Locally non-geometric fluxes in M-theory and missing momenta

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Günaydin, Lüst, EM arXiv:1607.06474;
Lüst, EM, Syväri arXiv:1710.05919

Missing wrapping modes, missing momenta

- Geometric spaces in general do not allow all possible brane wrapping modes.
- Duality: wrapping $\longleftrightarrow$ momentum.
- Occurs in locally non-geometric backgrounds (c.f. Dieter Lüst's talk).
- Local non-geometry $\Rightarrow$ non-associativity [Blumenhagen, Plauschinn], [Lüst], [Blumenhagen, Deser, Lüst, Plauschinn, Rennecke], [Mylonas, Schupp, Szabo], ...

NS-NS string duality chain

- Toy model T^3 with H-flux [Kachru, Schulz, Tripathy, Trivedi]

$$H_{123} \xrightarrow{T_1} T^1{}_{23} \xrightarrow{T_2} Q^{12}{}_3 \xrightarrow{T_3} R^{123} .$$

- Generate globally non-geometric (Q-flux) and locally non-geometric (R-flux) spaces.
- Uplift to M-theory? 4-d model with U-dualities (3 directions).

$$T^1{}_{23} \xrightarrow{U_{234}} \text{locally non-geometric} .$$

- Dependence on dual $\tilde{x}_\alpha \longrightarrow \tilde{x}_{ij}$ coordinates.

M-theory duality chain

- Start with "geometric flux", e.g. $\mathcal{N}_3(N) \times S^1$
- Nilmanifold $\mathcal{N}_3(N) = \mathbb{R}^3 / \sim$ with

$$\left(x^1, x^2, x^3\right) \sim \left(x^1 + 1, x^2, x^3\right) \sim \left(x^1, x^2, x^3 + 1\right) \sim \left(x^1 - N x^3, x^2 + 1, x^3\right) .$$

Can choose metric

$$ds^2_{\mathcal{N}_3} = \left(dx^1 + N\, x^2\, dx^3\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 .$$

- Parallelisable $\Rightarrow$ well-defined 1-forms

$$e^{\bar{1}} = dx^1 + N x^2 dx^3 , \qquad e^{\bar{2}} = dx^2, \qquad e^{\bar{3}} = dx^3 .$$

$$de^{\bar{i}} = T^{\bar{i}}{}_{\bar{j}\bar{k}} e^{\bar{j}} \wedge e^{\bar{k}} .$$

- Geometric flux: $T^1{}_{23} = N$.

Local non-geometry in M-theory

- U-duality along x_2, x_3, x_4 gives

$$ds^2 = \left(1 + N\tilde{x}_{34}\right)^{-2/3} \left[\left(dx^2\right)^2 + \left(dx^3\right)^2 + \left(dx^4\right)^2\right] + \left(1 + N\tilde{x}_{34}\right)^{1/3} \left(dx^1\right)^2 ,$$

$$C_{(3)} = \frac{N\tilde{x}_{34}}{1 + N^2\tilde{x}_{34}} dx^2 \wedge dx^3 \wedge dx^4 .$$

- (g, C_3) patched by U-duality as $\tilde{x}_{34} \longrightarrow \tilde{x}_{34} + 1$.

$$\mathcal{M}_{MN} = \begin{pmatrix} g_{ij} + C_{ikl} C_j{}^{kl} & C_i{}^{jk} \\ C^{ij}{}_k & g^{i[k} g^{l]j} \end{pmatrix} = \begin{pmatrix} \hat{g}_{ij} & \Omega_i{}^{jk} \\ \Omega^{ij}{}_k & \hat{g}^{i[k} \hat{g}^{l]j} + \Omega^{ijm} \Omega_m{}^{kl} \end{pmatrix} .$$

- Non-geometric parameterisation:

$$d\hat{s}^2 = \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 + \left(dx^4\right)^2 , \qquad \Omega^{124} = -N\tilde{x}_{34} .$$

- Well-defined up to Ω gauge transformation: $\Omega^{ijk} \longrightarrow \Omega^{ijk} + 3\partial^{[ij}\chi^{k]}$.

