Generalised T-dualities and Integrable deformations

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String Dualities and Geometry

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Based roughly on :

1711.0084 (Demulder; Driezen; Sevrin DCT) and

1706.05322+forthcoming (Appadu, Hollowood, Price, DCT)

Overview and Motivation

We will look at classical and quantum features of some classes of integrable sigma models

- Old problem: Which spacetimes give integrable theory?
- Rich interplay of string dualities and holography
- Examples of generalised notions of T-duality
- Triggered advances in DFT (I-modified supergravity)

Plan

- λ and η Models
- λ, η and Poisson Lie Duality
- Quantum aspects of multi-pr η Model
- Quantum aspects of multi-pr λ Model

Setting: The Principal Chiral Model

$$S = \int \langle g^{-1}\partial_+ g | g^{-1}\partial_- g\rangle \qquad\qquad g : \Sigma \to G$$

- G_L x G_R global symmetry and classically integrable

$$\mathcal{L}(z) = \frac{1}{1-z^2} J + \frac{z}{1-z^2} \star J \ , \quad d\mathcal{L} - \mathcal{L}\wedge\mathcal{L} = 0 \ , \quad T(z) = Pexp \int d\sigma \mathcal{L}_\sigma$$

- Factorised scattering S-matrix [Zamolodchikov,Zamolodchikov] and exact results for the mass gap [Neidermayer, Hasenfratz]

$$\mathbb{S}[\theta] = S_{SU(2)} \otimes S_{SU(2)}$$

Non-Abelian Dualisation

- Non-Abelian T-dual [de la Ossa, Quevedo] theory obtained via Buscher procedure
  - Gauge the G_L isometry and add lag. multiplier for flat connection
  - Integrate out dual gauge fields for sigma model

- Useful tool for generating new SUGRA solutions in context of AdS/CFT (see talk of Lozano)

- Some puzzles:— apparent non-compactness of dual geometry, loss of control in global issues, quantum aspects

λ model as a regularisation

"Sfetsos procedure" constructs a <u>new</u> integrable theory

1. Double degrees of freedom: $\kappa^2 S_{PCM}[\tilde{g}] + k S_{WZW}[g]$

2. Gauge G_L in PCM and G_{Diag} in WZW

3. Fix PCM group element to identity

4. Integrate out gauge fields $\lambda = \dfrac{k}{\kappa^2 + k}$

$$S_\lambda = k S_{WZW} + \frac{k}{2\pi}\int Tr(g^{-1}\partial_+ g \frac{1}{\lambda^{-1} + Ad_g}\partial_- g g^{-1})$$

λ model limits

- Small λ marginally relevant deformation of WZW

$$S_\lambda|_{\lambda\to 0} \approx k S_{WZW} + \frac{k}{\pi}\int \lambda J_+^a J_-^a + \mathcal{O}(\lambda^2)$$

- Large deformations in scaling limit recover Non-Abelian T-dual; Sfetsos procedure reduces to Buscher procedure

$$S_\lambda|_{\lambda\to 1} \approx \frac{1}{\pi}\int \partial_+ X^a (\delta_{ab} + f_{ab}{}^c X_c)^{-1} \partial_- X^b + \mathcal{O}(k^{-1})$$

- An integrable model for all values of λ

η model

Klimcik introduced integrable models based on "modified classical Yang-Baxter equation"

$$S_\eta = \frac{1}{2\pi t}\int_\Sigma d^2\sigma Tr\left(g^{-1}\partial_+ g, \frac{1}{1-\eta\mathcal{R}} g^{-1}\partial_- g\right)$$

$$[\mathcal{R}A, \mathcal{R}B] - \mathcal{R}([\mathcal{R}A, B] + [A, \mathcal{R}B]) - [A, B] = 0\ , \quad \forall A, B \in \mathfrak{g}$$

η model

- Simplest example is the sigma model on squashed S^3

$$S = \frac{1}{2\pi}\int_\Sigma d^2\sigma Tr\left(g^{-1}\partial_+ g g^{-1}\partial_- g\right) + C\, J^3_+\, J^3_-$$

