CONSISTENT TRUNCATIONS AND EXCEPTIONAL GEOMETRY

[D. CASSANI, O. de FELICE, M.P, C. STRICKLAND-CONSTABLE, D. WALDRAM, arXiv:1605.00563, ...]

INTRODUCTION

- A major question in string theory/sugra is how to derive lower dimensional effective actions
  - given a spontaneous compactification of the type

    $$M_{10} = M_{10-d} \times M_d$$

    with M_d compact and M_{10-d} maximally symmetric
    - expand the 10d field in harmonics on M_d
    - obtain a $10-d$ action with infinite tower of KK modes
  - then need to truncate the theory to a finite set of fields in a consistent way
    - the dependence on the internal manifold factorises out in the eom and susy variations
    - recover the full non-linear interactions and symmetries for the lower dimensional fields
    - all solutions of the lower dimensional theory lift to solutions of the higher dimensional one

- Consistent reductions are not a mathematical curiosity
  - establish a map between sugra theories in different dimensions
  - insight on the higher dimensional origin of the lower dimensional gauge symmetries
  - powerful tool in AdS/CFT
  - embed into string theory AdS vacua, black holes, domain walls, and non-relativistic backgrounds

- In this talk I will focus on applications of Generalised Geometry to consistent truncations
  - summary of the main features of EGG for type IIA
  - g eneralise ordinary Scherk-Schwarz and G-structure reductions to EGG
    - maximally susy reductions
      - generalised garallelisability
      - generalised Scherk-Schwarz ansatz
      - application to sphere reductions in type IIA
    - Less susy reductions
      - generalised G-structures

GENERALISED GEOMETRY

[hitchin 02; gualtieri 04; hull 07; pacheco, waldram 08, ...]

- Geometrise the gauge symmetries of RR and NS potentials by enarging the tangent space

	Riemannian	G C G	E G G
tangent b.	TM	$TM \oplus T^*M$	$T \oplus T^* \oplus \Lambda^{\pm} \oplus \Lambda^5 T^* \oplus (T^* \otimes \Lambda^6 T^*)$
structure group	$SO(6)$	$O(6,6)$ T-duality	$E_{7(7)}$ U-duality

- charges under the various symmetries
- the transition functions involve RR and NS potentials as generalised diffeomorphims
- the structure group is the duality group on the internal manifold

EX: Exceptional Geometry for Type IIA

- The construction can be extended to include all potentials of massive IIA SUGRA
  - the IIA potentials (democratic formulation [bergshoeff et al. 01])

NS two-form	RR polyform	NS six-form
B	$C = \sum_{k=0}^{4} C_{2k+1}$	$\tilde{B}$
$H = \mathrm{d}B$	$F = \mathrm{d}C - H \wedge C + m e^{B}$	$\tilde{H} = \mathrm{d}\tilde{B} - \frac{1}{2}[s(F) \wedge C + m e^{-B} \wedge C]_{(7)}$

  and $m = F_0$ is the Romans mass

  - with gauge transformations

$$\delta_V B = \mathcal{L}_v B - \mathrm{d}\lambda$$

$$\delta_V C = \mathcal{L}_v C - e^{B} \wedge (\mathrm{d}\omega - m\lambda)$$

$$\delta_V \tilde{B} = \mathcal{L}_v \tilde{B} - (\mathrm{d}\sigma + m\,\omega_6) - \frac{1}{2}\left[e^{B} \wedge (\mathrm{d}\omega - m\lambda) \wedge s(C)\right]_6$$

