Supersymmetric brane field theories and non-linear instantons on curved backgrounds

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based on
arXiv: 1509.02926
arXiv: 1707.07002 with R. Minasian and D. Prins

Overview

1) SUSY field theories on curved backgrounds

2) Brane worldvolume theories in flux backgrounds

3) Worldvolume flux and nonlinear instantons

SUSY field theories on curved spaces

- Supersymmetric localization allows to compute partition functions on compact spaces from supersymmetric instanton solutions.

Witten ’83; Witten ’88; Pestun ’08; …

- These partition functions contain specific information of field theory, Kähler potential for *N=2* SCFTs:

$$Z_{S^4} \sim e^{K/12}$$

Benini, Cremonesi ‘12; Jockers, Kumar, Lapan, Morrison, Romo ’12; Gerchkovitz, Gomis, Komargodski ’14, …

- Partition functions of the theory on different curved spaces might yield to a number of refined observables

SUSY field theories on curved spaces

- By coupling the SUSY field theory to a non-dynamical ('*off-shell*') supergravity multiplet it can be defined on a variety of curved (compact) spaces. Festuccia, Seiberg '11, …
- Non-dynamical supergravity effectively means the limit:

$$m_{\mathrm{Pl}} \to \infty$$

- topological twist: gauge R-symmetry Johansen '94; Witten '94

$$\nabla_m \varepsilon + A_m \cdot \varepsilon = 0$$

- Coupling to off-shell supergravity allows for background profiles for fields that usually are *auxiliary.* Festuccia, Seiberg '11, …
- New minimal supergravity: Sohnius, West '81, '82

$$\delta\Psi_m = \nabla_m \varepsilon + A_m \cdot \varepsilon + V_n \gamma^n \gamma_m \varepsilon = 0 \qquad (V = * \, dB)$$

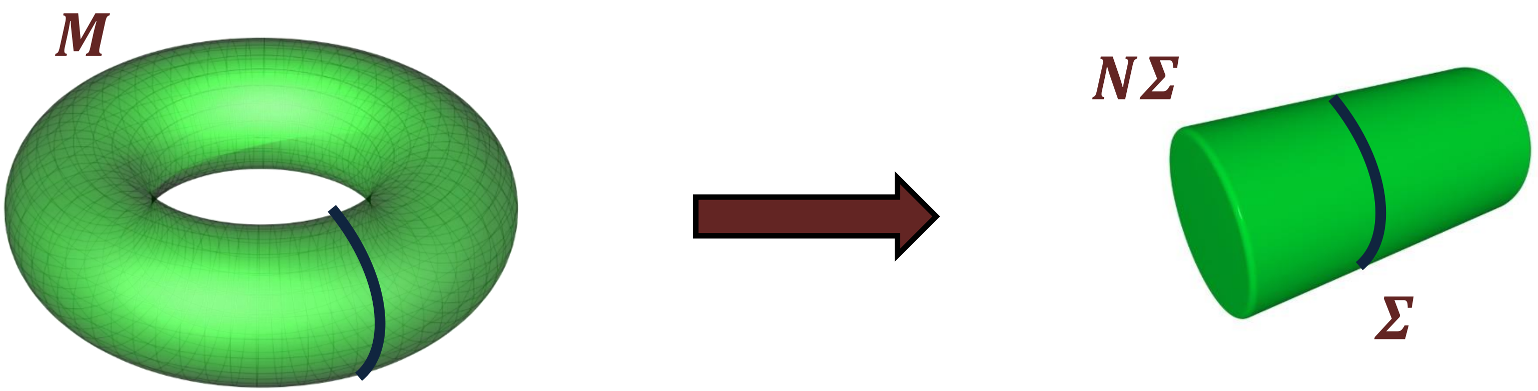
Can these supersymmetric field theories arise in string theory?

What can we learn from such a string theory embedding?

