Brane Wess-Zumino Terms in String Theory

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Outline

Branes and p-form Potentials

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Branes and p-form Potentials

'Exotic Branes' and Mixed-symmetry Potentials

Summary and Open Issues

Branes

Branes are extended objects with a number of worldvolume and transverse directions. They are an essential part of (non-perturbative) string theory

- The NS-NS 2-form B_2 suggests a half-supersymmetric string
- The 3-form C_3 of 11D sugra couples to a half-susy M2-brane

sugra potential $\leftrightarrow$ half-supersymmetric brane

Does it always work as simple as that?

‘Wess-Zumino term requirement’

the construction of a gauge-invariant WZ term requires, besides the embedding coordinates, the introduction of a number of extra worldvolume *p*-form potentials

We require worldvolume supersymmetry, i.e. these worldvolume fields must fit into a multiplet with 16 supercharges

Does the ‘WZ term requirement’ always lead to the rule that

$$\text{potential} \;\Leftrightarrow\; \text{half-susy brane ?}$$

Input from Supergravity

- The T-duality representations of all high-rank form potentials have been determined using three different techniques:

  - closure of the supersymmetry algebra

    de Roo, Hartong, Howe, Kerstan, Ortín, Riccioni + E.B. (2005-2010)

  - using the embedding tensor technique

    for a review, see de Wit, Nicolai, Samtleben (2008)

  - using the very extended Kac-Moody algebra E_{11}/E_{10}

    West (2001); Riccioni, West (2007); Nutma + E.B. (2007); Damour, Henneaux, Nicolai (2002)

A scaling symmetry

All potentials transform as a representation of the T-duality group O(d,d) and scale under a scaling symmetry

The scaling weight α determines the dependence of the brane tension T on the string coupling constant g_s via

$$\mathrm{T} \sim (g_s)^{\alpha}$$

This scaling weight is invariant under dimensional reduction

A universal pattern arises

α	potentials	branes
$\alpha = 0$	$B_{1,A}$, B_2	fundamental
$\alpha = -1$	$C_{2n+1,a}$, $C_{2n,\dot{a}}$	Dirichlet
$\alpha = -2$	D_{D-4}, $D_{D-3,A}$, D_{D-2,A_1A_2}, $D_{D-1,A_1\cdots A_3}$, $D_{D,A_1\cdots A_4}$	solitonic
$\vdots$	$\vdots$	$\vdots$

A $(a, \dot{a})$ are vector (spinor)-indices of T-duality

$\alpha = -3$: S-dual of D7-brane

$\alpha = -4$: S-dual of D9-brane

Branes with $\alpha < -4$ have no ten-dimensional brane origin!

Question

given a $(p+1)$-form potential which (components of its) T-duality repres. couple to a half-supersymmetric brane ?

Outcome Wess-Zumino Term Requirement

Riccioni + E.B. (2010)

There is a simple group-theoretical characterization of which (components of the) T-duality representation couple to a half-supersymmetric brane

- the (group-theoretical) details can be found in our papers
- Comparing branes in different dimensions an interesting set of wrapping rules emerge.

The Solitonic Wrapping Rules

the wrapping rules of solitonic branes are given by

$$\mathrm{T_S} \sim (g_s)^{-2} \ : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{undoubled} \\ \text{unwrapped} & \rightarrow \quad \text{doubled} \end{cases}$$

For instance, in 9D we have two solitonic 5-branes coming from an un-wrapped NS5-brane and a KK monopole

10D KK monopole :
- $5+1$ worldvolume directions
- 1 isometry direction
- 3 transverse directions

Counting Solitonic Branes with $T \geq 3$

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	
1					1	**10**		
2				1	8			
3			1	6				
4		1	4					
5	1/1	2						

S(D-5)-brane and S(D-4)-brane$_A$

Solitonic Branes with $T \leq 2$

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	84
1					1	10	60	280
2				1	8	40	160	560
3			1	6	24	80	240	
4		1	4	12	32	80		
5	1/1	2	4	8	16			

The red numbers follow from imposing the Wess-Zumino term requirement

A Numerical Coincidence?