• The generalised tangent bundle is $E \in \mathbf{56}$ of $E_{7(7)}$ and decomposes under $GL(6)$

$$E \simeq TM \oplus T^*M \oplus \Lambda^5 T^*M \oplus (T^* \otimes \Lambda^6 T^*) \oplus \Lambda^{\mathrm{even}} T^*M$$

has a non-trivial fibration structure: the patching of generalised vectors

$$V = e^{\tilde{B}}\, e^{-B}\, e^{C} \hat{V} \sim v + \lambda + \sigma + \tau + \omega$$

on $U_{(\alpha)} \cap U_{(\beta)}$ mply the IIA gauge transformations

$$\left.\begin{aligned} \delta_V B\big|_\alpha &= \delta_V B\big|_\beta \\ \delta_V C\big|_\alpha &= \delta_V C\big|_\beta \\ \delta_V \tilde{B}\Big|_\alpha &= \delta_V \tilde{B}\Big|_\beta \end{aligned}\right\} \iff V_{(\alpha)} = e^{\mathrm{d}\tilde{\Lambda}_{(\alpha\beta)}}\, e^{\mathrm{d}\Omega_{(\alpha\beta)} + m\, \Omega_{6(\alpha\beta)}}\, e^{-\mathrm{d}\Lambda_{(\alpha\beta)} - m\, \Lambda_{(\alpha\beta)}} \cdot V_{(\beta)}$$

• the patching further reduces the structure group to $GL(R,6) \ltimes \Lambda^{odd}(M) \rightarrow$ generalised diffemorphisms

• The Roman's mass deforms the gauge transformations of the potentials $\Rightarrow$ deform the patching conditions

- $F_0 = m$ has no potential $\Rightarrow$ it cannot be used to defiine E

Dorfman Derivative

- The ordinary Lie derivative generates diffeomorphisms

$$\mathcal{L}_v v'^m = v^n \partial_n v'^m - v'^n \partial_n v^m = v^n \partial_n v'^m - (\partial \times_{\text{ad}} v)^m{}_n v'^n$$

GL(6) adjoint action $\hookleftarrow$

- The Dorfmann derivative generates generalised diffeos: diffeos plus gauge transf.

$$L_V V'^M = V^N \partial_N V'^M - (\partial \times_{\text{ad}} V)^M{}_N V'^N + \underline{m}(V) \cdot V'$$

$\partial_N = (\partial_n, 0, 0, 0, 0) \in E^* \hookleftarrow$ $\hookrightarrow$ $E_{7(7)}$ adjoint action

  - natural in the untwisted Dorfman derivative

$$\hat{L}_{\hat{V}} \hat{V}' = \hat{L}^{(m=0)}_{\hat{V}} \hat{V}' - \iota_{\hat{v}} H + \hat{\omega} \wedge H - (\iota_{\hat{v}} F + \hat{\lambda} \wedge F) + \hat{\omega} \wedge F \qquad F = F_0 + F_2 + F_4 + F_6$$

with

$$\begin{aligned} L_V V' \quad = \quad & \mathcal{L}_v v' + \big(\mathcal{L}_v \lambda' - \iota_{v'} \mathrm{d}\lambda\big) + \big(\mathcal{L}_v \sigma' - \iota_{v'} \mathrm{d}\sigma + [s(\omega') \wedge \mathrm{d}\omega]_5\big) \\ & + \big(\mathcal{L}_v \tau' + j\sigma' \wedge \mathrm{d}\lambda + \lambda' \otimes \mathrm{d}\sigma + js(\omega') \wedge \mathrm{d}\omega\big) + \big(\mathcal{L}_v \omega' + \mathrm{d}\lambda \wedge \omega' - (\iota_{v'} + \lambda' \wedge)\mathrm{d}\omega\big) \end{aligned}$$

Generalised Frame and Metric

- Define a generalised metric containing all the bosonic degrees of freedom on M

$$\mathcal{G} \in \frac{E_{7(7)}}{SU(8)}$$

- given a frame and coframe on M

$$\{e^a\} \in TM \qquad \{e_a\} \in T^*M \qquad a = 1, \ldots, 6$$

define a generalised frame $E_A = e^{\tilde{B}} e^{-B} e^{C} e^{\Delta} e^{\phi} \cdot \hat{E}_A$

$$\hat{E}_A = \{\hat{e}_a\} \cup \{e^a\} \cup \{e^{a_1 \ldots a_5}\} \cup \{e^{a, a_1 \ldots a_6}\} \cup \{1\} \cup \{e^{a_1 a_2}\} \cup \{e^{a_1 \ldots a_4}\} \cup \{e^{a_1 \ldots a_6}\}$$

for the generalised tangent bundle

$$E \simeq TM \oplus T^*M \oplus \Lambda^5 T^*M \oplus (T^* \otimes \Lambda^6 T^*) \oplus \Lambda^{\text{even}} T^*M$$

