How to uplift
Gauged [maximal] supergravities

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Generalised Scherk–Schwarz reductions

Higher-dim. (super)gravity

↓

Non-linear field redefinitions

↓

$$\mathcal{M}_{MN}(x,y) = (\det \hat{E})^2 M_{AB}(x) \hat{E}_M^A(y) \hat{E}_N^B(y)$$

$$\mathcal{A}_\mu^M(x,y) = A_\mu^A(x) \hat{E}_A^M(y)$$

⋮

↓

Lower-dim. gauged (super)gravity

$$\left.\begin{matrix} g_{\hat{\mu}\hat{\nu}} \\ B_{\hat{\mu}\hat{\nu}} \\ \phi \\ C_{(p)} \\ \dots \end{matrix}\right\} \left\{\begin{matrix} g_{\mu\nu}(x,y) \\ \mathcal{M}_{MN}(x,y) \\ \mathcal{A}_\mu^M(x,y) \\ \mathcal{B}_{\mu\nu}(x,y) \\ \dots \end{matrix}\right.$$

[Aldazabal Baron Marques Nunez; Geissbuhler; Grana Marques; Berman Musaev Thompson; Aldazabal Grana Marques Rosabal; Berman Blair Malek Perry; Godazgar^2 Nicolai; Lee Strickland-Constable Waldram, Hohm Samtleben, Baron Dall'Agata; Ciceri Gl Guarino; Cassani De Felice Petrini Strickland Constable Waldram; Baguet Pope Samtleben; Gl Samtleben Trigiante; Hassler, du Bosque Hassler Luest, ...]

How to find **all** generalised Scherk–Schwarz reductions leading to a specific gauged supergravity

Generalised Lie derivative

Generalised parallelisation (twist matrix)

$$\left(\mathbb{L}_{\hat{E}_A} + \hat{F}_0(\hat{E}_A)\right) \hat{E}_B = -X_{AB}{}^C \hat{E}_C$$

Mass/gauged deformation (eg Romans mass)

[Ciceri Guarino GI]
[Cassani De Felice Petrini Strickland-Constable Waldram]

Embedding tensor (constant)

Exhaustive uplift procedure for *maximal* gauged sugras

Group theoretical/algebraic (no integration!)

Still applies to non-maximal. E.g. ½ max.: DFT, SL(2)-DFT, ...

[Ciceri Dibitetto Guarino GI Melgarejo]
[ALSO: Str.Constable; Hohm Musaev Samtleben]

How to find **all** generalised Scherk–Schwarz reductions leading to a specific gauged supergravity

$$\left(\mathbb{L}_{\hat{E}_A} + \hat{F}_0(\hat{E}_A)\right)\hat{E}_B = -X_{AB}{}^C\,\hat{E}_C$$

- Generalised Lie derivative → $\mathbb{L}_{\hat{E}_A}$
- Generalised parallelisation (twist matrix) → $\hat{E}_B$
- Mass/gauged deformation (eg Romans mass) → $\hat{F}_0$
- Embedding tensor (constant) → $X_{AB}{}^C$

Restrictions:

- discrete quotients (same as standard S-S)
- gauging of matter symmetries (e.g. hypers)
- D=3 with 'ancillary transformations' [technical details change, *WIP with Ciceri*]

[Hohm Samtleben, Hohm Musaev Samtleben]

(see also talks by Bossard, Cederwall)

Math:

- What is the generalised geom. analogue of a group manifold?
- Is there a generalised Lie-Cartan theorem?

$$[K_a\,,\ K_b]_{Lie} = f_{ab}{}^c K_c \qquad \mathrm{Jac}(f_{ab}{}^c) = 0 \qquad \Rightarrow \quad \text{Group mfd.}$$

Physics of gauged supergravities:

- Solution generation
  - (susy) AdS vacua $\longrightarrow$ holography applications (gauged sugra as cft subsector)
  - dS, Minkowski vacua; BHs, domain walls, ...
  - non-susy solutions

Other techniques: talks by Lozano, Tomasiello

Talks by Dall'Agata, Guarino

Talk by Dibitetto

- How to focus only on models embeddable in 10d/11d?