- Integrable [Cherednik 81] but with G_R broken to U(1)

- Non-local charges recover classical version of $U_q(SU(2))$ [Kawaguchi et al 12]

$$\{Q^+_R, Q^-_R\}_{P.B.} = \frac{q^{Q^3_R} - q^{-Q^3_R}}{q - q^{-1}}\,, \quad q = \exp\left(\frac{\sqrt{C}}{1+C}\right)$$

- Same structure for arbitrary group [Delduc Magro Vicedo et al]

Deforming AdS_5xS^5

- Significant interest following discovery of η deformations for cosets and AdS_5 S^5 superstring [Delduc Magro Vicedo]
  - Conjectured q-deformation of holography?
  - TsT can also be thought of as Yang-Baxter deformation
  - Not local Weyl invariant, solves “modified” SUGRA [Arutyunov et al]
  - Relation to DFT established [Sakamoto et al; Baguet et al]

- λ deformations also for cosets PSU(2,2|4) superstring [Hollowood]
  - Conjectured to be q root-of-unity deformation [Hollowood]
  - Do give rise to genuine (albeit ugly) supergravity solutions [Driezen Sfetsos Thompson; Borsato and Wulff]
  - Squashing of “conformal-coset” (gWZW) backgrounds

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η λ connection

η and λ models are related via Poisson-Lie T-duality + analytic continuation **[Tseytlin Hoare; Siampos Sfetsos DCT; Klimcik]**

- η model broke G_R in a very special way:

$$d \star J_a = \tilde{f}^{\ bc}{}_{a} \ \star J_b \wedge \star J_c$$

- Here we find structure constants for dual algebra:

$$\mathfrak{g}_{\mathcal{R}} \qquad [A,B]_{\mathcal{R}} = [\mathcal{R}A, B] + [A, \mathcal{R}B]$$

- So structure constants pure gauge in dual algebra. Suggestive of a duality transformation?

$$\mathfrak{g} \oplus \mathfrak{g}_{\mathcal{R}} \simeq \mathfrak{g}^{\mathbb{C}}$$

- In fact we have the structure of a Drinfeld double

η λ connection

- Not isometric but still dualisable (not via Buscher) in Poisson-Lie sense [Klimcik and Severa]

- Classical canonical transformation between two non-isometric sigma models

$$S[g] = \frac{1}{2\pi t}\int d^2\sigma L_+^T (E-\Pi)^{-1} L_- \,, \quad g \in \mathcal{G}\,,$$
$$\tilde{S}[\tilde{g}] = \frac{1}{2\pi t}\int d^2\sigma \tilde{L}_+^T (E^{-1}-\tilde{\Pi})^{-1} \tilde{L}_- \,, \quad \tilde{g} \in \tilde{\mathcal{G}}\,.$$

$$a_a{}^b = \langle g^{-1} T_a g, \tilde{T}^b\rangle\,, \quad b^{ab} = \langle g^{-1}\tilde{T}^a g, \tilde{T}^b\rangle\,, \quad \Pi = b^T a$$

- η model is of this form $E = \eta^{-1} - \mathcal{R}$

- Most examples are pretty ugly…. something nice coming up

Doubled Approach

- PL pairs follow from a parent Doubled Worldsheet sigma model [[Klimcik Severa, Sfetsos, Hull & Reid-Edwards]

$$S = \int_\Sigma d^2\sigma - \mathcal{H}_{AB}\mathbb{L}^A_\sigma\mathbb{L}^B_\sigma + \eta_{AB}\mathbb{L}^A_\sigma\mathbb{L}^B_\tau + \int_{\mathcal{M}_3} f_{AB}{}^D\eta_{DC}\mathbb{L}^A\wedge\mathbb{L}^B\wedge\mathbb{L}^C$$

$$\langle T_a|T_b\rangle = \langle\tilde{T}^a|\tilde{T}^b\rangle = 0 \Rightarrow \langle T_A|T_B\rangle = \eta_{AB}$$

- PST covariantisation exists
- Duality manifest one-loop conformal invariance conditions [Avramis, Derendinger, Prezas; Sfetsos-Siampos-DT]]
- DFT interpretation later today (Hassler)
- H depends on X <u>and</u> dual directions — beyond section!