Field theory limit for branes

- Supersymmetric field theories naturally appear on the worldvolume of calibrated branes in string theory
- Gravity is decoupled by making normal directions very big:

$$m_{\mathrm{Pl}}{}^{p-1} \sim \frac{\mathrm{vol}(M)}{\mathrm{vol}(\Sigma)} m_{(10)}{}^{8} \to \infty$$



- Effectively, in this limit M just becomes the normal bundle $N\Sigma$

Calibration condition

- In the vicinity of the brane:

$$TM|_{\Sigma} \quad = \quad T\Sigma \quad \oplus \quad N\Sigma$$

- indices: $\quad M \quad \rightarrow \quad m \qquad\qquad a$

$$SO(1,9) \rightarrow SO(1,p) \times SO(9-p)$$

roughly: Lorentz group $\times$ R-symmetry group

- Calibration condition:

$$\Gamma_{\kappa}\varepsilon = \varepsilon$$

kappa symmetry

- In general Γ_{κ} is complicated function of worldvolume flux, but if that flux is zero we have

$$\Gamma_{\kappa} = \gamma_{(p+2)} P_p$$

(where P_p in sl(2,Z))

Example: Branes in Calabi-Yau manifolds

- The best-understood supersymmetric string theory backgrounds are Calabi-Yau manifolds without fluxes

$$\widehat{\boldsymbol{\nabla}}_M \hat{\boldsymbol{\varepsilon}} = \mathbf{0}$$

- It is known that D3-branes wrapping *holomorphic* four-cycles in a Calabi-Yau threefold are supersymmetric

- The resulting worldvolume theory is a topologically twisted supersymmetric field theory on a Kähler manifold

Bershadsky, Vafa, Sadov ‘95

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Let's prove it!

Example: Branes in Calabi-Yau manifolds

- Combine $\hat{\nabla}_m \varepsilon = 0$ and $\gamma_{(p+2)} P_p \varepsilon = \varepsilon$

$$[\hat{\nabla}_m, \gamma_{(p+2)}]\varepsilon = 0 \quad \Rightarrow \quad (\omega_m)_{na} = 0$$

Thus connection $\hat{\nabla}_m$ is block-diagonal at brane

$$\{\hat{\nabla}_m, \gamma_{(p+2)}\}\varepsilon = 0 \quad \Rightarrow \quad \nabla_m \varepsilon + A_m \cdot \varepsilon = 0$$

with topological twist $A_m = (\omega_m)_{ab} \hat{\gamma}^{ab}$

- **Note:** $\hat{\nabla}_a \varepsilon = 0$ determines embedding of the brane

Where do the auxiliary fields come from?

Branes in flux backgrounds

- In general (type IIB) flux backgrounds the supersymmetry condition is much more involved:

$$\delta\Psi_M = \hat{\nabla}_M \varepsilon + \not{\hat{H}}_M P \varepsilon + e^{\Phi} \sum_n \not{\hat{F}}_{(2n+1)} \Gamma_M P_n \varepsilon$$

$$\delta\lambda = (\partial_M \Phi)\Gamma^M \varepsilon + \not{\hat{H}} P \varepsilon + e^{\Phi} \sum_n \not{\hat{F}}_{(2n+1)} \Gamma_M P_n \varepsilon$$

where P, P_1 and $P_2 = P_0$ generate Sl(2,R).

- We can combine those equations again with the calibration condition

$$\gamma_{(p+2)} P_p \varepsilon = \varepsilon$$

Branes in flux backgrounds

The resulting equations split into three kinds:

- algebraic constraints that eliminate some of the components of the 10d fields on the worldvolume
- differential conditions on the embedding of the brane into the ambient geometry
- differential conditions that determine whether the worldvolume field theory is supersymmetric.

