Connecting low-energy Dark Matter searches with high-energy physics: *the role of operator mixing*

Bradley J. Kavanagh - LPTHE (Paris)

F. D'Eramo, **B. J. Kavanagh**, P. Panci, JHEP (2016) 2016: 111, arXiv:1605.04917

F. D'Eramo, **B. J. Kavanagh**, P. Panci, PLB 771 (2017) 339-348, arXiv:1702.00016

IRN Terascale, Montpellier - 4th July 2017

erc NewDark

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Complementarity of Dark Matter searches: Direct detection vs Indirect detection vs LHC

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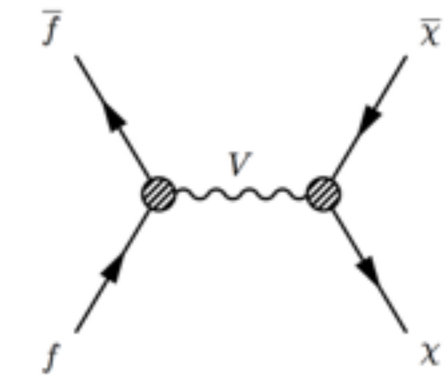
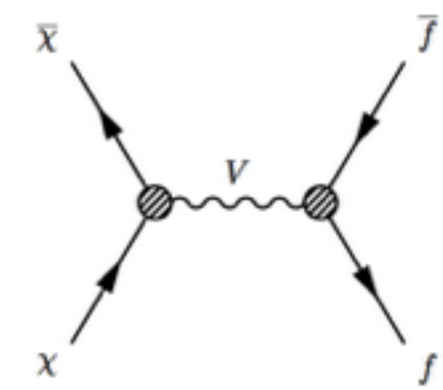
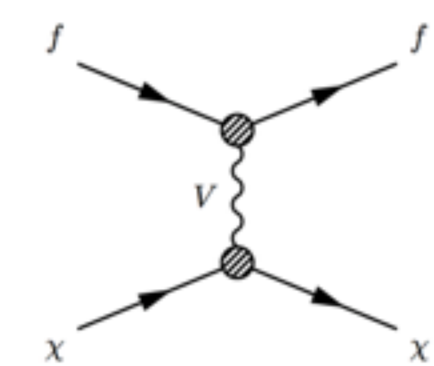
Collider searches

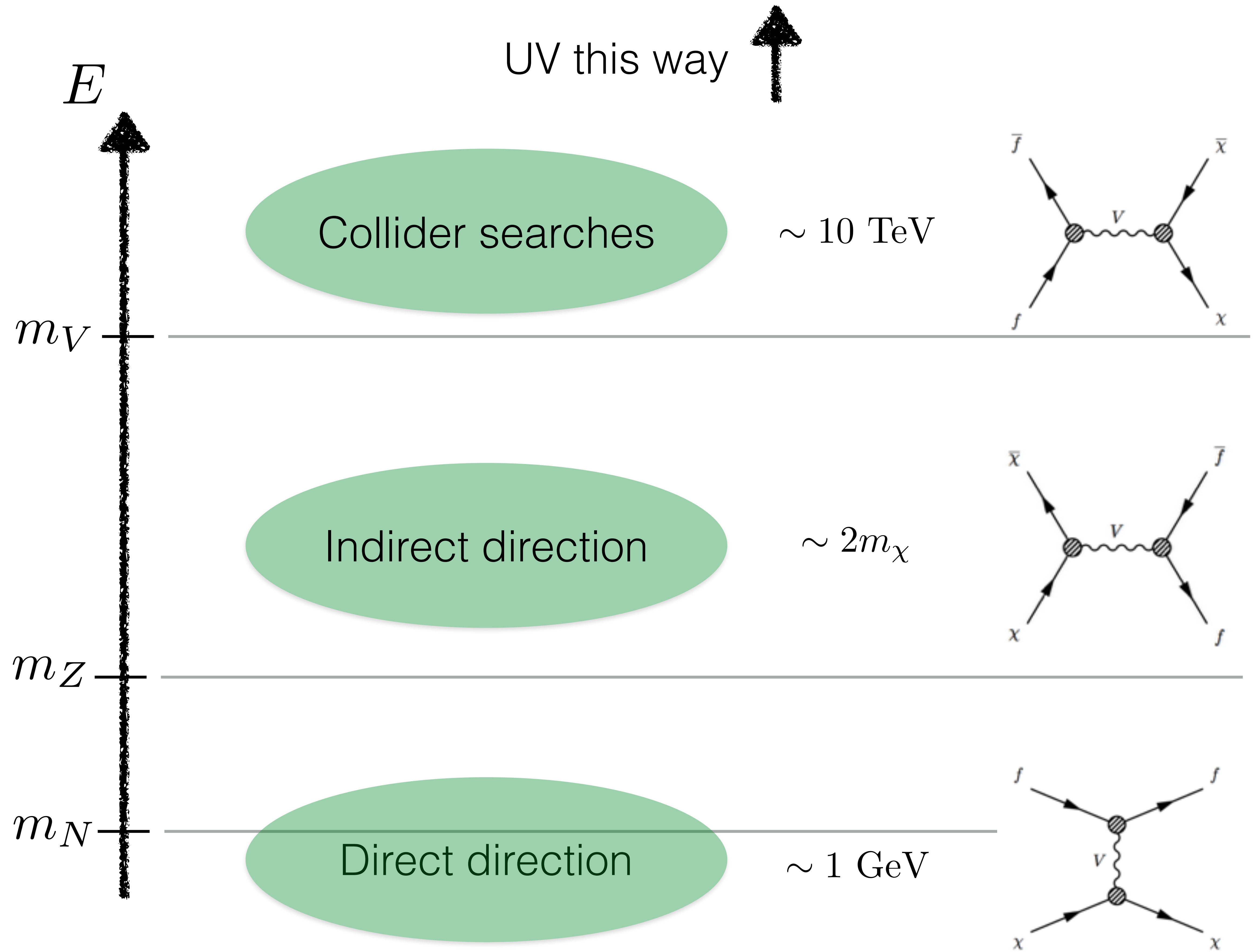


Indirect direction



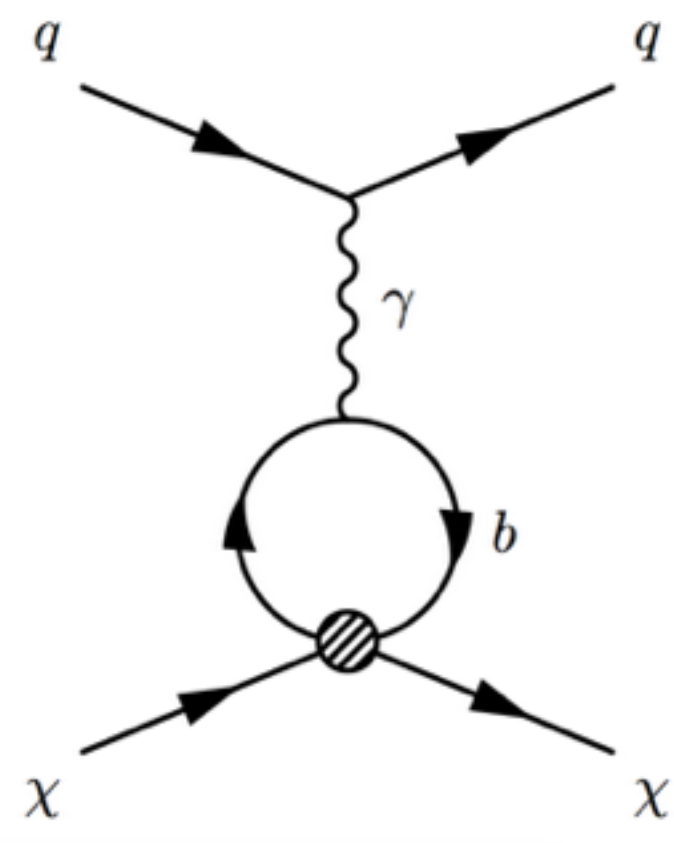
Direct direction




UV this way
E
m_V
m_Z
m_N
Collider searches
~ 10 TeV
Indirect direction
$\sim 2m_\chi$
Direct direction
~ 1 GeV
$\bar{f}$ $\bar{\chi}$ V f χ
$\bar{\chi}$ $\bar{f}$ V χ f
f f V χ χ

Connect scales carefully and consistently [RUNDM code] accounting for RG and operator mixing in the *SM sector*

e.g.



Gives corrections to direct detection bounds

e.g. mapping from LHC to DD

Gives bounds you never thought were there

e.g. leptophilic DM

Outline

RG effects in Simplified Models

Crivellin, D'Eramo, Procura [1402.1173]; D'Eramo, Procura [1411.3342]

Direct detection constraints on Simplified Models

D'Eramo, Procura [1411.3342]; D'Eramo, BJK, Panci [1605.04917]

LHC vs Direct Detection

D'Eramo, BJK, Panci [1605.04917]

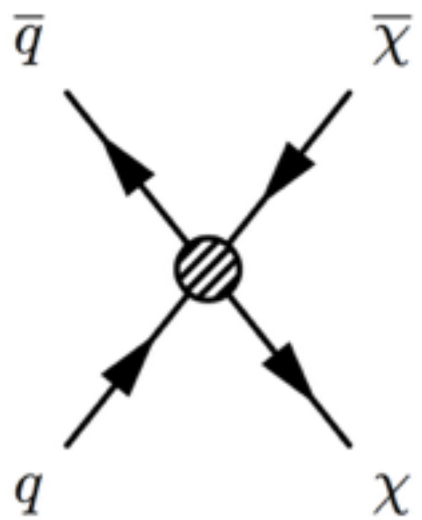
Leptophilic Dark Matter

D'Eramo, BJK, Panci [1702.00016]

Simplified Models

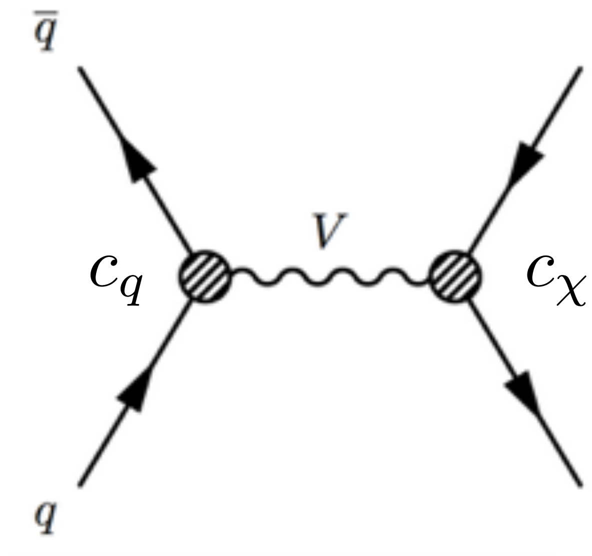
Review: De Simone, Jacques [1603.08002]

EFT



m_χ, Λ

Simplified Model



c_q, c_χ, m_χ, m_V

For direct detection, momentum transfer is small, q ~ 100 MeV: EFT okay

For colliders, momentum transfer is large, q ~ O(TeV): EFT may break down!

Frandsen et al. [1204.3839], Buchmueller et al. [1407.8257], Malik et al. [1409.4075], Abdallah et al. [1506.03116], and many others…

Our Simplified Model

Assume fermion Dark Matter χ and a new (massive) vector mediator V

$$\mathcal{L} = \mathcal{L}_\mathrm{SM} + \mathcal{L}_\mathrm{DM} + \mathcal{L}_\mathrm{V} + J^\mu_\mathrm{DM} V_\mu + J^\mu_\mathrm{SM} V_\mu$$

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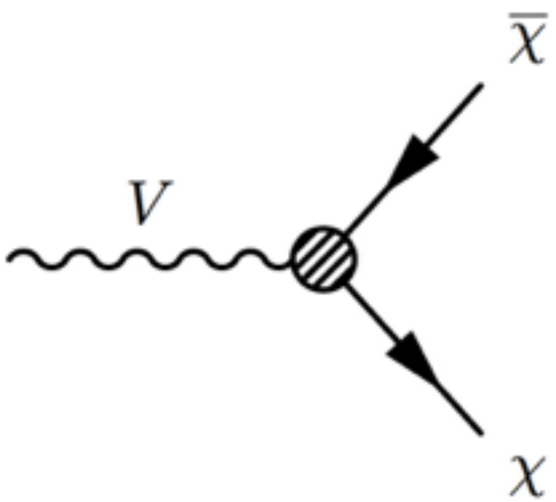
$$J^\mu_\mathrm{DM} = c_{\chi V} \overline{\chi} \gamma^\mu \chi + c_{\chi A} \overline{\chi} \gamma^\mu \gamma^5 \chi$$



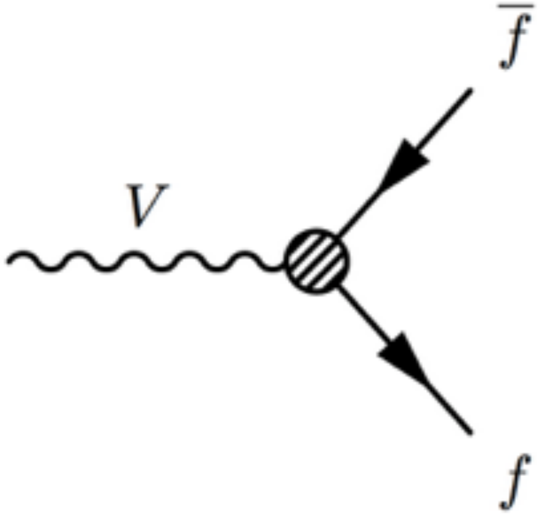
Our Simplified Model

Assume fermion Dark Matter χ and a new (massive) vector mediator V

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm DM} + \mathcal{L}_{\rm V} + J^\mu_{\rm DM} V_\mu + J^\mu_{\rm SM} V_\mu$$

$$J^\mu_{\rm SM} = \sum_{i=1}^{3} \left[c_q^{(i)}\,\overline{q^i_L}\gamma^\mu q^i_L + c_u^{(i)}\,\overline{u^i_R}\gamma^\mu u^i_R + c_d^{(i)}\,\overline{d^i_R}\gamma^\mu d^i_R + c_l^{(i)}\,\overline{l^i_L}\gamma^\mu l^i_L + c_e^{(i)}\,\overline{e^i_R}\gamma^\mu e^i_R\right]$$

