The exceptional sigma model

Chris Blair

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VRIJE UNIVERSITEIT BRUSSEL

Fonds Wetenschappelijk Onderzoek Vlaanderen
Opening new horizons

Motivation

- Doubled sigma model describes strings in doubled geometry of DFT [Duff; Tseytlin; Hull; ...]
- How to describe strings (and branes) in exceptional geometry of EFT? [Duff,Lu; ...]

The doubled sigma model

- Idea: coordinates $(X^\mu, Y^i) \to (X^\mu, Y^M)$
- Constraint: $dY^M = \star \mathcal{M}^{MN} \eta_{NP} dY^P$
- Following [Hull], implement by gauging shift symmetry in dual directions $\to$ auxiliary worldsheet one-form V_α^M such that $V_\alpha^M \partial_M = 0$.
- V_α^M needed to respect generalised diffeomorphism symmetry: "ds^2" $= \mathcal{M}_{MN} dY^M dY^N$ not invariant $\Rightarrow$ replace $dY^M \to dY^M + V^M$ [Lee, Park]

The doubled sigma model

- Doubled sigma model:

$$S = -\frac{T_{F1}}{2}\int d^2\sigma\Big(\sqrt{-\gamma}\gamma^{\alpha\beta}\Big(g_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu + \frac{1}{2}\mathcal{M}_{MN}D_\alpha Y^M D_\beta Y^N\Big) + \epsilon^{\alpha\beta}\big(B_{\mu\nu}\partial_\alpha X^\mu\partial_\beta X^\nu + \eta_{MN}A_\mu{}^M\partial_\alpha X^\mu D_\beta Y^N + \eta_{MN}\partial_\alpha Y^M V_\beta{}^N\big)\Big)$$

  where $D_\alpha Y^M = \partial_\alpha Y^M + \partial_\alpha X^\mu A_\mu{}^M + V_\alpha^M$.
- Section: $\partial_i \neq 0$, $V_\alpha^i = 0$, integrate out $V_{\alpha i}$ $\rightarrow$ usual string + total derivative (cancel via topological term):

$$S \supset -\frac{T_{F1}}{2}\int d^2\sigma(-\epsilon^{\alpha\beta}\partial_\alpha Y^i\partial_\beta\tilde{Y}_i)$$

$$S_{top} = \frac{T_{F1}}{2}\int d^2\sigma\epsilon^{\alpha\beta}\Omega_{MN}\partial_\alpha Y^M\partial_\beta Y^N \quad , \quad \Omega_{MN} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} .$$

Generalising the doubled string

- Wess-Zumino term:

$$B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu + \eta_{MN} A_\mu{}^M \partial_\alpha X^\mu D_\beta Y^N + \eta_{MN} \partial_\alpha Y^M V_\beta{}^M$$

  involves $A_\mu^M \in R_1 = \mathbf{2d}$, $B_{\mu\nu} \in R_2 = \mathbf{1}$.

- Generic DFT/EFT tensor hierarchy $(A_\mu, B_{\mu\nu}, \dots)$.

- $\bullet : (R_1 \otimes R_1)_{sym} \to R_2$, e.g. $(A_1 \bullet A_2) = \eta_{MN} A_1^M A_2^N$.

- Conjecture Wess-Zumino term in EFT:

$$q \cdot (B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu + \partial_\alpha X^\mu A_\mu \bullet D_\beta Y + \partial_\alpha Y \bullet V_\beta)$$

  Need charge $q \in \bar{R}_2$ such that $q \cdot B$ a scalar.

The exceptional sigma model

- Can always write $B \in R_2$ as $B_{\mu\nu}$ as $B_{\mu\nu}^{MN} = B_{\mu\nu}^{(MN)}$, q as q_{MN}.
  e.g. DFT $B_{\mu\nu}{}^{MN} = B_{\mu\nu}\eta^{MN}$, $q_{MN} = \frac{1}{2d}\eta_{MN} T_{F1}$
  e.g. E_6, $R_1 = \mathbf{27}$, $R_2 = \mathbf{\overline{27}}$, $B_{\mu\nu}{}^{MN} = d^{MNP} B_{\mu\nu P}$, $q_{MN} = d_{MNP} q^P$.

