CLASSIFICATION OF DOUBLED-YET-GAUGED SPACETIME

String Dualities and Geometry, Centro Atomico Bariloche, January 2018

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Prologue

– Generalized metric appeared prior to DFT, taking the particular form:

$$\mathcal{H}_{MN} \equiv \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

Giveon-Rabinovici-Veneziano '89, Duff '90

– But, DFT is valid for *any* generalized metric which satisfies two defining properties:

$$\mathcal{H}_{MN} = \mathcal{H}_{NM}\,, \qquad \mathcal{H}_M{}^K \mathcal{H}_N{}^L \mathcal{J}_{KL} = \mathcal{J}_{MN} \qquad \text{where} \qquad \mathcal{J}_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

– Similarly, SDFT can be formulated by DFT-vielbeins satisfying defining properties:

$$V_{Mp}V^M{}_q = \eta_{pq}\,, \qquad \bar{V}_{M\bar{p}}\bar{V}^M{}_{\bar{q}} = \bar{\eta}_{\bar{p}\bar{q}}\,, \qquad V_{Mp}\bar{V}^{M\bar{q}} = 0\,, \qquad V_{Mp}V_N{}^p + \bar{V}_{M\bar{p}}\bar{V}_N{}^{\bar{p}} = \mathcal{J}_{MN}\,.$$

– **This talk is about the classification of generalized metric/vielbeins, with applications.***

– Henceforth I will refer to the above parametrization as the Riemannian DFT-metric.

Before presenting the main result, I need to review some of my earlier works.

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Doubled-yet-gauged

- Let $\mathcal{F} := \{\, d, \mathcal{H}_{MN}\,, \xi^{M}\,, \partial_{N} d\,, \partial_{L}\mathcal{H}_{MN}, \cdots \}$ be the set of all the functions in DFT.
  - It contains not only the covariant physical fields, $d, \mathcal{H}_{MN}$, and local symmetry parameters, ξ^{A}, but also their arbitrary derivatives and products.
  - It is closed under additions, products and derivatives: if $\Phi_i, \Phi_j \in \mathcal{F}$ then

$$a\,\Phi_i + b\,\Phi_j \in \mathcal{F}\,, \qquad \Phi_i\Phi_j \in \mathcal{F}\,, \qquad \partial_A\Phi_i \in \mathcal{F}\,,$$

    where $a, b \in \mathbb{R}$.
  - The section condition,

$$\partial_M\partial^M\Phi_i = 0\,, \qquad \partial_M\Phi_i\partial^M\Phi_j = 0\,,$$

    is mathematically equivalent to certain translational invariance:

$$\Phi_i(x) = \Phi_i(x + \Delta)\,, \qquad \Delta^M = \Phi_j\partial^M\Phi_k\,,$$

    where Δ^M is said to be *derivative-index-valued.*
  - 'Physics' should be invariant under such shifts of the doubled coordinates in DFT.

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Doubled-yet-gauged spacetime

JHP 1304.5946

Doubled coordinates, $x^M = (\tilde{x}_\mu, x^\nu)$, are gauged through an equivalence relation,

$$x^M \sim x^M + \Delta^M(x)\,,$$

where Δ^M is derivative-index-valued.

Each equivalence class, or gauge orbit in $\mathbb{R}^{D+D}$, corresponds to a single physical point in $\mathbb{R}^D$.

– If we solve the section condition by letting $\tilde{\partial}^\mu \equiv 0$, and further put

$$\Delta^M = c_\mu \partial^M x^\mu \quad : \quad \text{derivative}-\text{index}-\text{valued}\,,$$

we obtain explicitly,

$$(\tilde{x}_\mu\,,\,x^\nu) \;\sim\; (\tilde{x}_\mu + c_\mu\,,\,x^\nu) \quad : \quad \tilde{x}_\mu\text{'s are gauged and } x^\nu\text{'s form a section.}$$

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– In DFT, the usual infinitesimal one-form, $\mathrm{d}x^M$, is neither diffeomorphic covariant,

$$\delta x^M = \xi^M \,, \qquad \delta(\mathrm{d}x^M) = \mathrm{d}x^N \partial_N \xi^M \neq \mathrm{d}x^N(\partial_N \xi^M - \partial^M \xi_N) \,,$$

nor invariant under the coordinate gauge symmetry,

$$\mathrm{d}x^M \quad \longrightarrow \quad \mathrm{d}\big(x^M + \Delta^M\big) \neq \mathrm{d}x^M \,.$$