- the generalised metric is

$$\mathcal{G}^{-1} = \delta^{AB} E_A . \otimes E_B$$

CONSISTENT TRUNCATIONS AND EGC

- Consistent truncations are rare and non-trivial
  - often the truncation ansatz is ensured by symmetry
    - | reductions on group manifolds and coset
  - sphere reductions are interesting
    - consistency is not guaranteed by symmetry
    - only S^1, S^3, anf S^7 are parallelisable
    - understanding of such reduction requires explicit use of the U-duality symmetry
      - reformulation with manifest SU(8) symmetry [de Wit and Nicolai 87;...]
      - generalised Scherck-Schwarz reductions in EGG or EFT [lee, strickland-constable, waldram 14; hohm, samtleben 14, ...]
- Higher dimensional orgin of the gauge symmetries in the reduced theory
  - Killing symmetries of the internal manifold
  - p-form potentials

$\Rightarrow$ consider diffeomorphisms and p-form gauge transformations in a unified fashion

ORDINARY SCHERK-SCHWARZ REDUCTiON

- Consider a compactification on a group manifold M_d

$$M_{10} = M_{10-d} \times M_d$$

- the left-invariant vector fields $\{\hat{e}_a\}$ satisfy the algebra

$$\mathcal{L}_{\hat{e}_a} \hat{e}_b = f_{ab}{}^c \hat{e}_c \qquad\qquad f_{ab}{}^c \text{ constant}$$

- $\{\hat{e}_a\}$ generate the (right) isometries of the metric
- $\{\hat{e}_a\}$ give a globally defined frame $\rightarrow M_d$ is parallelisable

- Truncation ansatz
  - define the twisted frame and the internal metric

$$\hat{e}'_a{}^m(x,z) = U_a{}^b(x)\,\hat{e}_b{}^m(z) \qquad U_a{}^b \in GL(d)$$

$$g^{mn}(x,z) = \mathcal{M}^{ab}(x)\,\hat{e}_a{}^m(z)\,\hat{e}_b{}^n(z) \qquad \mathcal{M}^{ab} = \delta^{cd}\,U_c{}^a U_d{}^b \in GL(d)/\,SO(d)$$

  - write the 10d SUGRA fields as

$$\mathrm{d}s^2 = g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathcal{M}_{ab}(x)(e^a - \mathcal{A}^a_\mu(x)\mathrm{d}x^\mu)(e^b - \mathcal{A}^b_\nu(x)\mathrm{d}x^\nu)$$

$$C_1(x,z) = C_\mu(x)\mathrm{d}x^\mu + C_a(x)(e^a - \mathcal{A}^a_\mu(x)\mathrm{d}x^\mu) + c_1$$

$$C_3(x,z) = \dots$$

  where $g_{\mu\nu}$, $\mathcal{M}_{ab}$, $\mathcal{A}^a_\mu$, etc are $D-d$-dimensional fields

- Features of the truncated theory
  - the reduction is consistent by symmetry
  - being parallelisable M_d has globally defined spinors $\Rightarrow$ maximal SUSY
  - the gauge group comes from Killing symmetries and gauge transformations of the potentials.

GENERALISED SCHERCK-SCHWARZ REDUCTION

- Extend to EGG the notion of parallelisability $\rightarrow$ Generalised Leibniz parallelisation
  - $\exists$ a globally defined frame $\{E_A\}$ for the $E_{d+1(d+1)} \times \mathbb{R}^+$ generalised tangent bundle on M_d.
  - the frame must satisfy the algebra