15 independent, $SU(2)_L \times U(1)_Y$ gauge-invariant couplings

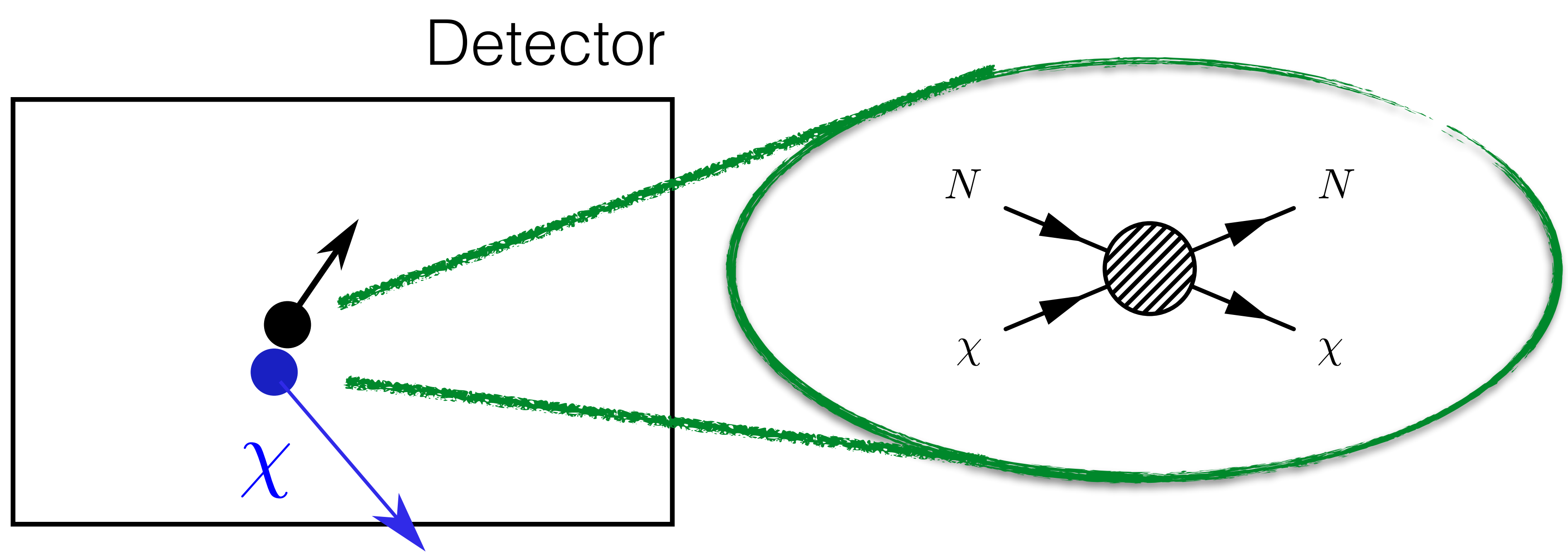


Direct detection

Look for low energy - O(keV) - recoils of detector nuclei…

$m_\chi \gtrsim 1$ GeV

$v \sim 10^{-3}$



Rate driven by coupling of DM to light quarks (u, d, s):

$$\overline{\chi}\gamma^\mu\chi\,\overline{q}\gamma_\mu q \qquad \overline{\chi}\gamma^\mu\chi\,\overline{q}\gamma_\mu\gamma^5 q$$

$$\overline{\chi}\gamma^\mu\gamma^5\chi\,\overline{q}\gamma_\mu q \qquad \overline{\chi}\gamma^\mu\gamma^5\chi\,\overline{q}\gamma_\mu\gamma^5 q$$

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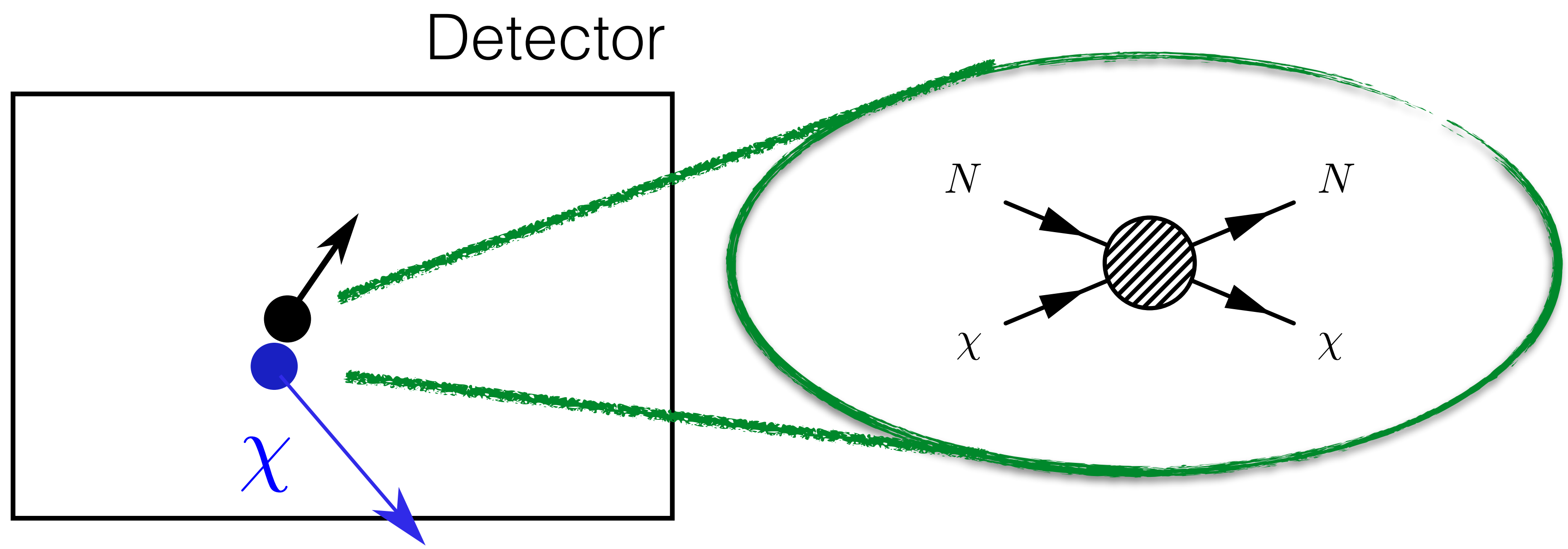

Rate driven by coupling of DM to light quarks (u, d, s):

Standard SI $\overline{\chi}\gamma^\mu\chi\,\overline{q}\gamma_\mu q$

$\overline{\chi}\gamma^\mu\chi\,\overline{q}\gamma_\mu\gamma^5 q$

$\overline{\chi}\gamma^\mu\gamma^5\chi\,\overline{q}\gamma_\mu q$

$\overline{\chi}\gamma^\mu\gamma^5\chi\,\overline{q}\gamma_\mu\gamma^5 q$ Standard SD

Direct detection

Look for low energy - O(keV) - recoils of detector nuclei…

$$m_\chi \gtrsim 1\ \mathrm{GeV}$$
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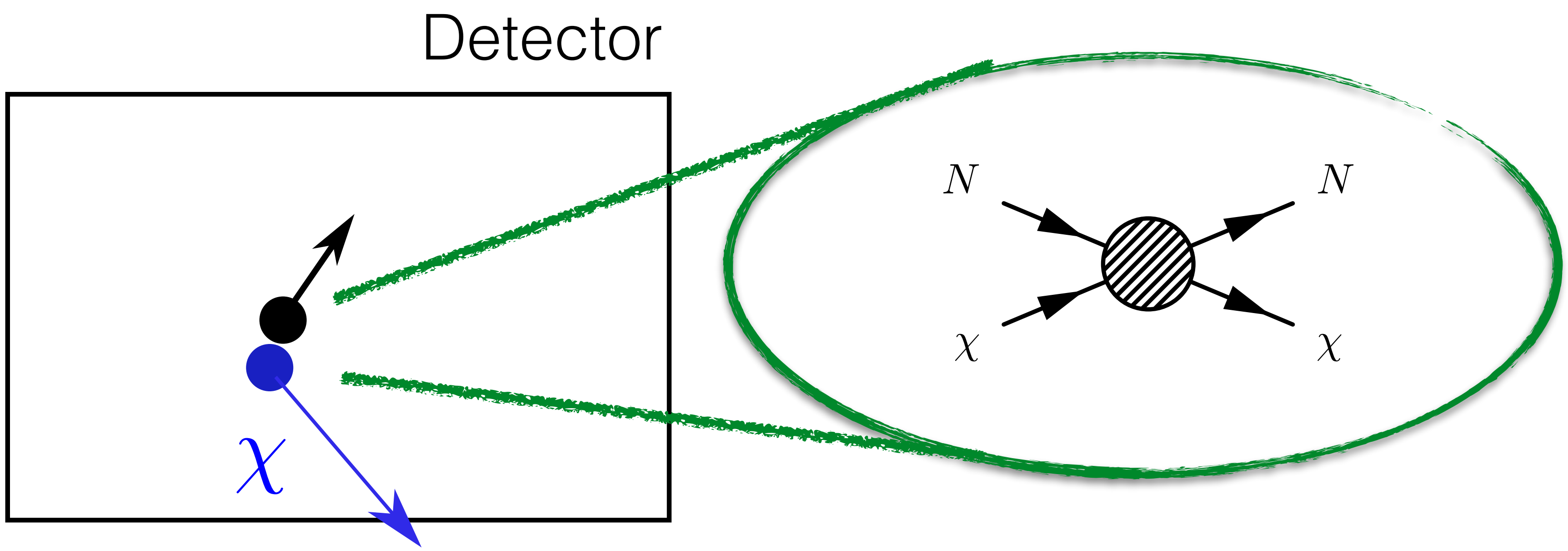

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$$\overline{\chi}\gamma^\mu\gamma^5\chi\, \overline{q}\gamma_\mu q \qquad \overline{\chi}\gamma^\mu\gamma^5\chi\, \overline{q}\gamma_\mu\gamma^5 q$$

Velocity suppressed

Connecting high and low scales

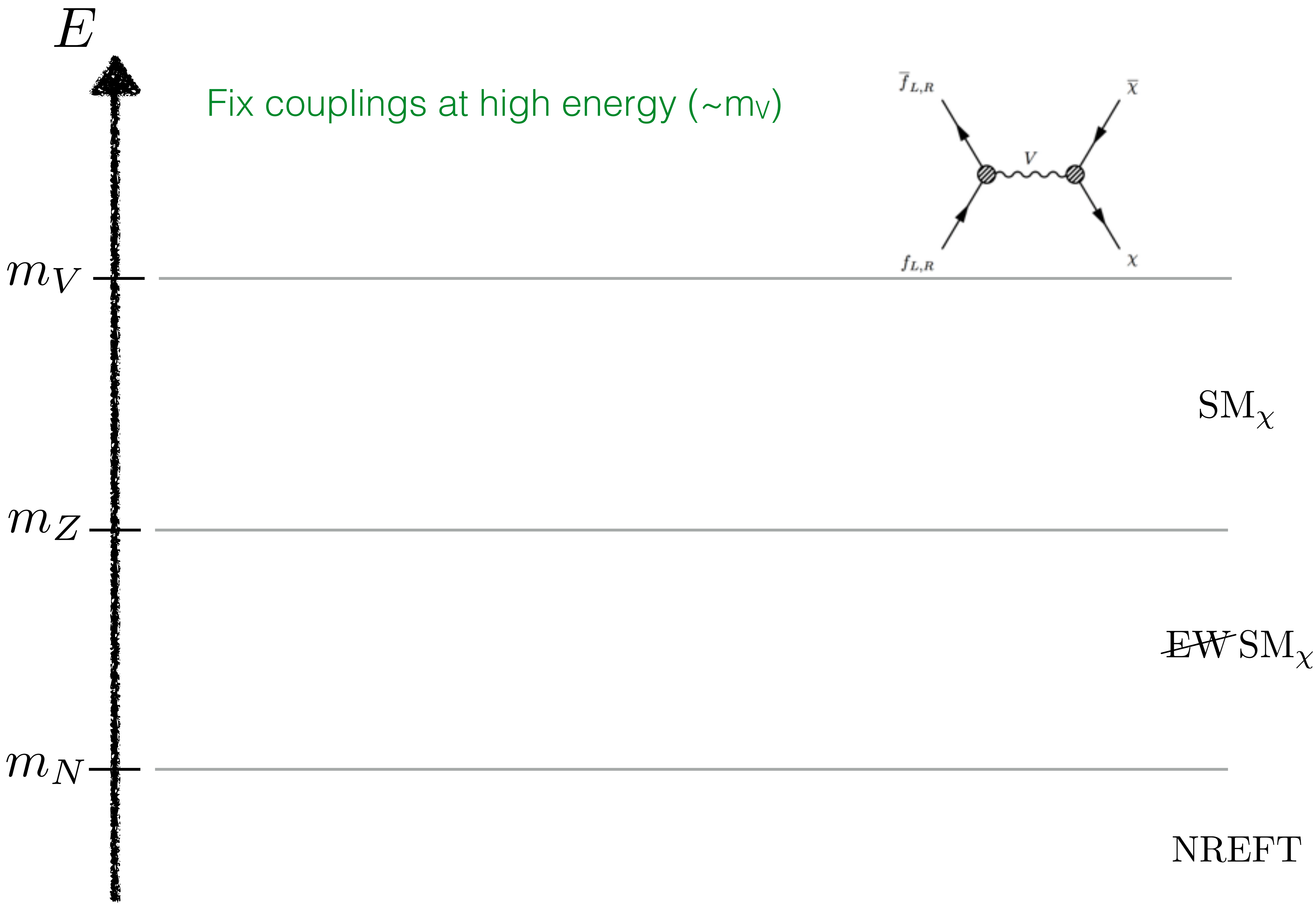
Define couplings at high energy scale (mediator mass), but need to calculate direct detection rate at low energy

Running can change the DD rate by orders of magnitude. Examples in specific models:

