Non-Abelian T-duality in AdS/CFT: The CFT side

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1. Introduction & motivation

Non-Abelian T-duality (NATD) has proved to be very useful as a solution generating technique in AdS/CFT

Many interesting AdS backgrounds (some of which evading existing classifications) have been constructed

What NATD does to the CFT remains less understood

Examples so far show that NATD may change the CFT dual to the AdS backgrounds in which it is applied

This may happen because, contrary to its Abelian counterpart, NATD has not been proved as a String Theory symmetry

In this talk we will discuss different examples for which the CFT realization of NATD has been studied

In all of them the NATD background will be associated to QFT living on (Dp,NS5) Hanany-Witten brane set-ups

These examples have been studied in:

- Y.L., Carlos Núñez, 1603.04440
- Y.L., Niall Macpherson, Jesús Montero, Carlos Núñez, 1609.09061
- Georgios Itsios, Y.L., Jesús Montero, Carlos Núñez, 1705.09661

2. Basics of NATD

Using the string sigma-model Rocek and Verlinde proved that Abelian T-duality is a symmetry to all orders in g_s and α'

(Buscher'88; Rocek, Verlinde'92)

The extension to arbitrary wordsheets determines the global properties of the dual variable:

$$\theta \in [0, 2\pi] \quad \stackrel{\text{T}}{\longrightarrow} \quad \tilde{\theta} \in [0, 2\pi]$$

In the non-Abelian case neither proof works

Variables living in a group manifold are substituted by variables living in its Lie algebra

$$g \in SU(2) \quad \stackrel{\text{NAT}}{\longrightarrow} \quad \chi \in \mathbb{R}^3$$

In the absence of global information the new variables remain non-compact

Still, NATD has been proved to be a very useful solution generating technique (Sfetsos and Thompson (2010))

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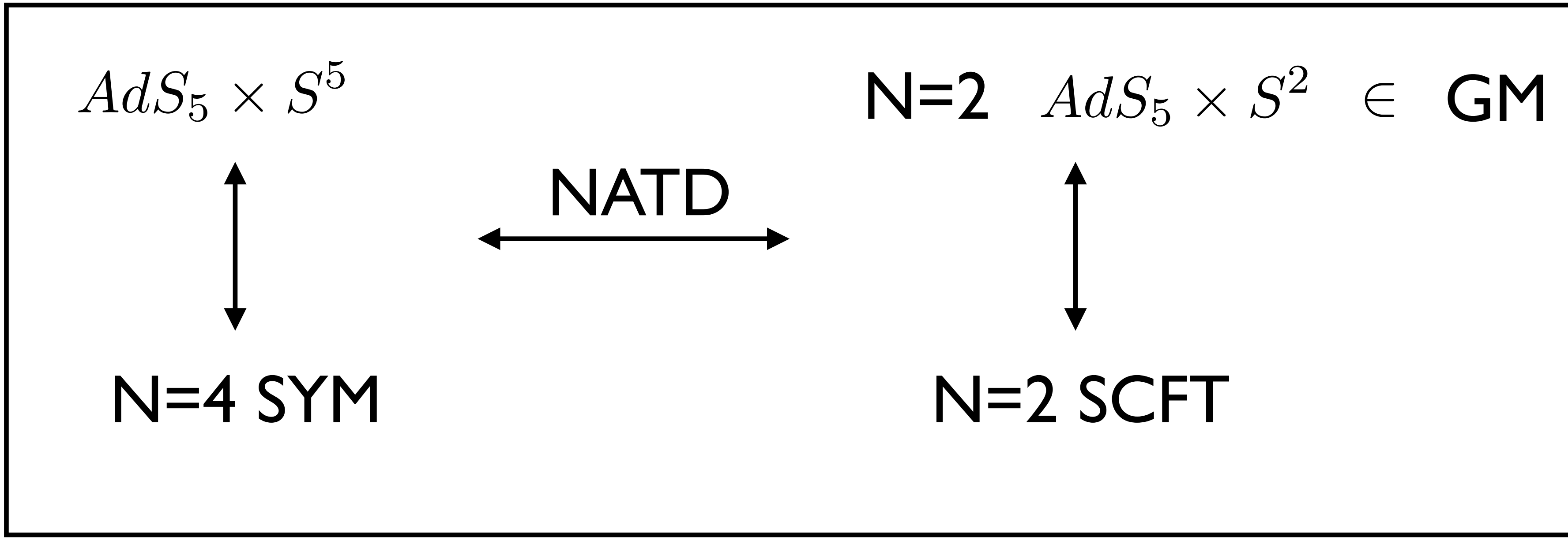
But we will have to see how we interpret the non-compact directions in the context of AdS/CFT