Branes in flux backgrounds

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Example: Supersymmetry on D3-brane

- Differential conditions on D3-brane:

$$\nabla_m \varepsilon^i + A_m{}^i{}_j \varepsilon^j + T^{ij}{}_{np} \gamma^{np} \gamma_m \varepsilon_j + \gamma_m \eta^i = 0$$

$$\frac{1}{\sqrt{\tau_2}} (\partial_m \tau) \gamma^m \varepsilon_i + E_{ij} \varepsilon^j + \epsilon_{ijkl} T^{kl}{}_{np} \gamma^{np} \varepsilon^j = 0$$

- Extra terms are coming from fluxes:

$$T^{ij}{}_{mn} = e^{\Phi} (F_{amn} - \tau H_{amn})^{(ASD)} \hat{\gamma}^{a\, ij}$$

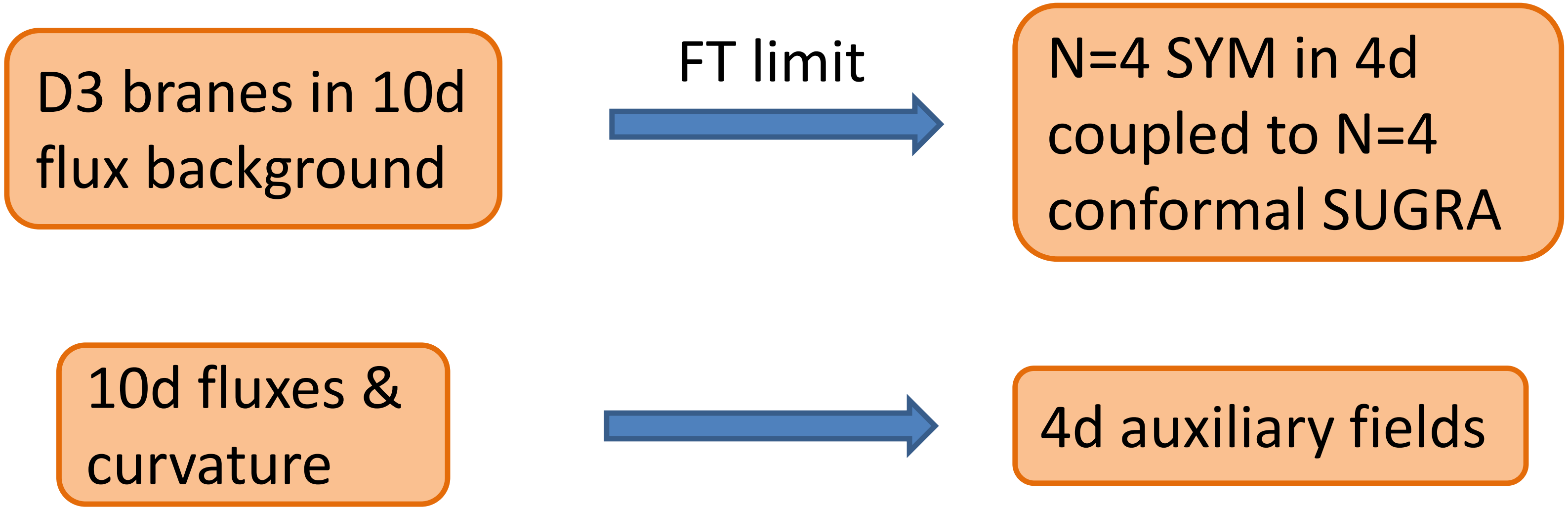
$$E_{ij} = e^{\Phi} (F_{abc} - \tau H_{abc}) \hat{\gamma}^{abc}_{ij}$$

$$\tau = c_0 + \mathrm{i} e^{-\Phi}$$

- Agrees with SUSY variations of N=4 conformal SUGRA
- The η term is special superconformal transformation.

SCFT coupled to conformal supergravity

- SUSY equations and field content match N=4 conformal SUGRA



- All other components of 10d fields **decouple** from field theory
- By introducing other sources in 10d, this can easily be generalized to theories with less supersymmetry.

