# A DFT $O(d, d, \mathbb{Z})$ invariant action including (some) winding modes.

(Work in progress)

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# Outline

1. Motivation
   - Toroidal compactification
   - DFT and momentum/winding states
2. Circle example
3. Problems in higher dimensions
4. DFT description
5. Summary

# Toroidal compactification of the Bosonic String

Compactification of the bosonic string $\mathcal{M} \times \mathcal{T}^d$

$$S = \frac{1}{4\pi\alpha'} \int_M d\tau d\sigma \left(\sqrt{g} g^{\alpha\beta} G_{mn} + \epsilon^{\alpha\beta} B_{mn}\right) \partial_\alpha Y^m \partial_\beta Y^n$$

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  Gauge group: $(U(1)_L \times U(1)_R)^d$
- A special point: $n^i \neq 0, w^i \neq 0$.
  Gauge group enhancement: $(U(1)_L \times U(1)_R)^d \to G_L \times G_R$

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- $O(d, d, \mathbb{Z})$ T-duality symmetry not present in this DFT description (not always)
- How to get an action with this symmetry?

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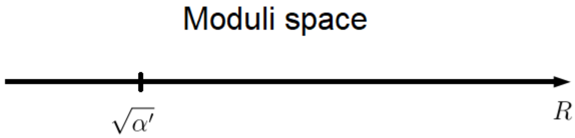
# Circle example

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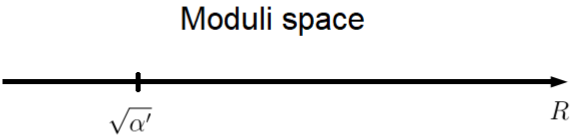


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$R = \sqrt{\alpha'}$: gauge group $SU(2)_L \times SU(2)_R$.

Vertex Operators:

$V^{3\mu}(\mathbf{p}) = \int d^2z \bar{\partial} X^\mu e^{ip^\nu X_\nu} \partial Y$

$V^{\pm\mu}(\mathbf{p}) = \int d^2z \bar{\partial} X^\mu e^{ip^\nu X_\nu} e^{\pm i p_L Y_L}$, with $p_L = n + w$ and $n = w = 1$

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$O(1,1,\mathbb{Z})$ symmetry: $\frac{R}{\sqrt{\alpha'}} \leftrightarrow \frac{\sqrt{\alpha'}}{R}$, $n \leftrightarrow w$, $Y_L \to Y_L$, $Y_R \to -Y_R$

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$O(1,1,\mathbb{Z})$ over vertex operators:

Left sector untouched

Right sector:

$$\bar{V}^{3\mu} \to -\bar{V}^{3\mu}$$
$$\bar{V}^{+\mu} \leftrightarrow \bar{V}^{-\mu}$$

Automorphism of the gauge algebra

# Circle example-DFT

[1510.07644, A.G.I.M.N.R]

Take a new metric in the coset $\frac{O(D+3,D+3,\mathbb{R})}{O(D+3,\mathbb{R})\times O(D+3,\mathbb{R})}$. dim$=D^2+6D+9$

$$\underbrace{D}_{extended\ coordinates} + \underbrace{1}_{compactified:\ d} + \underbrace{2}_{''ladder''}$$

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$\mathcal{H}^{MN} = \delta^{AB}\mathcal{E}_A{}^M\mathcal{E}_B{}^N = \mathcal{H}^{AB}E_A{}^M E_B{}^N$,

$\mathcal{E}_{\hat{A}}(x,y,\tilde{y}) = \mathcal{U}_{\hat{A}}{}^{\hat{A}'}(x)E_{\hat{A}'}(y,\tilde{y})$

Internal part: $\mathcal{E}_A{}^M(x,y,\tilde{y}) = \Phi_A{}^B(x)E_B{}^M(y,\tilde{y})$

Ansatz

$E_A{}^M = \frac{i}{\sqrt{2}}\mathrm{diag}\left(e^{ip_L Y_L}, e^{-ip_L Y_L}, 2, 2, -e^{ip_R Y_R}, -e^{-ip_R Y_R}\right)$

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Generalized fluxes

$$\mathcal{L}_{E_A}E_B = f_{AB}{}^C E_C.$$

# Circle example-DFT

$O(1,1,\mathbb{Z})$ symmetry?

$$\tilde{\Lambda} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \in O(3,3,\mathbb{R})$$

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$O(d, d, \mathbb{Z})$ symmetry maps massless states to massive ones.

Can't have the symmetry unless we incorporate them in some way [G.P.R. hep-th/9401139].

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- Find new gauge group that includes these structure constants.

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How to get a DFT action?

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Fluxes as

$$(\tilde{\mathcal{L}}_{E_A} E_B)^M = (\mathcal{L}_{E_A} E_B)^M + \Omega_{AB}{}^C E_C{}^M = f_{AB}{}^C E_C$$

where $\Omega_{ABC}$ contains the information about cocycles [1704.04242,G.I.N.C].
Consistency conditions:

$$\Omega_{ABC} = \Omega_{[ABC]}\,, \qquad \Omega_{[AB}{}^D \Omega_{C]D}{}^E = 0\,, \qquad \Omega_{ABC}\partial^C \cdots = 0$$

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Thanks!