3. The NATD of $AdS_5 \times S^5$



(Sfetsos, Thompson'10)

(Y.L., Nunez, '16)

- Gaiotto-Maldacena geometries encode the information about the dual CFT
- Useful example to study the CFT realization of NATD

- Take the $AdS_5 \times S^5$ background

$$ds^2 = ds^2_{AdS_5} + L^2\Big(d\alpha^2 + \sin^2\alpha d\beta^2 + \cos^2\alpha ds^2(S^3)\Big)$$

$$F_5 = 8L^4 \sin\alpha \cos^3\alpha d\alpha \wedge d\beta \wedge \mathrm{Vol}(S^3)$$ + Hodge dual

- Dualize it w.r.t. one of the $SU(2)$ symmetries

In spherical coordinates adapted to the remaining $SU(2)$:

$$ds^2 = ds^2_{AdS_5} + L^2\Big(d\alpha^2 + \sin^2\alpha d\beta^2\Big) + \frac{d\rho^2}{L^2\cos^2\alpha} + \frac{L^2\cos^2\alpha\rho^2}{\rho^2 + L^4\cos^4\alpha} ds^2(S^2)$$

$$B_2 = \frac{\rho^3}{\rho^2 + L^4\cos^4\alpha}\mathrm{Vol}(S^2)\,, \qquad e^{-2\phi} = L^2\cos^2\alpha(L^4\cos^4\alpha + \rho^2)$$

$$F_2 = L^4 \sin\alpha\cos^3\alpha d\alpha \wedge d\beta, \qquad F_4 = B_2 \wedge F_2$$

- New Gaiotto-Maldacena geometry
- What about ρ ?
  - Background perfectly smooth for all $\rho \in \mathbb{R}^+$
  - No global properties inferred from the NATD
  - How do we interpret the running of ρ to infinity in the CFT?
- Singular at $\alpha = \pi/2$ where the original S^3 shrinks (due to the presence of NS5-branes)

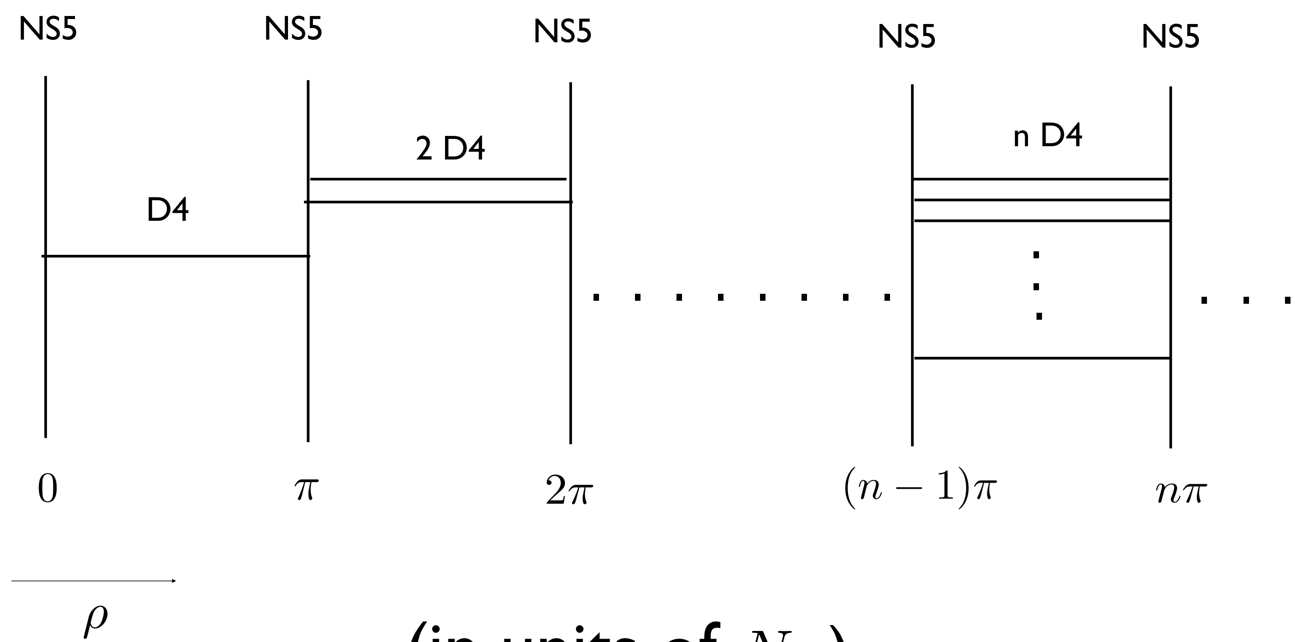
  This is the tip of a cone with S^2 boundary $\rightarrow$ We have to care about large gauge transformations
- Large gauge transformations modify the quantized charges such that $N_4 = nN_6$ in each $[(n-1)\pi, n\pi]$ interval

