AdS classifications

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Bariloche, 19/1/2018

Introduction

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  - backreacted (unsmeared) orientifolds
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  - backreacted (unsmeared) orientifolds
  - inspire several new AdS_4 solutions
- AdS/CFT {not today}

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I. AdS7

II. AdS5

III. AdS4

I. AdS7

[Apruzzi,Fazzi,Rosa,AT'13; Gaiotto, AT '14; Apruzzi,Fazzi,Passias,Rota'15;Cremonesi,AT'15]

Most general solution:

$$ds^2 = 8\sqrt{-\frac{\ddot\alpha}{\alpha}}ds^2_{\mathrm{AdS}_7} + \sqrt{-\frac{\alpha}{\ddot\alpha}}dz^2 + \frac{\alpha^{3/2}(-\ddot\alpha)^{1/2}}{\sqrt{2\alpha\ddot\alpha-\dot\alpha^2}}ds^2_{S^2}$$

$$e^\phi \propto \frac{(-\alpha/\ddot\alpha)^{3/4}}{\sqrt{\dot\alpha^2-2\alpha\ddot\alpha}}$$

$$B = \pi\left(-z + \frac{\alpha\dot\alpha}{\dot\alpha^2 - 2\alpha\ddot\alpha}\right)\mathrm{vol}_{S^2}$$

$$F_2 = \left(\frac{\ddot\alpha}{162\pi^2} + \frac{\pi F_0 \alpha\dot\alpha}{\dot\alpha^2 - 2\alpha\ddot\alpha}\right)\mathrm{vol}_{S^2}$$

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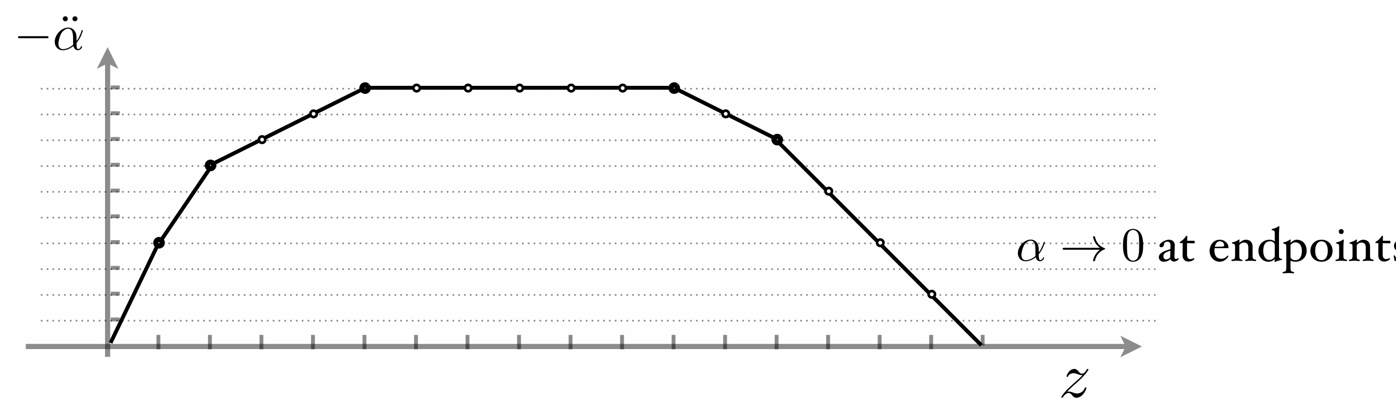

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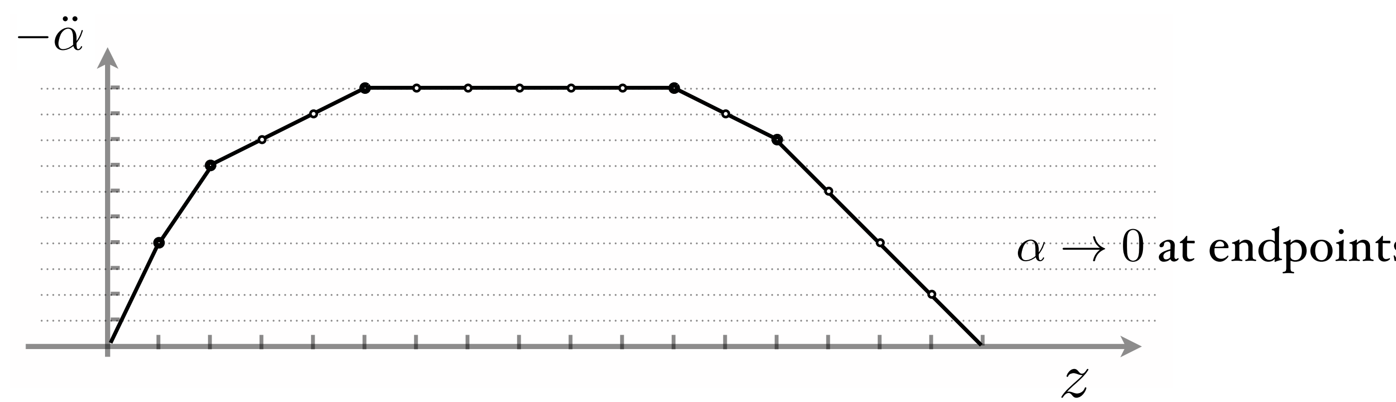