Local non-geometry in M-theory

- Characterised by M-theory R-flux
- Generalised metric + generalised diffeos:

$$\delta_\xi \Omega^{ijk} = L_\xi \Omega^{ijk} - 3\partial^{[ij}\xi^{k]} \,.$$

- $R^{i,jklm} = 4\hat{\partial}^{i[j}\Omega^{klm]}$ is spacetime tensor. (c.f. [Andriot, Hohm, Larfors, Lüst, Patalong])
- Also appears in embedding tensor of 7-D gSUGRA.
- $R^{4,1234} = N$.

$$T^1{}_{23} \xrightarrow{U_{234}} R^{4,1234} \,.$$

- Beyond 4-d compactifications?

Embedding tensor and non-geometric fluxes

- Fluxes generate gaugings of lower-dim gSUGRAs.
- 4-D embedding tensor $\in \mathbf{912} \oplus \mathbf{56}$ of $\mathrm{E}_{7(7)}$.
- Under $\mathrm{GL}(7)$,

$$\begin{aligned} \mathbf{912} \oplus \mathbf{56} \longrightarrow\ & \mathbf{1}_{-14} \oplus \mathbf{35}_{-10} \oplus \overline{\mathbf{140}}_{-6} \oplus 2 \cdot \overline{\mathbf{7}}_{-6} \oplus \mathbf{224}_{-2} \oplus 2 \cdot \mathbf{21}_{-2} \\ & \oplus \mathbf{28}_{-2} \oplus \overline{\mathbf{28}}_{2} \oplus \overline{\mathbf{21}}_{2} \oplus \overline{\mathbf{21}}_{2} \oplus \overline{\mathbf{224}}_{2} \oplus 2 \cdot \mathbf{7}_{6} \oplus \mathbf{140}_{6} \\ & \oplus \overline{\mathbf{35}}_{10} \oplus \mathbf{1}_{14} \,. \end{aligned}$$

- Construct from

$$\begin{aligned} \mathbf{133} &\longrightarrow e^{i}{}_{\bar{i}} \in \mathbf{49}_0 \,, \quad \Omega^{ijk} \in \mathbf{35}_4 \,, \quad \Omega^{ijklmn} \in \overline{\mathbf{7}}_8 \,, \quad \ldots \\ \partial_M &\longrightarrow \partial_i \in \overline{\mathbf{7}}_{-6} \,, \quad \partial^{ij} \in \mathbf{21}_{-2} \,, \quad \partial_{ij} \in \overline{\mathbf{21}}_2 \,, \quad \partial^i \in \mathbf{7}_6 \,. \end{aligned}$$

Locally non-geometric fluxes in M-theory

- $\mathrm{E}_{7(7)}$ generalised metric + generalised diffeos:

$$\delta_\xi \Omega^{ijk} = L_\xi \Omega^{ijk} - 3\partial^{[ij}\xi^{k]} \,,$$
$$\delta_\xi \Omega^{ijklmn} = L_\xi \Omega^{ijklmn} - \frac{1}{10}\epsilon^{ijklmnp}\partial_{pq}\xi^q - 6\Omega^{[ijk}\partial^{lm}\xi^{n]} \,.$$

- Spacetime tensors

$$R^{i,jklm} = 4\hat{\partial}^{i[j}\Omega^{klm]} - e_{\bar{i}}{}^{[j}\epsilon^{klm]inpq}\hat{\partial}_{pq}e^{\bar{i}}{}_n \,,$$
$$R^{ij}{}_k = \hat{\partial}_{kl}\Omega^{ijl} - \frac{1}{72}\epsilon_{klmnpqr}\hat{\partial}^{ij}\Omega^{lmnpqr} + \ldots \,,$$
$$R^i = \hat{\partial}_{jk}\Omega^{ijk} - 4e_{\bar{i}}{}^j\hat{\partial}^i e^{\bar{i}}{}_j - 8e_{\bar{i}}{}^i\hat{\partial}^j e^{\bar{i}}{}_j \,,$$
$$R^{ijklmnp} = \hat{\partial}^{[i}\Omega^{jklmnp]} - 2\Omega^{[ijk}\hat{\partial}^m\Omega^{lnp]} \,,$$
$$R^{ijkl} = \frac{5}{8}\hat{\partial}^{[i}\Omega^{jkl]} + \frac{1}{2}\hat{\partial}_{pq}\Omega^{pqijkl} + \frac{1}{4}\Omega^{[ijk}\hat{\partial}_{pq}\Omega^{l]pq} \,.$$