We have also N_5 charge, such that every time we cross a π interval one unit of NS5 charge is created

This is compatible with a D4/NS5 brane set-up:

D4: $\mathbb{R}^{1,3}, \rho$

NS5: $\mathbb{R}^{1,3}, \alpha, \beta$



(in units of N_6)

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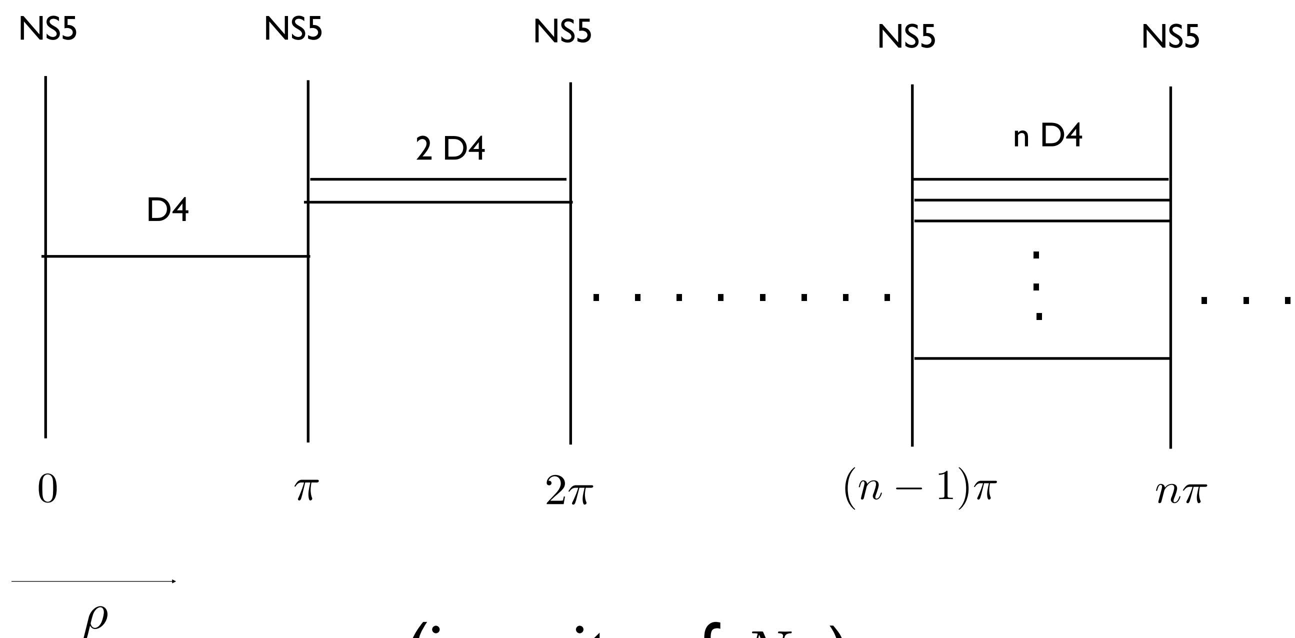
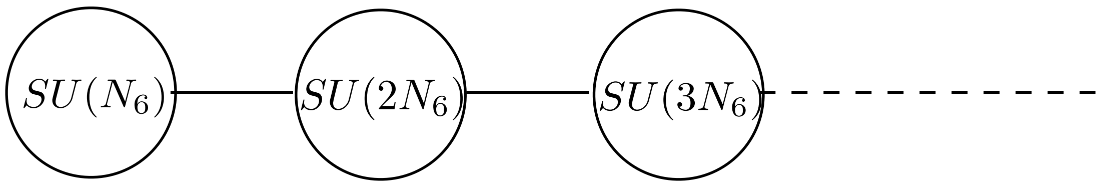