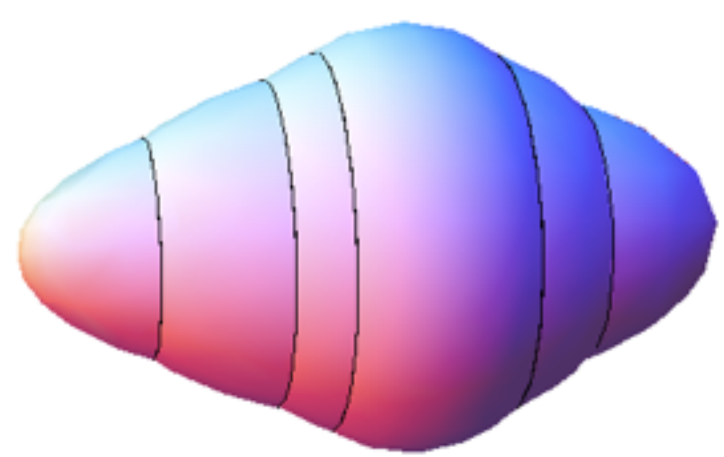

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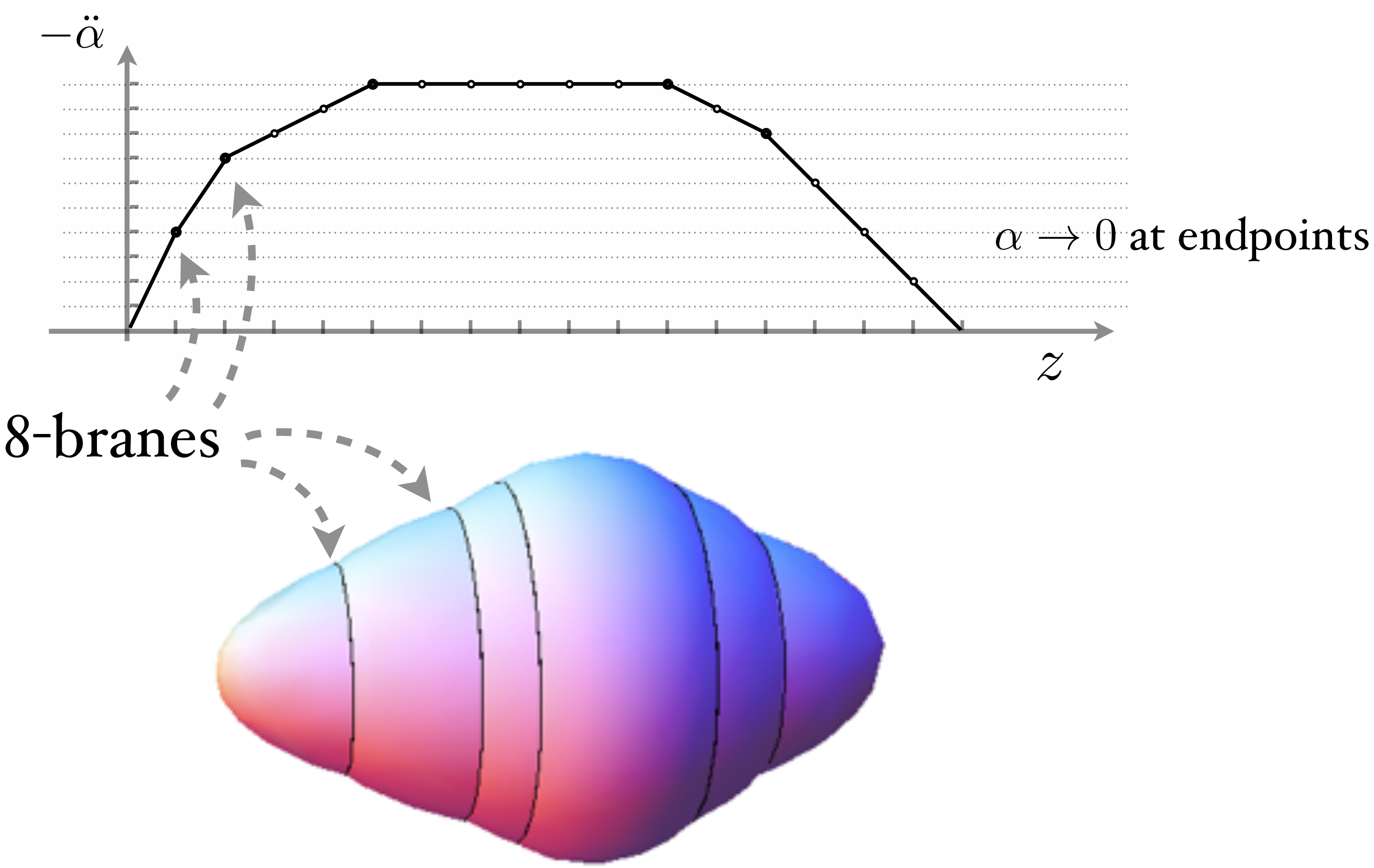

