Let’s talk about windings again

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Our recent work

- We were interested in constructing a double field theory which contains winding degrees of freedom.
- In order to do it, we'll take advantage of the enhancement in string theory because it points on which appear new massless states with winding number.
- In particular we give a higgsing enhancing-beaking from the double field theory perspective.

Review of basic ideas

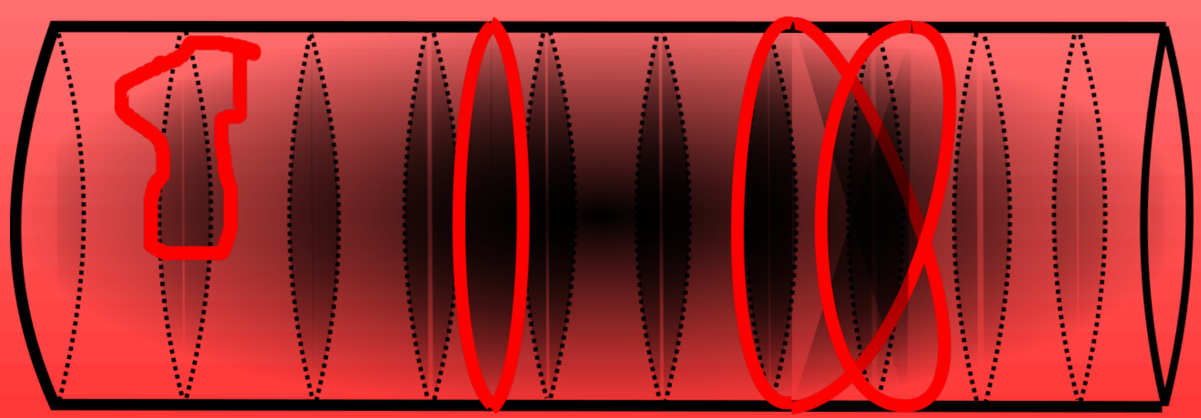
First of all: I will always speak about closed string theory compactified to a torus !!!

It's well known that when we compactify string theory to a torus a new property emerges: THE WINDING

Winding Number

Due to compatification, strings can "wind" around non-contractible circles.

Thus, there's a difference beetween strings which have different winding number



So, if we want to make a field description of string theory we should give to the fields in this theory the property of carry "windings"

winding property --------------- > stringy property

(extended object)

field theories --------------- > point particles

(naturaly)

On string theory, left and right operator positions can be thought as two different position, namely, the space-time watched by strings is double

$$(X_L, X_R) \qquad\qquad (X, \tilde{X})$$

Actually, zero-modes (mass center) of left/right operator positions should be different if we want to say it. But this only happens on compactified dimensions (only where winding exists)

Moreover, T-Duality, which exchange momentum and winding, also maps:

$$X \Leftrightarrow \tilde{X}$$

where now winding plays the role of momentum, i.e. the fourier transformed of the new coordinate

To achieve a field theory description of strings we could required:

- Double coordinates “entangled” by T-duality
- Fields covariants under T-duality

An that’s the beggining of Double Field Theory

In the construction of DFT there's also:

generalized diffeomorphism (GD)

- The action is constructed requiring to be invariant under GD
- When GD are required to close an algebra ....

..................They don't ! Unless, some constrain be imposed

Constrains are neccesary, they are physical:

- String theory is a constrained theory: LMC

$$p \cdot \tilde{p} = N - \bar{N} \quad \rightarrow \quad \partial_M \partial^M = N - \bar{N}$$

But, unfortunately, LMC is not enough....So we generalized it to the STRONG CONSTRAIN (or SC)

$$\partial_M \partial^M = 0$$

For all products fields

$N = \bar{N} = 0$

¡¡¡¡But SC destroy all the dependence on winding modes, the initial motivation !!!!

If one just cares about phenomenology, one makes a truncation of massive states (Planck scale) and no-winding

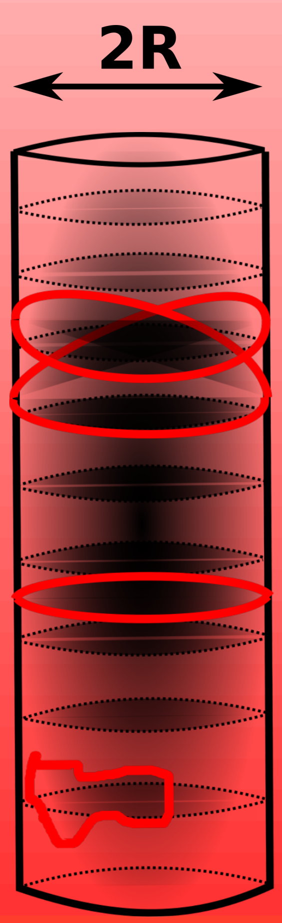
states are present ------ > so, everything is consistent