η λ connection summary

- After PL T-dualisation of the η theory one gets something…
- A further analytic continuation of some angles and parameters gives the λ model

$$\eta \to i\frac{1-\lambda}{1+\lambda}\,, \qquad t \to \frac{\pi(1+\lambda)}{k(1-\lambda)}$$

$$q = e^{\eta t} \leftrightarrow q = e^{\frac{i\pi}{k}}$$

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Yang-Baxter + WZ model

- Natural to search for more general models of this type e.g. by inclusion of a WZ term to the η model

$$\mathcal{S} = -\frac{1}{2\pi}\int \langle g^{-1}\partial_+ g, \left(\alpha\mathbb{I} + \beta\mathcal{R} + \gamma\mathcal{R}^2\right) g^{-1}\partial_- g\rangle + \frac{k}{24\pi}\int_{M_3}\langle \bar{g}^{-1}d\bar{g}, [\bar{g}^{-1}d\bar{g}, \bar{g}^{-1}d\bar{g}]\rangle$$

- locus of classical integrability [Delduc, Magro, Vicedo]

$$\beta^2 = \frac{\gamma}{\alpha}\left(\alpha^2 - \alpha\gamma - k^2\right)$$

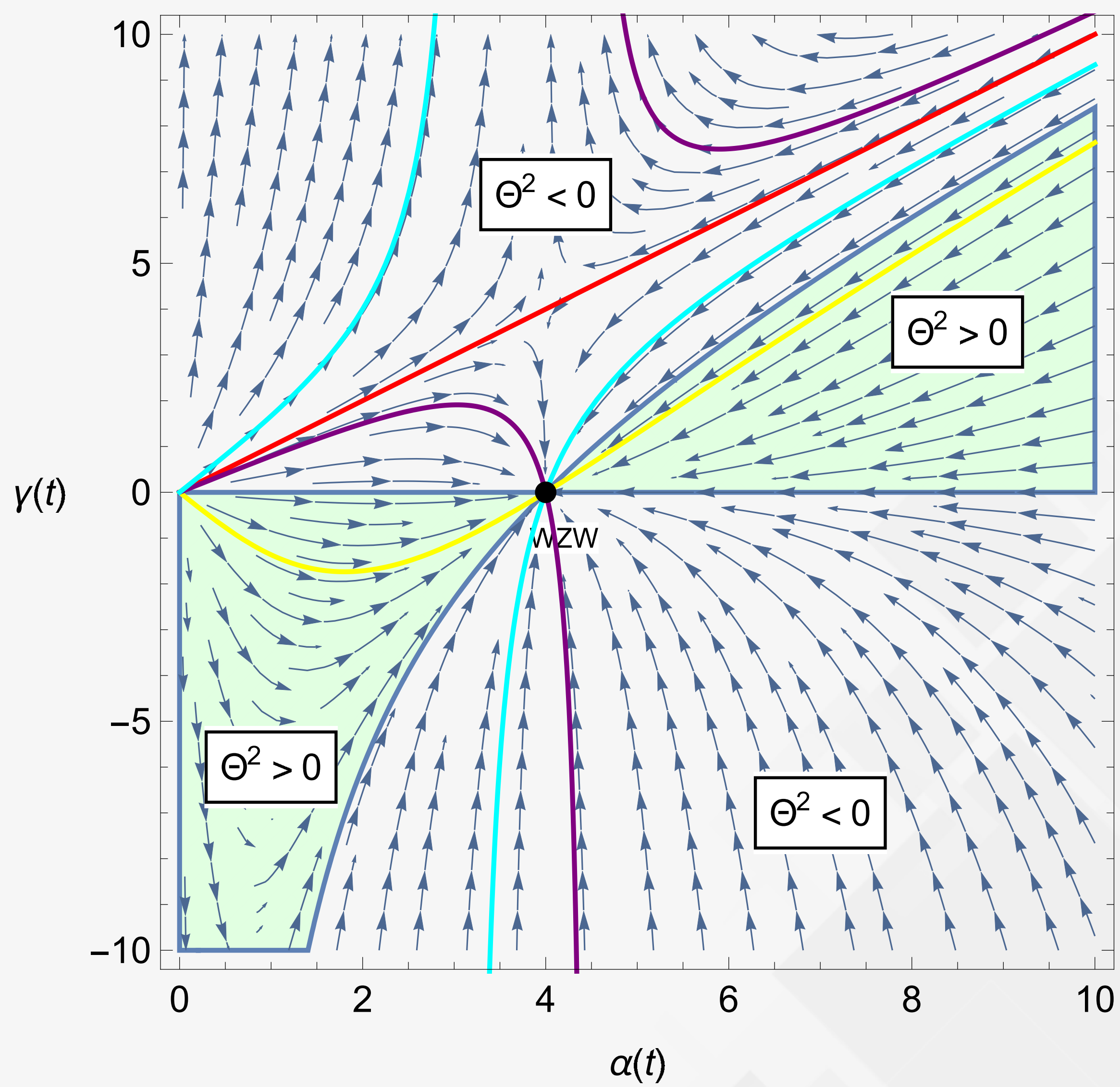
- SU(2) case exhibits q-deformed G_R symmetries [Kawaguci Orlando Yoshida]

