

# Non-Geometric Calabi-Yau Backgrounds

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A Dabolkar and CH, 2002

# Non-Geometric Calabi-Yau Geometries

- Non-geometric reductions to  $D=4$  Minkowski space
- For type II,  $N=2$  SUSY in  $D=4$ . Fixes many moduli
- Mirrorfold — Mirror symmetry transitions
- “New” gauged  $D=4$  sugras, escaping no-go
- At minimum of potential: SCFT — asymmetric Gepner model

- Suggestive of novel kind of doubling?
- Bigger landscape?
- **Kawai & Sugawara**: Non-susy mirrorfolds
- **Blumenhagen, Fuchs & Plauschinn**. Gepner models from non-geometric quotient of CY CFT. Fixed point, so intrinsically stringy
- **HIS**: Gepner from asymm quotient of  $K3 \times T^2$  CFT. Freely acting, so susy breaking scale not fixed at string scale. Sugra: good low energy description

# Scherk-Schwarz reduction of Supergravity

- Supergravity in D dims:

Global duality  $G$

Scalars:  $G/H$

- Reduce on  $T^n$

Monodromy for each  $S^1$        $M_i \in G$        $[M_i, M_j] = 0$

Conjugating gives equivalent theory

$$M'_i = g M_i g^{-1} \quad g \in G$$

Consistent truncation of sugra to give gauged sugra in  $D-n$  dims.

Fields that are twisted typically become massive

# Lifting to string theory

- Duality  $G$  broken to duality  $G(\mathbb{Z})$

CH&Townsend

$G(\mathbb{Z})$  is automorphism group of charge lattice

Moduli space  $G(\mathbb{Z}) \backslash G/H$

- Monodromies must be in  $G(\mathbb{Z})$

CH '98

- Compactification with duality twists

AD&CH '02

$G(\mathbb{Z})$  conjugacy classes    Masses quantized

- If  $D$ -dim theory comes from 10 or 11 dimensions by compactification on  $N$  (e.g. torus or  $K3$ ), this lifts to “bundle” of  $N$  over  $T^n$  with  $G(\mathbb{Z})$  transitions

# Torus Reductions with Duality Twists

If  $N=T^d$  then have  $T^d$  “bundle” over  $T^n$

For bosonic string  $G(\mathbb{Z})=O(d,d;\mathbb{Z})$

Monodromies in T-duality group: T-fold

String theory on  $T^d$ : natural formulation on doubled torus  
 $T^{2d}$  with  $O(d,d;\mathbb{Z})$  acting as diffeomorphisms

T-fold:  $T^{2d}$  bundle over  $T^n$

CH

Fully doubled:  $T^{2d}$  bundle over  $T^{2n}$

CH+ Reid-Edwards

Monodromies on doubled torus

# K3 Sugra Reductions

IIA on K3:  $G=O(4,20)$ ,  $H=O(4)\times O(20)$

(2,2) Supergravity in  $d=6$

IIA on  $K3\times T^2$ :  $G=O(6,22)$ ,  $H=O(6)\times O(22)$

N=4 Supergravity in  $d=4$

Scherk-Schwarz reduction:  $d=6$  theory reduced on  $T^2$   
with monodromies  $M_1, M_2 \in O(4,20)$

Gives gauged N=4 supergravity in  $d=4$

# Supersymmetry

Fermions: monodromies in  $\text{Pin}(4) \times O(20)$

Gravitini in  **$(2,1,1) \times (1,2,1)$**  of  $SU(2) \times SU(2) \times O(20)$

Preserving 16 SUSYs:  $M_i \in O(20)$

Preserving 8 SUSYs:  $M_i \in SU(2) \times O(20)$

Breaking all SUSY:  $M_i \in SU(2) \times SU(2) \times O(20)$



# IIA String on K3

$G=O(4,20;\mathbb{Z})$  Automorphism group of charge lattice

$$\Gamma_{4,20} = H^2(K3; \mathbb{Z}) \cong E_8 \oplus E_8 \oplus U \oplus U \oplus U \oplus U$$

U: 2-dim lattice of signature (1,1)

$$\mathcal{M}_\Sigma \cong O(\Gamma_{4,20}) \backslash O(4,20) / O(4) \times O(20)$$

$O(3,19;\mathbb{Z})$ : large diffeomorphisms of K3

$\mathbb{Z}^{3,19}$ : B-shifts

Rest of  $O(4,20;\mathbb{Z})$  non-geometric

Compactify on  $T^2$ , monodromies

$$M_1, M_2 \in O(\Gamma_{4,20})$$

# Heterotic String Dual

IIA string on K3  Heterotic string on  $T^4$   
CH&Townsend

IIA string on K3 “bundle” over  $T^2$



Heterotic string on  $T^4$  “bundle” over  $T^2$

Monodromies in heterotic T-duality group  $O(4,20;\mathbb{Z})$ :

**T-fold**

Doubled picture:  $T^{4,20}$  bundle over  $T^2$

# Compactification of String Theory with Duality Twists

AD&CH '02

Monodromies  $M_i \in G(\mathbb{Z})$

Points in moduli space that give Minkowski-space minima of  
Scherk-Schwarz scalar potential



