# $K(E_{10})$ as an R symmetry

***String Dualities and Geometry***
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*Based on:*

*A. Kleinschmidt, HN:* `JHEP1308(2013)041;`
`Phys.Lett.B747(2015)251;1602.04116`

*A. Kleinschmidt, HN and N.K.Chidambaram:* `PRD91(2015)8,085039`

*K. Meissner, HN:* `Phys.Rev.D91(2015)065029`

*and earlier work with T. Damour, M. Henneaux and A. Kleinschmidt*

# Main Points

- **Duality symmetries more important than space-time symmetries (general covariance, supersymmetry,...)**
- $E_{10}$**: a *symmetry based* proposal for (de-)emergence of space (and time) near cosmological singularity.**
- **Main focus of this talk: incorporating fermions and 'R symmetry' $K(E_{10}) \rightarrow$ perhaps the key challenge?**
- **A basic incompatibility between SUSY and $K(E_{10})$?**
  **But: distinction between space-time bosons and fermions may become meaningless in 'pre-geometric' regime.**
- **Finally: $56 - 8 = 3 \times 16 \;\Rightarrow$ is there a role to play for $K(E_{10})$ in 'real' physics??**

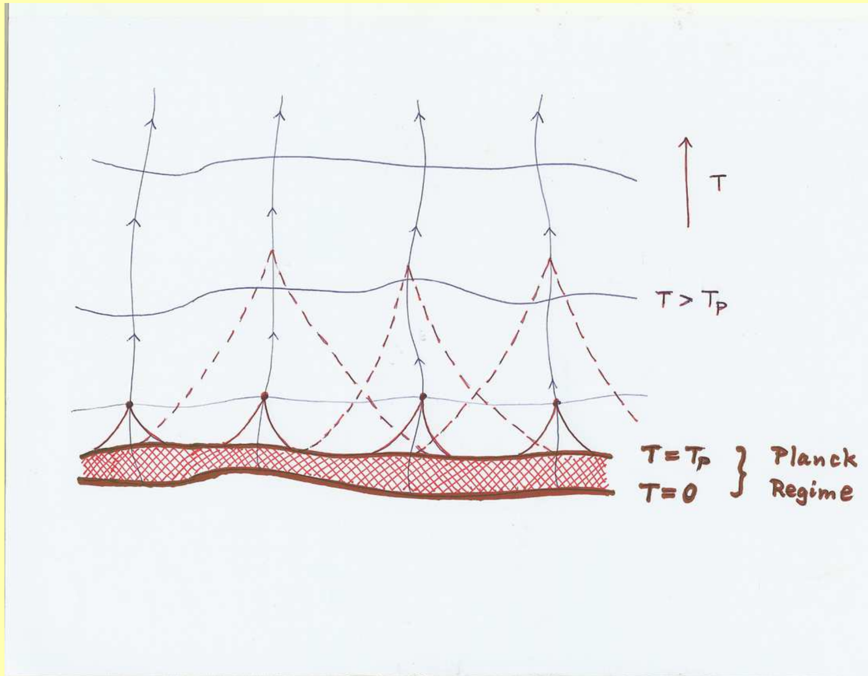
# Exceptionality and Maximal Supergravity

- **Maximal theories:** $E_{n(n)}$ **for** $D = 11 - n$ [Cremmer,Julia(1979)]
- **Coset structure** $E_{n(n)}/K(E_n)$ **for scalar manifold with** $K(E_n) \equiv$ **maximal compact subgroup of** $E_{n(n)}$

**Below** $D = 3$ **symmetries become** ***infinite-dimensional*****:**

- $E_{9(9)} \equiv E_8^{(1)}$**: a solution generating symmetry acting on** $\mathcal{M} = E_{9(9)}/K(E_9) =$ **moduli space of colliding plane wave solutions of maximal** $D = 2$ **supergravity.**
- **... suggests** $E_{10(10)}$ **for** $D = 1$**: no space, only time?**