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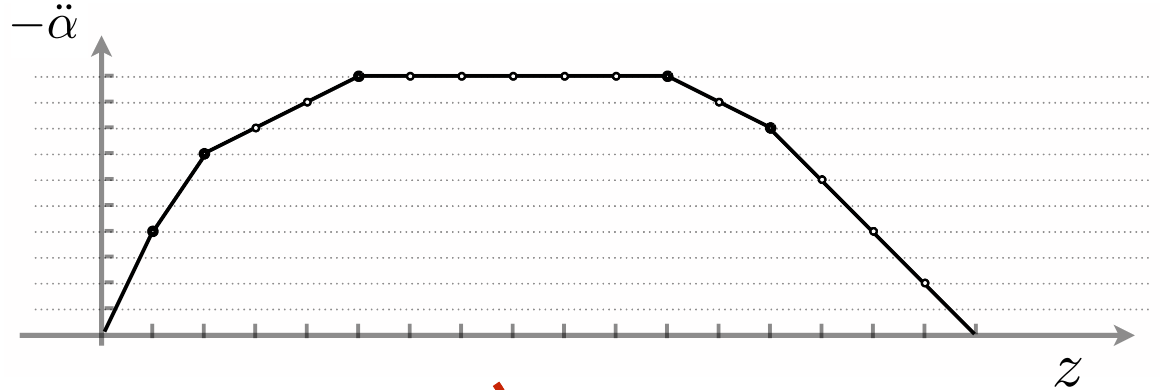

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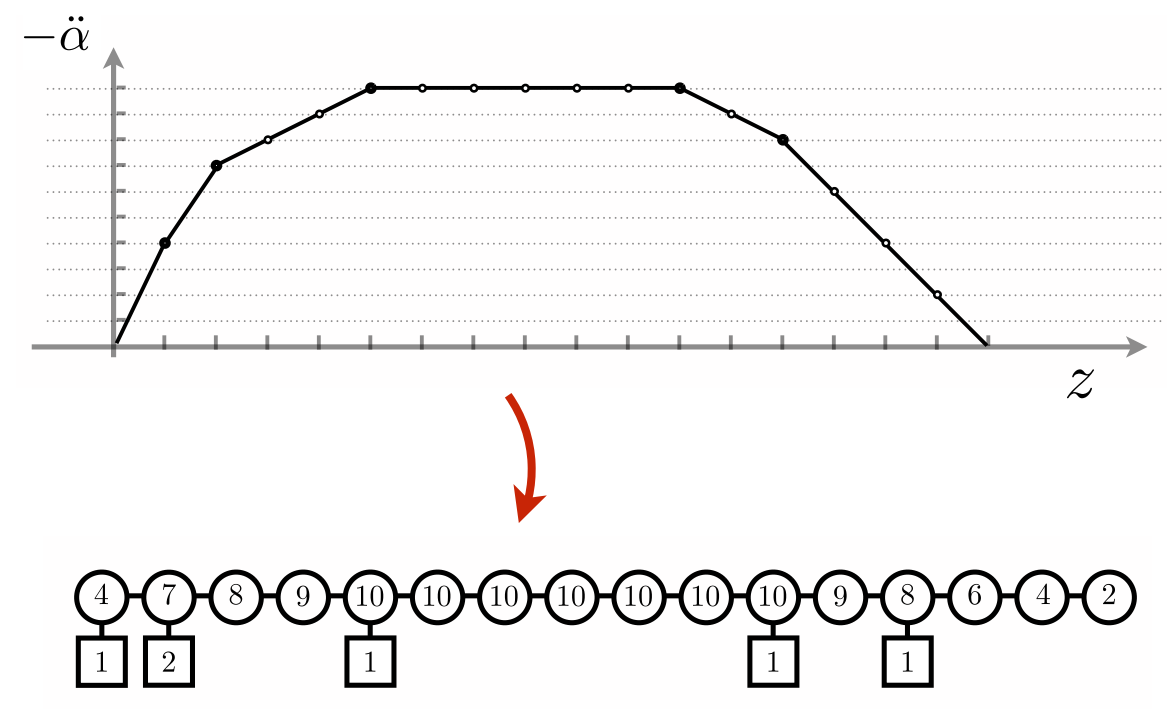
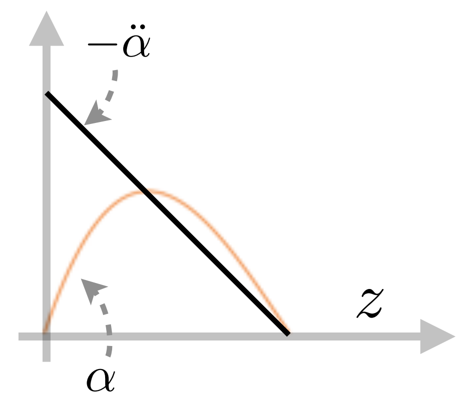
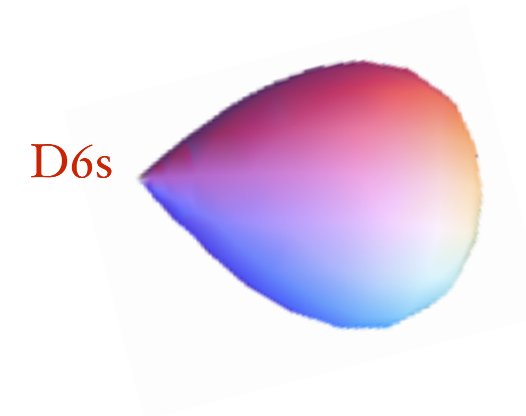