- $\mathrm{E}_{D(D)}$ exceptional sigma model action:

$$
\begin{aligned}
S = -\frac{1}{2}\int d^2\sigma \Big(T(\mathcal{M},q)\Big(&\sqrt{-\gamma}\gamma^{\alpha\beta} g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu \\
&+ \frac{1}{2}\sqrt{-\gamma}\gamma^{\alpha\beta}\mathcal{M}_{MN} D_\alpha Y^M D_\beta Y^N\Big) \\
&+ q_{MN}\epsilon^{\alpha\beta}\big(B_{\mu\nu}{}^{MN}\partial_\alpha X^\mu \partial_\beta X^\nu + \partial_\alpha X^\mu A_\mu{}^M D_\beta Y^N \\
&+ \partial_\alpha Y^M V_\beta^N\big)\Big)
\end{aligned}
$$

with

$$
D_\alpha Y^M = \partial_\alpha Y^M + \partial_\alpha X^\mu A_\mu{}^M + V_\alpha^M
$$

$$
T(\mathcal{M},q) = \sqrt{\frac{q_{MN} q_{PQ} \mathcal{M}^{MP}\mathcal{M}^{NQ}}{2(D-1)}}\,.
$$

Gauge invariance and constraint

- Gauge invariance under

$$\delta_\lambda B_{\mu\nu}{}^{MN} = 2D_{[\mu}\lambda_{\nu]}{}^{MN} + \ldots \quad \delta_\lambda A_\mu{}^M = Y^{MN}{}_{PQ}\partial_N\lambda_\mu{}^{PQ}$$

requires $\delta_\lambda V_\alpha^M = -Y^{MN}{}_{PQ}\partial_N\lambda_\mu{}^{PQ}\partial_\alpha X^\mu$ and constrains q_{MN}

$$q_{MN}Y^{NK}{}_{PQ}\partial_K = q_{PQ}\partial_M$$

- Equivalently $\mathcal{L}_\Lambda q = 0$, $q \otimes \partial|_{\bar{R}_3} = 0$.
- Solutions on solving the section condition:
  - M-theory sections: $q = 0 \Rightarrow$ no 11d string.
  - IIA sections: one component of q non-zero $\Rightarrow$ IIA F1.
  - IIB sections: $\mathrm{SL}(2)$ doublet of q non-zero $\Rightarrow$ IIB (m, n) string.

Reduction

- Solve section condition $\partial_i \neq 0$, $\partial_A = 0$, integrate out V_α^A and use EFT dictionaries $\to$ obtain 10-dimensional IIA F1 action or IIB (m, n) string action with tension

$$T_{(m,n)} = T_{F1}\sqrt{e^{-2\Phi}n^2 + (m + C_{(0)}n)^2}$$

  plus total derivative:

$$S \supset -\frac{1}{2}\int d^2\sigma \epsilon^{\alpha\beta} q_{Ai}\partial_\alpha Y^A \partial_\beta Y^i \,.$$

- Generalisation of topological term $\Rightarrow \Omega_{MN}$ with $\Omega_{Ai} = -\Omega_{iA} = q_{Ai}$

Covariance

- Also require "covariance" under generalised diffeomorphisms Λ^M

$$\bar{\delta}_\Lambda Y^M = \Lambda^M$$
$$\Rightarrow \bar{\delta}_\Lambda \mathcal{L}(X, Y, V; g, \mathcal{M}, A, B) = \mathcal{L}(X, Y, V; \delta_\Lambda g, \delta_\Lambda \mathcal{M}, \delta_\Lambda A, \delta_\Lambda B)$$

  $\Rightarrow$ *symmetry* if generalised Killing. Fixes $\bar{\delta}_\Lambda V_\alpha^M$ [Lee,Park; CB]