General branes

- We can use same techniques in *any dimension* and with *any amount of supercharges* in string and M-theory.
- Other cases of conformal branes: M2- and M5-branes (no Lagrangian description needed)
- **Example:** couplings of 6d (2,0) tensor theory in terms of M5-branes in M-theory flux backgrounds HT '15; Maxfield, Robbins, Sethi '15
- *Non-conformal* N=4 brane systems couple to N=4 background supergravity multiplet that was not known before.
- Are there new off-shell supergravity theories to be discovered?

What about worldvolume flux?

Calibrated branes with worldvolume flux

- If theory is SUSY on curved space for $F = 0$, then both

$$\Gamma_\kappa = N(F)^{-1} \cancel{\exp(F\,P)}\, \gamma_{(p+1)} P_p$$

where

$$N(F) = (\det(\iota^*(g) + F)/\det(\iota^*(g)))^{1/2}$$

- In general the supersymmetry preserved by the brane depends on F and differs from the one for $F = 0$.

- To understand the calibration condition, we have to include both the supersymmetries ε_+ preserved by the FT vacuum and the ε_- that are broken in that vacuum.

$$\varepsilon_\pm = (1 \pm \gamma_{(p+1)} P_p)\varepsilon$$

Non-linear instantons

Minasian, Prins, HT ‘17

- For D3 branes: $\not{F}\varepsilon_+ = (1 + N(F) + \cancel{F\wedge F})\varepsilon_-$

  Gaugino variation: linear SUSY — non-linear SUSY

- This is the non-linear instanton equation already studied in

  Bagger, Galperin ’96; Seiberg, Witten ’99; Mariño, Minasian, Moore, Strominger ’99

- For $\varepsilon_- = 0$ this is the standard instanton equation.

- The non-linearity for ε_- can be understood as a spontaneous partial supersymmetry breaking and ‘explains’ the DBI action

  Bagger, Galperin ’96; Tseytlin, Rocek ’98

- In many flux backgrounds (not IIB on warped CY with ISD flux) ε_- is non-zero, for instance along baryonic branch that inter-polates between Klebanov-Strassler and Maldacena-Nuñez.

  Aharony ’01; Gubser, Herzog, Klebanov ’04; Butti, Graña, Minasian, Petrini, Zaffaroni ’04

Coupling to background supergravity

Minasian, Prins, HT '17

- Since we want to study arbitrary combinations of $\boldsymbol{\varepsilon_+}$ and $\boldsymbol{\varepsilon_-}$, both must be coupled to background SUGRA
- We can again find 4d SUSY equations for SUSY backgrounds starting from 10d SUSY variations. This rewriting is similar to the SU(8)-covariant rewriting of 11d SUGRA De Wit, Nikolai '86

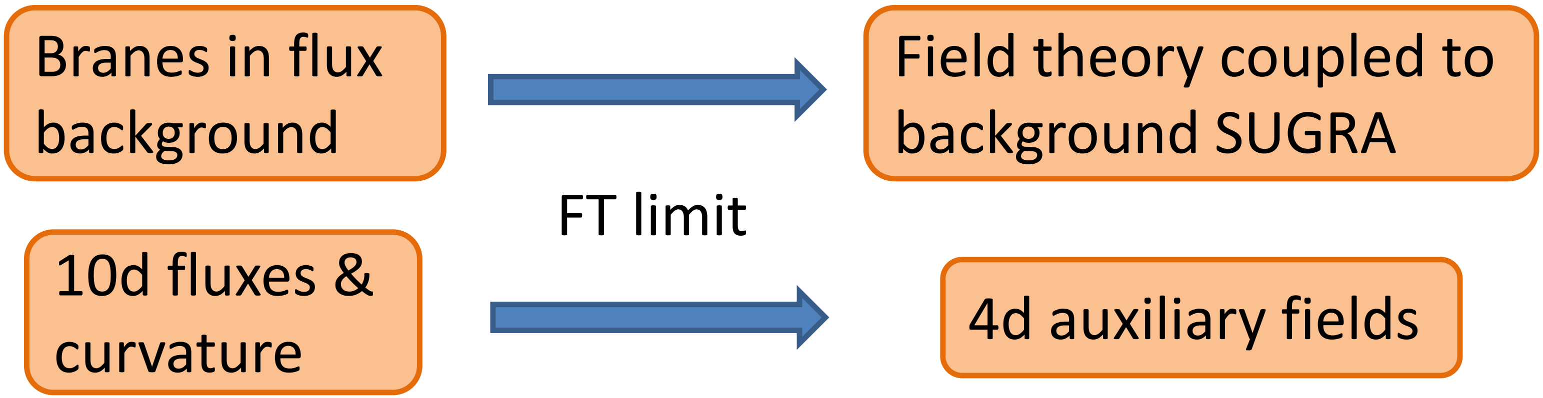
$$\boldsymbol{\nabla_m \varepsilon^i + A_m{}^i{}_j \varepsilon^j + T^{ij}{}_{np} \gamma^{np} \gamma_m \varepsilon_j + K \gamma_m \varepsilon^i = 0}$$