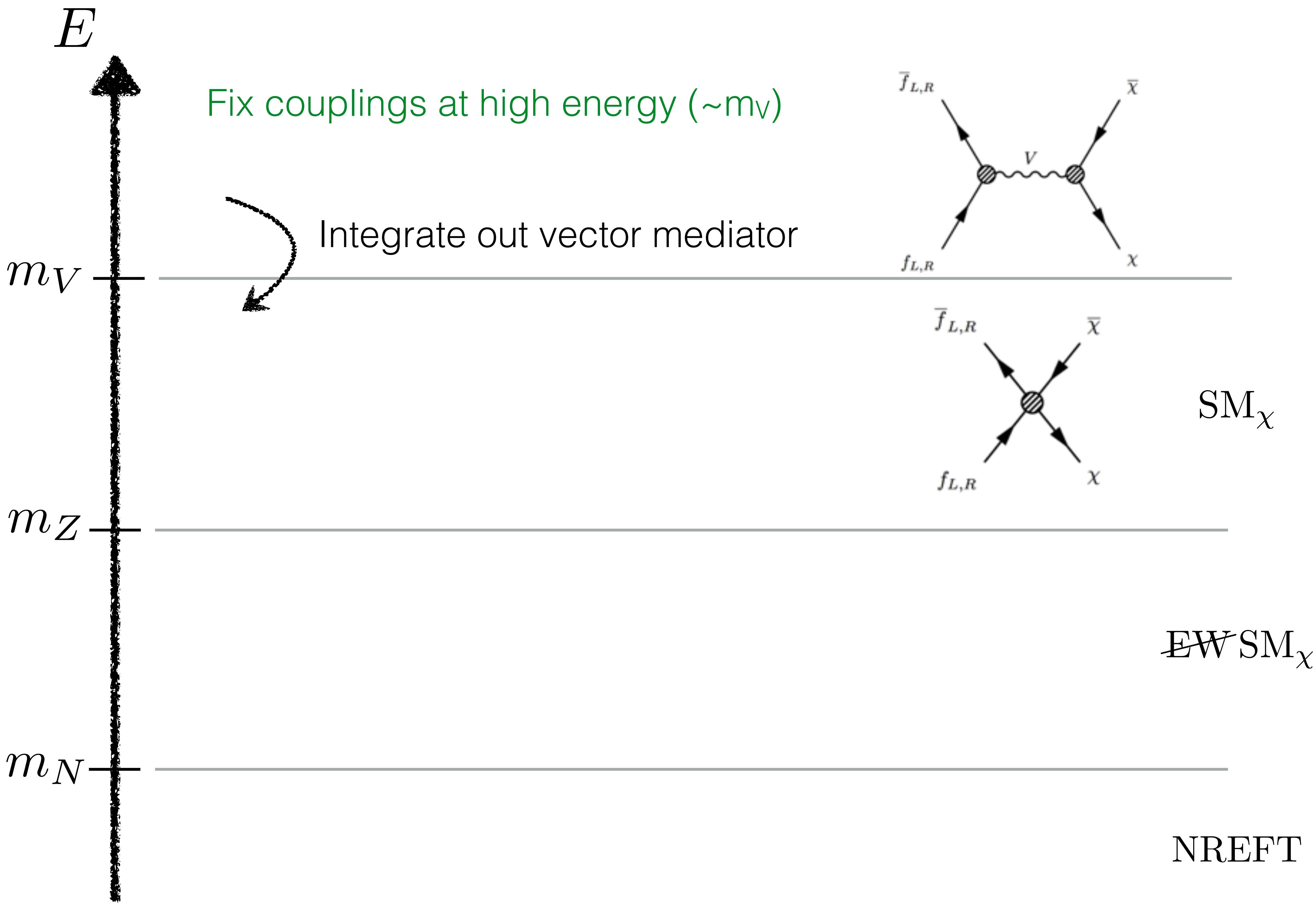
Kopp et al. [0907.3159], Frandsen et al. [1207.3971], Haisch, Kahlhoefer [1302.4454], Kopp et al. [1401.6457], Crivellin, Haisch [1408.5046]

- Use EFT techniques and RG flow to study the effects for general interactions
- Include all relevant DD interactions (not just naive ‘leading order’)

Fix couplings at high energy (~m_V)



D'Eramo, Procura [1411.3342]



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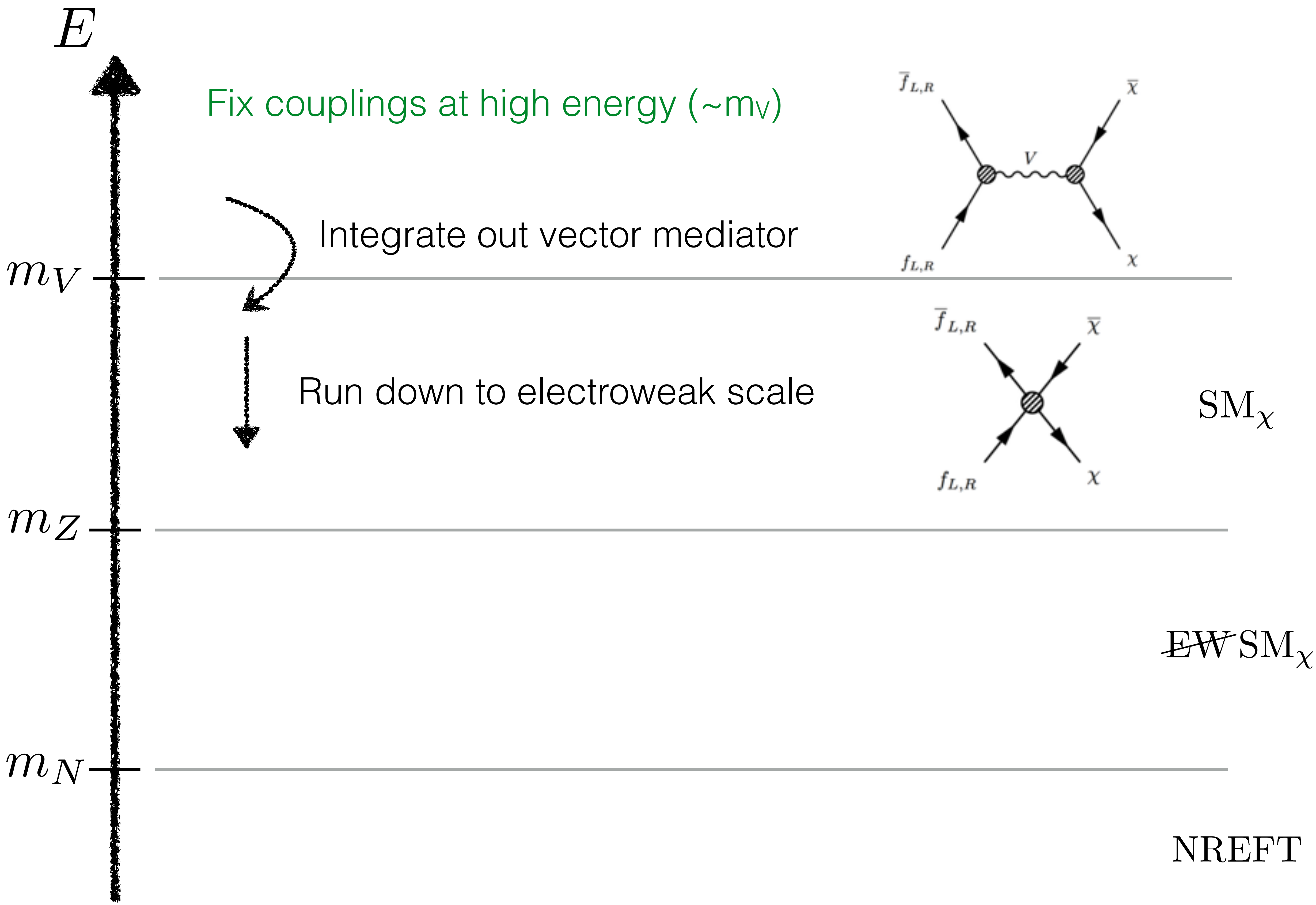

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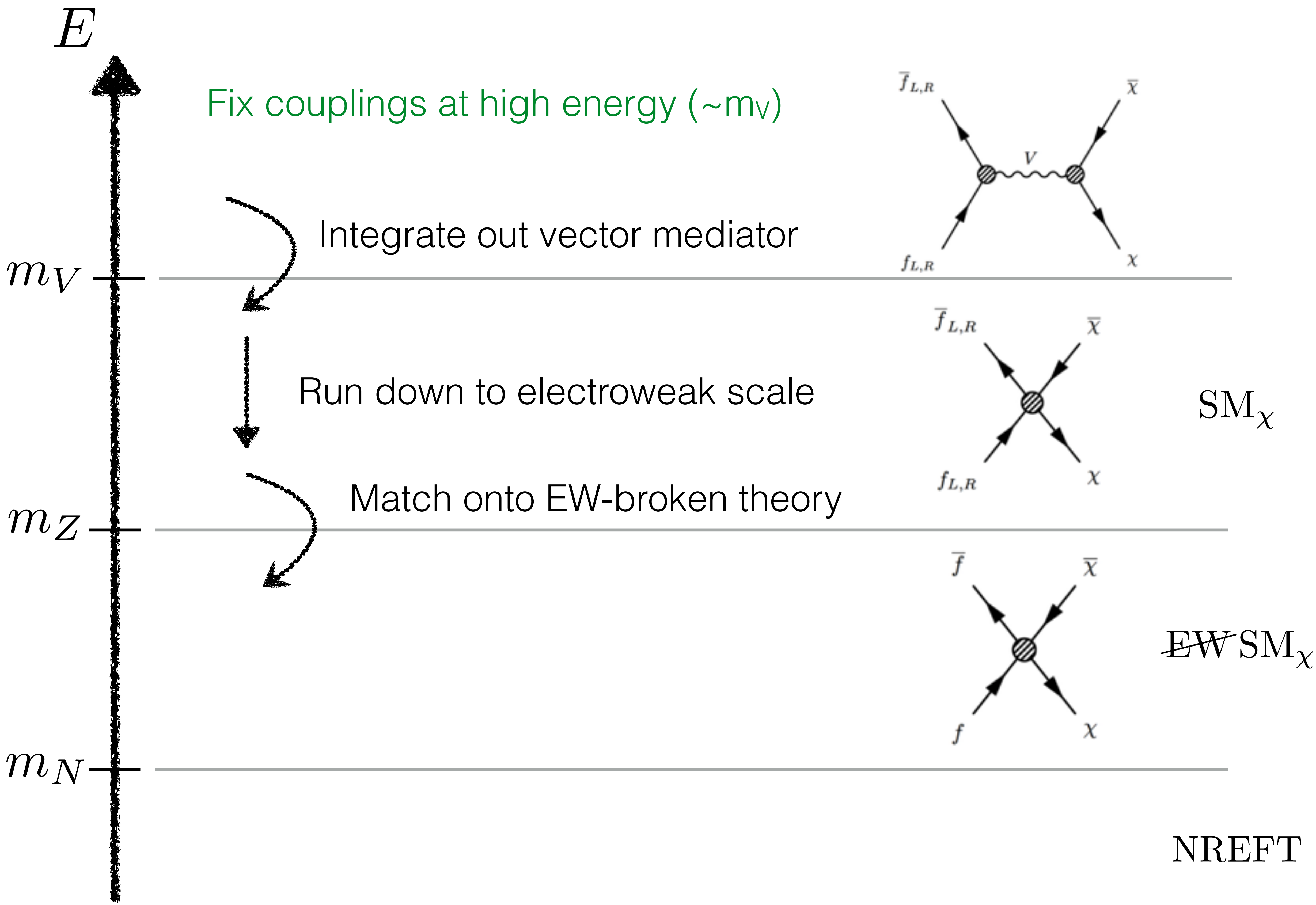

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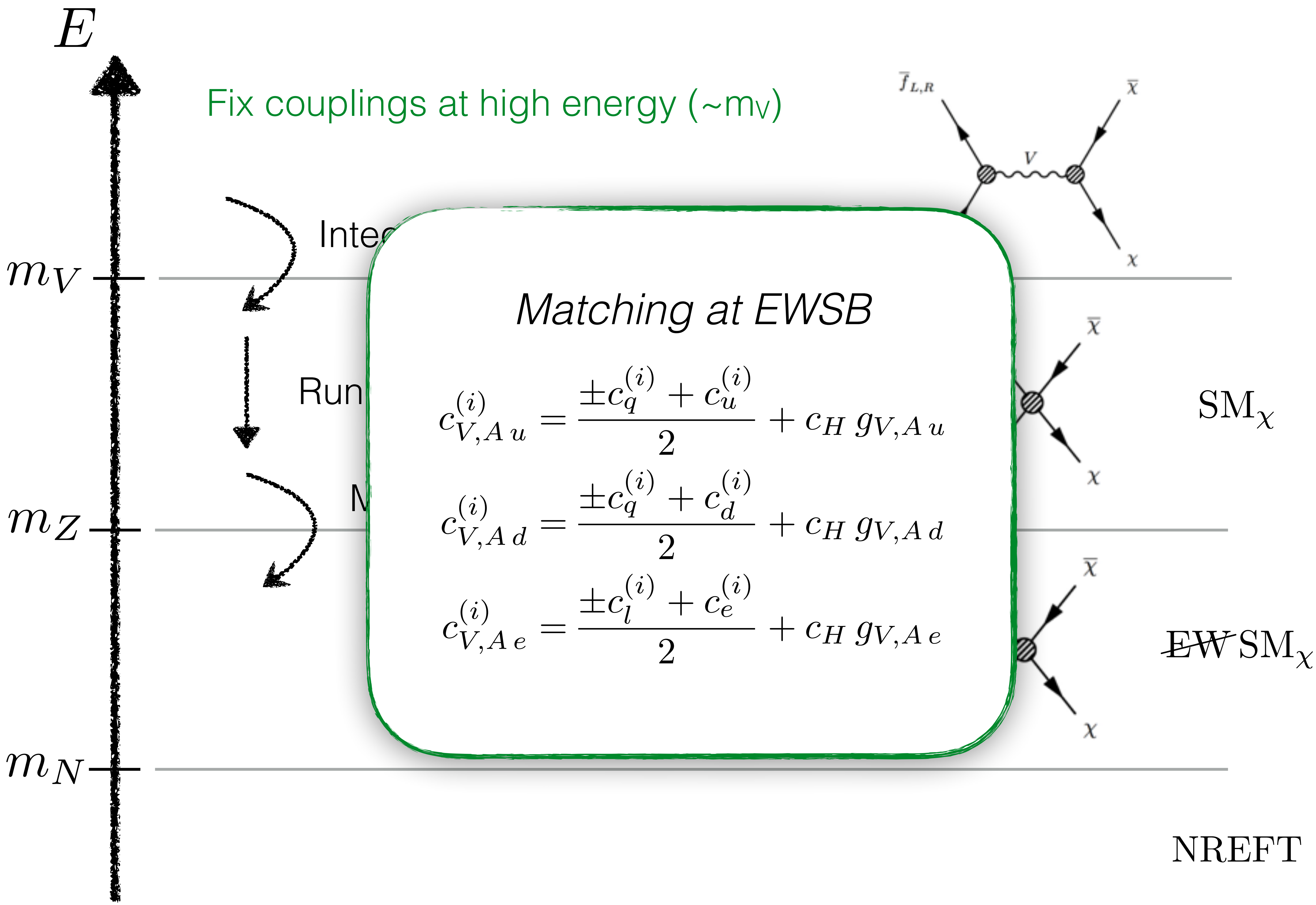