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The problems can be all cured by gauging the infinitesimal one-form explicitly,

$$Dx^M := \mathrm{d}x^M - \mathcal{A}^M .$$

Dx^M is a covariant vector in DFT

- The gauge potential should satisfy the same property as the coordinate gauge symmetry generator: it must be derivative-index-valued too, satisfying

$$\mathcal{A}^M\partial_M = 0 , \qquad \mathcal{A}_M\mathcal{A}^M = 0 .$$

- Essentially, half of the components are trivial, *e.g.* with $\tilde{\partial}^\mu \equiv 0$,

$$\mathcal{A}^M = A_\lambda\partial^M x^\lambda = (A_\mu , 0) , \qquad Dx^M = (\mathrm{d}\tilde{x}_\mu - A_\mu , \mathrm{d}x^\nu) .$$

- With the appropriate transformations of $\mathcal{A}^M$, the covariance of Dx^M is ensured:

$$\begin{array}{lll} \delta x^M = \Delta^M , \quad \delta\mathcal{A}^M = \mathrm{d}\Delta^M & \Longrightarrow & \delta(Dx^M) = 0 ; \\ \delta x^M = \xi^M , \quad \delta\mathcal{A}^M = \partial^M\xi_N(\mathrm{d}x^N - \mathcal{A}^N) & \Longrightarrow & \delta(Dx^M) = Dx^N(\partial_N\xi^M - \partial^M\xi_N) . \end{array}$$

c.f. natural extension to EFT by Blair 2017

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Doubled-yet-gauged spacetime

With $Dx^M = \mathrm{d}x^M - \mathcal{A}^M$, we can define **Proper Length** through a path integral,

$$\textbf{Length} := -\ln\left[\int \mathcal{D}\mathcal{A}\ \exp\left(-\int\sqrt{Dx^M Dx^N \mathcal{H}_{MN}}\right)\right].$$

which is gauged and covariant under $\mathbf{O}(D,D)$ and DFT-diffeomorphisms.

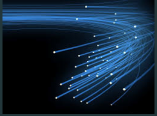
– For the Riemannian DFT-metric, we have a useful relation,

$$Dx^M Dx^N \mathcal{H}_{MN} \equiv \mathrm{d}x^\mu \mathrm{d}x^\nu g_{\mu\nu} + (\mathrm{d}\tilde{x}_\mu - A_\mu + \mathrm{d}x^\rho B_{\rho\mu})(\mathrm{d}\tilde{x}_\nu - A_\nu + \mathrm{d}x^\sigma B_{\sigma\nu})\, g^{\mu\nu}.$$

– Hence, after integrating out the gauge potential, A_μ, the above $\mathbf{O}(D,D)$ covariant definition of the proper length reduces to the conventional one,

$$\textbf{Length} \implies \int \sqrt{\mathrm{d}x^\mu \mathrm{d}x^\nu g_{\mu\nu}(x)}.$$

– Since it is independent of $\tilde{x}_\mu$, indeed it measures the distance between two gauge orbits, which is of course a desired feature.



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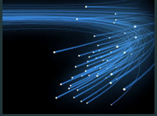

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Doubled-yet-gauged sigma models

The definition of the proper length readily leads to 'covariant' actions:

i) Particle Action Ko-JHP-Suh 2016

$$\mathcal{S}_{\rm particle} = \int {\rm d}\tau\ e^{-1}\, D_\tau x^M D_\tau x^N \mathcal{H}_{MN}(x) - \tfrac{1}{4} m^2 e$$

ii) String Action Lee-JHP 2013, *c.f.* Hull 2006

$$S_{\rm string} = \tfrac{1}{4\pi\alpha'}\int {\rm d}^2\sigma\ \ -\tfrac{1}{2}\sqrt{-h}h^{ij} D_i x^M D_j x^N \mathcal{H}_{MN}(x) - \epsilon^{ij} D_i x^M \mathcal{A}_{jM}$$

With the Riemannian DFT-metric plugged, after integrating out the auxiliary fields, the above actions reduce to the conventional ones:

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With the Riemannian DFT-metric plugged, after integrating out the auxiliary fields, the above actions reduce to the conventional ones:

$$S_{\rm particle} \Rightarrow \int {\rm d}\tau \; e^{-1} \dot{x}^\mu \dot{x}^\nu g_{\mu\nu} - \tfrac{1}{4} m^2 e \,,$$

$$S_{\rm string} \Rightarrow \tfrac{1}{2\pi\alpha'} \int {\rm d}^2\sigma \; -\tfrac{1}{2}\sqrt{-h} h^{ij} \partial_i x^\mu \partial_j x^\nu g_{\mu\nu} + \tfrac{1}{2}\epsilon^{ij}\partial_i x^\mu \partial_j x^\nu B_{\mu\nu} + \tfrac{1}{2}\epsilon^{ij}\partial_i \tilde{x}_\mu \partial_j x^\mu \,.$$

The scheme has been also extended to construct

iii) Doubled-yet-gauged Green-Schwarz superstring — JHP 2016

$$\mathcal{S} = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma \;\; -\tfrac{1}{2}\sqrt{-h}h^{ij}\Pi_i^M\Pi_j^N\mathcal{H}_{MN} - \epsilon^{ij}D_ix^M\left(\mathcal{A}_{jM} - i\Sigma_{jM}\right)\,,$$

where $\Pi_i^M := D_ix^M - i\Sigma_i^M$ and $\Sigma_i^M := \bar{\theta}\gamma^M\partial_i\theta + \bar{\theta}'\bar{\gamma}^M\partial_i\theta'$.