Non-unimodular geometric flux

- Generate all R-fluxes from $\mathcal{N}_3 \times T^4$ and $\mathcal{S}_2 \times T^5$.
- $\mathcal{S}_2 = \mathbb{R}^2 / \sim$ with

$$(x_1, x_2) \sim \left(x_1 + e^{-Nx_2}, x_2\right) ,$$

- Parallelisable $\Rightarrow$ well-defined 1-forms

$$\begin{aligned} e^{\bar{1}} &= dx_1 + N x_1 dx_2 \,, \\ e^{\bar{2}} &= dx_2 \,. \\ de^{\bar{1}} &= e^{\bar{1}} \wedge e^{\bar{2}} \Rightarrow H^2_{dR}(\mathcal{S}_2) = 0 \,. \end{aligned}$$

- Non-compact x_2 direction: ∞-long cylinder.
- Geometric flux: $T^2{}_{12} = N$.

Duality chains

- Two kinds of U-duality acting in 7-d:
  - U-duality taken along three directions (U_3)
  - U-duality taken along six directions (U_6)
- Read off "Buscher rules" by acting on generalised metric.

Duality chains

$$\mathbf{912}\quad {T^1}_{23} \xrightarrow{U_{234}} R^{4,1234} \xrightarrow{U_{156}} {R^{14}}_7\,,$$

$$\mathbf{912}\quad {T^1}_{23} \xrightarrow{U_{234567}} R^{4567} \xrightarrow{U_{123}} R\,,$$

$$\mathbf{56}\quad {T^1}_{12} \xrightarrow{U_{234}} {Q_2}^{234} \xrightarrow{U_{125}} R^{5,1234} = -R^{2,1345} \xrightarrow{U_{267}} {R^{25}}_5$$

$$\xrightarrow{U_{134}} R^2 = {R^{26}}_6 = {R^{27}}_7\,.$$

Missing wrapping modes

$\mathcal{N}_3 \times T^4$ has missing wrapping modes

- e.g. $e^{\bar{2}} \wedge e^{\bar{3}} = \frac{1}{N} de^{\bar{1}} \quad (de^{\bar{i}} = T^{\bar{i}}{}_{\bar{j}\bar{k}} e^{\bar{j}} \wedge e^{\bar{k}})$,
- $\mathcal{N}_3$ closed, compact, orientable $\Rightarrow w^{23} = 0$.
- Similarly $w^{23456} = w^{23457} = w^{23567} = w^{23467} = w^{14567} = 0$.
- $w^2_{KK} = w^3_{KK} = 0$ (e.g. from self-duality of $\mathcal{N}_3 \times T^4$ under U_{124}).
- Dual to Freed-Witten anomaly in IIB (T^6 with H-flux).

$\mathcal{S}_2 \times T^5$ missing wrapping states

- x^2 non-compact.
- $w^{12} = w^{12345} = w^{12346} = \ldots = w^2_{KK} = 0$.

U-duality $\Rightarrow$ missing momentum modes!

Missing momenta

$(w^{23} = w^{23456} = w^{23457} = w^{23467} = w^{23567} = w^{14567} = w^2_{KK} = w^3_{KK} = 0)$

$T_{23}{}^1 \xrightarrow{U_{234}} R^{4,1234} \xrightarrow{U_{156}} R^{14}{}_7$

- $R^{4,1234} = N$ has $p_4 = 0$.
- $R^{14}{}_7 = N$ has $p_4 = p_1 = 0$.

Missing momenta

$(w^{23} = w^{23456} = w^{23457} = w^{23467} = w^{23567} = w^{14567} = w^2_{KK} = w^3_{KK} = 0)$

$T_{23}{}^1 \xrightarrow{U_{234}} R^{4,1234} \xrightarrow{U_{156}} R^{14}{}_7$

- $R^{4,1234} = N$ has $p_4 = 0$.
- $R^{14}{}_7 = N$ has $p_4 = p_1 = 0$.