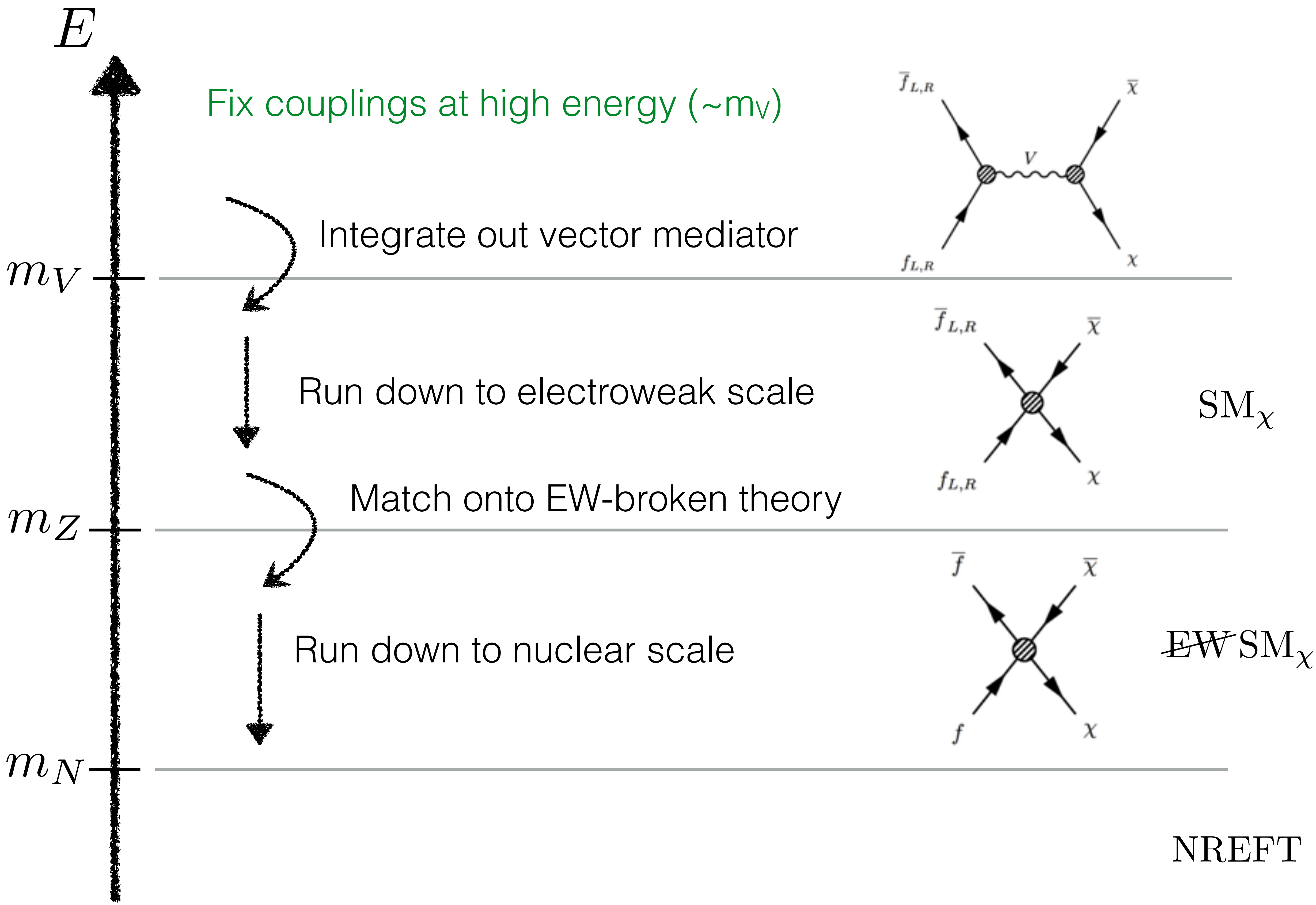
Matching at EWSB

$$c_{V,A\,u}^{(i)} = \frac{\pm c_q^{(i)} + c_u^{(i)}}{2} + c_H\, g_{V,A\,u}$$

$$c_{V,A\,d}^{(i)} = \frac{\pm c_q^{(i)} + c_d^{(i)}}{2} + c_H\, g_{V,A\,d}$$

$$c_{V,A\,e}^{(i)} = \frac{\pm c_l^{(i)} + c_e^{(i)}}{2} + c_H\, g_{V,A\,e}$$

D'Eramo, Procura [1411.3342]



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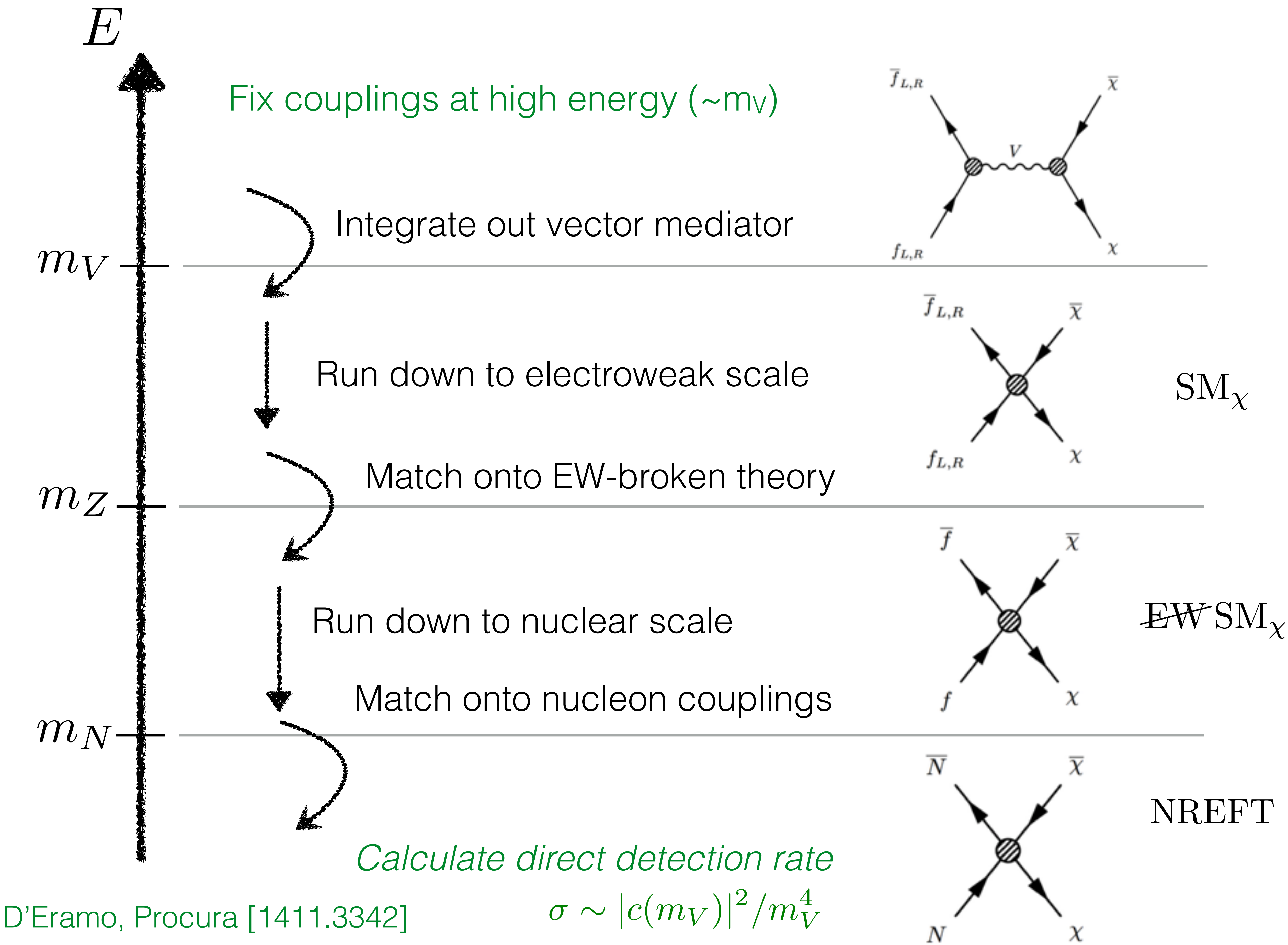
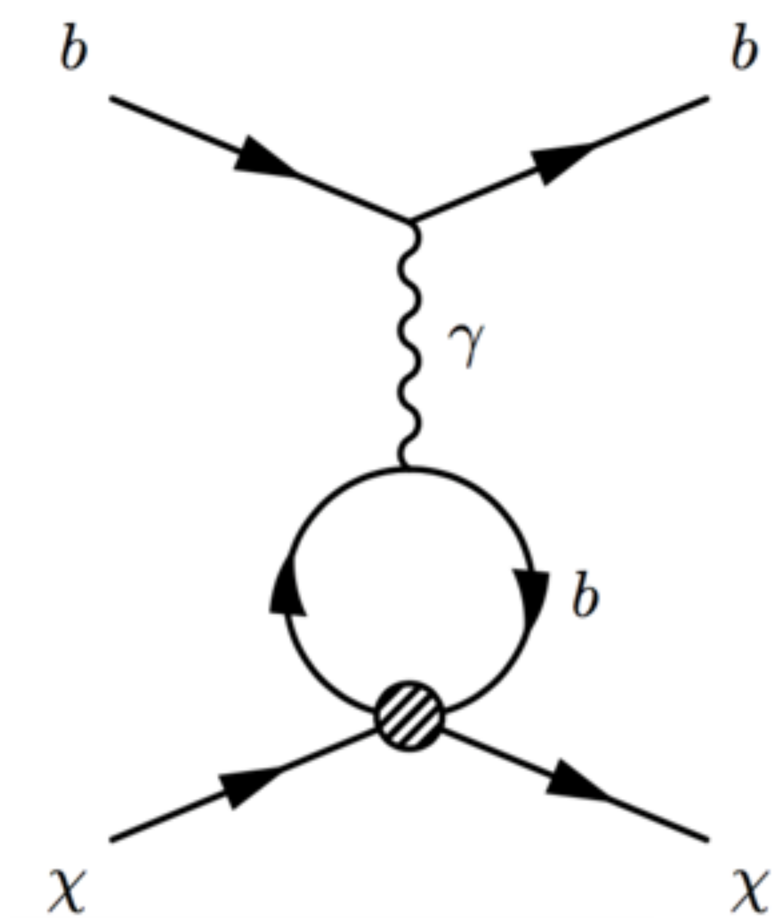
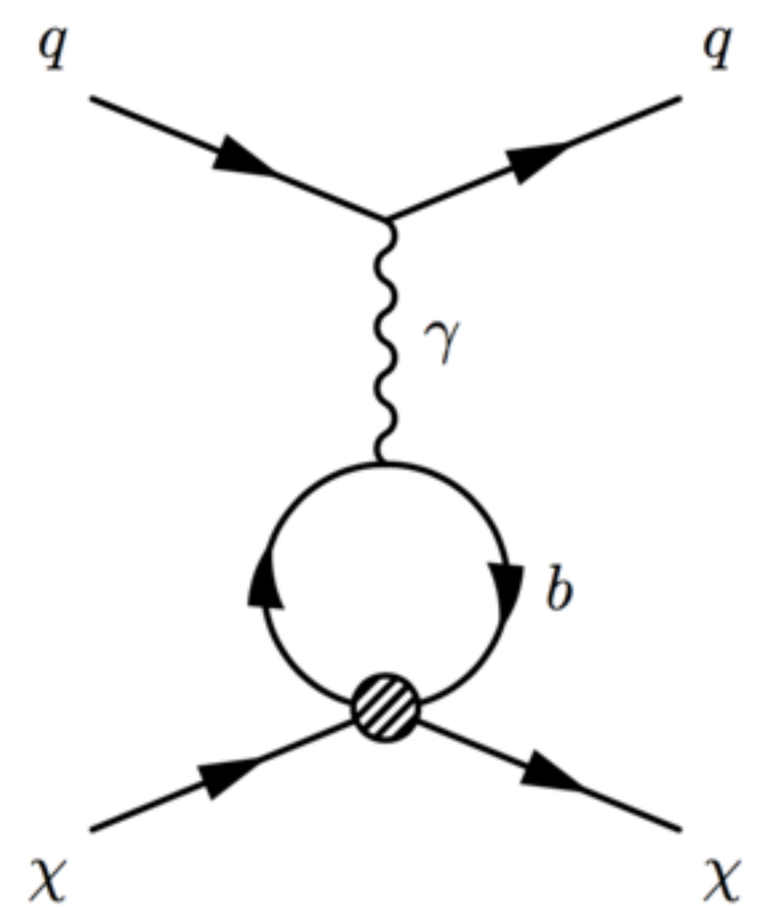

E
Fix couplings at high energy (~mV)
Integrate out vector mediator
m_V
Run down to electroweak scale
Match onto EW-broken theory
m_Z
Run down to nuclear scale
Match onto nucleon couplings
m_N
Calculate direct detection rate
$\sigma \sim |c(m_V)|^2/m_V^4$
D'Eramo, Procura [1411.3342]
$\bar{f}_{L,R}$
$\bar{\chi}$
V
$f_{L,R}$
χ
SM_χ
$\bar{f}$
f
~~EW~~ SM_χ
$\bar{N}$
N
NREFT

RG effects

As we move between the different scales, we have to take into account the running of the couplings, due only to loops of Standard Model particles, e.g.



Self-renormalisation



Operator mixing

The running doesn't depend on the properties of the Dark Sector.