(in units of N_6)

These D4/NS5 brane set-ups realize 4d $\mathcal{N} = 2$ field theories with gauge groups connected by bifundamentals (Witten'97)

Our case corresponds to an infinite linear quiver:



which satisfies the conditions for conformal invariance

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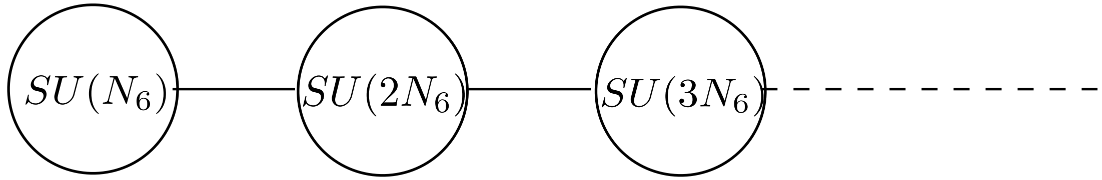


which satisfies the conditions for conformal invariance

Gaiotto-Maldacena geometries were built to study the CFTs associated to these brane set-ups (Gaiotto, Maldacena'09)

We will now see that, as a GM geometry, the dual quiver associated to the NATD solution is the same above

The NATD and GM geometries

GM geometries are described in terms of a function $V(\sigma, \eta)$ solving a Laplace eq. with a given charge density $\lambda(\eta)$

Regularity and quantization of charges impose strong constraints on the allowed form of $\lambda(\eta)$, which encodes the information of the dual CFT.

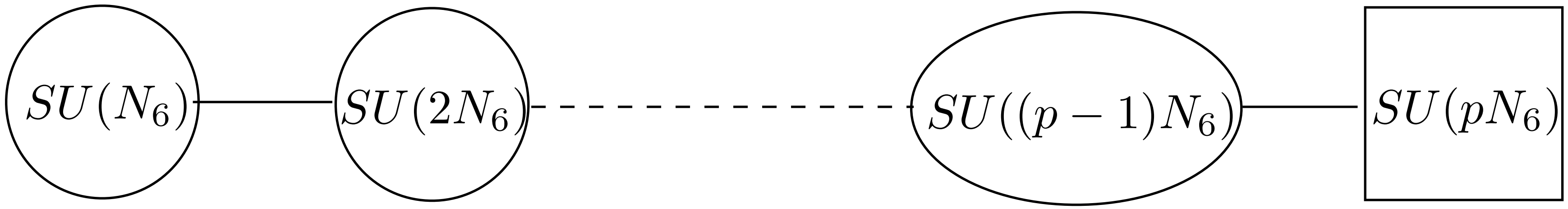
The NATD solution has $\lambda(\eta) = \eta$, $\eta \sim \rho \,,$ which corresponds to an infinite linear quiver with gauge groups of increasing rank, as predicted by the brane set-up

This brings strong evidence to the brane set-up predicted from the solution

However, the associated CFT is *infinite*.