But it does NOT work always due to:

ENHANCEMENT

Enhancement Example: SU(2) x SU(2)

- Circle case $$m^2 = (\frac{p}{R})^2 + (\frac{\tilde{p}}{\tilde{R}})^2 + \frac{2}{\alpha'}(N + \bar{N} - 2)$$



When $R = \sqrt{\alpha'}$ one finds new massless states

Vector and Scalar with: $N = 1\ , \bar{N} = 0,\ \ p = \tilde{p} = \pm 1$

Vector and Scalar with: $N = 0\ , \bar{N} = 1,\ \ p = -\tilde{p} = \pm 1$

More Scalar with: $$\begin{cases} N = \bar{N} = 0,\ \ p = \pm 2\ , \tilde{p} = 0 \\ N = \bar{N} = 0,\ \ p = 0\ , \tilde{p} = \pm 2 \end{cases}$$

Question: How to include those states in DFT ?

Extended Tangent Space

- When we count deegrees of freedom we realized:

  4 new vectors + 8 new scalars + old degrees of freedom are exactly the number of d.o.f encoded on the coset

$$\frac{O(D+3,D+3)}{O(D+3)\times O(D+3)}$$

  where D is the number of non-compact dimentions.

- 3 is exactly the number of direction of the tangent space of

SU(2)

- It may suggest that we could include new massless states growing up the tangent space of the DFT,
i.e. now the vielbeing $E_A{}^M$ of DFT will have D+3 directions instead D+1

- It was done in [1] (succesfully) giving an ansatz by the $E_A{}^M$ wich depends on the double internal coordinates (of the double circle) violating the Strong Constrain

But how exactly ?......

[1] "Enhancing gauge symmetry and winding modes in Double Field Theory" G. Aldazabal, M. Graña, S. Iguri, M. Mayo, C. Nuñez, J. A. Rosabal

Remember fields on DFT

$$E_A{}^M$$

And the Coset

$$\frac{O(D+3,D+3)}{O(D+3)\times O(D+3)}$$

Now demand that $E_A{}^M$ be a representation of

- In practice, it means that the double index now has 2 more directions because of the enhancement of the "tangent space" (but no more coordinates)

- Now go to the DFT action and make a SS reduction of the vielbeins

$$E_A(x, Y, \tilde{Y}) = \mathcal{U}_A{}^{A'}(x) E'_{A'}(Y, \tilde{Y})$$

$$
\begin{aligned}
S_{eff} \;=\; \frac{1}{2\kappa_d^2}\int d^dx\sqrt{g}e^{-2\varphi}\Big[&\mathcal{R}+4\partial^\mu\varphi\partial_\mu\varphi-\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}\\
&-\frac{1}{8}\mathcal{H}_{IJ}F^{I\mu\nu}F^J_{\mu\nu}+\frac{1}{8}(D_\mu\mathcal{H})_{IJ}(D^\mu\mathcal{H})^{IJ}\\
&-\frac{1}{12}f_{IJ}{}^K f_{LM}{}^N\left(\mathcal{H}^{IL}\mathcal{H}^{JM}\mathcal{H}_{KN}-3\,\mathcal{H}^{IL}\eta^{JM}\eta_{KN}+2\,\eta^{IL}\eta^{JM}\eta_{KN}\right)-\Lambda\Big]
\end{aligned}
$$

where

$$
\begin{aligned}
F^I &= dA^I+\frac{1}{\sqrt{2}}f_{JK}{}^I A^J\wedge A^K\\
H &= dB+F^I\wedge A_I,
\end{aligned}
$$

$$
\mathcal{H}_{\mathcal{C}}=\begin{pmatrix}1_n+MM^T & M\\ M^T & 1_n+M^TM\end{pmatrix}+O(M^3)
$$

$$
(D_\mu\mathcal{H})_{IJ}=(\partial_\mu\mathcal{H})_{IJ}+\frac{1}{\sqrt{2}}f^K{}_{LI}A^L_\mu\mathcal{H}_{KJ}+\frac{1}{\sqrt{2}}f^K{}_{LJ}A^L_\mu\mathcal{H}_{IK}
$$

$$
\mathcal{L}_{E'_A}E'_B=\frac{1}{2}[E'_A{}^P\partial_PE'_B{}^M-E'_B{}^P\partial_PE'_A{}^M+\eta^{MN}\eta_{PQ}\partial_NE'_A{}^PE'_B{}^Q]D_M
$$