$T_{23}{}^1 \xrightarrow{U_{234567}} R^{4567} \xrightarrow{U_{123}} R^{1234567}$

- $R^{4567} = N$ has $p_4 = p_5 = p_6 = p_7 = 0$.
- $R^{1234567} = N$ has all $p_i = 0$.

Missing momenta

$(w^{23} = w^{23456} = w^{23457} = w^{23467} = w^{23567} = w^{14567} = w^2_{KK} = w^3_{KK} = 0)$

$T_{23}{}^1 \xrightarrow{U_{234}} R^{4,1234} \xrightarrow{U_{156}} R^{14}{}_7$

- $R^{4,1234} = N$ has $p_4 = 0$.
- $R^{14}{}_7 = N$ has $p_4 = p_1 = 0$.

$T_{23}{}^1 \xrightarrow{U_{234567}} R^{4567} \xrightarrow{U_{123}} R^{1234567}$

- $R^{4567} = N$ has $p_4 = p_5 = p_6 = p_7 = 0$.
- $R^{1234567} = N$ has all $p_i = 0$.

$T_{12}{}^1 \xrightarrow{U_{234}} Q_2{}^{234} \xrightarrow{U_{125}} R^{5,1234}, R^{2,1345} \xrightarrow{U_{267}} R^{25}{}_5 \xrightarrow{U_{134}} R^2, R^{26}{}_6, R^{27}{}_7$

- $R^{5,1234} = -R^{2,1345} = N$ has $p_2 = p_5 = 0$.
- $R^{25}{}_5 = N$ has $p_2 = p_5 = 0$.
- $R^2 = R^{26}{}_6 = R^{27}{}_7 = N$ has $p_1 = p_2 = p_3 = p_4 = p_5 = 0$.

Missing momenta

- These can be summarised as:

$$
\begin{aligned}
R^{i,jklm}\, p_i &= 0\,, \\
\left(R^{ij}{}_k - 2R^{[i}\delta^{j]}_k\right) p_i &= 0\,, \\
R^{ijkl}\, p_i &= 0\,, \\
R^{ijklmnp}\, p_i &= 0\,.
\end{aligned}
$$

- In 4-d $R^{4,1234} p_4 = 0$ reduces to IIA strings as no $D0$-branes.
- Matches expectations from non-associativity $\Rightarrow$ minimal volume element $\Rightarrow$ no point particles.
- In 4-d, this leads to 7-dim phase space $\Rightarrow$ non-associativity based on imaginary octonions.
- More general consequences for non-associativity?

Outlook

- Solutions rather than toy model.
- Beyond $\mathcal{N}_3$, e.g. not parallelisable.
  Geometric flux $\Rightarrow$ 1st Chern class of $U(1)$-fibration.
- New R-branes [Bakhmatov, Berman, Kleinschmidt, Musaev, Otsuki].
- Topological result. More general derivation from EFT? Exceptional cohomology controlling allowed momentum / winding states?
- Implications for non-associative theories?
  Higher-bracket structures, L_∞?

Duality chains

- Two kinds of U-duality acting in 7-d:
  - U-duality taken along three directions (U_3)
    - M-theory $\xrightarrow{S^1 \longrightarrow 0}$ IIA $\xrightarrow{S^1 \longrightarrow 0}$ IIB
    - M-theory $\xrightarrow{T^2 \longrightarrow 0}$ IIB
    - M-theory $\xrightarrow{T^3 \longrightarrow 0}$ M-theory
  - U-duality taken along six directions (U_6)

Freed-Witten anomaly

- Dual IIB geometry: T^6 with H-flux, $H_{123} = N$.
- Freed-Witten: no wrapping states $w^{123} = w^{12345} = \ldots = 0$.
- M-theory $\longleftrightarrow$ IIB duality maps these to missing M-theory states.
- $\mathcal{N}_3 \times T^n$ is an F-theory description of the Freed-Witten anomaly.