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	84
1					1	10	60	280
2				1	8	40	**160**	**560**
3			1	6	24	80	**240**	
4		1	4	12	32	80		
5	1/1	2	4	8	16			

Precisely the same numbers are reproduced by the solitonic wrapping rule!

Question

what is the 10D origin of the solitonic branes with $T \leq 2$?

Note: extra input is needed to fill up the T-duality representations!

standard supergravity is not sufficient!

Outline

Mixed-Symmetry Potentials

- At the level of (linearized) supergravity T-duality can be recovered by assuming that these theories can be extended with a set of mixed-symmetry potentials with an underlying E_{10}/E_{11}-symmetry

- To recover T-duality at the level of branes we assume that these mixed symmetry potentials are in one to one correspondence with extended objects called 'exotic branes'. They have worldvolume, transverse and special isometry directions

see, e.g., Obers, Pioline (1999); Lozano-Tellechea, Ortín (2001)

see also work by de Boer and Shigemori (2010, 2012) → 'T-folds'

A 7D Example

$\alpha = -2$	$D_3,\ D_{4,A},\ D_{5,[AB]},\ D_{6,[ABC]},\ D_{7,[ABCD]}$	$D_{6+n,n}\ (n = 0,1,2,3,4)$

The 7D solitonic domain wall 6-forms $D_{6,[ABC]}$ $(A = 1, \ldots, 6)$ transform as 20 under $SO(3,3)$. These 6-forms are dual to (constant) fluxes

10D origin	mixed-symmetry	flux (a=1,2,3)
NS5 (5_2)	D_6	H_{abc} (1)
KK5 (5_2^1)	$D_{7,1}$	$f^a{}_{bc}$ (9)
5_2^2	$D_{8,2}$	$Q^{ab}{}_c$ (9)
5_2^3	$D_{9,3}$	R^{abc} (1)
5_2^4	$D_{10,4}$	$--$

see also Haßler, Lüst (2013); Kimura, Sasaki (2013); Chatzistavrakidis, Gautason, Moutsopoulos, Zagermann (2013)

Brane WZ Terms and DFT

Where does $D_6 \equiv B_6$ fits into DFT ?

In SUGRA one can dualize B_2 into B_6 without dualizing the metric tensor $g_{\mu\nu}$ but in DFT B_2 is part of the generalized metric $\mathcal{H}_{MN}$!

Standard Dualization of Kalb-Ramond Field

standard : $\mathrm{S[b]} \; = \; -\frac{1}{12}\int \mathrm{d^D x \, H_{abc} \, H^{abc}} \, , \qquad \mathrm{H_{abc}} = 3\partial_{\mathrm{[a}}\mathrm{b_{bc]}}$

first – order : $\mathrm{S[D,H]} \; = \; \int \mathrm{d^D x}\Big(- \tfrac{1}{12}\mathrm{H_{abc}H^{abc}} + \mathrm{D^{abcd}}\partial_{\mathrm{a}}\mathrm{H_{bcd}}\Big)$

$$\delta D^{abcd} = \partial_e \Sigma^{eabcd}$$

$$\text{(E.o.M)} \quad \partial_{[a} G_{bcd]} \; = \; 0 \, , \qquad G^{abc} = \partial_d D^{dabc}$$

$$\text{(B.I.)} \quad \partial_a G^{abc} \; = \; 0$$

Exotic Dualization

Hull (2001); Boulanger, Sundell, West (2015)

$$S[b] = -\tfrac{1}{12}\int d^Dx\, H^{abc}H_{abc} = -\tfrac{1}{4}\int d^Dx\,\left(\partial^a b^{bc}\,\partial_a b_{bc} - 2\,\partial_a b^{ab}\,\partial^c b_{cb}\right)$$

$$S[Q,D] = \int d^Dx\Big(-\tfrac{1}{4}\,Q^{a|bc}Q_{a|bc} + \tfrac{1}{2}\,Q_{a|}{}^{ab}Q^{c|}{}_{cb} - \tfrac{1}{2}\,D^{ab|cd}\,\partial_a Q_{b|cd}\Big)$$

$$\partial_{[a}Q_{b]|cd} = 0 \quad\Rightarrow\quad Q_{a|bc} = \partial_a b_{bc}$$

We now have a mixed-symmetry potential $D^{ab|cd} \overset{D=10}{\sim} D_{8,2}$!