While this action reduces consistently to the original undoubled one, it features the desired symmetries :

- **O(D, D) T-duality**
- **DFT-diffeomorphisms**
- **Worldsheet diffeomorphisms plus Weyl symmetry**
- **Coordinate gauge symmetry :** $x^M \sim x^M + \Delta^M \quad (\Delta^M\partial_M = 0)$
- **twofold Lorentz symmetry, Spin$(1,9)_L \times$ Spin$(9,1)_R$** $\Rightarrow$ **Unification of IIA & IIB**
- **Maximal 16+16 SUSY & kappa symmetry upon flat background**

All the above actions are formulated with $\mathcal{H}_{MN}$, V_{Mp}, $\bar{V}_{M\bar{p}}$ which satisfy the defining properties only, not necessarily parametrized by the Riemannian metric/vielbein.

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Classification

Classification scheme

- $\mathcal{J}_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ naturally decomposes the DFT-metric,

$$\mathcal{H}_{MN} = \begin{pmatrix} H^{\mu\nu} & \mathcal{H}^{\mu}{}_{\lambda} \\ \mathcal{H}_{\kappa}{}^{\nu} & \mathcal{H}_{\kappa\lambda} \end{pmatrix} .$$

The defining properties, $\mathcal{H}_{MN} = \mathcal{H}_{NM}$, $\mathcal{H}_M{}^K\mathcal{H}_K{}^N = \delta_M{}^N$, give algebraic relations:

$$H^{\mu\nu} = H^{\nu\mu} , \qquad \mathcal{H}_{\mu\nu} = \mathcal{H}_{\nu\mu} , \qquad \mathcal{H}_{\mu}{}^{\nu} = \mathcal{H}^{\nu}{}_{\mu} ,$$
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The classification task is to find the most general solutions to these conditions.

- Keeping the choice of the section $\tilde{\partial}^{\mu} \equiv 0$ in mind, we focus on $H^{\mu\nu}$.
- If $H^{\mu\nu}$ is invertible, $H^{\mu\nu} = g^{\mu\nu}$ and the Riemannian DFT-metric is most general.
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Result

The most general form of the DFT-metric is characterized by two non-negative integers, $(n, \bar{n})$, $0 \leq n + \bar{n} \leq D$, and assumes the form:

$$\mathcal{H}_{AB} = \begin{pmatrix} H^{\mu\nu} & -H^{\mu\sigma}B_{\sigma\lambda} + Y_i^\mu X_\lambda^i - \bar{Y}_{\bar{\imath}}^\mu \bar{X}_\lambda^{\bar{\imath}} \\ B_{\kappa\rho}H^{\rho\nu} + X_\kappa^i Y_i^\nu - \bar{X}_\kappa^{\bar{\imath}} \bar{Y}_{\bar{\imath}}^\nu & K_{\kappa\lambda} - B_{\kappa\rho}H^{\rho\sigma}B_{\sigma\lambda} + 2X_{(\kappa}^i B_{\lambda)\rho} Y_i^\rho - 2\bar{X}_{(\kappa}^{\bar{\imath}} B_{\lambda)\rho} \bar{Y}_{\bar{\imath}}^\rho \end{pmatrix}$$

i) Symmetric and skew-symmetric fields: $H^{\mu\nu} = H^{\nu\mu}$, $K_{\mu\nu} = K_{\nu\mu}$, $B_{\mu\nu} = -B_{\nu\mu}$;

ii) The kernels of H and K, with $i, j = 1, 2, \cdots, n$ and $\bar{\imath}, \bar{\jmath} = 1, 2, \cdots, \bar{n}$,

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$$\mathcal{H}_{AB}=\left(\begin{array}{cc}1&0\\B&1\end{array}\right)\left(\begin{array}{cc}H & Y_{i}(X^{i})^{T}-\bar{Y}_{\bar{\imath}}(\bar{X}^{\bar{\imath}})^{T}\\ X^{i}(Y_{i})^{T}-\bar{X}^{\bar{\imath}}(\bar{Y}_{\bar{\imath}})^{T} & K\end{array}\right)\left(\begin{array}{cc}1&-B\\0&1\end{array}\right).$$

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We now insert the $(n, \bar{n})$ DFT-metric into the doubled-yet-gauged sigma models, and analyze the particle and the string dynamics separately.