# BKL and Spacelike Singularities

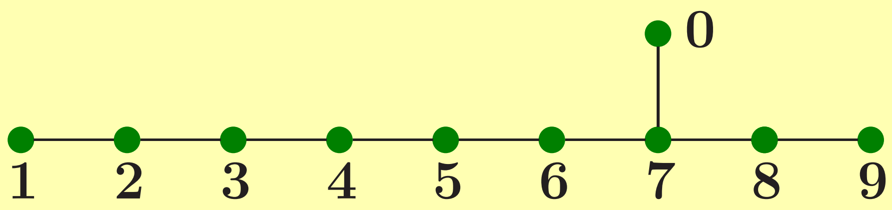



**For $T \to 0$ spatial points decouple and the system is effectively described by a continuous *superposition of one-dimensional systems* $\to$ effective dimensional reduction to $D = 1$!** [Belinski,Khalatnikov,Lifshitz (1972)]

# What is $E_{10}$?

The nice thing about it is that no one knows .... [Murat Günaydin, unpublished]

**$E_{10}$ is the 'group' associated with the Kac-Moody Lie algebra $\mathfrak{g} \equiv \mathfrak{e}_{10}$ defined via the Dynkin diagram** [e.g. Kac]




**Defined by generators $\{e_i, f_i, h_i\}$ and relations via Cartan matrix $A_{ij}$ ('Chevalley-Serre presentation')**

$$
\begin{aligned}
[h_i, h_j] &= 0, \qquad & [e_i, f_j] &= \delta_{ij} h_i, \\
[h_i, e_j] &= A_{ij} e_j, \qquad & [h_i, f_j] &= -A_{ij} f_j, \\
(\mathrm{ad}\, e_i)^{1-A_{ij}} e_j &= 0 \qquad & (\mathrm{ad}\, f_i)^{1-A_{ij}} f_j &= 0.
\end{aligned}
$$

**$\mathfrak{e}_{10}$ is the free Lie algebra generated by $\{e_i, f_i, h_i\}$ modulo these relations → infinite dimensional as $A_{ij}$ is *indefinite* → Lie algebra of *exponential growth* !**

## $SL(10)$ level decomposition of $E_{10}$

- **Decomposition w.r.t. $SL(10)$ subgroup in terms of $SL(10)$ tensors $\rightarrow$ *level expansion***

$$\alpha = \ell\alpha_0 + \sum_{j=1}^{9} m^j \alpha_j \quad \Rightarrow \quad E_{10} = \bigoplus_{\ell\in\mathbb{Z}} E_{10}^{(\ell)}$$

- **Up to $\ell \leq 3$ basic fields of $D = 11$ SUGRA together with their magnetic duals (spatial components)**

| | | |
|---|---|---|
| $\ell = 0$ | $G_{mn}$ | **Graviton** |
| $\ell = 1$ | $A_{mnp}$ | **3-form** |
| $\ell = 2$ | $A_{m_1\ldots m_6}$ | **dual 6-form** |
| $\ell = 3$ | $h_{m_1\ldots m_8\|n}$ | **dual graviton** |

- **Analysis up to level $\ell \leq 28$ yields 4 400 752 653 representations (Young tableaux) of $SL(10)$** [Fischbacher,HN:0301017]

- **Lie algebra structure (structure constants, etc.) understood only up to $\ell \leq 4$. Also: no matter where you stop it will get even more complicated beyond!**

## The $E_{10}/K(E_{10})$ $\sigma$-model

**Basic Idea:** **map evolution according to $D = 11$ SUGRA equations of motion onto null geodesic motion of a point particle on $E_{10}/K(E_{10})$ coset manifold**

$$\mathcal{V}(t) = \exp\left( h_{ab}(t)S^{ab} + \frac{1}{3!}A_{abc}(t)E^{abc} + \frac{1}{6!}A_{abcdef}(t)E^{abcdef} + \cdots \right)$$

**and then work out Cartan form $\partial_t \mathcal{V}\mathcal{V}^{-1} = Q + P$ with associated $\sigma$-model $\rightarrow$ $E_{10}/K(E_{10})$ $\sigma$-model dynamics up to $\ell \leq 3$ matches with supergravity equations of motion when truncated to first order spatial gradients.**

**Conjecture:** **information about spatial dependence gets 'spread' all over $E_{10}$ Lie algebra. More specifically:**

**Infinite tower of $\sigma$-model fields $\leftrightarrow$ SUGRA fields and their non-local descendants (duals)** ***at fixed spatial point?***

**Hint: level expansion contains complete set of** ***gradient representations*** **for all $D = 11$ fields and their duals.**