One can also have D6s:





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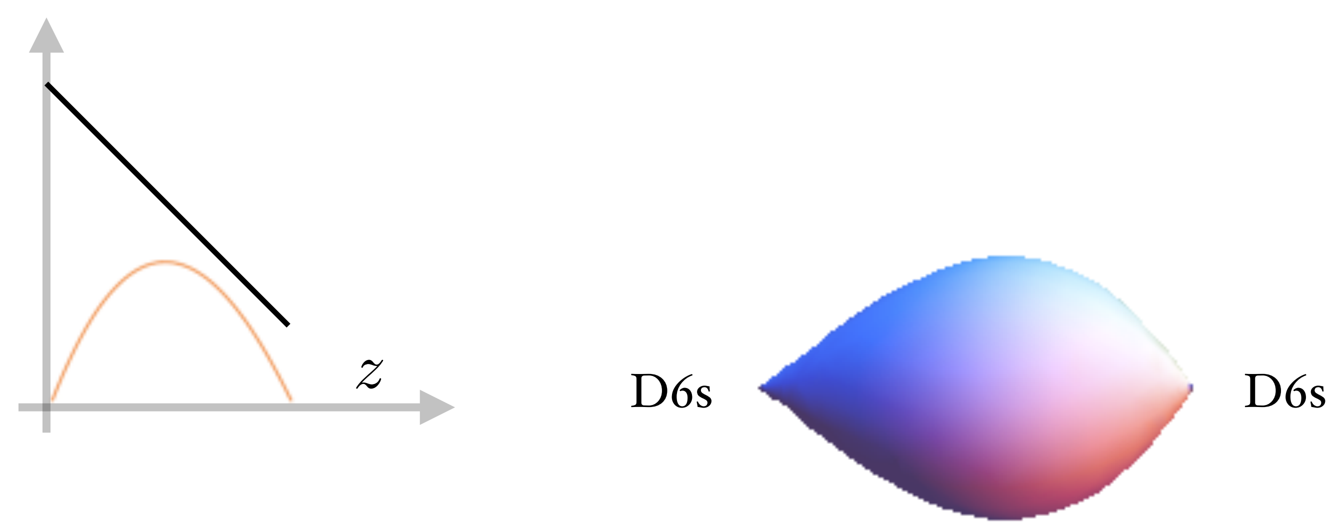
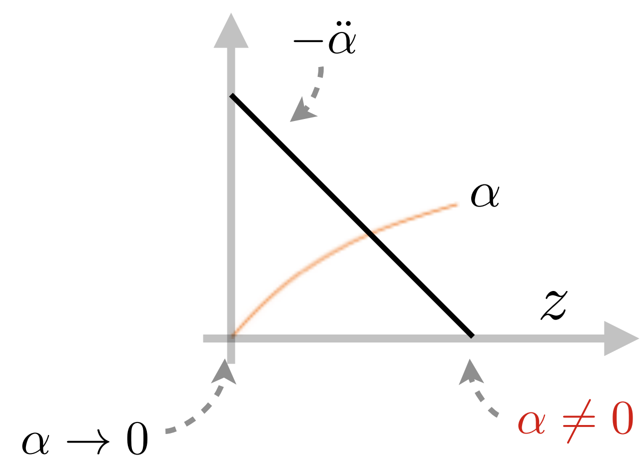
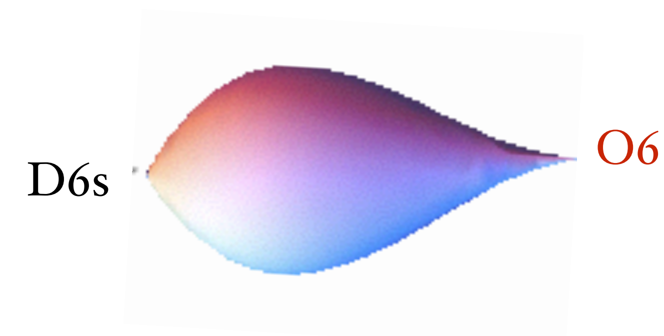