$$L_{E_A} E_B = X_{AB}{}^C E_C \qquad\qquad X_{AB}{}^C \text{ constant}$$

  - $X_{AB}{}^C$ generate the gauge algebra

$$[X_A, X_B] \;=\; -X_{AB}{}^C X_C$$

  and are related to the embedding tensor of the $D-d$ gauged supergravity

$$X_{AB}{}^C = \Theta_A{}^\alpha (t_\alpha)_B{}^C$$

  embedding tensor $\hookleftarrow$ $\hookrightarrow$ U-duality generators

  - GLP implies the manifold is a coset $M_d \sim G/H$ and maximal SUSY

- Truncation ansatz
  - decompose all 10d fields according to $SO(1,9) \to SO(1,9-d) \times SO(d)$

$$
\begin{aligned}
\mathrm{d}s^2 &= e^{2\Delta} g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}(x,z) Dz^m Dz^n \\
B &= \tfrac{1}{2} B_{m_1 m_2}(x,z) Dz^{m_1} \wedge Dz^{m_2} + \overline{B}_{\mu m}(x,z)\mathrm{d}x^\mu \wedge Dz^m + \tfrac{1}{2}\overline{B}_{\mu\nu}(x,z)\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \\
C_1 &= C_m Dz^m + \overline{C}_\mu \, \mathrm{d}x^\mu \\
&\dots
\end{aligned}
$$

$$
Dz^m = \mathrm{d}z^m - h_\mu^m(x,z)\mathrm{d}x^\mu
$$

  - field redefinition in the lower dimensional theory $\to$ covariance under gen diffeos

$$
\begin{aligned}
B_\mu &= \overline{B}_\mu & B_{\mu\nu} &= \overline{B}_{\mu\nu} + \iota_{h_{[\mu}} B_{\nu]} \\
C_\mu &= e^{-B} \wedge \overline{C}_\mu & \tilde{B}_{\mu\nu} &= \overline{\tilde{B}}_{\mu\nu} - \tfrac{1}{2}\big[\,\overline{C}_{\mu\nu} \wedge s(C)\,\big]_4 + \iota_{h_{[\mu}} \tilde{B}_{\nu]} \\
\tilde{B}_\mu &= \overline{\tilde{B}}_\mu - \tfrac{1}{2}[\,\overline{C}_\mu \wedge s(C)\,]_5 & C_{\mu\nu} &= e^{-B} \wedge \overline{C}_{\mu\nu} + \iota_{h_{[\mu}} C_{\nu]} + B_{[\mu} \wedge C_{\nu]}
\end{aligned}
$$

    - B, C are field on the internal manifold
    - reproduce the gauge transformations of the lower-dimensional supergravity

- arrange fields with the same external indices in $E_{d+1(d+1)} \times \mathbb{R}^+$ tensors
  - scalars $\rightarrow$ gen metric

$$\{g_{mn},\, B_{m_1m_2},\, \tilde{B}_{m_1\ldots m_6},\, C_m,\, C_{m_1m_2m_3},\, C_{m_1\ldots m_5}\}$$

  - vectors $\rightarrow$ gen vector ${\mathcal{A}_\mu}^M \in E$

$${\mathcal{A}_\mu}^M = \{{h_\mu}^m,\, B_{\mu m},\, \tilde{B}_{\mu m_1\ldots m_5},\, C_\mu,\, C_{\mu m_1m_2},\, C_{\mu m_1\ldots m_4},\, C_{\mu m_1\ldots m_6}\}$$

  - two-forms $\rightarrow$ ${\mathcal{B}_{\mu\nu}}^{MN} \in N' \subset (E \otimes E)_{\text{sym}}$

$${\mathcal{B}_{\mu\nu}}^{MN} = \{B_{\mu\nu},\, \tilde{B}_{\mu\nu m_1\ldots m_4},\, C_{\mu\nu m},\, C_{\mu\nu m_1m_2m_3},\, C_{\mu\nu m_1\ldots m_5}\}$$