We will see next that we can complete the quiver to produce a well-defined 4d CFT, and, using holography, complete the geometry

A natural way to complete the quiver is by adding fundamentals:



This completion reproduces correctly the value of the holographic central charge:

From the geometry:

$$c_{NATD} \sim \int_0^{\eta_*} f(\eta) d\eta = \frac{N_6^2 N_5^3}{12}$$

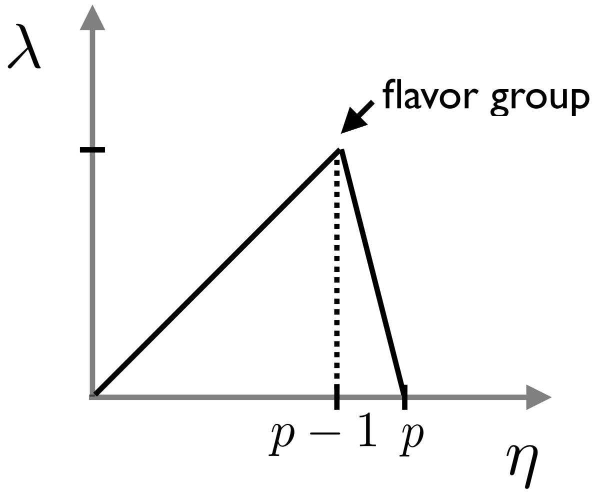
(Klebanov, Kutasov, Murugan'08)

In the field theory we can use: $c = \frac{1}{12}(2n_v + n_h)$ (Shapere, Tachikawa'08)

This gives

$$c = \frac{N_6^2 p^3}{12}\left[1 - \frac{1}{p} - \frac{2}{p^2 N_6^2} + \frac{2}{N_6^2 p^3}\right] \approx \frac{N_6^2 p^3}{12}$$

In the geometry, the completed quiver corresponds to



$$\frac{\lambda(\eta)}{N_6} = \begin{cases} \eta & 0 \leq \eta \leq p-1 \\ (1-p)\eta + (p^2 - p) & (p-1) \leq \eta \leq p \end{cases}$$

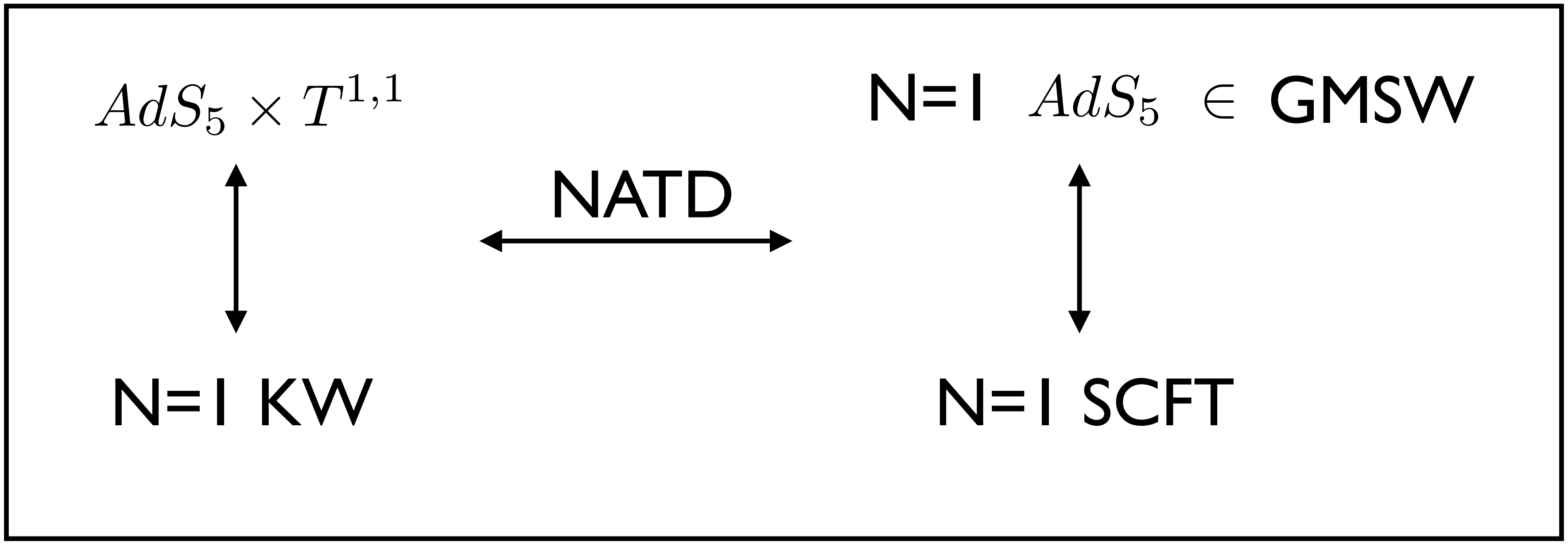
The solution to the Laplace equation that gives rise to this charge density *completes* the non-Abelian T-dual geometry, and resolves its singularity