Boundary conditions for α are also important





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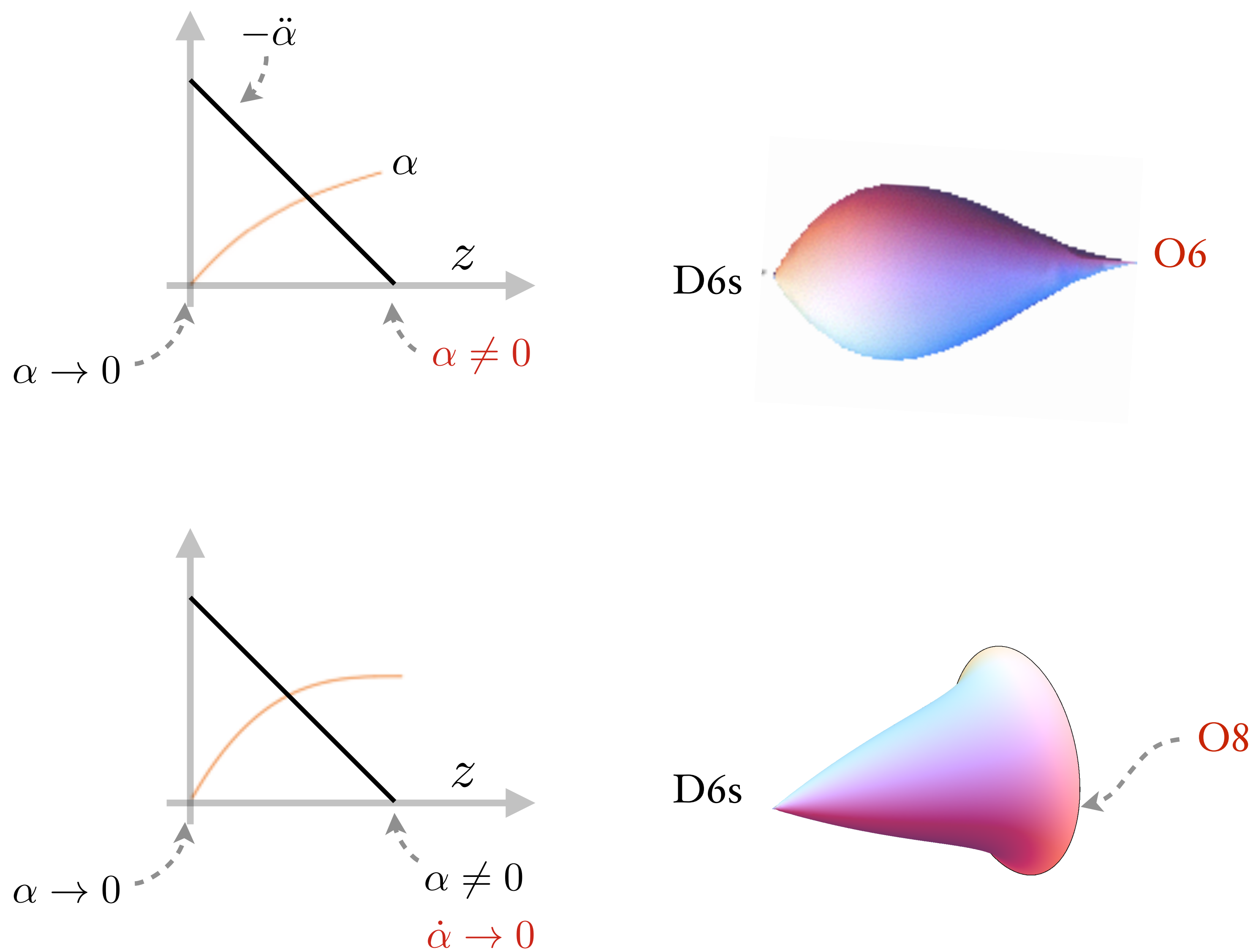

Any of these solutions can be compactified

To any of our solutions

$$e^{2A} ds^2_{\mathrm{AdS}_7} + dr^2 + v^2 ds^2_{S^2}$$

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$$\frac{3}{4} e^{2A} (ds^2_{\mathrm{AdS}_5 \times \Sigma_2}) + dr^2 + \frac{v^2}{1-4v^2} e^{2A} ds^2_{S^2}$$

dual to $\mathrm{CFT}_4 \cong \mathrm{CFT}_6/\Sigma_2$

[twisted compactification]

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[twisted compactification]

[Rota, AT '15]

$$\frac{5}{8}e^{2A}(ds^2_{\mathrm{AdS}_4\times\Sigma_3}) + dr^2 + \frac{v^2}{1-6v^2}e^{2A}ds^2_{S^2}$$

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in particular we can have
AdS4 solutions with localized O6s and O8s

Even more generally:

To any of our solutions

$$e^{2A}ds^2_{\mathrm{AdS}_7} + dr^2 + v^2 ds^2_{S^2}$$

$$e^{2A}ds^2_7 + dr^2 + \frac{v^2}{1+16(X^5-1)v^2}e^{2A}ds^2_{S^2}$$

an Ansatz for a consistent truncation!