- twist the generalised frame and metric

$$E'_A{}^M(x,z) = U_A{}^B(x) E_B{}^M(z) \qquad \mathcal{G}^{MN}(x,z) = \mathcal{M}^{AB}(x) E_A{}^M(z) E_B{}^N(z)$$

$$E_{d+1(d+1)} \hookleftarrow \qquad \qquad \hookrightarrow \frac{E_{d+1(d+1)}}{K}$$

with K maximally compact subgroup of $E_{d+1(d+1)}$

- write the 10d fields as

$$\mathcal{A}_\mu{}^M(x,z) = \mathcal{A}_\mu{}^A(x) E_A{}^M(z)$$
$$\mathcal{B}_{\mu\nu}{}^{MN}(x,z) = \frac{1}{2} \mathcal{B}_{\mu\nu}{}^{AB}(x) (E_A \otimes_{N'} E_B)^{MN}(z)$$

a similar construction should work for higher rank forms in $D-d$-dimensions

- Partial check of the consistency of the reduction $\rightarrow$ recover the gauge transformations of the lower dimensional gauged SUGRA

GENERALISED SPHERE REDUCTIONS

• Apply generalised Scherk-Schwarz reduction to sphere compactifications in massless and massive type IIA

	$m = 0$		$m \neq 0$	
S^6	$ISO(7)$	[guarino,varela 15]	$ISO(7)_m$	[guarino,varela 15]
S^4	$SO(5)$	[cowdall 88; cvetic et al 00]	–	
S^3	$ISO(4)$	[malek, samtleben 15]	–	
S^2	$SO(3)$	[salam, sezgin 85]	–	

$\Rightarrow$ only S^6 admits a parallelisation for massive IIA

- Sphere backgrounds and generalised parallelisable [lee, strickland-constable, waldram 14]
  - a $S^d = SO(d+1)/SO(d)$ backgrounds has a d-form $F_d = \mathrm{d}A_{d-1}$
  - define the gen. tangent bundle where the twisting is given by A_{d-1}

$$E_{GL(d+1)} = T \oplus \Lambda^{d-2}T^*$$

    - $E_{GL(d+1)}$ is in $\frac{1}{2}d(d+1)$ dim. bivector representation of $GL(d+1,\mathbb{R})$
      $\Rightarrow GL(d+1,\mathbb{R})$ generalised geometry
    - $E_{GL(d+1)}$ can be see as

$$E_{GL(d+1)} = W \wedge W \qquad W \sim (\det T^*)^{1/2}(T + \Lambda^d T)$$

W is the fundamental of $GL(d+1)$.

  - $E_{GL(d+1)}$ always admits a globally defined frame

$$E_{ij} = v_{ij} + \frac{R^{d?2}}{(d-2)!}\epsilon_{ijk_1\ldots k_{d-1}} y^{k_1}\mathrm{d}y^{k_2} \wedge \ldots y^{k_{d-1}} + \iota_{v_{ij}} A$$

$SO(d+1)$ killing vectors $\hookleftarrow$ $\hookrightarrow$ constrained coordinates on $\mathbb{R}^d$ $\delta_{ij}y^i y^j = 1$

such that $E_{ij} = E_i \wedge E_j$ with E_j frame on W

- For massless IIA the full $E_{d+1(d+1)}$ tangent bundle is parallelisable
  - define a frame

$$E_A = E_{ij} + \sum E_B \qquad E_k \in \text{ other } GL(d+1) \text{ reprs}$$

  such that

$$L_{E_A} E_B = X_{AB}{}^C E_C \;, \qquad [X_A, X_B] = -X_{AB}{}^C X_C$$

- For massive IIA the frame the algebra of the massless frame does not close

$$X^C_{AB} \to X^C_{AB} + E_A{}^M E_B{}^N E^C{}_P m^P_{MN}$$

  - to have a GLP we would need

$$E_A{}^M E_B{}^N E^C{}_P m^P_{MN} = const$$

  - this is possible if $E_A \in G_m$ (stabilises the Romans mass) $\to$ true only for S^6

d	6	5	≤ 4
G_m	$GL(7)$	$SL(5) \times SL(2) \times \mathbb{R}^+$	$GL(d) \times \mathbb{R}^+$