Points in moduli space fixed under action of  $M_i \in G(\mathbb{Z})$

$M_i \in G(\mathbb{Z})$  has fixed point



$M_i \in G(\mathbb{Z})$  in elliptic conjugacy class

$$G = SL(2, \mathbb{R})$$

$SL(2, \mathbb{Z})$  Elliptic conjugacy classes  
of order 2,3,4,6

$$M_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad M_3 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}, \quad M_4 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad M_6 = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}_6$$

$$G = SL(2, \mathbb{R})$$

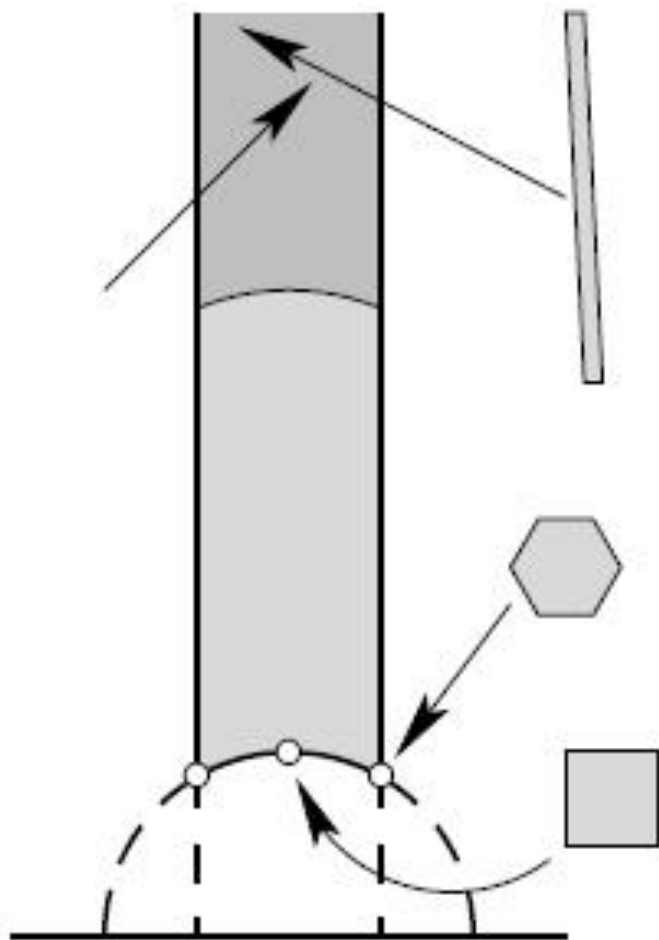
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$$\mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}_6$$

Corresponding fixed points in

$$SL(2, \mathbb{Z}) \backslash SL(2, \mathbb{R}) / U(1)$$



# Minkowski Vacua and Orbifolds

AD&CH '02

At fixed point  $M_i$  generates  $\mathbb{Z}^{p_i}$   $(M_i)^{p_i} = 1$

At this point in moduli space, construction becomes an orbifold, quotient by  $M_i \times s_i$

$G(\mathbb{Z})$  transformation together with shift in  $i$ 'th  $S^1$

$$s_i : y_i \rightarrow y_i + 2\pi/p_i$$

Geometric monodromies: orbifolds

T-duality monodromies: asymmetric orbifolds

K3 SCFT automorphisms: (asymmetric) Gepner models

Israel & Thierly

- Reduction with duality twist becomes orbifold at minima of potential, with explicit SCFT construction
- Reduction with duality twist gives extension of orbifold construction to whole of moduli space, identifies effective supergravity theory
- General point in moduli space not critical point. No Minkowski solution there but often e.g. domain wall solutions

# String Constructions with Minkowski Vacua with N=2 SUSY

Need monodromies in elliptic conjugacy classes  
of  $O(20,4;\mathbb{Z})$ : i.e. in

$$M_i \in [O(4) \times O(20)] \cap O(4, 20; \mathbb{Z})$$

$$\text{SUSY} \quad M_i \in [SU(2) \times O(20)] \cap O(4, 20; \mathbb{Z})$$

Any such monodromies will give Minkowski  
vacuum with N=2 SUSY

But finding such conjugacy classes is very hard  
open problem

Algebraic geometry constructs solutions



# K3 Mirror Symmetry

Moduli space doesn't factorise

$$\frac{O(4, 20)}{O(4) \times O(20)}$$

No mirror symmetry: all K3's diffeomorphic

For algebraic K3, moduli space of CFTs factorises

$$\mathcal{M}_{complex} \times \mathcal{M}_{Kahler} = \frac{O(2, 20 - \rho)}{O(2) \times O(20 - \rho)} \times \frac{O(2, \rho)}{O(2) \times O(\rho)}$$

Picard number  $\rho$

Mirror symmetry interchanges factors

# Mirrored Automorphisms

$$\hat{\sigma}_p := \mu^{-1} \circ \sigma_p^T \circ \mu \circ \sigma_p \quad \text{CH, Israel and Sarti}$$

$\mu : X \rightarrow \tilde{X}$  Mirror map for algebraic K3

$\sigma_p$  Diffeomorphism of  $X$   $(\sigma_p)^p = 1$

$\sigma_p^T$  Diffeomorphism of  $\tilde{X}$   $(\sigma_p^T)^p = 1$

For suitable  $X, \sigma_p$  this acts on charge lattice by an  $O(4,20;\mathbb{Z})$  transformation that is elliptic and SUSY

Use such automorphisms for monodromies

# Non-Geometric CY Vacua

- Minkowski vacuum with  $N=2$  SUSY
- Asymmetric Gepner model of Israel & Thiercy
- Explicit SCFT with Landau-Ginsburg formulation, asymmetric orbifold with discrete torsion
- $D=4$  gauged  $N=4$  SUGRA, outside classification of Horst, Louis, Smyth
- Massless sector:  $N=2$  SUSY, STU model, or STU plus small number of hypermultiplets

# Conclusions

- Non-geometries giving supersymmetric Minkowski vacua of string theory with few massless moduli
- Further non-geometries? Landscape? Physics?
- Mirrored automorphism involves K3 and its mirror. Some bigger picture? e.g.  $X \times \tilde{X}$
- General mathematical structure? Generalised CY?