$$
[E'_I,E'_J]=\mathcal{L}_{E'_I}E'_J=f_{IJ}{}^KE'_K\ .
$$

$$
\partial_A=(0,\ 0,\ \partial_{y_L},\ 0,\ 0,\ \partial_{y_R}),
$$

Now, if some ansatz for vielbeins gives the appropriate structure constants, namely:

$$f_{IJ}{}^{K} = \begin{cases} (\frac{2}{\alpha'})^{\frac{1}{2}} \epsilon_{ijk} \\ -(\frac{2}{\alpha'})^{\frac{1}{2}} \bar{\epsilon}_{ijk} \end{cases} .$$

We will obtain the following action:

$$\begin{aligned} S &= \frac{1}{2\kappa_d^2} \int d^d x \sqrt{g} e^{-2\varphi} \left(\mathcal{R} + 4\partial^\mu \varphi \partial_\mu \varphi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \\ &- \frac{1}{8} \left(\delta_{ij} F^{i\mu\nu} F^j_{\mu\nu} + \delta_{ij} \bar{F}^{i\mu\nu} \bar{F}^j_{\mu\nu} - \frac{1}{2} g_d \sqrt{\alpha'} M_{ij} F^i_{\mu\nu} \bar{F}^{j\mu\nu} \right) \\ &- D_\mu M_{ij} D_\nu M^{ij} g^{\mu\nu} + \frac{16 g_d}{\sqrt{\alpha'}} \det M \end{aligned}$$

Effective action for massless states of string theory compactify on the selfdual radius

But....Which's the vielbein ?

$$
\begin{aligned}
E_{\pm} &= c(e^{\mp i\frac{2}{\sqrt{\alpha'}}y_L}, ie^{\mp i\frac{2}{\sqrt{\alpha'}}y_L}, 0,0,0,0), \quad E_3 = -c(0,0,1,0,0,0,0) \\
\bar{E}_{\hat{\pm}} &= c(0,0,0,e^{\mp i\frac{2}{\sqrt{\alpha'}}y_R}, ie^{\mp i\frac{2}{\sqrt{\alpha'}}y_R}, 0) \quad \bar{E}_{\hat{3}} = -c(0,0,0,0,0,1)
\end{aligned}
$$

They depend explicit on the double internal coordinates of the circle !!!

Moreover

- The reason of how we realized those vielbeins going to work is very long, but it was inspired on vertex operator of the states
- Therefore, when we saw the vertex operator outside of the selfdual point we asked ourself if we could change the vielbeins in order to obtain fluxes which give an action outside of the selfdual point

- Doing it, we obtained "structure constants" outside of the selfdual point !!! and when we replace them in the action we obtained

  the effective action of string outside of the self dual point

  Where all fields have mass terms with the exactly value of the mass. Moreover, all coupling are exactly like they should be by string theory

$$
\begin{aligned}
S_{eff} &= \frac{1}{2\kappa_d^2}\int d^dx\sqrt{g}e^{-2\varphi}\Big[\mathcal{R}+4\partial^\mu\varphi\partial_\mu\varphi-\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \\
&\qquad -\frac{1}{8}\mathcal{H}_{IJ}F^{I\mu\nu}F^J_{\mu\nu}+\frac{1}{8}(D_\mu\mathcal{H})_{IJ}(D^\mu\mathcal{H})^{IJ} \\
&\qquad -\frac{1}{12}f_{IJ}{}^K f_{LM}{}^N\left(\mathcal{H}^{IL}\mathcal{H}^{JM}\mathcal{H}_{KN}-3\,\mathcal{H}^{IL}\eta^{JM}\eta_{KN}+2\,\eta^{IL}\eta^{JM}\eta_{KN}\right)-\Lambda\Big]
\end{aligned}
$$