LESS SUPERSYMMETRY

- G-structure on TM
  - given the structure group $K \subset GL(d)$ and the metric structure $SO(d)$
    - scalar manifold $H \in \frac{C_{GL(d)}(K)}{C_{SO(d)}(K)}$ $C_G(K) \to$ commutant of K in G
    - vector fields $A^a k_a$ $k_a \to$ globally defined vectors on TM
    - gauge group $[k_a, k_b] = f^c_{ab} k_c$ $f^c_{ab} \to$ K-singlets of the intrinsic torsion
  - Examples

	K	H	k_a
group manifold	$\mathbb{I} \subset GL(d)$	$H \in \frac{GL(d)}{SO(d)}$	$\hat{e}_a$ frame on TM
Sasaki Einstein	$SU(n) \subset GL(2n+1)$	$H \in \frac{\mathbb{C} \times \mathbb{R}^+}{U(1)}$	$\xi = \partial_\psi$ Reeb vector

- Do the same for generalised structure on E

- Example: Half-maximal truncation of $d = 11$ supergravity on tri-Sasakian manifolds
  - tri-Sasakian manifolds are $3d$ foliations over a Quaternionic Kähler space
  - $SU(2)$ structure manifold

3 vectors	$\rightarrow$	ξ_I			$\iota_{\xi_I}\eta^J = \delta^{IJ}$
3 dual one-forms	$\rightarrow$	η^I	$I = 1,2,3$	such that	$\iota_{\xi_I}J^J = 0$
3 two-forms	$\rightarrow$	J^I			$J^I \wedge J^J = 2\delta^{IJ}\mathrm{vol}_{\mathrm{QK}}$

and

$$[\xi_J, \xi_K] = 2\,\epsilon^I{}_{JK}\xi_I \qquad \begin{aligned} \mathrm{d}\eta^I &= 2J^I - \epsilon^I{}_{JK}\eta^J \wedge \eta^K \\ \mathrm{d}J^I &= 2\epsilon^I{}_{JK}J^J \wedge \eta^K \end{aligned}$$

  - the consistent truncation gives $\mathcal{N} = 4$ gauged sugra in 4d with three vector multiplets and $SO(3) \ltimes_{\mathbf{3}\oplus\mathbf{3}}$ nil$_{(6,3)}$ gauging [Cassani, Koerber 11]

- Embed the truncation in exceptional geometry
  - determine the scalar manifold
    - generalised tangent space of M_7

$$E = TM \oplus \Lambda^2 T^* M \oplus \Lambda^5 T^* M \oplus T^* M \otimes \Lambda^7 T^* M$$

    - structure group $E_{7(7)}$
    - generalised metric defines an $SU(8)$ structure in $E_{7(7)}$
    - compute the commutant of $SU(2)$ in $SU(8)$ and in $E_{7(7)}$

$$\frac{SL(2,\mathbb{R})}{SO(2)} \times \frac{SO(6,3)}{SO(6) \times SO(3)}$$

scalar manifold for $\mathcal{N} = 4$ sugra with three vector multiplets

- determine the gauge fields → generalised vectors that are singlets under $SU(2)$
  - $SU(2)$ embeds as

$$E_{7(7)} \to SU(8) \to SU(6) \to SU(3)_{diag} \to SU(2)$$

  - the **56** of $E_{7(7)}$ splits as

$$\mathbf{58} = \mathbf{28} + \overline{\mathbf{28}} \qquad \Longleftrightarrow \qquad \text{electric } A_\mu^{M+} \text{ and magnetic } A_\mu^{M-} \text{ vectors in } d=4 \text{ sugra}$$

  - each **28** gives

$$\mathbf{28} = \mathbf{1} + \mathbf{6} + \overline{\mathbf{6}} + \mathbf{15} = 2\times\mathbf{1} + 3\times\mathbf{3} + 3\times\overline{\mathbf{3}} + \mathbf{8} = 9\times\mathbf{1} + 8\times\mathbf{2} + \mathbf{3}$$

  - the generlised vectors are

$$\hat{k}_{M\alpha} \quad \to \begin{cases} \text{fundamental of } SO(6,3) \\ \text{doublets of } SO(1,1) \subset SL(2,\mathbb{R}) \end{cases}$$