$$q = \exp\left[\frac{8\pi\Theta}{\Theta^2 + k^2}\right] \qquad \Theta^2 = \frac{\beta^2\alpha^2}{\gamma^2}$$

YB+WZ: RG

- Study RG flow equations "geometrically"

$$\mu\frac{d}{d\mu}G_{\mu\nu} = \hat{\beta}^G_{\mu\nu} = \alpha'\left(R_{\mu\nu} - \frac{1}{4}H^2_{\mu\nu} + \nabla_{(\mu}W_{\nu)}\right) + O(\alpha')^2 ,$$
$$\mu\frac{d}{d\mu}B_{\mu\nu} = \hat{\beta}^B_{\mu\nu} = \alpha'\left(-\frac{1}{2}\nabla^\lambda H_{\lambda\mu\nu} + (\iota_W H)_{\mu\nu}\right) + O(\alpha')^2$$



- General groups integrable locus is both preserved and required to avoid generating new couplings

- Quantum group parameter q is RG invariant

unlike λ model here WZW is IR fixed point: solvable marginally irrelevant deformation

YB+WZ: Poisson - Lie

- YB+WZ can, on integrable locus, be obtained as a PL sigma model from its parent 1st order formalism [Klimcik] Same algebra g^{C} but a deformed inner product for the Drinfeld double

$$(z_1, z_2)_{\mathfrak{d}} = \frac{k^2+\Theta^2}{8\pi\Theta} \mathrm{Im} \, \langle \frac{k+i\Theta}{k-i\Theta} z_1, z_2 \rangle$$

- T-dual pairs found by picking out maximal isotropic subspaces
- Messy PL T-duality becomes a really simple radial inversion

$$\begin{aligned} \alpha \to \tilde{\alpha} &= \frac{k^2}{\alpha} \,, \\ \beta \to \tilde{\beta} &= -\beta \,, \\ \gamma \to \tilde{\gamma} &= \frac{k^2 + \alpha\gamma - \alpha^2}{\alpha} = -\frac{\beta^2}{\gamma} \,. \end{aligned}$$

Self dual point == IR conformal invariant fixed point

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λ XXZ model

- Sfetsos procedure starting not with P.C.M. but any other integrable model gives λ-type with many deformation parameters [Siampos Sfetsos Thompson]

$$S = kS_{WZW}[g] + \frac{k}{2\pi}\int Tr\left(g^{-1}\partial_+ g \frac{1}{\Lambda^{-1} + ad_g} g^{-1}\partial_- g g^{-1}\right)$$

