

Mathematical Physics at LPT

Matinée des théoriciens de la vallée

Orsay, 12 June 2017

Members of the group

Permanent staff

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- Vincent Rivasseau
- Jean-Christophe Wallet
- Robin Zegers

PhD Students/Post-docs

- Luca Lioni
- Timothé Poulain
- Vasily Sazonov

Emeriti

- Michel Dubois-Violette
- Jean Ginibre

Research

1. Noncommutative Geometry
2. Random tensors/Quantum gravity

1. Noncommutative geometry (1/2)

Starting point: a set of classic dualities stating equivalences

Geometric objects \rightsquigarrow Algebraic structures

E.g. Topological space $X \rightsquigarrow C^*$ -algebra $C^0(X, \mathbb{C})$

Duality	Geometry	Algebra
Gel'fand	Topological spaces	C^* -algebras
Gel'fand-Naimark	Measure space	Von Neumann algebra
Serre-Swan	Vector bundle	Projective Modules
:	:	:

Observation: On the algebraic side, everything is **commutative**.

Question: What if those algebras were **noncommutative**?

► Noncommutative geometry

1. Noncommutative geometry (2/2)

Noncommutative Geometry and NCQFT

- Investigate the properties of physically relevant NC geometric structure
NC differential calculus, NC fibre bundles, NC metric spaces etc
- Construct QFT on noncommutative spacetimes
Definition, renormalization etc

Quantum groups

- One more duality: Lie group \rightsquigarrow (co)commutative Hopf algebra
- Quantum groups = non-(co)commutative version
(careful: bad name, not a group! But *noncommutative group* already means something else!)
- Play the role of symmetry groups in NC geometry
- Representation theory ? Highly relevant to
 - NC geometry: provide many examples of NC spaces
(E.g. Symmetric q -spaces like quantum spheres etc)
 - Integrable systems: provide solutions of the Yang-Baxter equation

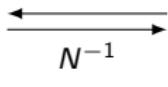
2. Random tensors/Quantum gravity (1/2)

Starting point: equivalence

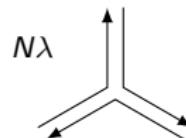
Random matrix models \leftrightarrow 2d quantum gravity

E.g. $e^{-F(\lambda)} = \int_{\text{Hermitian } N \times N} dM e^{-N\left(\frac{1}{2}\text{tr } M^2 + \lambda \text{tr } M^3\right)}$

Propagator:



Vertex:



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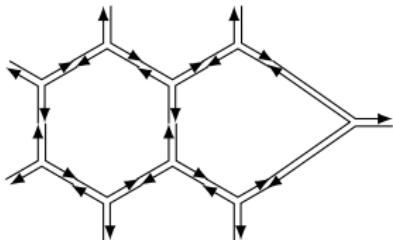
Propagator:

$$\xleftarrow[N^{-1}]{} \quad$$

Vertex:

$$N\lambda \quad \begin{array}{c} \uparrow \\ \diagdown \quad \diagup \\ \swarrow \quad \searrow \end{array}$$

► Feynman graphs = Ribbon graphs



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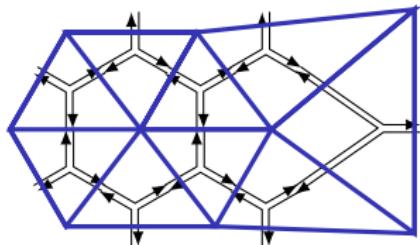
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► Feynman graphs = Ribbon graphs \leftrightarrow Triangulations of 2d Riemann surfaces



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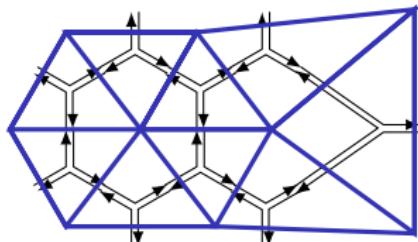
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► Feynman graphs = Ribbon graphs \leftrightarrow Triangulations of 2d Riemann surfaces



$$F(\lambda) = \sum_{g=0}^{+\infty} N^{2-2g} F_g(\lambda) \underset{N \rightarrow \infty}{\sim} N^2 F_0(\lambda)$$

$$\langle n \rangle = \frac{\partial F_0}{\partial \lambda} \sim \frac{1}{\lambda - \lambda_c} \underset{\lambda \rightarrow \lambda_c}{\rightarrow} \infty$$

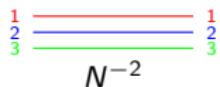
2. Random tensors/Quantum gravity (2/2)

Question : How do we get random $d > 2$ -dimensional topologies?

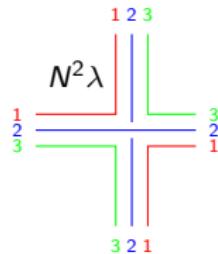
- (Rank d) random (colored) tensor models

E.g. $d = 3$, $e^{-F(\lambda)} = \int_{\text{Rank 3 tensors}} dT e^{-N^2 \left(\frac{1}{2} T_{i_1 i_2 i_3} T_{i_1 i_2 i_3} + \lambda T_{i_1 i_2 i_3} T_{i_1 j_2 j_3} T_{k_1 i_2 j_3} T_{k_1 j_2 i_3} \right)}$

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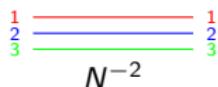
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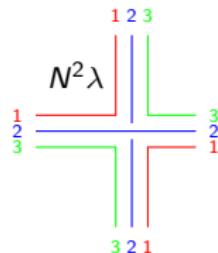
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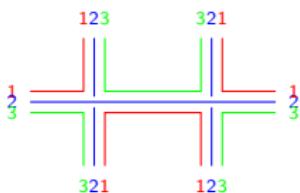
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► Feynman graphs



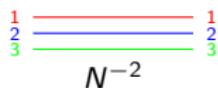
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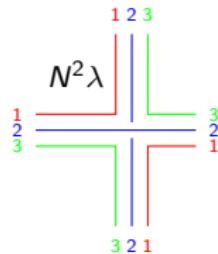
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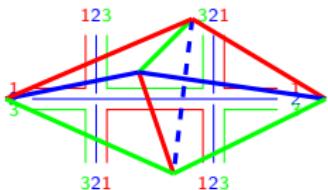
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► Feynman graphs \rightsquigarrow Tetrahedrizations of 3d manifolds



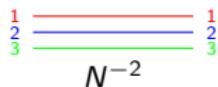
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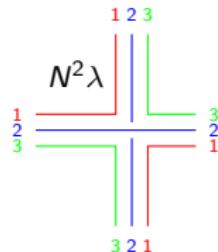
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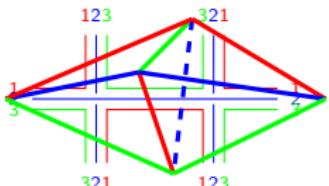
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- $1/N$ expansion
- Critical points

Thank you!