Linearized DFT

Use formulation with generalized fluxes $\mathcal{F}_{ABC}$

Aldazabal, Baron, Marques, Nunez (2011); Geissbuhler (2011)

Grana, Marques (2011); Geissbulher, Marques, Nunez, Penas (2013)

Duality leads to 4-form potential D^{ABCD}

Hohm, Penas, Riccioni + E.B. (2016)

$$
\begin{aligned}
D^{\mu_1\cdots\mu_4} &\rightarrow B_6\\
D^{\mu_1\cdots\mu_3}{}_{\mu_4} &\rightarrow h_{7,1}\\
D^{\mu_1\mu_2}{}_{\mu_3\mu_4} &\rightarrow D_{8,2}\\
D^{\mu_1}{}_{\mu_2\cdots\mu_4} &\rightarrow D_{9,3}\\
D_{\mu_1\cdots\mu_4} &\rightarrow D_{10,4}
\end{aligned}
$$

Can we define brane WZ terms in DFT?

Berkeley, Berman, Rudolph (2014); Bakhmatov, Berman, Kleinschmidt, Musaev and Otsukin (2017); Chatzistavrakidis, Gautason, Moutsopoulos, Zagermann (2014); Blair, Musaev (2017); Kleinschmidt, Ortín, Riccioni + E.B., work in progress

$\alpha = -2$: solitonic 5-brane WZ terms

Blair and Musaev (2017)

$$\int d^6\xi \, q_{M_1\ldots M_{10}} D^{M_1\ldots M_4} \mathcal{D}_{a_1} X^{M_5} \ldots \mathcal{D}_{a_6} X^{M_{10}} \epsilon^{a_1\ldots a_6} \qquad \text{with}$$

$$\mathcal{D}_a X^M = \partial_a X^M - K^{-2} K^M_{\alpha} K^{\alpha}_N \partial_a X^N \,,$$

$$q_{M_1\ldots M_{10}} = Q \epsilon_{\nu_1..\nu_{10}} \frac{\partial Z^{\nu_1}}{\partial X^{M_1}} \cdots \frac{\partial Z^{\nu_{10}}}{\partial X^{M_{10}}}$$

- NS 5-brane: $\quad q_{M_1\ldots M_{10}} = Q\epsilon_{m_1\ldots m_{10}}$ otherwise zero $\rightarrow D_6$
- KK-monopole: $\quad q_{M_1\ldots M_{10}} = Q\epsilon_{\nu_1\ldots\nu_{10}} K^{\nu_1}_{M_1} \delta^{\nu_2}_{M_2} \cdots \delta^{\nu_{10}}_{M_{10}} \rightarrow D_{6x,x}$

DFT Potentials

- $\alpha = 0: \quad B_{MN}$
- $\alpha = -1: \quad C_{\alpha}$
- $\alpha = -2: \quad D_{MNPQ}$
- $\alpha = -3: \quad E_{MN\alpha}$
- $\alpha = -4: \quad F^{+}_{M_1 \cdots M_{10}}\,, \qquad F_{M_1 \cdots M_4, N_1 N_2}\,, \qquad F_{M_1 \cdots M_7, N}$

$\vdots$

What are the brane WZ terms within DFT?

A. Kleinschmidt, T. Ortín, F. Riccioni + E.B. (work in progress)

Outline

Summary

- In this talk I reviewed the classification of the potentials and branes of maximal supergravity using the Wess-Zumino term requirement and showed how this suggests the introduction of mixed-symmetry potentials and exotic branes

- all branes satisfy simple wrapping rules!

- I reported on some progress in how to describe these Wess-Zumino terms within DFT. More work remains to be done!

Take Home Message

Can we understand the role of mixed-symmetry potentials better ?

See, e.g., Bunster, Henneaux (2013)

Thanks for your Attention !