‘minimal gauged 7d sugra’

fields: $g^{(7)}_{\mu\nu}$, A^i_μ, X

[Passias, Rota, AT ’15]

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- Old partial classifications in M-theory and IIB exist [Gauntlett, Martelli, Sparks, Waldram '04, '05]

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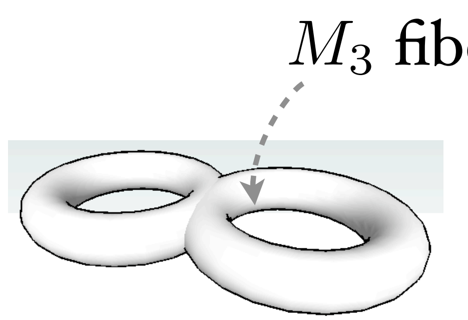
- IIA: schematically $ds^2 = e^{2A} ds^2_{\mathrm{AdS}_5} + e^{2f} ds^2_{\Sigma_g} + (d\psi + A)^2 + g$

[Bah, Passias, AT '15]

$$g \;/\!/\; -\partial_s D_s\, ds^2 - 2\partial_u D_s\, du ds - \partial_u D_u\, du^2$$

internal $M_5 =$ 

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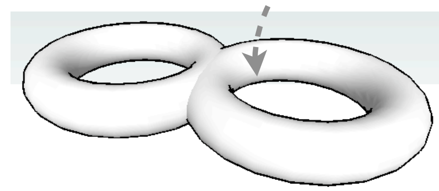
M_3 fiber

internal $M_5 =$ 

- potentials D_s, D_u satisfy 2 PDEs

a mix of Toda and Monge–Ampère…

$$\Delta_2 D_s = \partial_s(s\det(g)e^{D_s}) + \tfrac{F_0}{s}\partial_s e^{D_s}$$

Σ_g Laplacian

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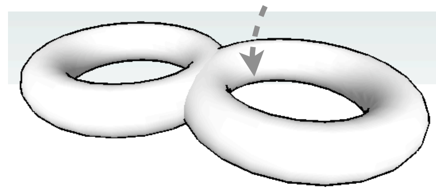

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- 'Compactification' Ansatz: D_s, D_u don't dep. on Σ_g & $f = A$ ⇨ AdS$_7$ comp. on Σ_g [Apruzzi, Fazzi, Passias, AT '15]

- More general 'Separation' Ansatz: D_s, D_u don't dep. on Σ_g & some guesswork...

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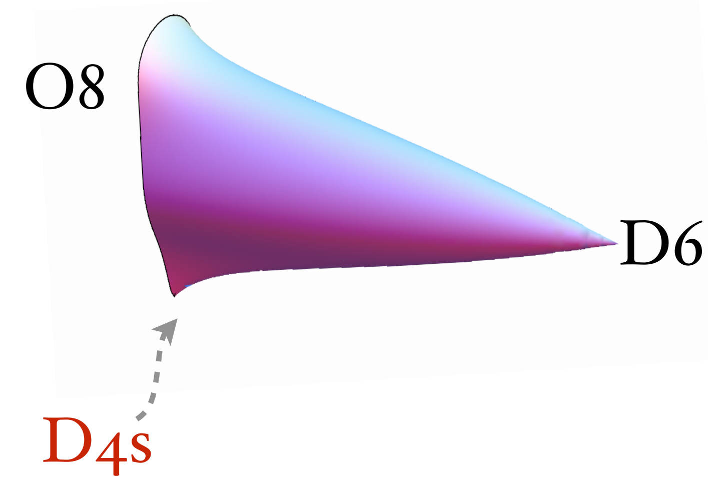
$\mathrm{AdS}_5 \times \Sigma_g \times$



$$ds_5^2 = ds^2(\Sigma_g) + \underbrace{\frac{3z dz^2}{p} + \frac{9z^3}{3p - zp'}\left[\frac{k dk^2}{1-k^3} + \frac{4}{3}\frac{(1-k^3)p}{3p - zp'(1-k^3)}\eta_\psi^2\right]}_{M_3}$$

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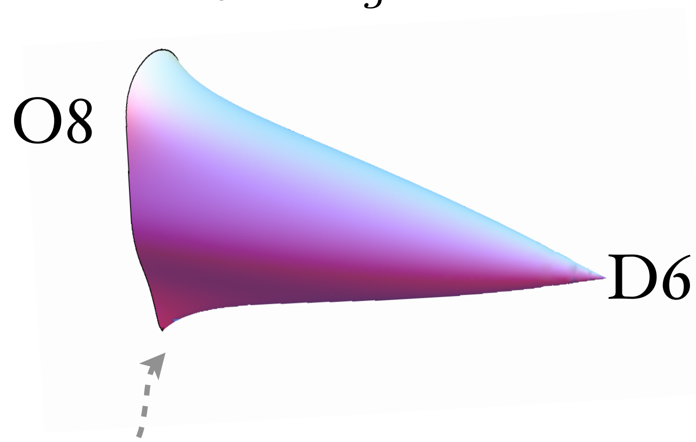