Close to the kink the behaviour of the completed geometry is that of D6-branes

The NATD solution arises when zooming-in around the origin

This idea also works in other examples

4. The NATD of Klebanov-Witten



(Itsios, Nunez, Sfetsos, Thompson'13) (Itsios, Y.L., Montero, Nunez, '17)

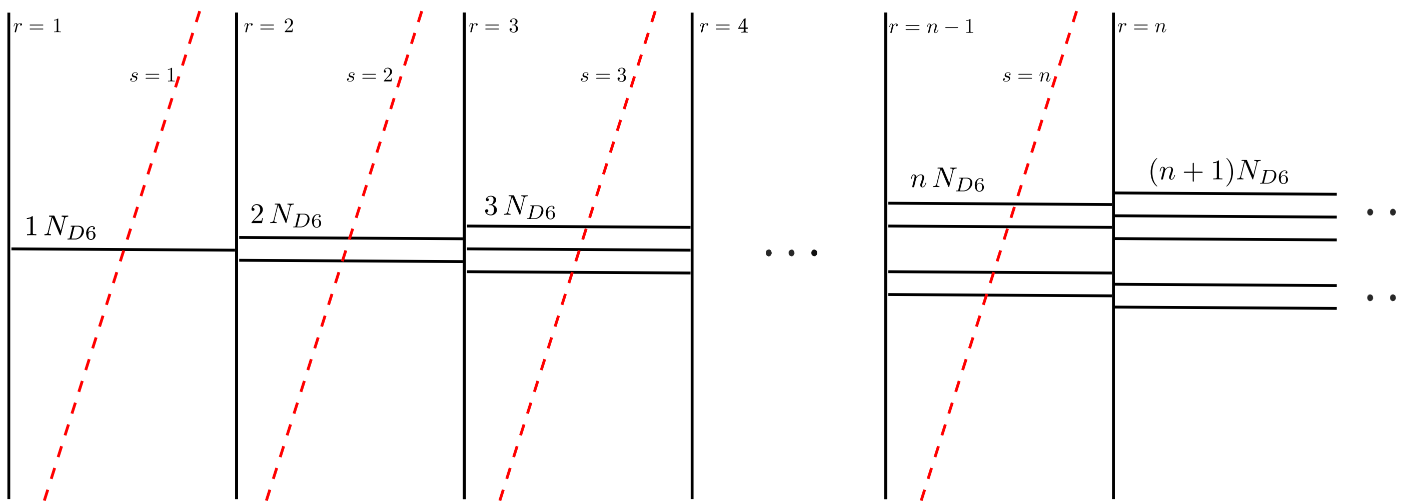
- GMSW geometries do not encode the information about the dual CFT
- Still, we can extract useful information using the relation with GM

The non-Abelian T-dual solution

IIA background with $B_2\,, F_2\,, F_4 = B_2 \wedge F_2$ fluxes

Two types of, orthogonal, NS5 and NS5' branes

Brane set-up:



Mutual rotation equivalent to a mass deformation in the CFT breaking $\mathcal{N}=2$ to $\mathcal{N}=1$

It generalizes the brane set-ups describing the linear quiver gauge theories of Bah-Bobev (Bah, Bobev'13)