- Same for external diffeomorphisms $\xi^\mu \Rightarrow$ cross-cancellation between Wess-Zumino and kinetic terms. Need identities:

$$\mathcal{M}^{MP} q_{PQ} \mathcal{M}^{QK} \partial_M \otimes \partial_K = 0 \,, \quad T^2 \partial_M = q_{MN} \mathcal{M}^{NP} q_{PQ} \mathcal{M}^{QK} \partial_K \,.$$

  generalises $\mathcal{M}^{MN} \eta_{NP} \mathcal{M}^{PQ} = \eta^{MQ}$ of DFT. Note constraint:

$$D_\alpha Y^M = \frac{\gamma_{\alpha\beta}\epsilon^{\beta\gamma}}{T\sqrt{-\gamma}} \mathcal{M}^{MN} q_{NP} D_\gamma Y^P$$

Interpretation

- In extended spacetime, string charged under $B_{\mu\nu} \in R_2$, extra worldsheet scalars $\sim$ dual directions,
- In 10-dims ($\partial_A = 0$), usual 1-branes on integrating out dual coords
- In n-dims ($\partial_M = 0$), full U-duality multiplet of strings $\sim$ wrapped branes
- As a "quasi-tensionless" string: encode q as momenta of worldsheet 1-form (c.f. $\mathrm{SL}(2)$ covariant string [Cederwall, Townsend] and EFT massless particle actions [CB])

Quasi-tensionless action

- Particles (wrapped branes) in n dimensions

$$S = \int d\tau \left(-\sqrt{p_M p_N \mathcal{M}^{MN}} \sqrt{-\det g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu} + p_M \dot{X}^\mu A_\mu{}^M \right)$$

uplift to EFT massless particle [CB] c.f. "branes are waves" [Berman, Rudolph]

$$S = \int d\tau \frac{\lambda}{2} \left(g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu + \mathcal{M}_{MN} (\dot{Y}^M + V^M + \dot{X}^\mu A_\mu{}^M)(\dot{Y}^N + V^N + \dot{X}^\mu A_\mu{}^N) \right)$$

- $\mathrm{E}_{D(D)}$ string uplifts to

$$S = \int d^2\sigma \frac{1}{2} \lambda \left(\det \left(g + \frac{1}{2}\mathcal{M} \right) + 2(D-1) \mathcal{M}_{MP} \mathcal{M}_{NQ} F^{MN} F^{NQ} \right)$$

$$F^{MN} = \epsilon^{\alpha\beta} \left(\partial_\alpha Z_\beta^{MN} + W_{\alpha\beta}^{MN} + \frac{1}{2} (\mathcal{L}_{WZ})_{\alpha\beta}{}^{MN} \right)$$

p	Charge	Coordinate	Gauge field
0	p_M	Y^M	$V_\alpha{}^M$
1	q_{MN}	Z_α^{MN}	$W_{\alpha\beta}^{MN}$
⋮			

c.f. tensor hierarchy extended "mega-space" of [Aldazabal, Graña, Marqués, Rosabal]

Conclusions

- Generalised double sigma model to exceptional sigma model, including Fradkin-Tseytlin term.
- General method for constructing branes coupling to tensor hierarchy of EFT.
- Add p-forms to extended geometry c.f. [Freidel, Rudolph, Svoboda]. Generalise to membranes? e.g. $\mathrm{SL}(5)$, $q_a \in \bar{\mathbf{5}}$, $q_{[a}\partial_{bc]} = 0$, M-theory section $\partial_{i5} \neq 0$, guess Ω_{MNP} from

$$S \supset -\frac{1}{2}\int d^3\sigma \epsilon^{\alpha\beta\gamma} q_5 \epsilon_{ijkl5} \partial_\alpha Y^{ij} \partial_\beta Y^{k5} \partial_\gamma Y^{l5} \,.$$

with Y^{ij}, Y^{k5}, Y^{l5} set of Nambu triples.

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- **Thanks for listening!**