## Fermions and $K(E_{10})$

**Important point: maximal supersymmetric theories *not* based on (hypothetical) superextensions of $E_n$:**

- **There is no proper superextension of $E_n$ for any $n$.**
- **For $D \geq 3$ supergravity fermions transform in *maximal compact subgroup* $K(E_n) \subset E_{n(n)}$, e.g.**

$K(E_7) \equiv SU(8)$ **fermions $\in$ 8 and 56**

$K(E_8) \equiv Spin(16)/Z_2$ **fermions $\in$ $\mathbf{16}_v$ and $\mathbf{128}_c$**

- **The associated (double-valued) fermion representations are not 'liftable' to $E_n$ representations**
- **Expect all of this to remain true for $E_9$ and $E_{10}$.**

# What is $K(E_{10})$?

**The nice thing about it is that no one knows ....**

**For $E_{10}$, the 'maximal compact' subalgebra is defined as the fixed point algebra of the Chevalley involution**

$$\omega(e_j) = -f_j \ , \quad \omega(f_j) = -e_j \ , \quad \omega(h_j) = -h_j$$

**together with invariance property** $[\omega(x), \omega(y)] = \omega([x, y])$

$$\Rightarrow \quad E_{10} = K(E_{10}) \oplus K(E_{10})^{\perp} \ , \qquad x = \omega(x) \text{ **for** } x \in K(E_{10})$$

**This definition is analogous to the corresponding one for the finite-dimensional case, e.g.** $x = \omega(x) \in \mathfrak{so}(n) \subset \mathfrak{sl}(n)$ **for** $\omega(x) = -x^T$**, with corresponding decomposition** $\mathfrak{sl}(n) = \mathfrak{so}(n) \oplus \mathfrak{so}(n)^{\perp}$

**Consequently, $K(E_{10})$ is generated by**

$$x_i := e_i - f_i = \omega(x_i) \qquad i, j, ... = 1, ..., 10$$

with Berman-Serre relations

$$\begin{aligned} \big[x_i\,,\,x_j\big] &= 0 && \textit{if } i \textit{ and } j \textit{ are non-adjacent} \\ \big[x_i,[x_i,x_j]\big] + x_j &= 0 && \textit{if } i \textit{ and } j \textit{ are adjacent} \end{aligned}$$

**Theorem:** ***each set of*** $\{x_i\}$ ***satisfying the above relations provides a realization of*** $K(E_{10})$**.** [S.Berman(1989)]

**Involutory subalgebra** $K(E_{10}) \subset E_{10}$ **is spanned by** $\{J^r_\alpha\}$

$$J^r_\alpha \equiv E^r_\alpha - E^r_{-\alpha}\,, \quad \alpha \in \Delta_+(E_{10})\,, \quad r = 1, ..., \mathrm{mult}(\alpha)$$

**But:** $K(E_{10})$ **is $\infty$-dimensional and a very strange beast!**

- $K(E_{10})$ **has finite-dimensional (unfaithful) representations**
- $\Rightarrow K(E_{10})$ **is *not* simple ($\equiv$ has non-trivial ideals)**
- **No faithful ($\infty$-dimensional) fermionic representations known**

# Unfaithful representations

$\Longleftrightarrow$ **existence of non-trivial ideals $\mathfrak{i}_V$ in $K(E_{10})$.**

**More precisely: for a given unfaithful representation $V$ the associated ideal is**

$$\mathfrak{i}_V := \left\{ x \in K(E_{10}) \,\middle|\, x \cdot v = 0 \;\forall v \in V \right\} \subset K(E_{10})$$

**For known examples, $\mathfrak{i}_V$ has *finite* co-dimension in $K(E_{10})$**

$\Rightarrow$ $\mathfrak{i}_V^\perp \equiv K(E_{10}) \ominus \mathfrak{i}_V$ **is *not* a subalgebra of $K(E_{10})$!**

**... but rather a *distribution space*** [Kleinschmidt,Palmkvist,HN:JHEP(2007)051]

**Analysis of fermionic sector of $D\!=\!11$ SUGRA** $\Rightarrow$

**Spin-$\frac{1}{2}$ ('Dirac representation' $V_D$):** [deBuyl,Henneaux,Paulot(2005)]

$$J^{(0)}_{ab}\chi = \frac{1}{2}\Gamma_{ab}\chi, \qquad J^{(1)}_{abc}\chi = \frac{1}{2}\Gamma_{abc}\chi$$