[Similar direction trough gEFT: Hassler; du Bosque Hassler Luest]

Some outstanding examples:

- **11d sugra on S^7 gives 4d SO(8) gauged maximal supergravity**

  de Wit, Nicolai;

- **IIB sugra on S^5 gives 5d SO(6) gauged maximal supergravity**

  Full Proof only given in terms of Exc. Gen. Geometry!

  Lee, Strict.-Const., Waldram;
  Hohm & Samtleben

- **IIA on S^6 gives 4d ISO(7) gauged maximal supergravity**

  But there are two inequivalent such theories! Dall'Agata, GI Marrani

  The deformation parameter is the **Romans mass** Guarino, Jafferis, Varela

[Also see: Nicolai Pilch; Ciceri de Wit Varela; Natase Vaman van Nieuwehnhiuzen; Cvetic Lu Pope Sadrazdeh Tran; ....]

$$Y^{MN}{}_{PQ}\partial_M\partial_N = 0$$

`No section constraints have been violated`
`in the making of this talk.`

PLAN

- Three prerequisites
- General bottom-up construction
  - Vector components (detailed)
  - p-form components (brief)
- Examples

Prereq. 1: Embedding tensor & central charges

[Nicolai Samtleben
de Wit Samtleben Trigiante
....]

Sugra with global symmetries $\mathcal{G} \times \mathbb{R}^+$ (e.g. $\mathrm{E}_{7(7)} \times \mathbb{R}^+$)

gauge $(X_A)_B{}^C \in \mathcal{L}\mathrm{ie}\,\mathcal{G} + \mathbb{R} \longrightarrow \mathrm{G}_{\mathrm{gauge}}$

↑ $1, \ldots, \#$ of vector fields

$$[X_A\,,\; X_B] = -X_{AB}{}^C X_C$$

On *potentials* A_μ^A, central extension $\widehat{\mathrm{G}}_{\mathrm{gauge}}$

Defined by $\quad X_{(AB)}{}^C\,\widehat{X}_C = 0, \quad [\widehat{X}_A\,,\; \widehat{X}_B] = -X_{AB}{}^C\,\widehat{X}_C$

$$A_\mu^{\hat{a}} \equiv A_\mu{}^A\;\widehat{\Theta}_A{}^{\hat{a}}$$

Prereq. 2: splits & twists

[Coimbra Strickland Constable Waldram; Berman Cederwall Kleinschmidt Thompson]

$$\mathbb{L}_\Lambda V^M = \Lambda^P \partial_P V^M - V^P \partial_P \Lambda^M + Y^{MN}{}_{PQ} \partial_N \Lambda^P V^Q$$

Choice of section: $\partial_M \equiv \mathcal{E}_M{}^m \partial_m \qquad Y^{MN}{}_{PQ} \mathcal{E}_M{}^m \mathcal{E}_N{}^n = 0$

Structure group

$$\mathcal{E}_M{}^m \text{ breaks } \mathcal{G} \times \mathbb{R}^+ \quad \to \quad (\mathrm{GL}(d) \times \underbrace{\mathcal{G}_0 \times \mathbb{R}^+_0}) \ltimes \mathcal{P}$$

Higher dim.
global symmetries

p-form shifts

$$C_M{}^N \mathcal{E}_N{}^m = \mathcal{E}_M{}^m \quad \Leftrightarrow \quad C_M{}^N \in (\mathcal{G}_0 \times \mathbb{R}^+_0) \ltimes \mathcal{P}$$

Prereq. 3: background fluxes, massive & gauged deformations

$$\mathbb{L}_\Lambda V^M \quad \to \quad \mathbb{L}_\Lambda V^M - \Lambda^P V^Q F_{PQ}{}^M$$

[Ciceri Guarino GI
Ciceri Dibitetto Guarino GI Melgarejo]
[Hohm Kwak; Grana Marques]

- Background p-form fluxes: $C_p = C_p^{\text{dyn}} + C_p^{\text{bkg}}$
- Massive deformations (e.g. massive IIA)
- Gauged deformations (e.g. gauged IIA)