The auxiliary 'coordinate gauge symmetry' potential decomposes into three parts:

$$A_\mu = K_{\mu\rho} H^{\rho\nu} A_\nu + X^i_\mu Y^\nu_i A_\nu + \bar{X}^{\bar{\imath}}_\mu \bar{Y}^\nu_{\bar{\imath}} A_\nu \,.$$

– The first part appears quadratically, which leads to Gaussian integral.

– The second and third parts appear linearly, as Lagrange multipliers, to prescribe

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DFT-vielbeins

- Given the most general form of the DFT-metric, one can easily derive the corresponding DFT-vielbeins:

$$P_{MN} := \tfrac{1}{2}(\mathcal{J}_{MN} + \mathcal{H}_{MN}) = V_M{}^p V_N{}^q \eta_{pq}\,, \qquad \bar{P}_{MN} := \tfrac{1}{2}(\mathcal{J}_{MN} - \mathcal{H}_{MN}) = \bar{V}_M{}^{\bar{p}} \bar{V}_N{}^{\bar{q}} \bar{\eta}_{\bar{p}\bar{q}}\,.$$

- In particular, the 'doubled' spin group for η_{pq}, $\bar{\eta}_{\bar{p}\bar{q}}$ turns out to be generically heterotic:

$$\mathbf{Spin}(t+n, s+n) \times \mathbf{Spin}(s+\bar{n}, t+\bar{n})\,,$$

where (t, s) is the signature of H and K.

  - It is not the full but the sub-signature, (t, s), that matters for unitarity, since $K_{\mu\nu}$ becomes the effective metric remaining after the dynamical 'freezings' :

$$S_{\mathrm{particle}} \Rightarrow \int \mathrm{d}\tau\; e^{-1}\dot{x}^\mu \dot{x}^\nu K_{\mu\nu} - \tfrac{1}{4}m^2 e\,,$$

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DFT-vielbeins

- Given the most general form of the DFT-metric, one can easily derive the corresponding DFT-vielbeins:

$$P_{MN} := \tfrac{1}{2}(\mathcal{J}_{MN} + \mathcal{H}_{MN}) = V_M{}^p V_N{}^q \eta_{pq}\,, \qquad \bar{P}_{MN} := \tfrac{1}{2}(\mathcal{J}_{MN} - \mathcal{H}_{MN}) = \bar{V}_M{}^{\bar{p}} \bar{V}_N{}^{\bar{q}} \bar{\eta}_{\bar{p}\bar{q}}\,.$$

- In particular, the 'doubled' spin group for η_{pq}, $\bar{\eta}_{\bar{p}\bar{q}}$ turns out to be generically heterotic:

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Kaluza-Klein ansatz for DFT

- The Kaluza-Klein ansatz for Riemannian metric:

$$\hat{g}=\left(\begin{array}{cc} g'+aga^T & ag \\ ga^T & g \end{array}\right)=\exp[\hat{a}]\left(\begin{array}{cc} g' & 0 \\ 0 & g \end{array}\right)\exp\left[\hat{a}^T\right]\,,\qquad \hat{a}_{\hat{\mu}}{}^{\hat{\nu}}=\left(\begin{array}{cc} 0 & a_{\mu'}{}^{\nu} \\ 0 & 0 \end{array}\right),$$

- Similarly, with the decomposition of $\hat{D}=D'+D$,

$$\mathbf{O}(\hat{D},\hat{D})\quad\rightarrow\quad\mathbf{O}(D',D')\times\mathbf{O}(D,D)\,,\qquad \hat{\mathcal{J}}=\left(\begin{array}{cc} \mathcal{J}' & 0 \\ 0 & \mathcal{J} \end{array}\right),$$

the Kaluza-Klein ansatz for the DFT-metric, $\hat{\mathcal{H}}_{\hat{M}\hat{N}}$, reads

$$\hat{\mathcal{H}}=\exp\left[\hat{W}\right]\left(\begin{array}{cc} \mathcal{H}' & 0 \\ 0 & \mathcal{H} \end{array}\right)\exp\left[\hat{W}^T\right]\,,$$

where $\hat{W}$ is an off-block-diagonal $\mathbf{so}(\hat{D},\hat{D})$ element,

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  – Our classification applies to $\mathcal{H}'$ and $\mathcal{H}$, as $(n',\bar{n}')$ and $(n,\bar{n})$.

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Application

One must be prepared to follow up the consequence of theory, and feel that one just has to accept the consequences no matter where they lead.

– Paul Dirac –

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