- E.g. start with squashed S^3 we get integrable λ-XXZ model

$$\Lambda = \begin{pmatrix} \xi & & \\ & \xi & \\ & & \lambda \end{pmatrix}$$

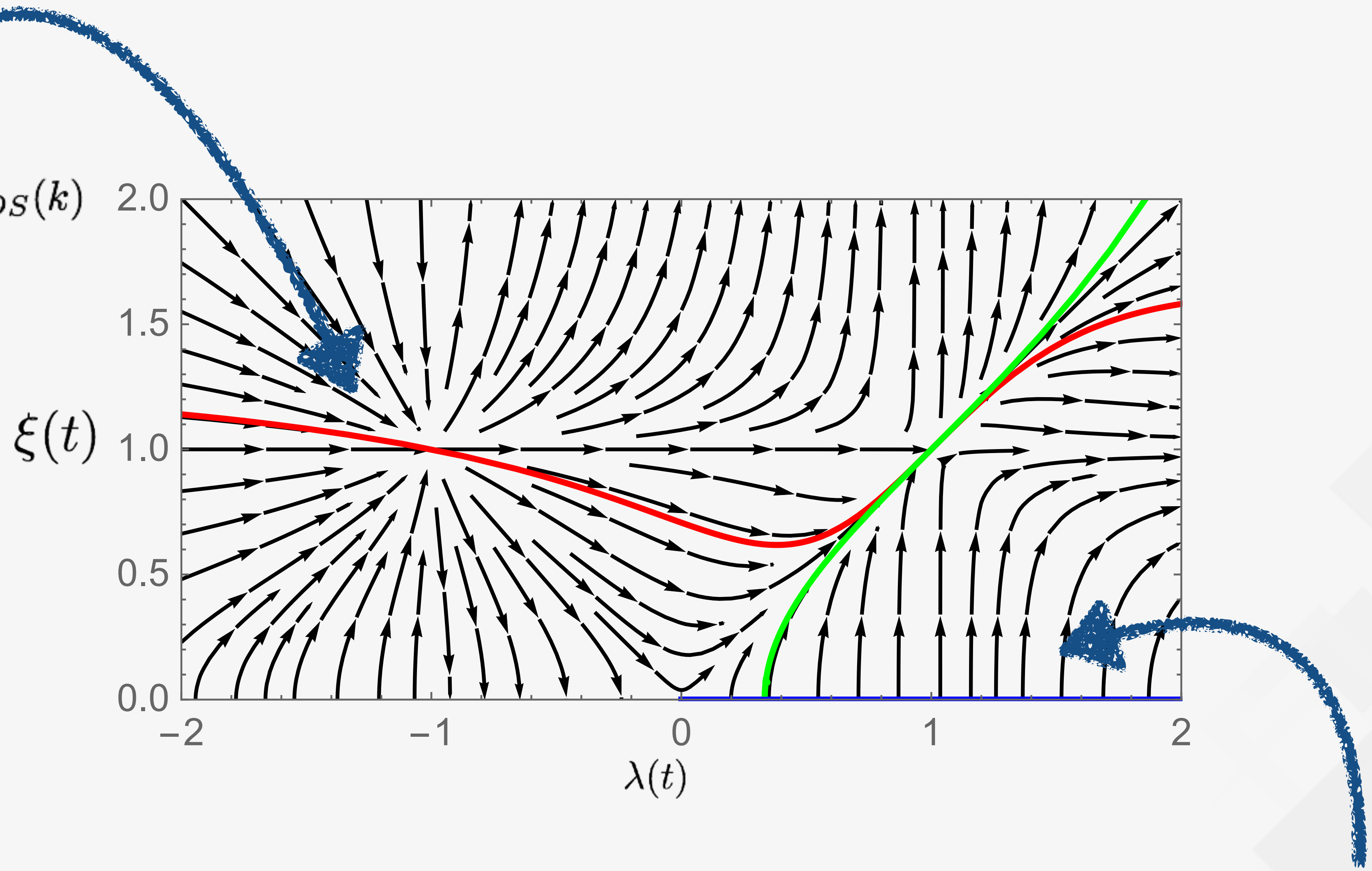
- Has a classical Uq(SU(2)) (affine — principal gradation)