Consistency:

$$F_{MN}{}^P \mathcal{E}_P{}^m = 0\,,$$

$$\left(F_{(PQ)}{}^M \delta_S^N - Y^{MN}{}_{TS} F_{(PQ)}{}^T - \tfrac{1}{2} Y^{TN}{}_{PQ} F_{TS}{}^M\right)\mathcal{E}_N{}^m = 0\,,$$

$$[F_M\,,\,F_N]_P{}^Q - F_{MN}{}^R F_{RM}{}^Q + \partial_M F_{NP}{}^Q - 2\partial_{[N} F_{|M|P]}{}^Q - Y^{QR}{}_{SP}\partial_R F_{MN}{}^S = 0$$

$\int$-bility: $\quad F = \text{torsion}[C_M{}^N] + C \cdot \hat{F}_0$

p-1 form potentials 0 form (mass/gauging)

How to solve $\left(\mathbb{L}_{\hat{E}_A} + \hat{F}_0(\hat{E}_A)\right) \hat{E}_B = -X_{AB}{}^C \hat{E}_C$

Focus on vector components: $\hat{E}_A{}^M \mathcal{E}_M{}^m \equiv K_A{}^m$

$$\hat{E}_A{}^M = \begin{pmatrix} K_A{}^m & p\text{-form trfs etc.} \end{pmatrix}$$

$$\left(\mathbb{L}_{\hat{E}_A} + \hat{F}_0(\hat{E}_A)\right) \hat{E}_B = -X_{AB}{}^C \hat{E}_C \quad \Rightarrow \quad [K_A\,,\,K_B]_{Lie} = -X_{AB}{}^C K_C$$

$$\Rightarrow K_A \text{ generate } \widehat{\mathrm{G}}_{\mathrm{gauge}}$$

K_A determine transitive action of $\widehat{\mathsf{G}}_{\mathsf{gauge}}$

$$\mathcal{M} = \widehat{H} \backslash \widehat{\mathsf{G}}_{\mathsf{gauge}}$$

Choose coset space. $K_A{}^m$ constructed from coset representatives

$$L(y) g = h(y') L(y') \,, \quad g \in \hat{\mathrm{G}} \,, \ h(y) \in \hat{\mathrm{H}} \,.$$

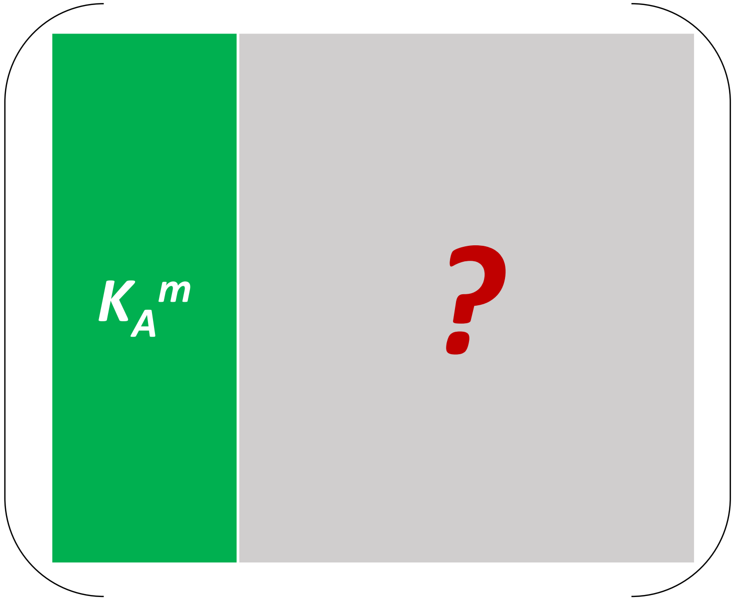
$$\hat{\Theta}_A{}^{\hat{a}} K_{\hat{a}}{}^m = (L X_A L^{-1})|^{\underline{m}} \mathring{e}_{\underline{m}}{}^m = L^{-1}{}_A{}^B \hat{\Theta}_B{}^{\underline{m}} \mathring{e}_{\underline{m}}{}^m$$

Reference Vielbein

$$Y^{AB}{}_{CD} \hat{\Theta}_A|_{\text{coset}} \ \hat{\Theta}_B|_{\text{coset}} = 0 \quad \Rightarrow \xi \equiv \hat{\Theta}|$$

$$\left(\mathbb{L}_{\hat{E}_A} + \hat{F}_0(\hat{E}_A)\right)\hat{E}_B = -X_{AB}{}^C\,\hat{E}_C$$