**Spin-$\frac{3}{2}$ ('Rarita-Schwinger representation' $V_{RS}$)** [DKN,dBHP(2006)]

$$J^{(0)}_{ab}\psi_c = \frac{1}{2}\Gamma_{ab}\psi_c + 2\delta_c^{[a}\psi^{b]}\ , \qquad J^{(1)}_{abc}\psi_d = \frac{1}{2}\Gamma_{abc}\psi_d + 4\delta_d^{[a}\Gamma^b\psi^{c]} - \Gamma_d{}^{[ab}\psi^{c]}.$$

**In both examples multiple commutators generate full $K(E_{10})$ algebra:**

$$\left[J^{(1)}_{abc}\,,\,J^{(1)}_{def}\right] = J^{(2)}_{abcdef} + \delta^{[de}_{[ab}J^{(0)\,f]}_{c]} \qquad etc.$$

**Quotient algebras:**

$$K(E_{10})/\mathfrak{i}_{V_D} = \mathfrak{so}(32) \not\subset K(E_{10})$$

$$K(E_{10})/\mathfrak{i}_{V_{RS}} = \mathfrak{so}(288,32) \not\subset K(E_{10})$$

**Rarita-Schwinger equation can be reformulated as a (kind of) '$K(E_{10})$ covariant Dirac equation'.** [DKN: 0606105]

**Subalgebras of $K(E_{10})$** [cf. 1602.04116]

| | | |
|---|---|---|
| $(a)$ | $\mathfrak{so}(10)$ | **SUGRA in** $D = 11$ |
| $(b)$ | $\mathfrak{so}(2) \oplus \mathfrak{so}(16)$ | **SUGRA in** $D = 3$ |
| $(c)$ | $\mathfrak{so}(9) \oplus \mathfrak{so}(2)$ | **IIB SUGRA in** $D = 10$ |
| $(d)$ | $\mathfrak{so}(9) \oplus \mathfrak{so}(9)$ | **mIIA SUGRA in** $D = 10$ |

# Decomposing the spin-$\frac{3}{2}$ representation

$$\begin{aligned}
\mathbf{320} &\xrightarrow{a} \mathbf{288} \oplus \mathbf{32} \\
&\xrightarrow{b} \left(\frac{1}{2}, \mathbf{128}_c\right) \oplus \left(\frac{1}{2}, \mathbf{16}_v\right) \oplus \left(\frac{3}{2}, \mathbf{16}_v\right) \\
&\xrightarrow{c} \left(\mathbf{16}, \frac{3}{2}\right) \oplus \left(\mathbf{128}, \frac{1}{2}\right) \oplus \left(\mathbf{16}, \frac{1}{2}\right) \\
&\xrightarrow{d} (\mathbf{9}, \mathbf{16}) \oplus (\mathbf{16}, \mathbf{9}) \oplus (\mathbf{1}, \mathbf{16}) \oplus (\mathbf{16}, \mathbf{1})
\end{aligned}$$

**In particular: decompositions of $K(E_{10})$ w.r.t. $\mathfrak{so}(10)$, $\mathfrak{so}(9) \oplus \mathfrak{so}(2)$ and $\mathfrak{so}(9) \oplus \mathfrak{so}(9)$ yield correct fermion assignments for $D = 11$, mIIA and IIB supergravity.**

$\Rightarrow$ **$K(E_{10})$ unifies known R symmetries.** [KN: hep-th/0603205]

# $\Gamma$-matrices for $K(E_{10})$

**Wall basis for roots** $\alpha = \sum p_{\mathsf{a}} e^{\mathsf{a}}$ , $\beta = \sum q_{\mathsf{a}} e^{\mathsf{a}}$ **with simple roots**

$$\alpha_1 = (1\,-100000000)\,,\cdots,\,\alpha_9 = (000000001\,-1)\,,\,\alpha_0 = (0000000111)$$

**and** $\alpha \cdot \beta = G^{\mathsf{ab}} p_{\mathsf{a}} q_{\mathsf{b}} \;\Rightarrow\; \alpha_i \cdot \alpha_j = A_{ij}$ **(**$\equiv$ **Cartan matrix of** $E_{10}$**).**