$$
\begin{aligned}
S_{R\neq\tilde{R}} &= \int d^dx\sqrt{g}e^{-2\varphi}\left[\frac{1}{2\kappa_d^2}(\mathcal{R}+4(\partial_\mu\varphi)^2-\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho})\right.\\
&- \frac{1}{8}F^3_{\mu\nu}F^{\mu\nu 3}-\frac{1}{8}\bar{F}^3_{\mu\nu}\bar{F}^{\mu\nu 3}\\
&- \frac{1}{8}F'^{+}_{\mu\nu}F'^{\mu\nu -}-\frac{1}{4}m_-^2A'_\mu A'_\nu g^{\mu\nu}-\frac{1}{8}\bar{F}'^{+}_{\mu\nu}\bar{F}'^{\mu\nu -}-\frac{1}{4}m_-^2\bar{A}'_\mu\bar{A}'_\nu g^{\mu\nu}\\
&- \partial_\mu M_{33}\partial^\mu M_{33}-\frac{1}{2}D_\mu M_{\pm\pm}D^\mu M_{\mp\mp}-\frac{1}{2}D_\mu M_{\pm\mp}D^\mu M_{\mp\pm}\\
&+ ig_d\frac{\sqrt{\alpha'}m_+}{2}A'^{+\mu}A'^{-\nu}F^3_{\mu\nu}+ig_d\frac{\sqrt{\alpha'}m_-}{2}A'^{+\mu}A'^{-\nu}\bar{F}^3_{\mu\nu}\\
&+ ig_d\frac{\sqrt{\alpha'}m_+}{2}\bar{A}'^{+\mu}\bar{A}'^{-\nu}\bar{F}^3_{\mu\nu}+ig_d\frac{\sqrt{\alpha'}m_-}{2}\bar{A}'^{+\mu}\bar{A}'^{-\nu}F^3_{\mu\nu}\\
&+ g_d\,\frac{m_+\sqrt{\alpha'}}{2}A'^{+\mu}A'^{-}_{\mu}M_{33}m_-+g_d\,\frac{m_+\sqrt{\alpha'}}{2}\bar{A}'^{+\mu}\bar{A}'^{-}_{\mu}M_{33}m_-\\
&- g_d\sqrt{\alpha'}\frac{1}{8}F'^{\pm}_{\mu\nu}\bar{F}'^{\pm\mu\nu}M_{\mp\mp}-g_d\sqrt{\alpha'}\frac{1}{8}F'^{\pm}_{\mu\nu}\bar{F}'^{\mp\mu\nu}M_{\mp\pm}-g_d\sqrt{\alpha'}\frac{1}{2}F^3_{\mu\nu}\bar{F}^{3\mu\nu}M_{33}\\
&- \frac{4g_d}{\sqrt{\alpha'}}M_{+-}M_{-+}M_{33}(\frac{\sqrt{\alpha'}}{\tilde{R}})^2+\frac{4g_d}{\sqrt{\alpha'}}M_{++}M_{--}M_{33}(\frac{\sqrt{\alpha'}}{R})^2\\
&+ \left.\frac{4m_-}{\tilde{R}}\frac{1}{2}M_{+-}M_{-+}-\frac{4m_-}{R}\frac{1}{2}M_{++}M_{--}\right]
\end{aligned}
$$

$$
\begin{aligned}
F'^{\pm}_{\mu\nu} &= 2\partial_{[\mu}A'^{\pm}_{\nu]} \mp ig_d\frac{\sqrt{\alpha'}m_+}{2}2A^3_{[\mu}A'^{\pm}_{\nu]} \mp ig_d\frac{\sqrt{\alpha'}m_-}{2}2\bar{A}^3_{[\mu}A'^{\pm}_{\nu]} \\
\bar{F}'^{\pm}_{\mu\nu} &= 2\partial_{[\mu}\bar{A}'^{\pm}_{\nu]} \mp ig_d\frac{\sqrt{\alpha'}m_+}{2}2\bar{A}^3_{[\mu}\bar{A}'^{\pm}_{\nu]} \mp ig_d\frac{\sqrt{\alpha'}m_-}{2}2A^3_{[\mu}\bar{A}'^{\pm}_{\nu]} \\
F^3_{\mu\nu} &= 2\partial_{[\mu}A^3_{\nu]}
\end{aligned}
$$

$$
D_\mu M_{\pm\pm} = [\partial_\mu + i(\pm)g_d\frac{\sqrt{\alpha'}}{R}A^3_\mu + i(\pm)g_d\frac{\sqrt{\alpha'}}{R}\bar{A}^3_\mu]M_{\pm\pm}
$$

$$
D_\mu M_{\pm\mp} = [\partial_\mu + i(\pm)g_d\frac{\sqrt{\alpha'}}{\tilde{R}}A^3_\mu - i(\pm)g_d\frac{\sqrt{\alpha'}}{\tilde{R}}\bar{A}^3_\mu]M_{\pm\mp}
$$

That means we have a higg mechanism from the DFT perspective !

Can it be generalized to another torus compactifications ?

Namely, can we enhanced the tangent space in order to describe enhancement points of string theory in a generic way ?

Yes

$$\frac{O(d+n,d+n)}{O(d+n)\times O(d+n)}$$

Where "n" is the dimention of the group of the enhacement

But it would imply give an ansatz for vielbein for all enhancement points in string theory......unless.....