8 **63** **28** **36** SU(8)-reps

- Coupling to background SUGRA treats $\boldsymbol{\varepsilon_+}$ and $\boldsymbol{\varepsilon_-}$ in same way.
- Can describe SUSY FTs where SUSY is spont. broken, but non-linear SUSY instantons exist. Localization possible?
- For anti-branes: Only $\boldsymbol{\varepsilon_-}$ preserved by gravitational background

Conclusions

- For *any* dimension and number of supercharges:



- Both linear and non-linear supercharges are coupled to supergravity background in exactly the same way.
- Possible supergravity backgrounds seem to be more general than known off-shell formulations of supergravity.

Discussion

- "Landscape" of SUSY flux vacua means there should be a rich class of supersymmetric field theories on curved backgrounds
- Describe field theories of wrapped branes in this way?
- Impact of string dualities on field theories?
- Non-linear instantons might help to use localization for theories with spontaneous SUSY breaking
- Does this help to better understand anti-branes?

Thank you!

Backup slides

Match with conformal N=4 supergravity

- bosonic fields:

4d Weyl multiplet	10d fields	d.o.f.
$e^{m}{}_{\mu}$	$E^{m}{}_{\mu}$	6-1
$A_{m}{}^{i}{}_{j}$	$(\omega_m)_{ab}, F_m, F_{mabcd}$	45
$T^{ij}{}_{mn}$	H_{amn}, F_{amn}	36
E_{ij}	H_{abc}, F_{abc}	20
D^{ij}_{kl}	$\mathrm{R}_{amb}{}^{m}$	20
τ	c_0, Φ	2
		128 ✓

Match with conformal N=4 supergravity

- fermionic fields:

4d Weyl multiplet	10d fields	d.o.f.
ψ^i_m	Ψ_m	48-16
λ^i	λ	16
χ^i_{jk}	$\gamma^m \nabla_m \Psi_a$	80
		128 ✓

- Also supersymmetry variations can be matched precisely

- **Note:** There is an additional projection for $\nabla_m \Psi_a$ and $\mathrm{R}_{am\,bn}$.

  Without this projection we would get larger multiplet.

  Are there larger N=4 conformal multiplets?

M5-branes and conformal (2,0) supergravity

6d Weyl multiplet	11d fields	d.o.f.
$e^m{}_\mu$	$E^m{}_\mu$	15-1
$A_m{}^i{}_j$	$(\omega_m)_{ab}, G_{mabc}$	50
$T^{ij}{}_{mnp}$	G_{amnp}	50
D^{ij}_{kl}	R_{ambm}	14
ψ^i_m	Ψ_m	80-16
χ^i_{jk}	$\gamma^m \nabla_m \Psi_a$	64

128+128 ✓