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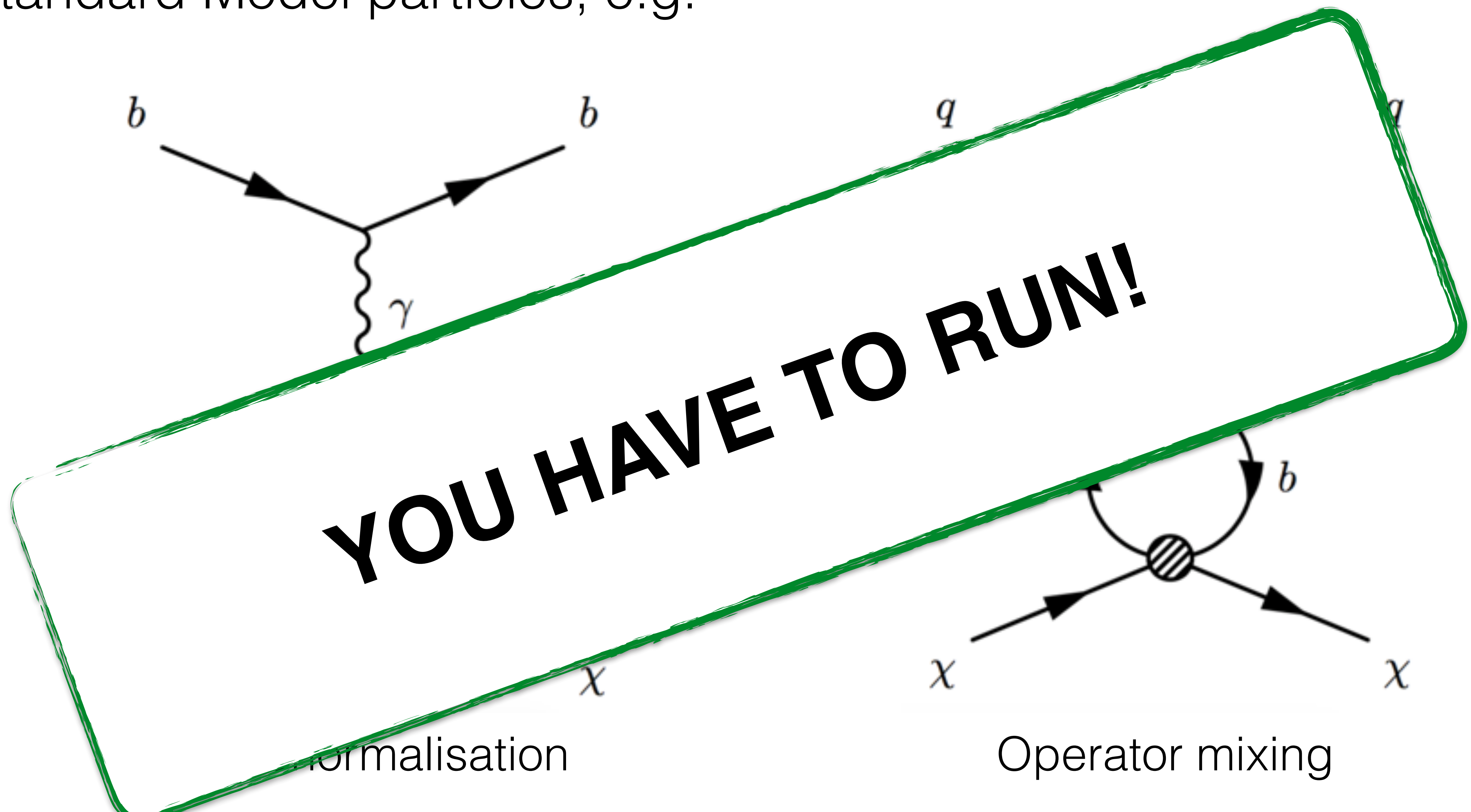
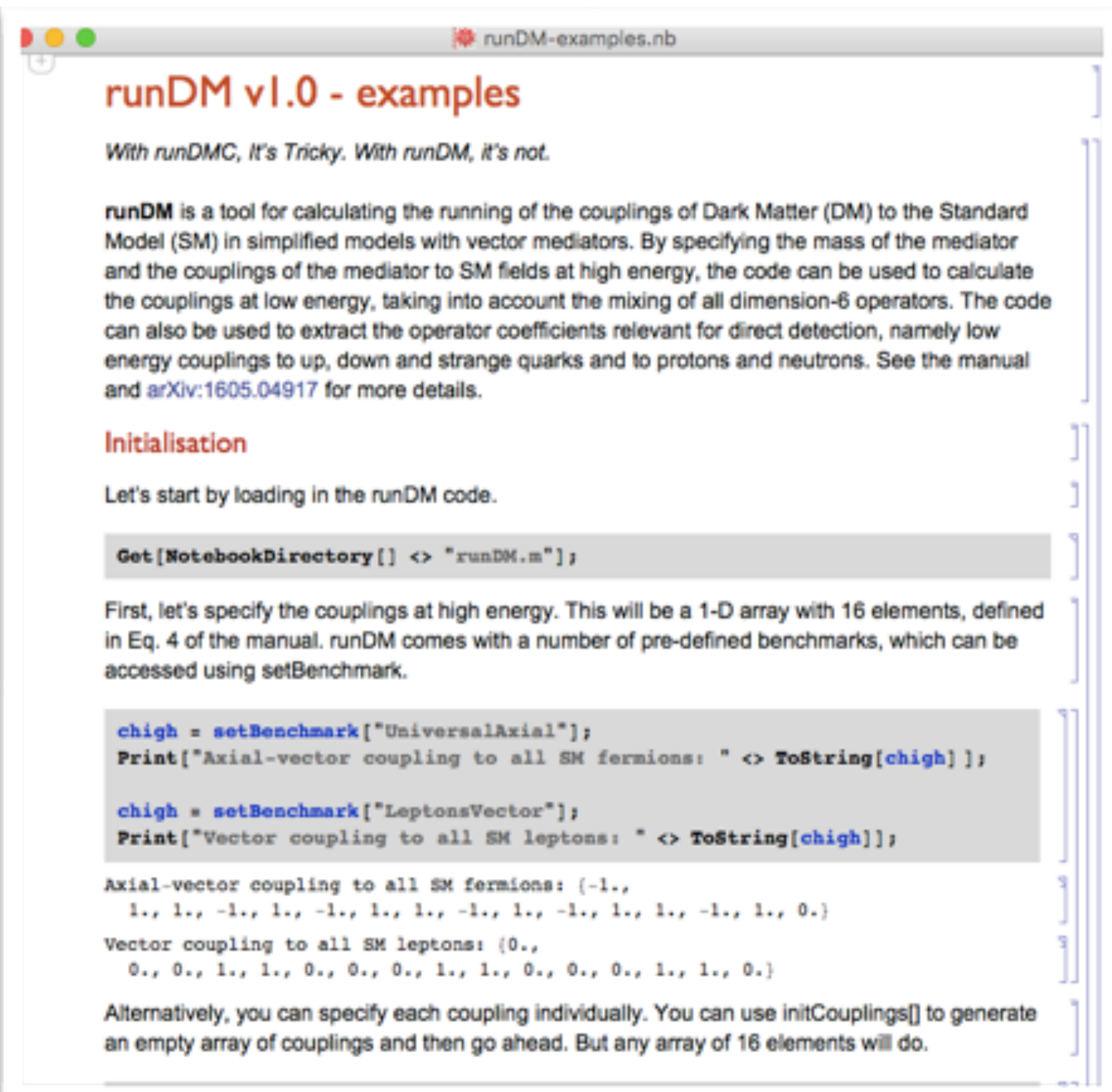


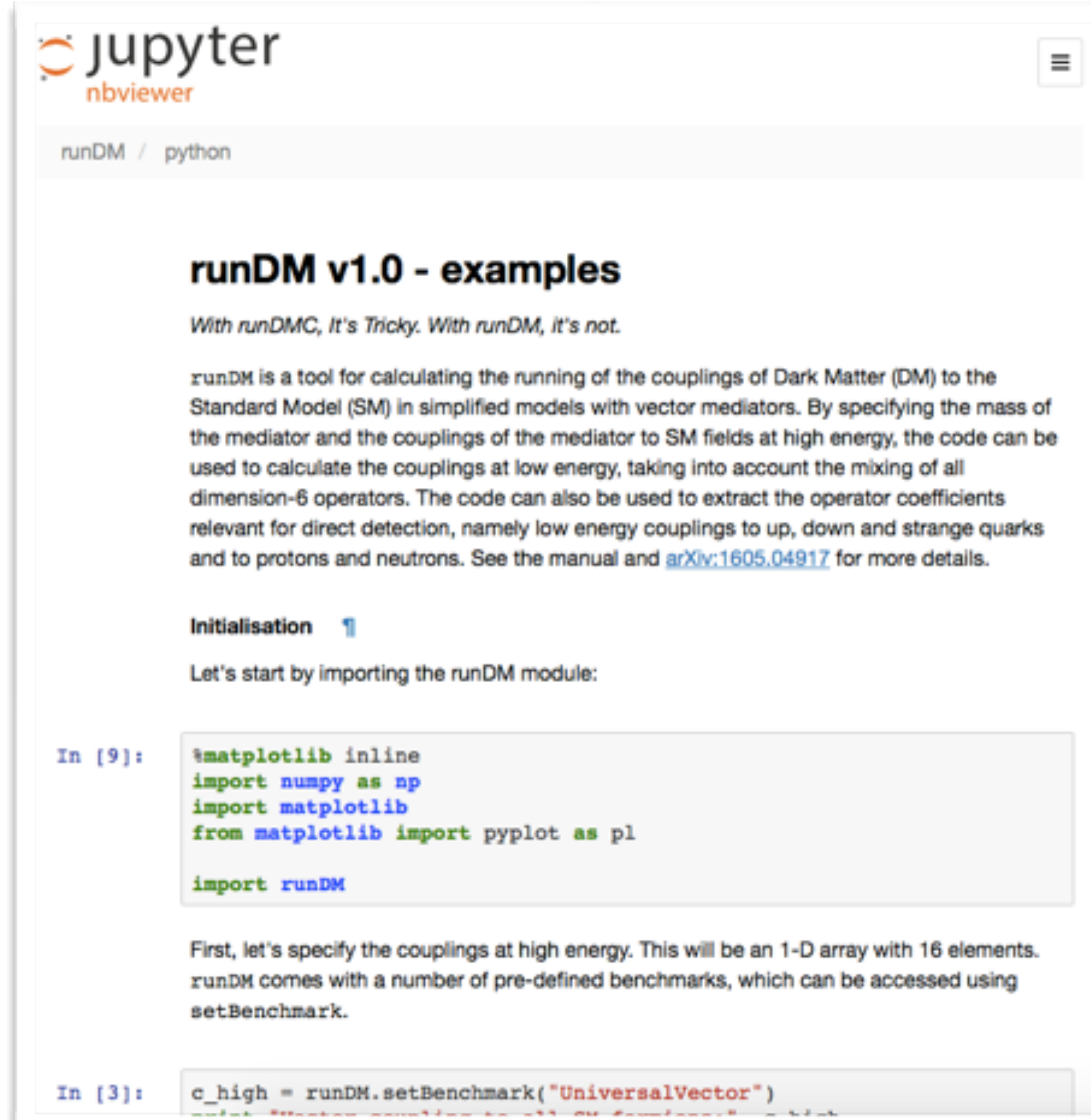
ormalisation

Operator mixing

The running doesn't depend on the properties of the Dark Sector.

runDM - a code for the RGE



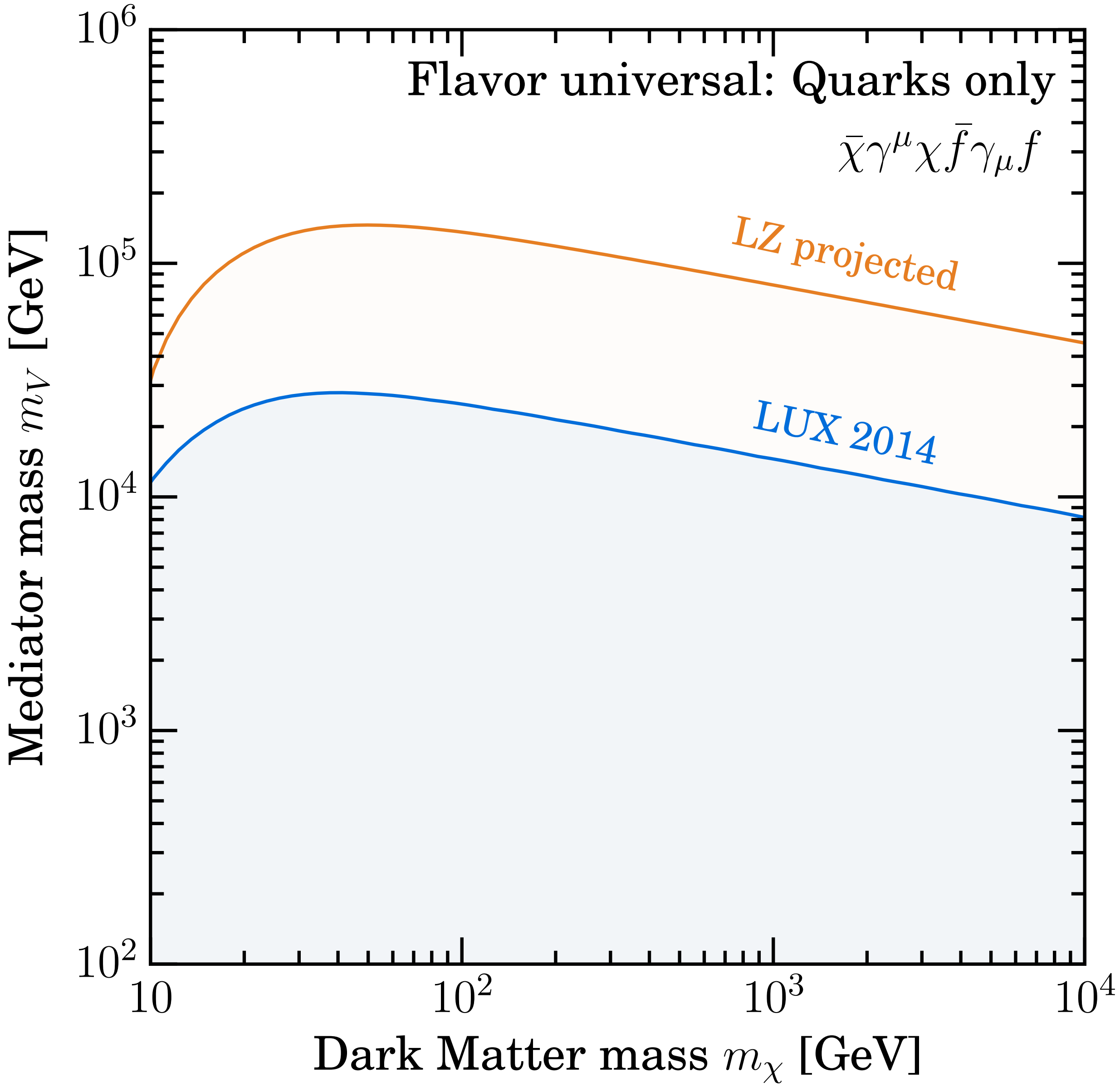


Mathematica and Python versions available at:

https://github.com/bradkav/runDM/

Results I - quarks vector

$$\mathcal{L}_{\mathrm{EFT}} = -\frac{1}{m_V^2}\, J_{\mathrm{DM}\,\mu} \sum_{i=1}^{3} \left[\overline{u^i}\gamma^\mu u^i + \overline{d^i}\gamma^\mu d^i\right]$$



