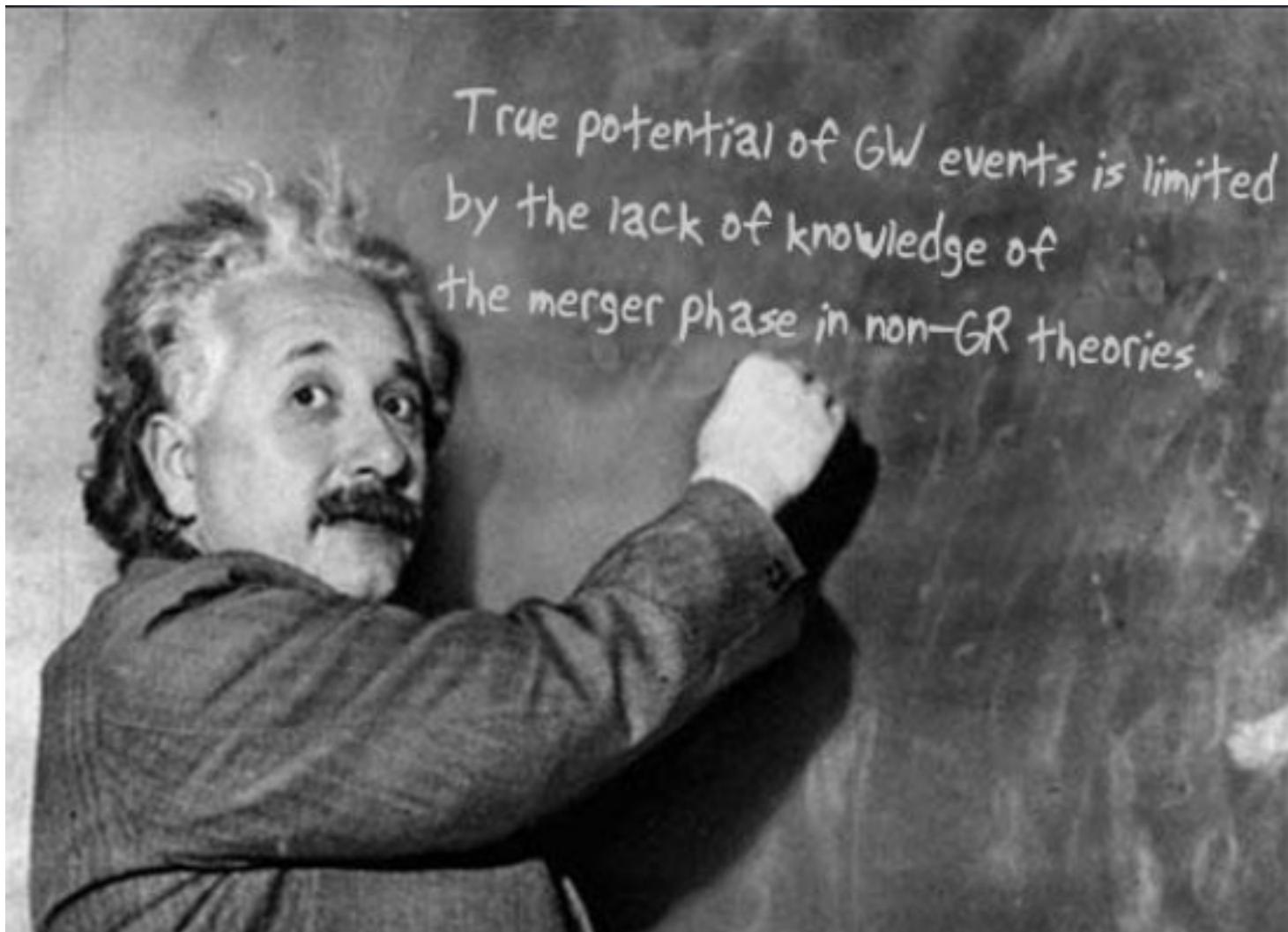
II. G-evolution beyond General Relativity

III. Neutron stars and gravitational waves

IV. Prospects and Conclusions

Conclusions and Prospects

- TOV-like dynamical system in $f(R)$ theories with **use of realistic neutron fluid EoS**
- **Physical mass** assigned to each solution. Oscillations of the curvature scalar are damp enough to start retrieving Schwarzschild solution
- Apparent masses larger than in GR and usually above observational values. **R^2 and Hu-Sawicki models**
- Energy available for gravitational-wave emission as well as the total mass can well exceed what is assumed in Einsteinian gravity. **LIGO claims can be weakened**
Viable $f(R)$ theories may accommodate a $3\text{-}4M_{\text{Sun}}$ emission in the BNS merger
- Dynamical strong-field gravity has little direct observational confirmation to date. Further insight may come from
 - Ground-based detectors, pulsar timing, the Event Horizon Telescope,
 - CMB polarization, surveys of the transient sky, SKA,
 - ...



True potential of GW events is limited
by the lack of knowledge of
the merger phase in non-GR theories.