$\hat{E}_A{}^M =$



$\hat{F}_0 = ?$

In short: everything else is determined by $K_A{}^m$

Int. Metric, scalars,
p-form potentials

Mass/gauge
deformation

$$\text{Finding } \hat{E}_A\,,\ \hat{F}_0 = \text{finding } \widetilde{E}_A\,,\ F$$

Int. Metric, scalars

p-form fluxes,
Mass/gauge defos

$$\widetilde{E}_A \overset{\text{GLOBALLY}}{\in} \mathcal{T} \oplus \Lambda^{p-1}\mathcal{T}^* \oplus \ldots$$

$$\widetilde{E}_A{}^M = \hat{E}_A{}^N C_N{}^M \qquad F = \text{torsion}[C_M{}^N] + C \cdot \hat{F}_0$$

CLAIM:* $\widetilde{E}_A{}^M = (\widetilde{L}^{-1})_A{}^B \mathring{e}_B{}^M$ extends globally

coset representative

'*minus*' $\mathcal{P}$-valued component

*: *un*gauged higher-dimensional theory (11d, IIA, IIB, mIIA, ...). Otherwise, gauged patching.

$$F = \mathrm{torsion}[C_M{}^N] + C\cdot \hat{F}_0 \quad = X - \mathrm{torsion}[\widetilde{E}]$$

Flux integrability (BI):

$$C[X]_{FCD}{}^{AB}\Theta_B{}^{\underline{m}} + \frac{1}{4}(Y^{AH}{}_{CD}\delta^I_F - Y^{AI}{}_{JF}Y^{JH}{}_{CD})\big(X_{HI}{}^B + 2\Theta_{(H}{}^{\underline{m}}t_{\underline{m}I)}{}^B\big)\Theta_B{}^{\underline{m}} = 0\,,$$

Often automatic! (e.g. for uplifts to 11d, IIA/IIB, mIIA, DFT, …)

Summary

- Gauged sugra has emb. tensor $X_{AB}{}^C = \Theta_A{}^a t_{aB}{}^C$, determines gauge group G
- Choose $\hat{\mathrm{H}} \subset \hat{\mathrm{G}}$ such that $\hat{\Theta}|_{\text{coset gen.s}}$ satisfies section constraint:

$$Y^{MN}{}_{PQ}\hat{\Theta}_M{}^m\hat{\Theta}_N{}^n = 0\,.$$

- extra constraint above *if necessary*
- E_A and $F_{MN}{}^P$ contain same information as $\hat{E}_A$ and $\hat{F}^0$.

Examples

Known* uplifts

If H\G = group manifold, recover standard Sch.–Schw.

If $\mathrm{H}\backslash\mathrm{G} = \frac{G\times G}{G_{diag}}$, recover *consistent Pauli reductions* in DFT.

$\mathrm{CSO}(p,q,r)$ and dyonic $\mathrm{CSO}(p,q,r)$ gaugings liftable on (products of)

$$\frac{\mathrm{SO}(p,q)}{\mathrm{SO}(p,q-1)} = H^{p,q}\,, \qquad \frac{\mathrm{CSO}(p,q,r)}{\mathrm{CSO}(p,q,r-1)} = \mathbb{R}^{p+q}$$

* : the $\mathbb{R}^{p+q}$ uplifts Based on 'locally geometric flux' upon torus compactification

Group manifold case: $$\frac{\mathrm{CSO}(3,0,5)}{\mathbb{R}^{15}} \simeq S^3$$

Example w/ a no-go

11d sugra on S^7 gives SO(8) $D = 4$ gauged max. sugra.
But there are many SO(8) gauged max sugras (all with $\mathcal{N} = 8$ AdS vacuum)

$$\Theta_{\text{original}} \to \text{Rot}(\omega)\Theta_{\text{original}}\,, \qquad \text{Rot}(\omega) \notin \text{E}_{7(7)}$$

$$\text{G} = \text{SO}(8)\,,\ \text{H} = \text{SO}(7) \ \longrightarrow \frac{\text{SO}(8)}{\text{SO}(7)} = S^7.$$

$\Theta_{\text{original}}|_{\text{coset}}$ satisfies section. **all others don't**.
(no-go's from de Wit–Nicolai and Lee–Str.Constrable–Waldram)

Non-compact: $\text{SO}(p, q)$, similar ω-deformation.
New no-go: again, only $\omega = 0$ liftable.