λ XXZ model: RG and S-matrix

Cyclic RG?!?

$$\mathbb{S}[\theta] = S_{cyclic}(\sigma) \times S_{RSOS}(k)$$

$$q = \exp(\pi/\sigma)$$



UV safe regime

$$q = e^{-i\pi/\gamma'}$$

$$\mathbb{S}[\theta] = S_{SG}(\gamma') \times S_{RSOS}(k)$$

λ XXZ as a spin chain

Can we prove conjectured S-matrix?

- Would like to find a lattice regularisation that can be quantised
- Hard to maintain integrability due to “non-ultra-locality” i.e. Schwinger/central term in classical current algebra structure

$$\{J_+^a(\sigma), J_+^b(\sigma')\} = f^{abc} J_+^c(\sigma)\delta(\sigma - \sigma') + \frac{k}{2\pi}\delta'(\sigma - \sigma')$$

- Heuristic idea of Faddeev-Reshetikhin to take a limit

$$k \to 0 \; , \quad k/\xi, k/\lambda, \; \text{fixed}$$

- Central term drops

λ XXZ as a spin chain

- We can now place on a light-cone lattice Destri Devega
- Currents lie on null links at $x_n = n L$ and can be quantised

$$[J_n^a J_m^b] = \frac{i}{L} f^{abc} J_m^c \delta_{mn}$$

- Pick the spin s= k/2 representation at each node
- Now able to apply Integrability ideas:
  - Classical Lax Wilson Line uniquely defines quantum R-matrix obeying the Yang-Baxter
  - Spin s XXZ spin chain + impurities

$$H_{XXZ} = \sum_n S_n^1 S_{n+1}^1 + S_n^2 S_{n+1}^2 + \Delta S_n^3 S_{n+1}^3 + \dots$$

$$\Delta = \cos(\gamma)$$

λ XXZ as a spin chain

- Bethe Ansatz techniques can be used to establish the ground state — a sea of 2s-Bethe strings with density ρ

- Excitation above the ground state are holes in the sea with density ρ_H

$$\rho(z) + \rho_h(z) + K_{2s,2s} * \rho(z) = \frac{d}{dz} p_{2s}(z)$$

- Implies scattering Kernel for excitations

$$K_h = -\frac{K_{2s,2s}}{1 + K_{2s,2s}}$$

λ XXZ as a spin chain

- Integrating the scattering kernel gives the S-matrix element

$$K_h = d_z \log S_h(z) \qquad S_h(z) = \exp 2i \int \frac{d\omega}{\omega} \sin(\omega z) \left(1 - \frac{\sinh(\pi\omega/2)\tanh(\gamma\omega/2)}{2\sinh(s\gamma\omega)\sinh((\pi - 2s\gamma)\omega/2)}\right)$$

- Precisely matches conjectured S-matrix $\gamma' = \frac{\pi}{\gamma}$

- Continuum limit:

$$L \to 0 \quad \nu \to \infty \quad M = \frac{1}{L}\exp(-\frac{\pi\nu}{\gamma}) \,, \quad \text{fixed}$$

- Relativistic dispersion relation emerges for particle of mass M
- Note mass generated from cut off and dimensionless parameter
- Beta functions recovered

$$\mu \frac{d}{d\mu} = \frac{\gamma}{\pi} \quad \mu = L^{-1}$$

- *no continuum limit in cyclic regime*

Conclusion

- η and λ provide paradigms for the interplay of generalised duality and integrability
  - Interesting potential application in holography
- Simple example where Poisson-Lie is really Buscher rule like
- Can harness integrability to probe quantum aspects…
  - …Can we harness this to learn more about duality?