  - in terms of $SU(2)$ invariant tensors ($I, J, K = 1,2,3$)

$$\hat{k}_{I,+} = J_I\,, \qquad \hat{k}_{3+I,+} = \eta_I \wedge 2\mathrm{vol}_{\mathrm{QK}}\,, \qquad \hat{k}_{6+I,+} = \xi_I$$
$$\hat{k}_{I,-} = \eta_1 \wedge \eta_2 \wedge \eta_3 \wedge J_I\,, \qquad \hat{k}_{3+I,-} = -2\,\eta_I \otimes \mathrm{vol}_{3\mathrm{S}}\,, \qquad \hat{k}_{6+I,-} = \tfrac{1}{2}\epsilon_I{}^{JK}\eta_J \wedge \eta_K\,,$$

- determine the gauge group → Dorfman derivative
  - the gauge generators

$$D_\mu = \nabla_\mu - g A_\mu^{M\alpha} \Theta_{\alpha M}{}^{NP} t_{NP} + g A_\mu^{M(\alpha} \epsilon^{\beta)\gamma} \xi_{\gamma M} t_{\alpha\beta}$$
$$D_\mu = \nabla_\mu - g A_\mu^{M\alpha} X_{M\alpha} + g A_\mu^{M\alpha} Y_{M\alpha}$$

are determined by embedding tensor of $\mathcal{N} = 4$ sugra [schon, weidner 06]

$$(\xi_{\alpha M}, f_{\alpha MNP}) \quad \Longleftrightarrow \quad \begin{cases} \Theta_{\alpha MNP} = f_{\alpha MNP} - \xi_{\alpha[N}\eta_{P]M} \\ \hat{f}_{\alpha MNP} = f_{\alpha MNP} - \xi_{\alpha[M}\eta_{P]N} - \frac{3}{2}\xi_{\alpha N}\eta_{MP} \end{cases}$$

  - Dorfman derivative

$$L_{\hat{k}_{M\alpha}} \hat{k}_{N\beta} = X_{(M\alpha)(N\beta)}{}^{(P\gamma)} \, \hat{k}_{P\gamma}$$

gives the embedding tensor

$$(X_{M\alpha})_{N\beta}{}^{P\gamma} = X_{(M\alpha)(N\beta)}{}^{(P\gamma)} = -\delta^\gamma_\beta f_{\alpha MN}{}^P \qquad (\xi_{M\alpha} = 0)$$

  - in a basis of only electric gaugings the algebra is $so(3) \ltimes_{\mathbf{3}\oplus\mathbf{3}} \mathrm{nil}_{(6,3)}$

$$[X_I, X_J] = 2\,\epsilon_{IJ}{}^K X_K\,, \qquad . \qquad [X_I, \tilde{X}_{(J+6)}] = 2\,\epsilon_{IJ}{}^K \tilde{X}_{(K+6)}$$
$$[\tilde{X}_{(I+3)}, \tilde{X}_{(J+3)}] = 2\,\epsilon_{IJ}{}^K \tilde{X}_{(K+6)} \qquad [X_I, \tilde{X}_{(J+3)}] = 2\,\epsilon_{IJ}{}^K \tilde{X}_{(K+3)}$$

SUMMARY AND OUTLOOK

- Generalised Geometry is unified framework to study
  - compactifications, effective actions, gauge/gravity duality, consistent truncations
- We developed an exceptional geometry for type IIA
  - introduction of the Romans mass as deformation of the Dorfman derivative
- Construction of generalised Scherk-Schwarz reduction of type IIA supergravity
  - explicit ansatz for vectors and two-forms
  - partial proof of consistency $\rightarrow$ factorisation of gauge transformations of the bosonic fields
  - apply it to sphere reductions
    - hints at a no-go theorem for massive truncations with maximal SUSY for $d \leq 6$

[see also Ciceri, Guarino, Inverso, 1604.08602 in EFT]

- Extension of the formalism to truncations with less SUSY
  - use of generalised structures [see also Malek, 1707.00714 in EFT]
  - interesting applications to AdS/CFT