**For any** $E_{10}$ **root** $\alpha$ **(or any element of** $E_{10}$ **root lattice) we define**

$$\Gamma(\alpha) := (\Gamma_1)^{p_1} \cdots (\Gamma_{10})^{p_{10}}$$

**Then** $\Gamma(\alpha)\Gamma(\beta) = \varepsilon_{\alpha,\beta}\,\Gamma(\alpha \pm \beta)$ **with cocycle** $\varepsilon_{\alpha,\beta} \equiv (-1)^{\sum_{\mathsf{a}<\mathsf{b}} q_{\mathsf{a}} p_{\mathsf{b}}} \;\Rightarrow$

$$\alpha \cdot \beta \in 2\mathbb{Z} \Longrightarrow \begin{cases} \big[\Gamma(\alpha), \Gamma(\beta)\big] &= 0 \\ \{\Gamma(\alpha), \Gamma(\beta)\} &= 2\epsilon_{\alpha,\beta}\Gamma(\alpha \pm \beta) \end{cases}$$

$$\alpha \cdot \beta \in 2\mathbb{Z}+1 \Longrightarrow \begin{cases} \big[\Gamma(\alpha), \Gamma(\beta)\big] &= 2\epsilon_{\alpha,\beta}\Gamma(\alpha \pm \beta) \\ \{\Gamma(\alpha), \Gamma(\beta)\} &= 0 \end{cases}$$

**Then** $x_i \to \frac{1}{2}\Gamma(\alpha_i)$ **provides a realization of Serre-like relations!**

**Multiple commutation shows that** $\frac{1}{2}\Gamma(\alpha)$ **provides realisation *for all* real roots of** $E_{10}$ **(of which there are infinitely many)!**

# Higher spin realizations of $K(E_{10})$

**$\rightarrow$ fermionic representations *beyond supergravity.***

**But first need to re-write spin-$\frac{3}{2}$ by means of crucial redefinition** [Damour,Hillmann:0906.3116]

$$\phi_A^{\mathsf{a}} \equiv \sum_{B=1}^{32} \Gamma^a_{AB} \psi^a_B \quad \textbf{(no sum on } a\textbf{!)}$$

**Re-definition breaks manifest Lorentz symmetry, but:**

$$\{\psi^a_A\,,\,\psi^b_B\}_{\mathrm{Dirac}} = \delta^{ab}\delta_{AB} - \frac{1}{9}(\Gamma^a\Gamma^b)_{AB} \quad \Rightarrow \quad \{\phi^{\mathsf{a}}_A\,,\,\phi^{\mathsf{b}}_B\} = G^{\mathsf{ab}}\delta_{AB}$$

**$\Rightarrow$ manifest $SO(1,9) =$ invariance group of mini-superspace WDW Hamiltonian with DeWitt metric $G_{\mathsf{ab}}$ instead.**

**From analysis of known $K(E_{10})$ transformation acting in RS representation we extract a *second quantised realisation* of $\hat{J}(\alpha)$ *for all real roots* $\alpha \in \Delta(E_{10})$:**

$$\hat{J}(\alpha) = \left(-\frac{1}{2}\alpha_{\mathsf{a}}\alpha_{\mathsf{b}} + \frac{1}{4}G_{\mathsf{ab}}\right)\,\phi^{\mathsf{a}}\Gamma(\alpha)\phi^{\mathsf{b}} \qquad \forall \textbf{ roots obeying } \alpha^2 = 2$$

[**NB: formula also valid for** $K(AE_3)$ [Damour,Spindel,1406.1309] ]

**There exists a *new* realization with 'spin-$\frac{5}{2}$' fermionic operators** [Kleinschmidt,HN.:1307.0413]

$$\{\phi_A^{\mathsf{ab}}\,,\,\phi_B^{\mathsf{cd}}\} = G^{\mathsf{a(c}}G^{\mathsf{d)b}}\delta_{AB} \qquad (\phi_A^{\mathsf{ab}} = \phi_A^{\mathsf{ba}})$$

**$\rightarrow$ a fermionic Fock space $\mathcal{F}$ of dimension $2^{880}$!**

**Then, Serre-like relations are satisfied on $\mathcal{F}$ with**

$$\hat{J}(\alpha) = X(\alpha)_{\mathsf{ab\,cd}}\,\phi^{\mathsf{ab}}\Gamma(\alpha)\phi^{\mathsf{cd}}$$