LUX 2014 [1310.8214]

LZ (projected) [1509.02910]

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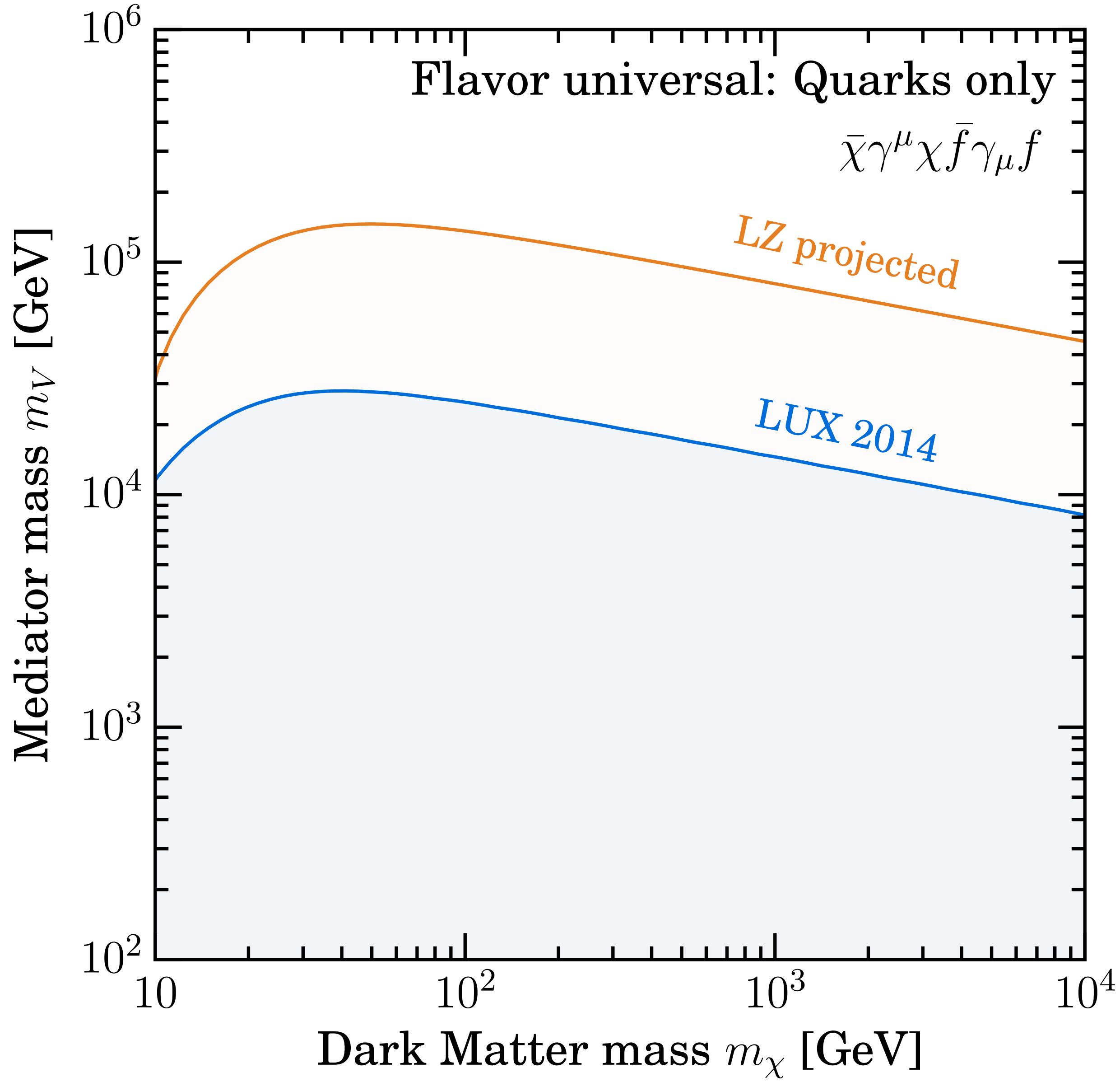

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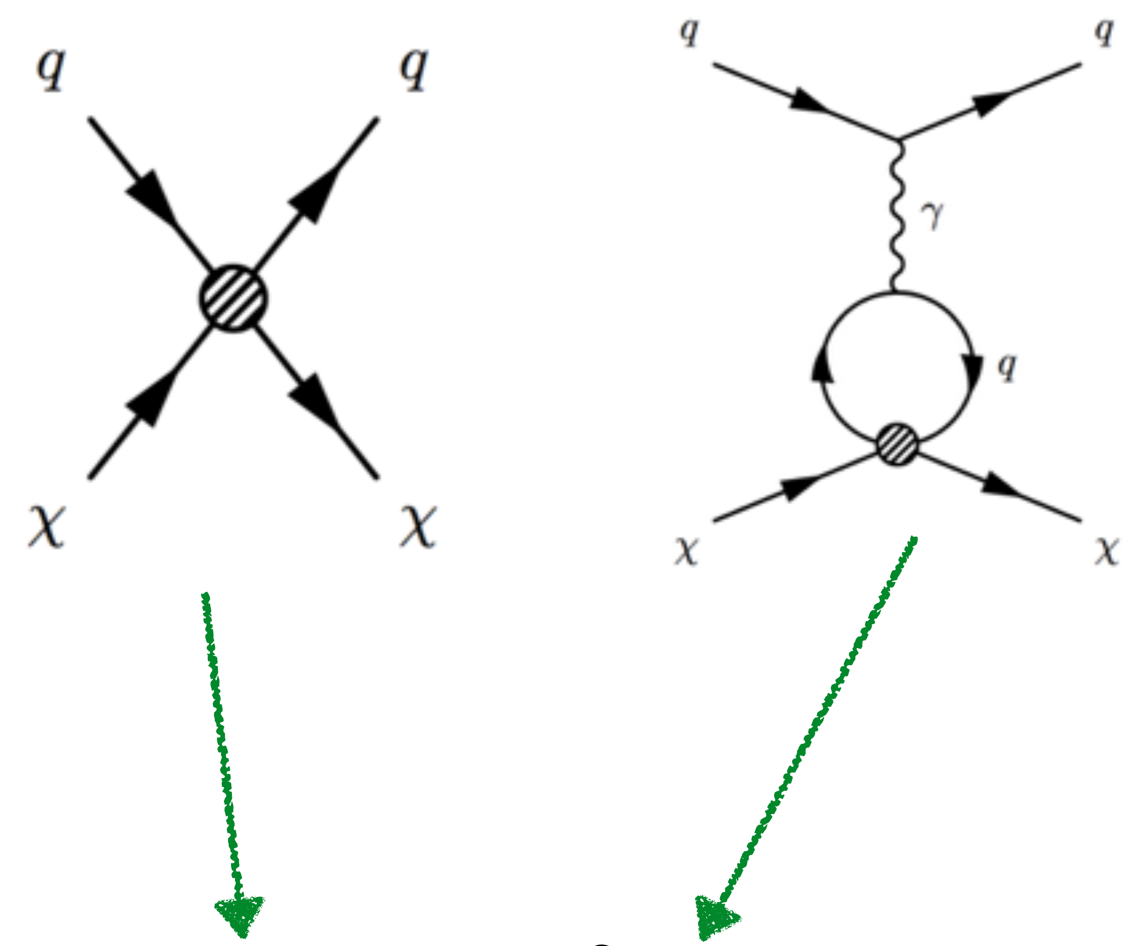

$$c_V^{(q)} \sim 1 + \# \frac{e^2}{16\pi^2} \ln(m_V/m_N)$$

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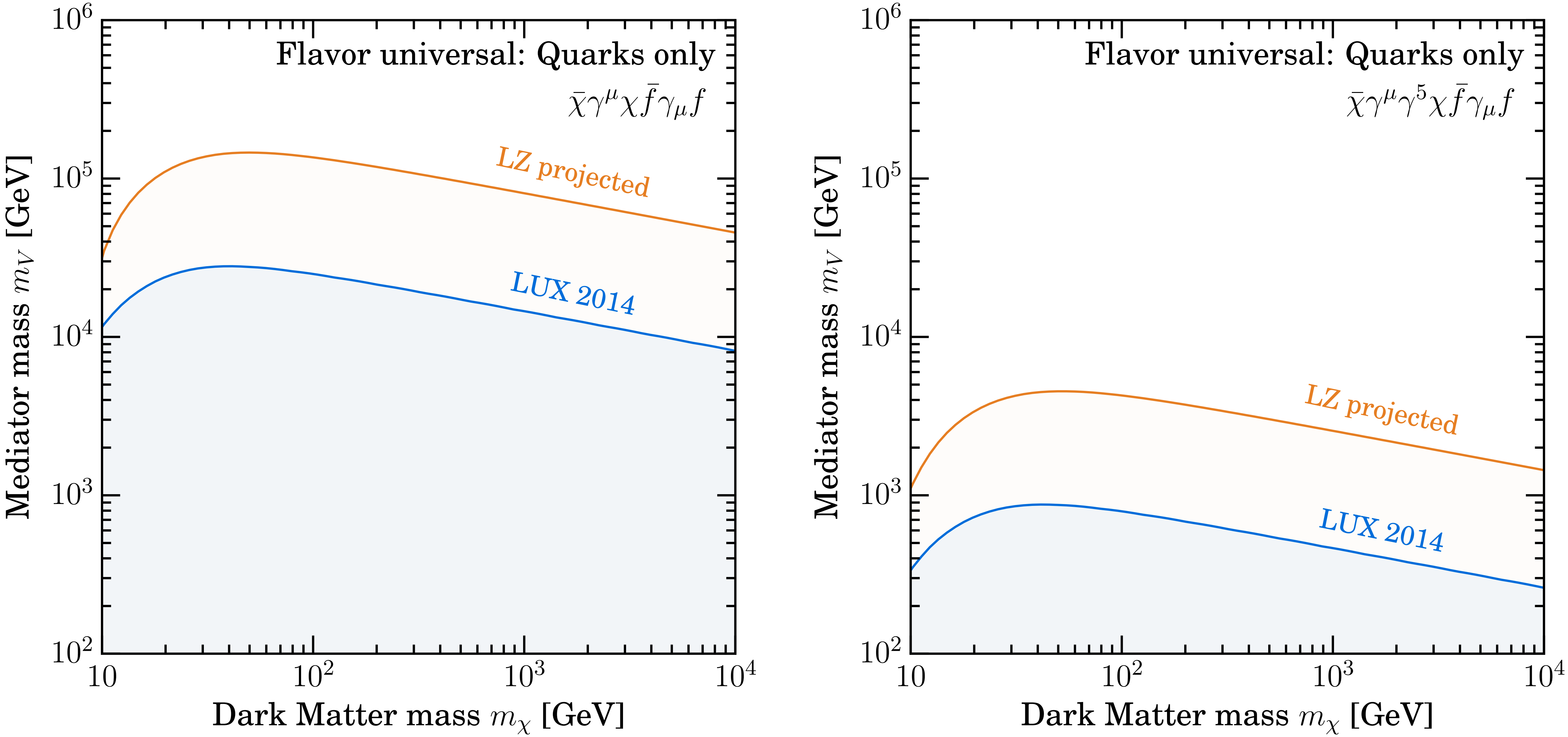

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Results II - quarks axial-vector