Go back to the action

$$
\begin{aligned}
S_{eff} \;=\; & \frac{1}{2\kappa_d^2}\int d^dx\sqrt{g}e^{-2\varphi}\Big[\mathcal{R}+4\partial^\mu\varphi\partial_\mu\varphi-\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \\
& -\frac{1}{8}\mathcal{H}_{IJ}F^{I\mu\nu}F^J_{\mu\nu}+\frac{1}{8}(D_\mu\mathcal{H})_{IJ}(D^\mu\mathcal{H})^{IJ} \\
& -\frac{1}{12}f_{IJ}{}^K f_{LM}{}^N\left(\mathcal{H}^{IL}\mathcal{H}^{JM}\mathcal{H}_{KN}-3\,\mathcal{H}^{IL}\eta^{JM}\eta_{KN}+2\,\eta^{IL}\eta^{JM}\eta_{KN}\right)-\Lambda\Big]
\end{aligned}
$$

And remember: Vielbeins were only used to give structure constants.

Now, let's suppose those vielbein exist. If we find how to express the structure constants in general way, most of the problem would be solved

Go back to string theory

And remember that structure constants on the selfdual point can be read just by computing scattering amplitudes of the massless vectors

Circle example

$$
\begin{aligned}
< V_L^i V_L^j V_L^k > \;&=\; \pi g_c \frac{i}{\sqrt{\alpha'}} \epsilon^{ijk} \left[(\epsilon_3^k \cdot K_1)(\epsilon_1^i \cdot \epsilon_2^j) - (\epsilon_2^j \cdot K_1)(\epsilon_1^i \cdot \epsilon_3^k) \right. \\
&+\; \left. (\epsilon_1^i \cdot K_2)(\epsilon_3^k \cdot \epsilon_2^j) \right]
\end{aligned}
$$

Same for right vectors and no mix between Left/Right

Actually, the only matters to know structure constant is calculate the expectation value of worldsheet currents presents in vertex operators

Left vector vertex:

$$V^{\pm,3}(z,\bar{z}) = i\frac{g_c'}{\alpha'^{1/2}}\epsilon_\mu^{\pm,3} : J^{\pm,3}(z)\bar{\partial}X^\mu e^{iK\cdot X} :$$

$$J^{\mp}(z) = e^{\mp\frac{2i}{\sqrt{\alpha'}}y^L(z)},\ J^3(z) = \partial_z Y(z)$$

Join left and right currents in one double current $\longrightarrow$ $J_I=(\bar{J}_a\text{-}, J_a)$

$$\langle J_I J_J J_K \rangle = f_{IJK}$$

$$f_{IJK} \equiv \eta_{KL} f_{IJ}{}^{L} = f_{[IJK]}\,, \qquad f_{[IJ}{}^{L} f_{K]L}{}^{R} = 0\,.$$

$$[E_\alpha, E_{-\alpha}] = k_L^{(\alpha)I} H_I + k_R^{(\alpha)\hat{I}} \hat{H}_{\hat{I}} \qquad [\hat{E}_{\hat{\alpha}}, \hat{E}_{-\hat{\alpha}}] = k_L^{(\hat{\alpha})I} H_I + k_R^{(\hat{\alpha})I} \hat{H}_I$$

$$[H_I, E_\alpha] = k_L^{(\alpha)I} E_\alpha \qquad [\hat{H}_{\hat{I}}, \hat{E}_{\hat{\alpha}}] = k_R^{(\hat{\alpha})\hat{I}} \hat{E}_{\hat{\alpha}}$$

$$[H_I, \hat{E}_{\hat{\alpha}}] = k_L^{(\hat{\alpha})I} \hat{E}_{\hat{\alpha}} \qquad [\hat{H}_I, E_\alpha] = k_R^{(\alpha)I} E_\alpha$$

And putting those fluxes in the action one recover exactly the action outside the selfdual point !!!

Conclusions

- It was able to include states with winding number into a DFT formalism violating the strong constrain and, at the same time, given a DFT description of the higgsing mechanism (at least on the circle case)
- The higgsing mechanism was easily extended to more general compactification and more complicated enhanced groups (despite of not know the vielbein ansatz always)
- In a more recent work, we showed the same for the Heterotic string theory, namely, a DFT description of the higgsing mechanism present at selfdual points,

Open questions

- The DFT description of the higgs mechanism is near each enhancement point, you can not move continuously from one enhancement point to another (because each group has a different dimension). So….

Is there a way to describe enhancement/breaking taking in to account all posible enhacement for a given number of compactify dimensions ?