**and**

$$X(\alpha)_{\mathsf{ab\,cd}} = \frac{1}{2}\alpha_{\mathsf{a}}\alpha_{\mathsf{b}}\alpha_{\mathsf{c}}\alpha_{\mathsf{d}} - \alpha_{(\mathsf{a}}G_{\mathsf{b})(\mathsf{c}}\alpha_{\mathsf{d})} + \frac{1}{4}G_{\mathsf{a(c}}G_{\mathsf{d)b}}$$

**again *for all* real roots $\alpha$!**

**$\Rightarrow$ novel realisation of $K(E_{10})$ *beyond supergravity.***

# ‘ Spin-$\frac{7}{2}$ ’

**Construction also works for spin-$\frac{7}{2}$ fermions:**

$$\left\{\phi_A^{\mathtt{abc}}, \phi_{\mathtt{def}\,B}\right\} = \delta_{(\mathtt{d}}^{(\mathtt{a}}\delta_{\mathtt{e}}^{\mathtt{b}}\delta_{\mathtt{f})}^{\mathtt{c})}\delta_{AB}$$

**Then ‘Serre-like’ relations are again obeyed with**

$$\hat{J}(\alpha) = X(\alpha)_{\mathtt{abc}\,\mathtt{def}}\,\phi^{\mathtt{abc}}\Gamma(\alpha)\phi^{\mathtt{def}}$$

**and**

$$\begin{aligned} X_{\mathtt{abc}}{}^{\mathtt{def}}(\alpha) \;=\; & -\frac{1}{3}\alpha_{\mathtt{a}}\alpha_{\mathtt{b}}\alpha_{\mathtt{c}}\alpha^{\mathtt{d}}\alpha^{\mathtt{e}}\alpha^{\mathtt{f}} + \frac{3}{2}\alpha_{(\alpha}\alpha_{\mathtt{b}}\delta_{\mathtt{c})}^{(\mathtt{d}}\alpha^{\mathtt{d}}\alpha^{\mathtt{e}}\alpha^{\mathtt{f})} - \frac{3}{2}\alpha_{(\mathtt{a}}\delta_{\mathtt{b}}^{(\mathtt{d}}\delta_{\mathtt{c})}^{\mathtt{e}}\alpha^{\mathtt{f})} \\ & + \frac{1}{4}\delta_{(\mathtt{a}}^{(\mathtt{d}}\delta_{\mathtt{b}}^{\mathtt{e}}\delta_{\mathtt{c})}^{\mathtt{f})} + \frac{1}{12}\big(2-\sqrt{3}\big)\,\alpha_{(\mathtt{a}}G_{\mathtt{bc})}G^{(\mathtt{de}}\alpha^{\mathtt{f})} \\ & \frac{1}{12}\big(-1+\sqrt{3}\big)\Big(\alpha_{\mathtt{a}}\alpha_{\mathtt{b}}\alpha_{\mathtt{c}}G^{(\mathtt{de}}\alpha^{\mathtt{f})} + \alpha_{(\mathtt{a}}G_{\mathtt{bc})}\alpha^{\mathtt{d}}\alpha^{\mathtt{e}}\alpha^{\mathtt{f}}\Big) \end{aligned}$$

**Fermionic Fock space has dimension** $\dim(\mathcal{F}) = 2^{3520}$**.**

**As before,** $\hat{J}(\alpha)$ **provides a realisation *for all* real roots.**

- **'Higher spin' *not* in ordinary space-time, but in (some variant of) Wheeler-DeWitt superspace.**
- **Restriction to** $\mathrm{E}_8 \subset E_{10}$ **must yield representations of** $\mathrm{K}(E_8) \equiv Spin(16)/Z_2$ $\rightarrow$ **for new realisations we find** $\mathbf{560}_v$ **for** $s = \frac{5}{2}$ **and** $\mathbf{1920}_s$ **for** $s = \frac{7}{2}$ $\rightarrow$ **implies strong restrictions beyond: e.g. no solution for** $s = \frac{9}{2}, \frac{11}{2}, \frac{13}{2}$ **!**
- **Another strange feature: decomposition under SO(10)** $\subset K(E_{10})$**: 1760** $\rightarrow$ **1120** $\oplus 2 \times$ **288** $\oplus 2 \times$ **32.** $\phi_A^{\mathsf{ab}} \rightarrow \psi_A^a$ **and** $\psi_A^{[ab]}$ **(= RS field strength?)**
- **Suggests nested structure of higher spin realizations that penetrate farther and farther into** $K(E_{10})$**...**
  **... but systematics (if any) is not known.**