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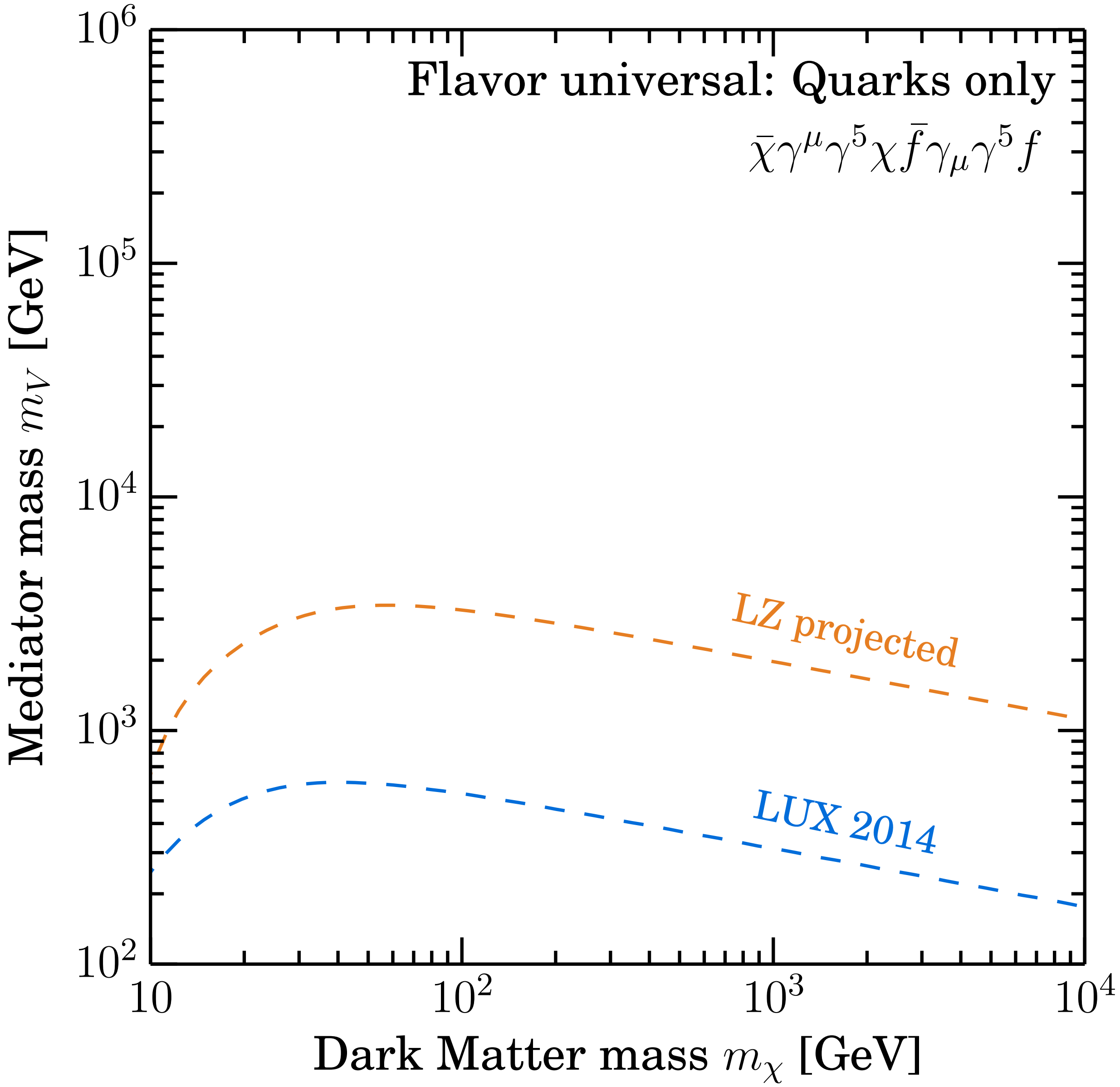
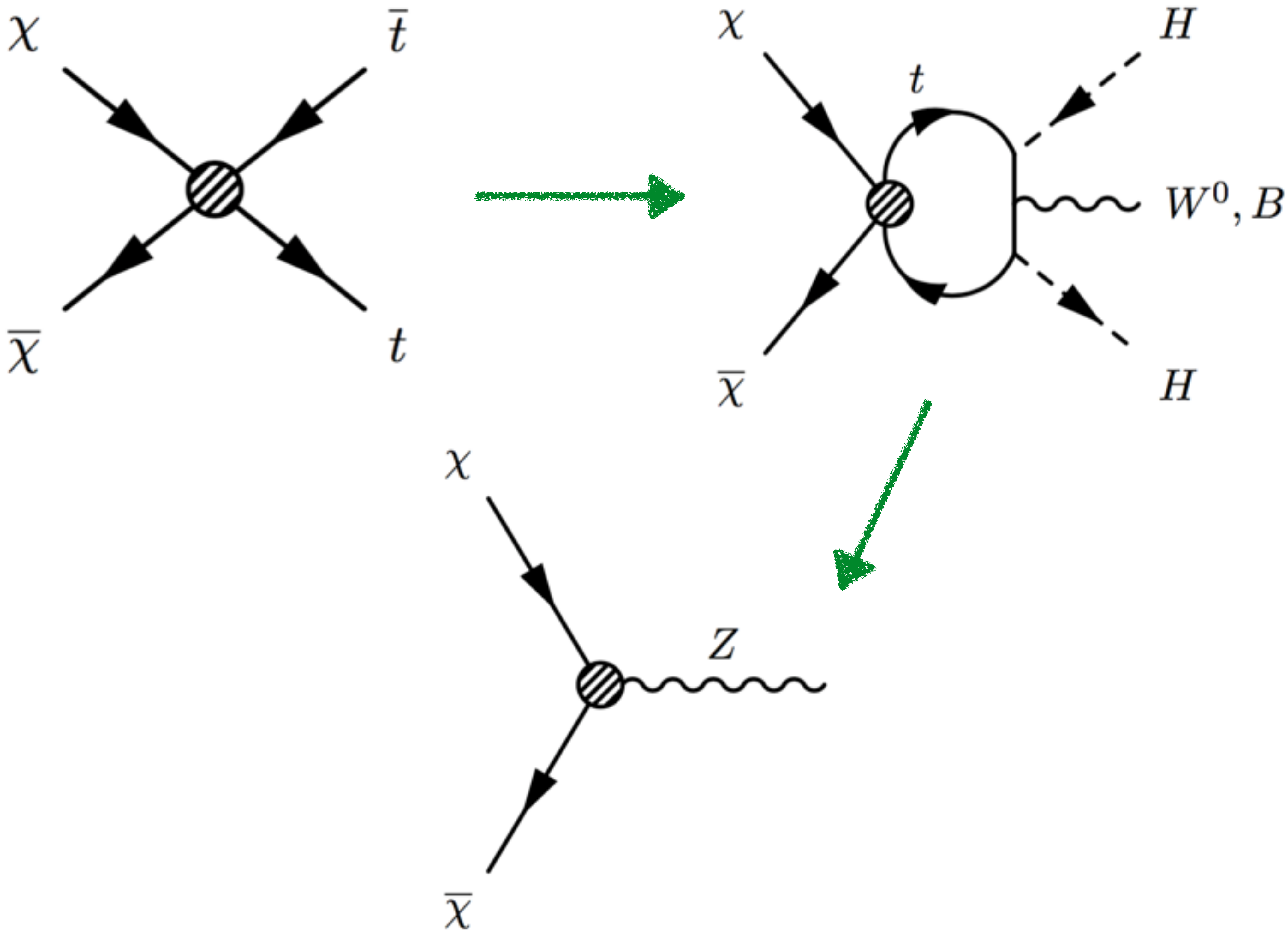


No running - - - - - -

Running ———

Aside: SM axial-vector current

$J_{\mathrm{DM}\,\mu}\,\overline{q}\gamma^{\mu}\gamma^{5}q$



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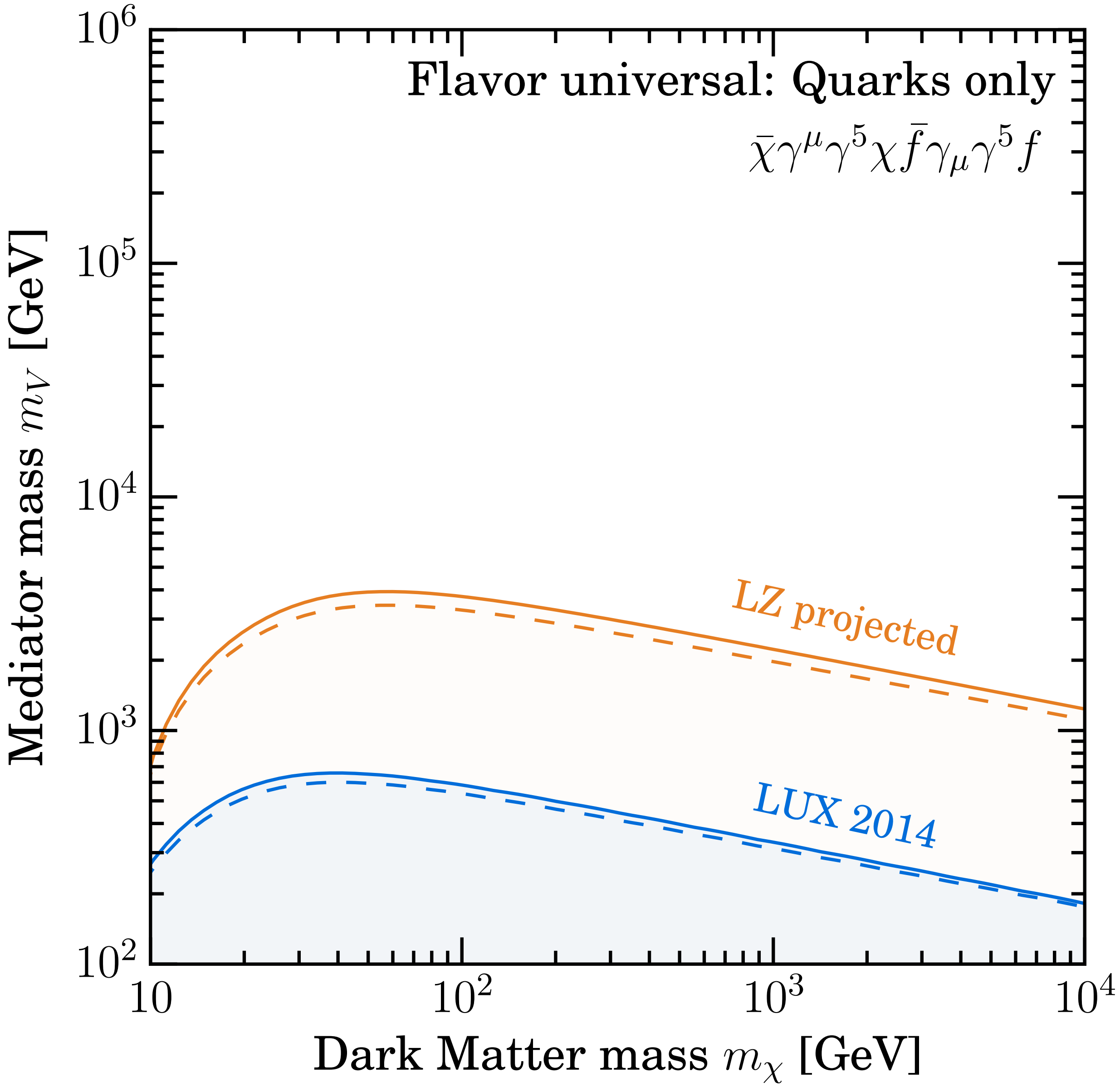

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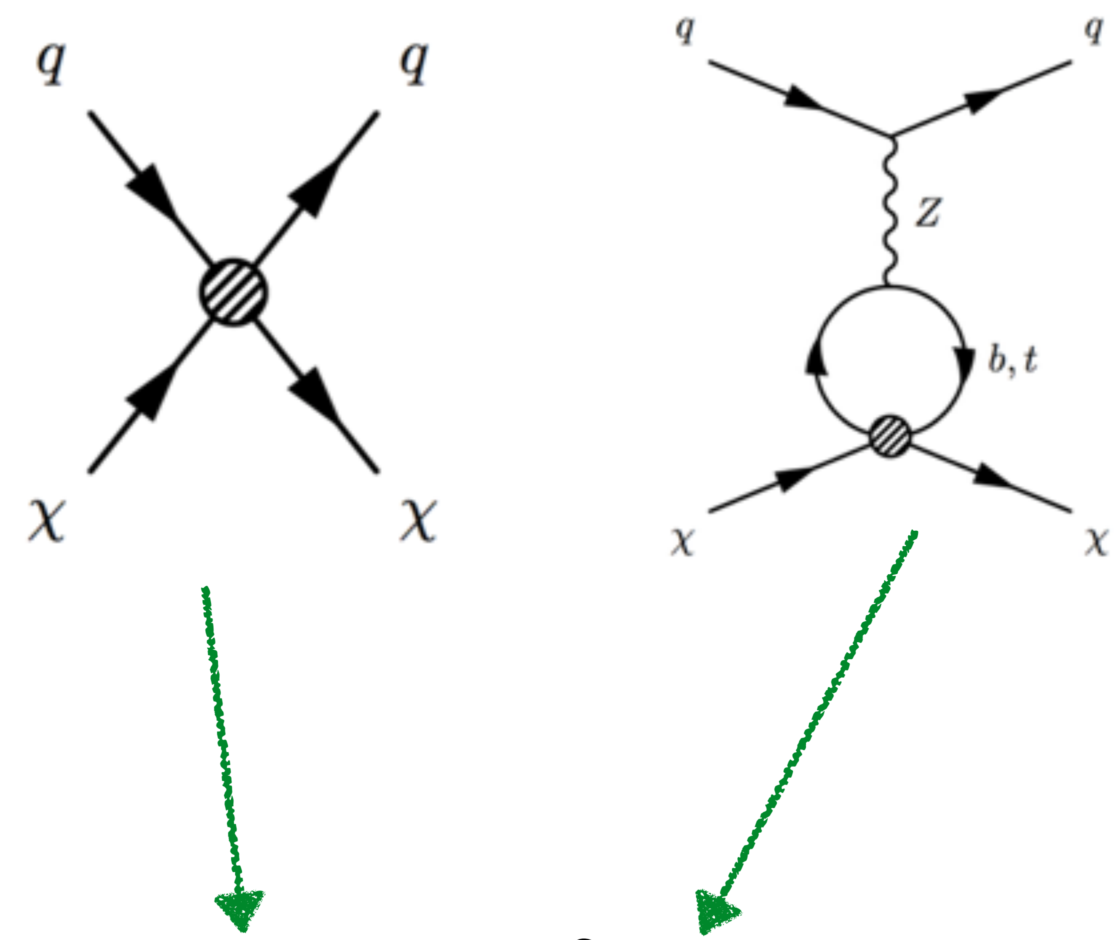
Running ———



$$c_A^{(q)} \sim 1 + \# \frac{\lambda_{b,t}^2}{16\pi^2} \ln(m_V/m_N)$$

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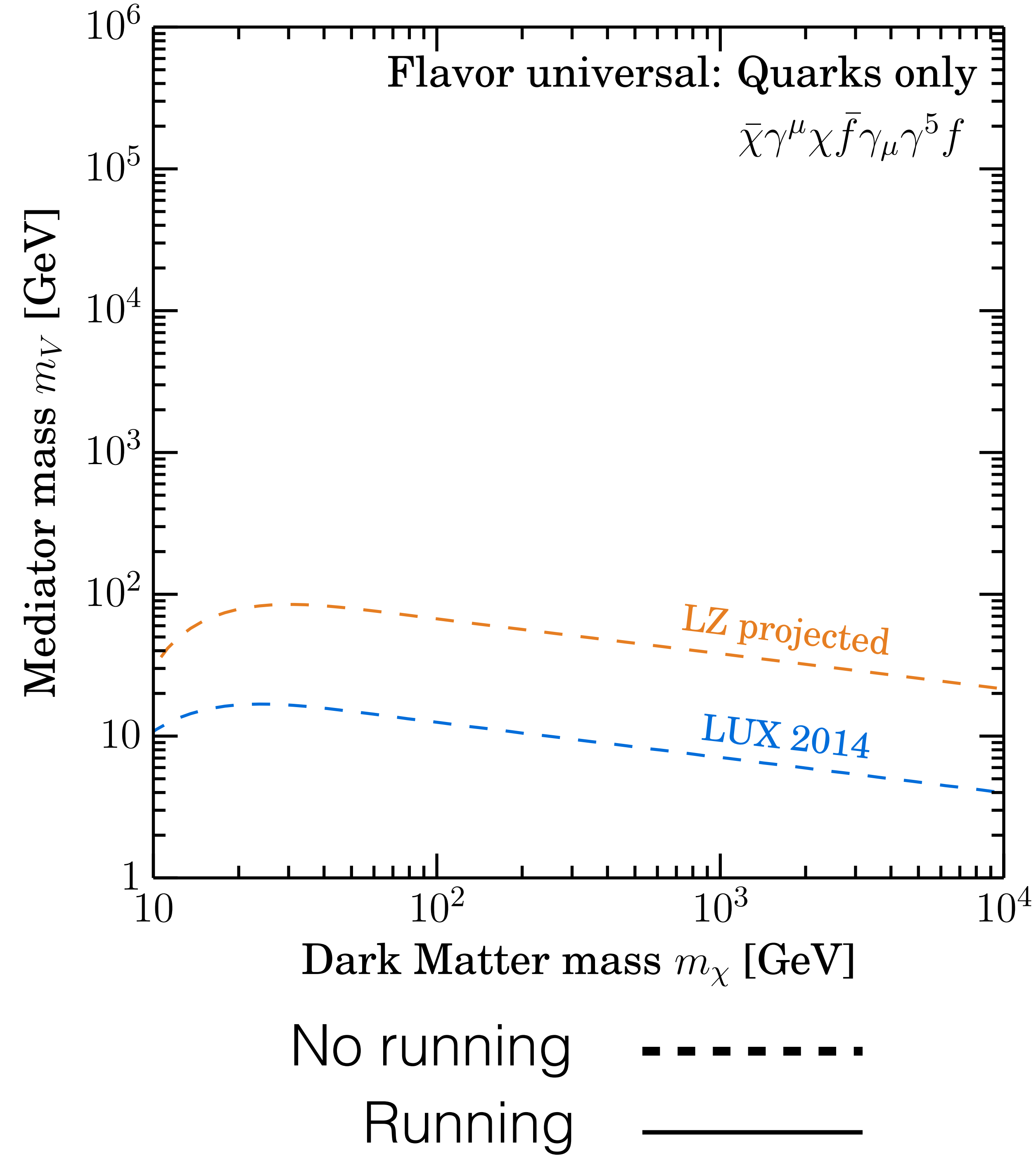