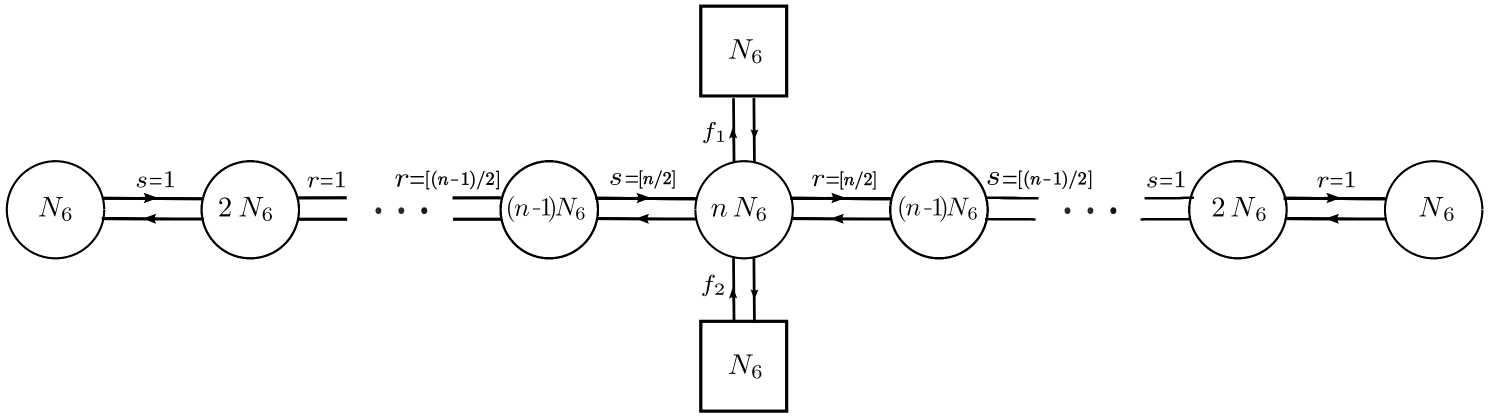
CFT dual to the NATD solution

Infinite linear quiver $\rightarrow$ Complete it, such that:

- Vanishing beta functions and R-symmetry anomalies
- Self-dual under Seiberg duality
- Mass deformation of the $\mathcal{N}=2$ quiver dual to the NATD of $AdS_5 \times S^5/\mathbb{Z}_2$:

$$
\begin{array}{ccc}
AdS_5 \times S^5/\mathbb{Z}_2 & \longrightarrow & \text{NATD of } AdS_5 \times S^5/\mathbb{Z}_2 \\
\downarrow \text{mass} & & \downarrow \text{mass} \\
AdS_5 \times T^{1,1} & \longrightarrow & \text{NATD of } AdS_5 \times T^{1,1}
\end{array}
$$

Our proposed *completed* quiver is:



A non-trivial check is that the central charges satisfy the Tachikawa-Wecht UV/IR relations:

$$c_{\mathcal{N}=1} = \frac{27}{32}\, c_{\mathcal{N}=2}$$ (Tachikawa, Wecht'09)