# SUSY and $K(E_{10})$: a basic incompatibility?

**SUSY Constraint from canonical analysis:**

$$\tilde{\mathcal{S}} \;=\; \Gamma^{ab}\Big[\partial_a\psi_b + \frac{1}{4}\omega_{acd}\Gamma^{cd}\psi_b + \omega_{abc}\psi_c + \frac{1}{2}\omega_{ac0}\Gamma^c\Gamma^0\psi_b\Big] + \frac{1}{4}\,F_{0abc}\Gamma^0\Gamma^{ab}\psi^c + \frac{1}{48}F_{abcd}\Gamma^{abcde}\psi_e$$

**Rewrite in terms of $E_{10}$ coset variables (up to $\ell = 3$)**

$$\mathcal{S} = \Big(P^{(0)}_{ab}\Gamma^a - P^{(0)}_{cc}\Gamma_b\Big)\,\Psi^b + \frac{1}{2}P^{(1)}_{abc}\Gamma^{ab}\Psi^c + \frac{1}{5!}P^{(2)}_{abcdef}\Gamma^{abcde}\Psi^f + \frac{1}{6!}\Big(P^{(3)}_{a|ac_1\cdots c_7}\Gamma^{c_1\cdots c_6}\Psi^{c_7} - \frac{1}{28}P^{(3)}_{a|c_1\cdots c_8}\Gamma^{c_1\cdots c_8}\Psi^a\Big)$$

**Rewrite as a partial sum over (real and null) $E_{10}$ roots:**

$$\mathcal{S}_A = \pi_{\mathsf{a}}\phi^{\mathsf{a}}_A + \sum_{\substack{\alpha^2=2\\ \ell\le 3,\alpha>0}} P_\alpha\Big(\Gamma(\alpha)\phi(\alpha)\Big)_A + \sum_{\substack{\delta^2=0\\ \ell=3}} P^r_\delta\Big(\Gamma(\delta)\phi(\epsilon^r)\Big)_A \quad (+\cdots ???)$$

**with** $\phi(v)_A \equiv v_{\mathsf{a}}\phi^{\mathsf{a}}_A \;\rightarrow$ **does not transform properly under $K(E_{10})$.**

$\rightarrow$ **need to extend sum to imaginary roots and all levels**

$\rightarrow$ **need higher-spin realisations to soak up polarisations?**

## $N = 8$ Supergravity: a strange coincidence?

$SO(8) \to SU(3) \times U(1)$ **breaking and 'family-color locking'**

$$
\begin{array}{lll}
(u\,,\,c\,,\,t)_L \ : & \mathbf{3}_c \times \bar{\mathbf{3}}_f \to \mathbf{8} \oplus \mathbf{1}\,, & Q = \frac{2}{3} - q \\
(\bar{u}\,,\,\bar{c}\,,\,\bar{t})_L \ : & \bar{\mathbf{3}}_c \times \mathbf{3}_f \to \mathbf{8} \oplus \mathbf{1}\,, & Q = -\frac{2}{3} + q \\
(d\,,\,s\,,\,b)_L \ : & \mathbf{3}_c \times \mathbf{3}_f \to \mathbf{6} \oplus \bar{\mathbf{3}}\,, & Q = -\frac{1}{3} + q \\
(\bar{d}\,,\,\bar{s}\,,\,\bar{b})_L \ : & \bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{6}} \oplus \mathbf{3}\,, & Q = \frac{1}{3} - q \\
(e^-, \mu^-, \tau^-)_L \ : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & Q = -1 + q \\
(e^+, \mu^+, \tau^+)_L \ : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & Q = 1 - q \\
(\nu_e\,,\,\nu_\mu\,,\,\nu_\tau)_L \ : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & Q = -q \\
(\bar{\nu}_e\,,\,\bar{\nu}_\mu\,,\,\bar{\nu}_\tau)_L \ : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & Q = q
\end{array}
$$

**Supergravity and SM assignments agree if we identify**

$SU(3)_{SUGRA} = \big[SU(3)_c \times SU(3)_f\big]_{\rm diag}$ **and** $q = \frac{1}{6}$ [Gell-Mann (1983)]

**Scheme is realized at** $SU(3) \times U(1)$ **stationary point.**

# Accommodating the spurion charge

[Meissner,HN: Phys.Rev.D91(2015)065029; Kleinschmidt,HN: 1504.01586]