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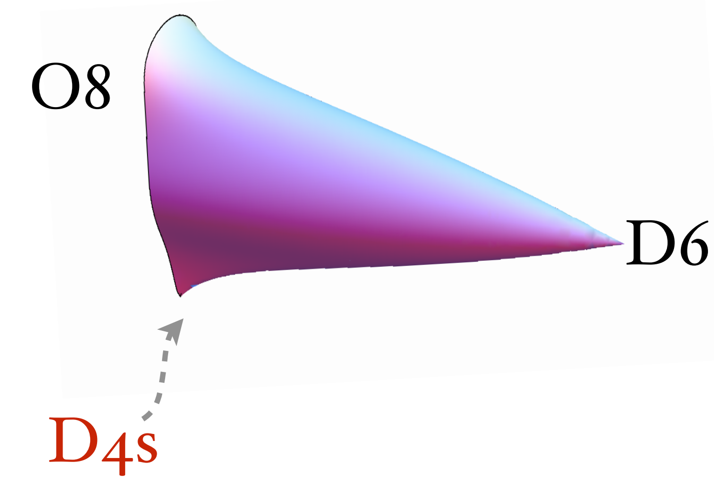

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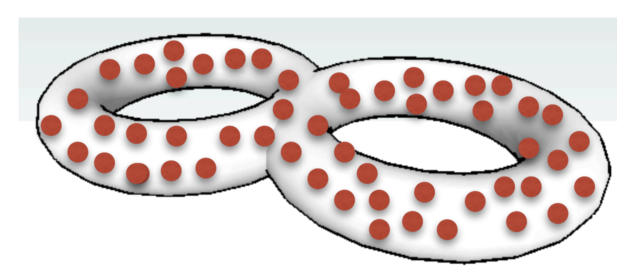

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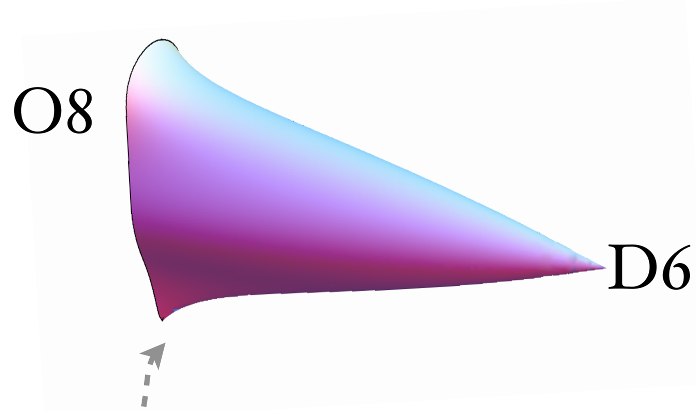
AdS$_5$ $\times$ Σ_g $\times$



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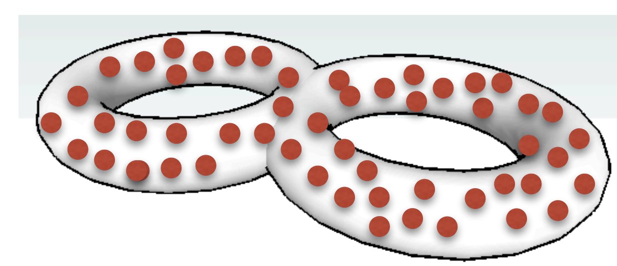
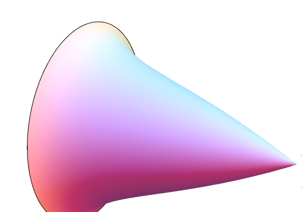


- when D4s absent, it becomes AdS5 comp. of AdS$_7$ $\times$



III. AdS_4

[IIB]

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$\mathcal{N}=1$ susy:

SU(3) $\times$ SU(3) structure satisfying

[Graña, Minasian, Petrini, AT '05]

$$d_H \Phi_- = 2e^{-A}\mathrm{Re}\Phi_+$$

$$d_H(e^A \mathrm{Im}\Phi_+) + 3\mathrm{Re}\Phi_- = e^{4A} * F$$

RR flux

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[IIB]

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RR flux

$\mathcal{N}=2$ susy:

identity structure

≀

matrix of pure spinor pairs $\Phi_\pm^{IJ}$

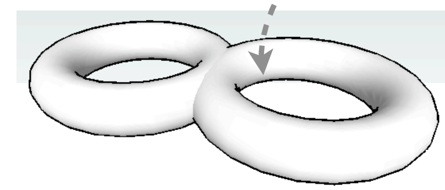
[Passias, Solard, AT '17]

$$d_H \Phi_-^{(IJ)} = 2e^{-A}\mathrm{Re}\Phi_+^{(IJ)}$$

$$d_H(e^A \mathrm{Re}\Phi_+^{[IJ]}) = 0$$

$$d_H(e^{-A}\mathrm{Im}\Phi_+^{[IJ]}) + 3e^{-2A}\mathrm{Im}\Phi_-^{[IJ]} = -e^A f \epsilon^{IJ} F$$

related to spinor norms

$\Rightarrow$ internal $M_6 =$ 

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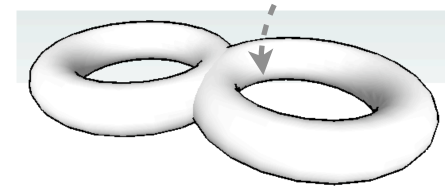
⇨ internal $M_6 =$ 