Four-parameter CSS uplift *[WIP with Dall'Agata, Spezzati]*

$D = 4$, $\mathcal{N} = 8$ $\qquad\qquad$ $\mathsf{G}_{\mathsf{gauge}} = \mathsf{U}(1) \ltimes \mathbb{R}^{24}$

Assuming $\mathbb{T}^7$ internal space:

$$\omega_{71}{}^{2} = -\omega_{72}{}^{1} = \tilde{m}_1\,,$$
$$\omega_{73}{}^{4} = -\omega_{74}{}^{3} = \tilde{m}_2\,,$$
$$\omega_{75}{}^{6} = -\omega_{75}{}^{5} = \tilde{m}_3\,,$$
$$\theta_{77} = -g_7(=\theta_{88}) = \tilde{m}_4\,.$$

"locally geometric"

$$\mathcal{M} = \widehat{H}\backslash\widehat{\mathsf{G}}_{\mathsf{gauge}} = \frac{\mathsf{U}(1) \ltimes \mathbb{R}^{24}}{\mathbb{R}^{18}} = \mathbb{T}^7$$

also with $\tilde{m}_4$!!

Talk by Dall'Agata

Dualisation of SS reductions *[WIP]*

DFT: $\eta_{MN} = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$, $X_{AB}{}^{C} \sim f_{[ABC]}$ $\quad$ $G_{\text{gauge}} = G \ltimes \mathfrak{g}$

$$\mathcal{M} = \frac{G \ltimes \mathfrak{g}}{\mathfrak{g}} = G$$

$$f|_{\text{coset}} \cdot \eta \cdot f|_{\text{coset}} = 0$$

$$f_a = \begin{pmatrix} \omega_{ab}{}^{c} & \\ & -\omega_{ac}{}^{b} \end{pmatrix} \qquad f^a = \begin{pmatrix} & \omega_{bc}{}^{a} \\ & \end{pmatrix}$$

Dualisation of SS reductions [WIP]

DFT: $\eta_{MN} = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$, $\quad X_{AB}{}^{C} \sim f_{[ABC]}$ $\qquad \mathrm{G}_{\mathrm{gauge}} = G \ltimes \mathfrak{g}$

$$\mathcal{M} = \frac{G \ltimes \mathfrak{g}}{H \ltimes (\mathfrak{g} \ominus \mathfrak{h})} \overset{\sim}{\cong} \frac{G}{H} \times \mathfrak{h}$$

$$f|_{\mathrm{coset}} \cdot \eta \cdot f|_{\mathrm{coset}} = 0$$

$$\left. \begin{matrix} f_{\mathfrak{g}\ominus\mathfrak{h}} \\ f_{\mathfrak{h}} \end{matrix} \right\} \quad f_a = \begin{pmatrix} \omega_{ab}{}^{c} & \\ & -\omega_{ac}{}^{b} \end{pmatrix} \qquad \left. \begin{matrix} f_{\mathfrak{g}\ominus\mathfrak{h}} \\ f_{\mathfrak{h}} \end{matrix} \right\} \quad f^a = \begin{pmatrix} & \omega_{bc}{}^{a} \\ & \end{pmatrix}$$

Applies for any reductive G/H!!!

Special case: G compact (semisimple), H abelian:
Normal bundle over regular adj orbit [Anderson]

Outlook

$$\mathcal{M} = \widehat{H} \backslash \widehat{\mathsf{G}}_{\mathsf{gauge}} \qquad Y^{AB}{}_{CD} \hat{\Theta}_A|_{\text{coset}}\ \hat{\Theta}_B|_{\text{coset}} = 0$$

- Full machinery in place, exploit it!
- standard SS of (super)gravity: quadratic algebraic eqs (Jacobi)
  - How close can we get to *explicit* classification?
- .....Having solved gSS, can we build non-susy preserving reductions?