**But need to go *beyond* $N=8$ supergravity.**

**Spurion charge shift can be realised via U(1)$_q$**

$$\mathcal{I} = \frac{1}{2}\big(T\wedge\mathbf{1}\wedge\mathbf{1}+\mathbf{1}\wedge T\wedge\mathbf{1}+\mathbf{1}\wedge\mathbf{1}\wedge T+T\wedge T\wedge T\big)$$

**acting on 56 fermions $\chi^{ijk}$ in 8 $\wedge$ 8 $\wedge$ 8 of SU(8), with $T=\varepsilon\otimes\mathbf{1}_4$ (imaginary unit in SU(3) $\times$ U(1) breaking).**

**$\mathcal{I}$ is *not* in SU(8) $\equiv K(\mathrm{E}_7)$ ... but it is in $K(E_{10})$!**

**The proof requires over-extended root of $E_{10}\Rightarrow$ no way to realise $q$-shift with finite-dimensional R symmetries!**

**It would be rather striking if $K(E_{10})$ were needed to relate $N=8$ supergravity to Standard Model fermions...**

# Summary and Outlook

- **Results obtained so far indicate that $E_{10}$ requires a setting beyond known concepts of space and time.**
- **In this case Standard Model fermions would have to emerge from 'pre-geometric' $E_{10}/K(E_{10})$ model $\rightarrow$ No 'detour' via field theoretic description (unlike superstring and $N = 1$ supergravity models).**
- **$K(E_{10})$ big enough to accommodate *chiral* transformations $\rightarrow$ can one incorporate electroweak quantum numbers (with composite $W^{\pm}$ and $Z$ bosons)?**
- **Need to resolve dichotomy between *finitely many fermionic* and *infinitely many bosonic* degrees of freedom $\rightarrow$ need *faithful* representations.**

# Summary and Outlook

- **Apparent incompatibility of $K(E_{10})$ and supersymmetry for imaginary (null and timelike) roots $\rightarrow$ a new way to break, or rather *avoid*, supersymmetry with *even more* symmetry?**
- **$\Rightarrow$ Can $E_{10}$ supersede SUSY as a unifying principle?**
- **Despite the existence of (at least) $10^{272000}$ string vacua**

  [most recent figures from: Taylor,Wang:1511.03209; Schellekens:1601.02462]

  **$N = 8$ Supergravity remains the only theory that (after complete breaking of supersymmetry) gives 48 spin-$\frac{1}{2}$ fermions, and nothing more $\rightarrow$ but may need $K(E_{10})$ to make contact with the real world.**

## Summary and Outlook

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**THANK YOU**

# SUSY constraint algebra

**Canonical constraint superalgebra** [Damour,Kleinschmidt,HN, CQG24(2007)046]

$$\{\mathcal{S}_A\,,\,\mathcal{S}_B\} = \delta_{AB}\mathcal{H} + \sum_\delta \mathfrak{L}_\delta\,\Gamma(\delta)_{AB} \;\; + \;\cdots$$

**Supergravity Hamiltonian $\mathcal{H}$ and $E_{10}$ Casimir $H$ agree up to $\ell = 2$, but start to differ for $\ell \geq 3$ $\;\to$ more $K(E_{10})$ invariants???**

**The other (bosonic) canonical supergravity constraints $\mathfrak{L}_\delta$ are all associated with null roots of $E_{10}$:** [Damour,Kleinschmidt,HN, CMP302(2011)755]

- **Diffeomorphisms:** $\delta = [0\,1\,2\,3\,4\,5\,6\,4\,2\,3]$ **= affine null root ($\ell = 3$).**
- **Gauss Constraint:** $\delta' = [1\,2\,3\,4\,5\,6\,7\,4\,2\,4]$ $(\ell = 4)$
- **'Dual Gauss Constraint' (Bianchi):** $\delta'' = [1\,2\,3\,4\,5\,7\,9\,6\,3\,5]$ $(\ell = 5)$
- **'Dual diffeomorphisms' (Bianchi):** $\delta''' = [1\,2\,3\,5\,7\,9\,11\,7\,3\,6]$ $(\ell = 6)$

**Recall affine Sugawara $\mathfrak{L}_{m\delta} \propto \sum :J^a_{m-n}J^a_n:$ and $\delta$ = affine null root**

$\to$ **is there a *hyperbolic* analog of the Sugawara construction?**