$\Delta_\lambda = y^2\partial^2_{\lambda_1} + y^{-2}\partial^2_{\lambda_2}$

Σ_g Laplacian

+4 PDEs

$$\Delta_\lambda \Sigma = -\Lambda_{,y}$$

$$\Delta_x \Sigma + \frac{1}{16}(e^{4\Sigma})_{,\lambda_1\lambda_1} = \frac{1}{4}y^2\left(e^{4\Sigma}y^{-2}U\right)_{,y}$$

$$\Delta_x \Lambda = -\frac{1}{4}\Delta_\lambda(e^{4\Sigma}U) - \frac{1}{4}(e^{4\Sigma}\Lambda)_{,\lambda_1\lambda_1}$$

$$U \equiv -y^{-1}(\Sigma_{,y}\Lambda + (y\Sigma_{,\lambda_1})^2 + (y^{-1}\Sigma_{,\lambda_2})^2)$$

[Passias, Solard, AT '17]

actually not so bad... many local solutions can be generated. Stay tuned

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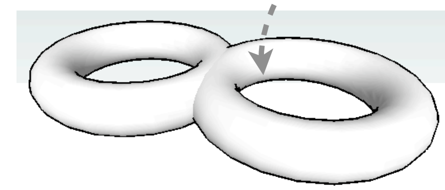

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+4 PDEs

$$\Delta_\lambda \Sigma = -\Lambda_{,y}$$

$$\Delta_x \Sigma + \frac{1}{16}(e^{4\Sigma})_{,\lambda_1\lambda_1} = \frac{1}{4} y^2 \left(e^{4\Sigma} y^{-2} U\right)_{,y}$$

$$\Delta_x \Lambda = -\frac{1}{4}\Delta_\lambda(e^{4\Sigma}U) - \frac{1}{4}(e^{4\Sigma}\Lambda)_{,\lambda_1\lambda_1}$$

$$U \equiv -y^{-1}(\Sigma_{,y}\Lambda + (y\Sigma_{,\lambda_1})^2 + (y^{-1}\Sigma_{,\lambda_2})^2)$$

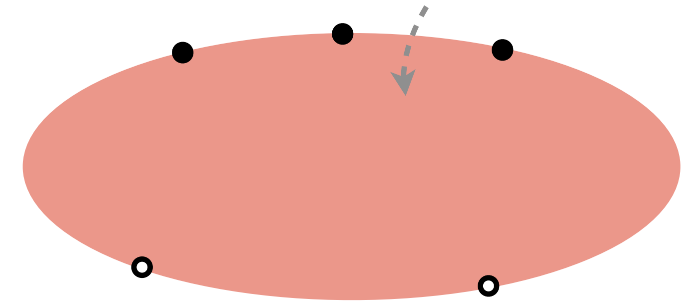
[Passias, Solard, AT '17]

actually not so bad... many local solutions can be generated. Stay tuned

- Right now only $\mathcal{N} = 4$ solutions exist

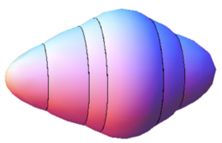
  with backreacted NS5s, D5s!

hopefully these can be generalized to lower susy.

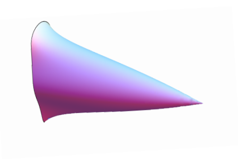


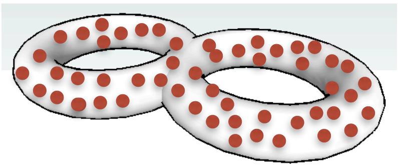
[Assel, Bachas, Estes, Gomis '11; d'Hoker, Estes, Gutperle '07]

Conclusions

- In higher dimensions, explicit classifications are emerging
    - AdS7 
    - Also AdS6: [d'Hoker, Gutperle, Karch, Uhlemann '16...]

- This is inspiring new solutions in lower dimensions





- Even in AdS4, new strategies to classify extended susy

Backup slides

To any of our solutions

$$e^{2A} ds^2_{\mathrm{AdS}_7} + dr^2 + v^2 ds^2_{S^2}$$

$$e^{2A} ds^2_7 + dr^2 + \frac{v^2}{1+16(X^5-1)v^2} e^{2A} ds^2_{S^2}$$

an Ansatz for a consistent truncation!

‘minimal gauged 7d sugra’

fields: $g^{(7)}_{\mu\nu}$, A^i_μ, X

[Passias, Rota, AT ’15]

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[Passias, Rota, AT '15]

One can use it to establish

- RG flows from AdS_7 to $\mathrm{AdS}_5 \times \Sigma_2$ and $\mathrm{AdS}_4 \times \Sigma_3$
- $\mathrm{AdS}_3 \times \Sigma_4$ solutions
- non-susy AdS_7 solution