and that the holographic central charge is reproduced

The completed solution is not known in this case

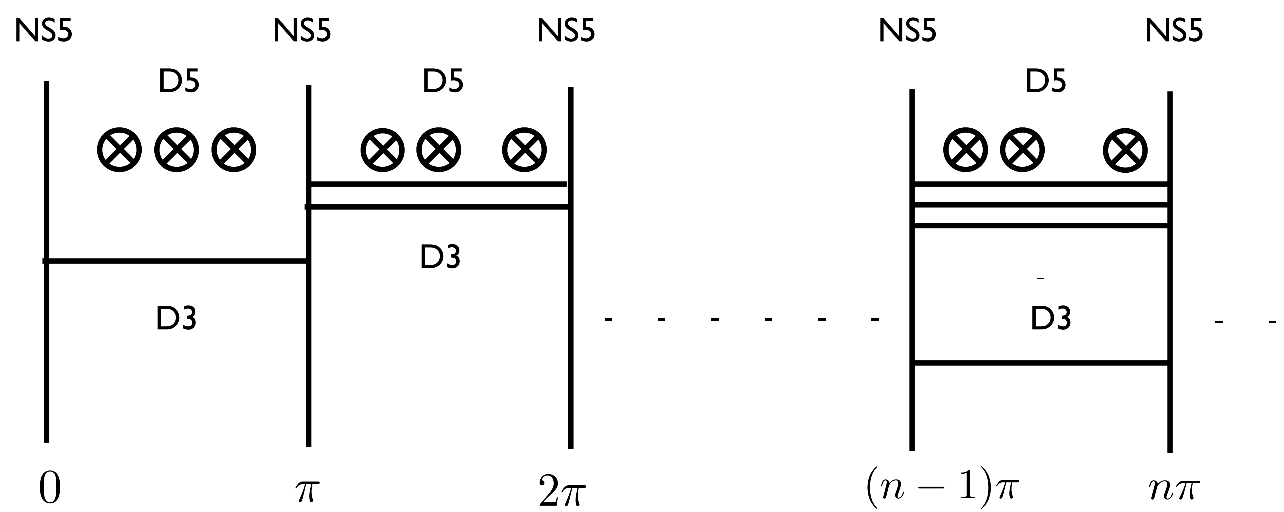
5. The $AdS_4 \times S^2 \times S^2$ example

(Y.L., Macpherson, Montero, Nunez, '16)

Non-Abelian T-duality on a reduction to IIA of $AdS_4 \times S^7/\mathbb{Z}_k$

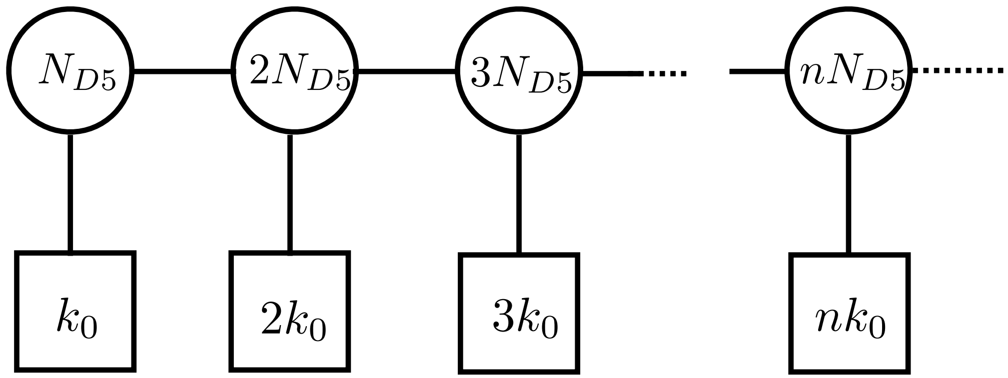
$\rightarrow$ IIB $AdS_4 \times S^2 \times S^2$ background, N=4 SUSY, in the classification of D'Hoker, Estes and Gutperle'07

Analysis of charges: (D3,NS5,D5) brane set-up:



Gaiotto and Witten'08: 3d N=4 $T_\rho^{\hat\rho}(N)$ theories

$T_{\rho}^{\hat{\rho}}(N)$ field theories flow to CFTs in the infrared if the partitions satisfy certain conditions, that are satisfied by our brane set-up



The holographic duals of these CFTs are known (Assel, Bachas, Estes and Gomis'11). They can be used to complete the geometry

This completion smoothes out the singularities and defines the geometry globally

The non-Abelian T-dual arises as a result of zooming-in in a particular region of the *completed* solution

6. Conclusions

NATD geometries dual to infinite linear quivers realized in different (Dp,NS5) Hanany-Witten brane set-ups

$\rightarrow$ Different CFTs after NATD

$\rightarrow$ Novel way of describing (Dp,NS5) CFTs holographically

The NATD geometry arises as the result of zooming-in in a particular patch (Penrose limit when NATD acts on AdS)

Other examples:

NATD of $AdS_6 \times S^4$: (D4,D8) system

(D5,NS5,D7) brane system candidate to describe 5d fixed points arising from (p,q) 5-brane webs

Candidate for $AdS_6 \times S^2 \times Y_2$ F-theory solution as in

Couzens, Lawrie, Martelli, Schafer-Nameki, Wong'17

THANKS!