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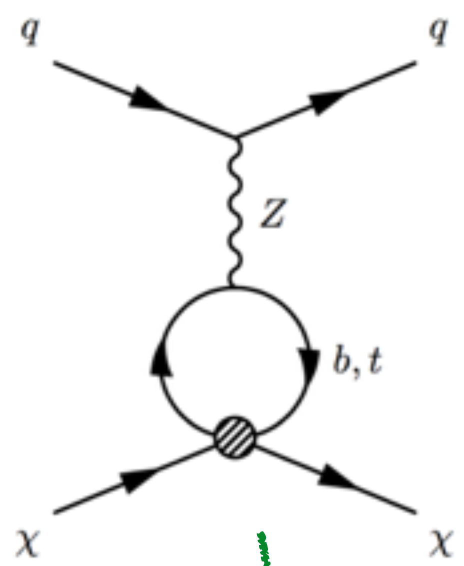
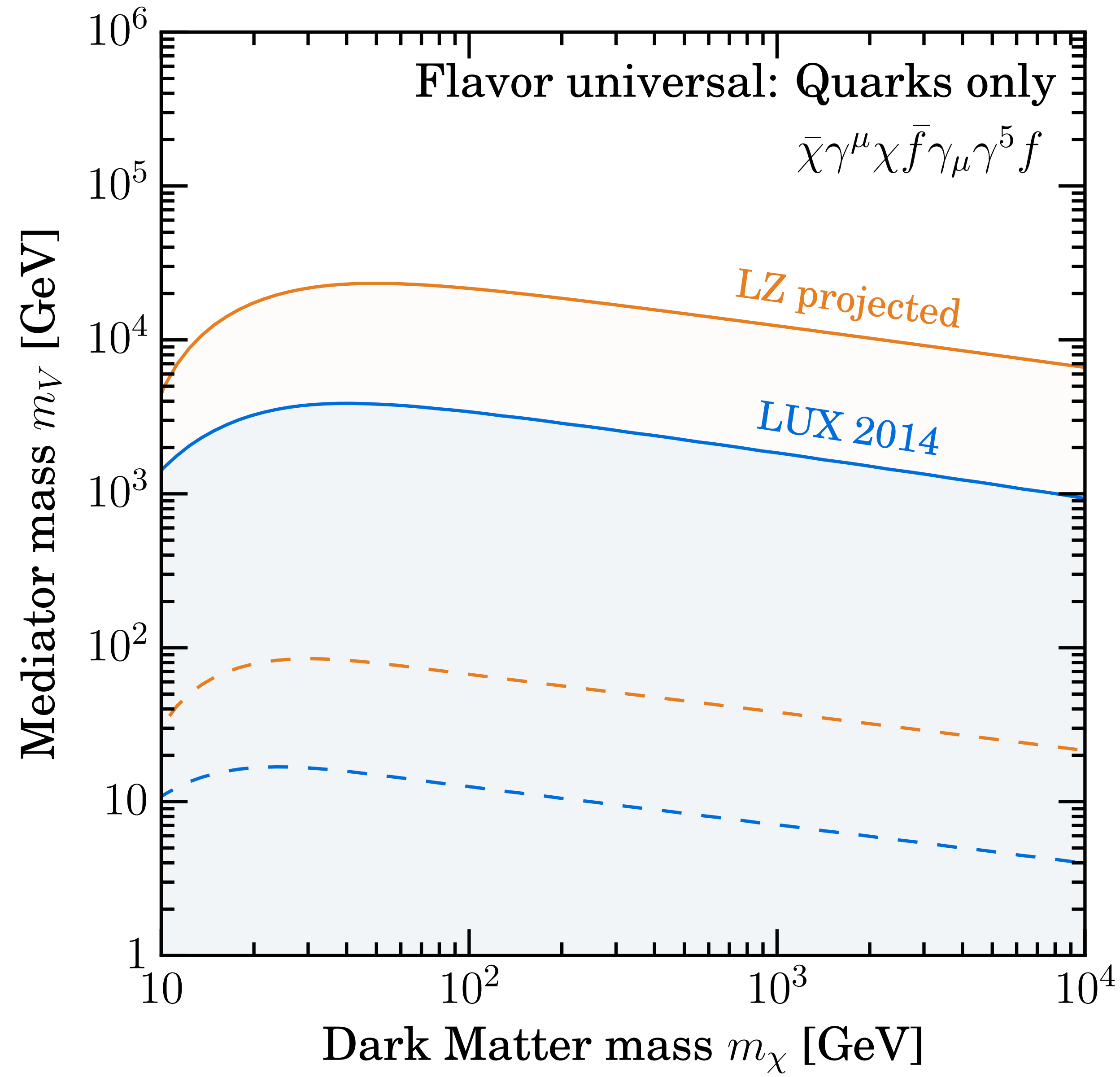


$$c_V^{(q)} \sim 0 + \# \frac{\lambda_{b,t}^2}{16\pi^2} \ln(m_V/m_N)$$



No running - - - - - -
Running ———

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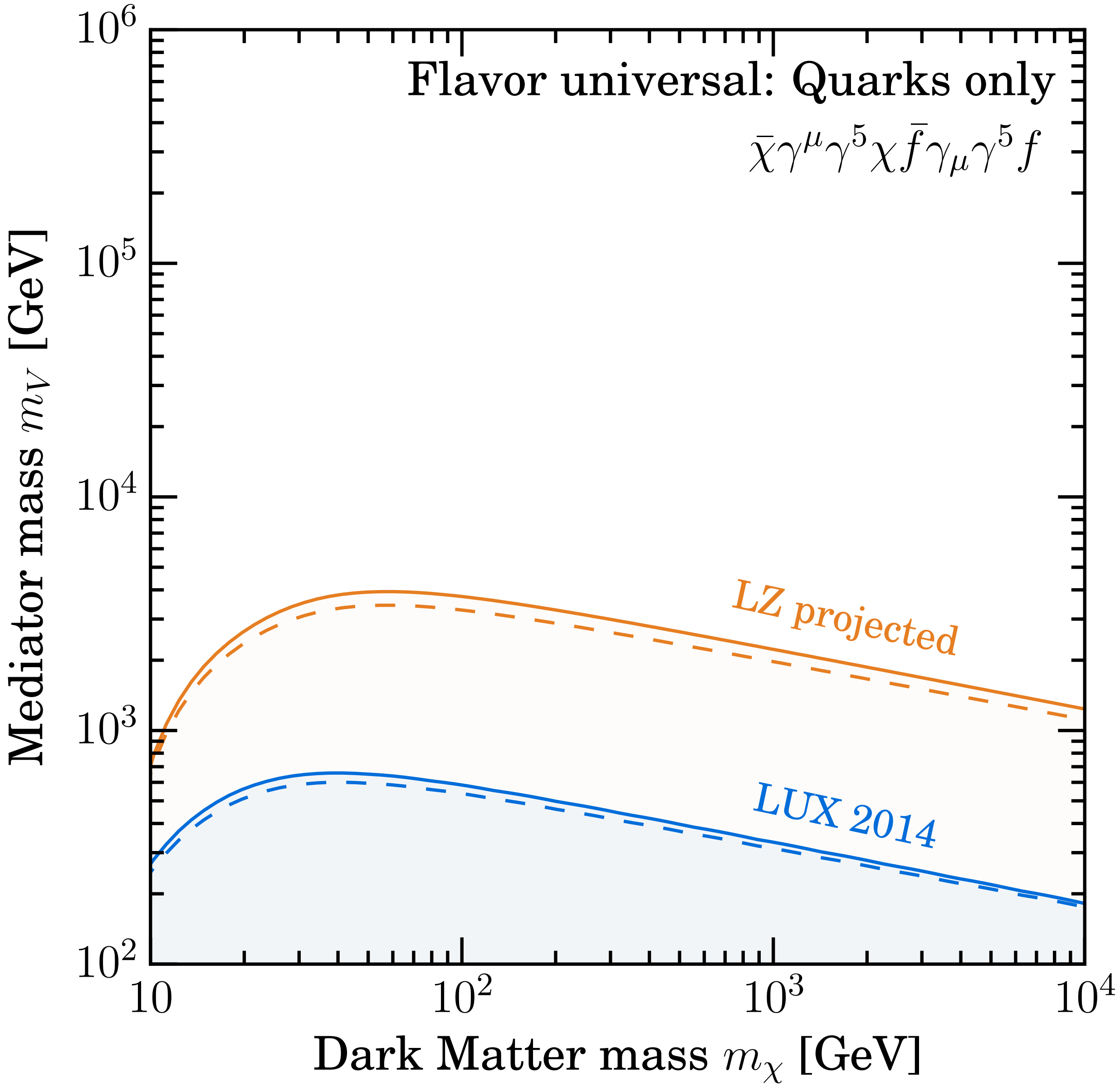
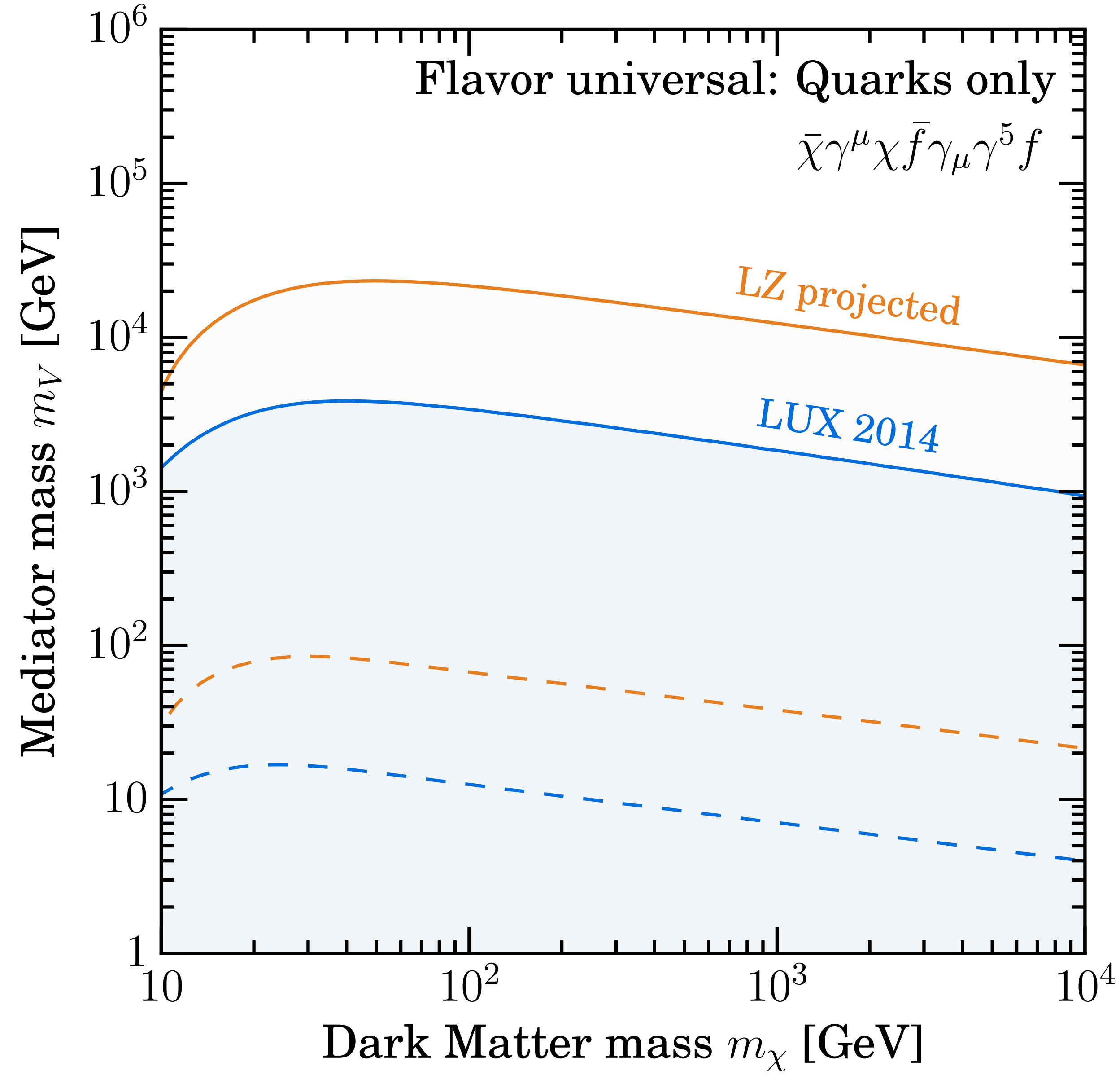




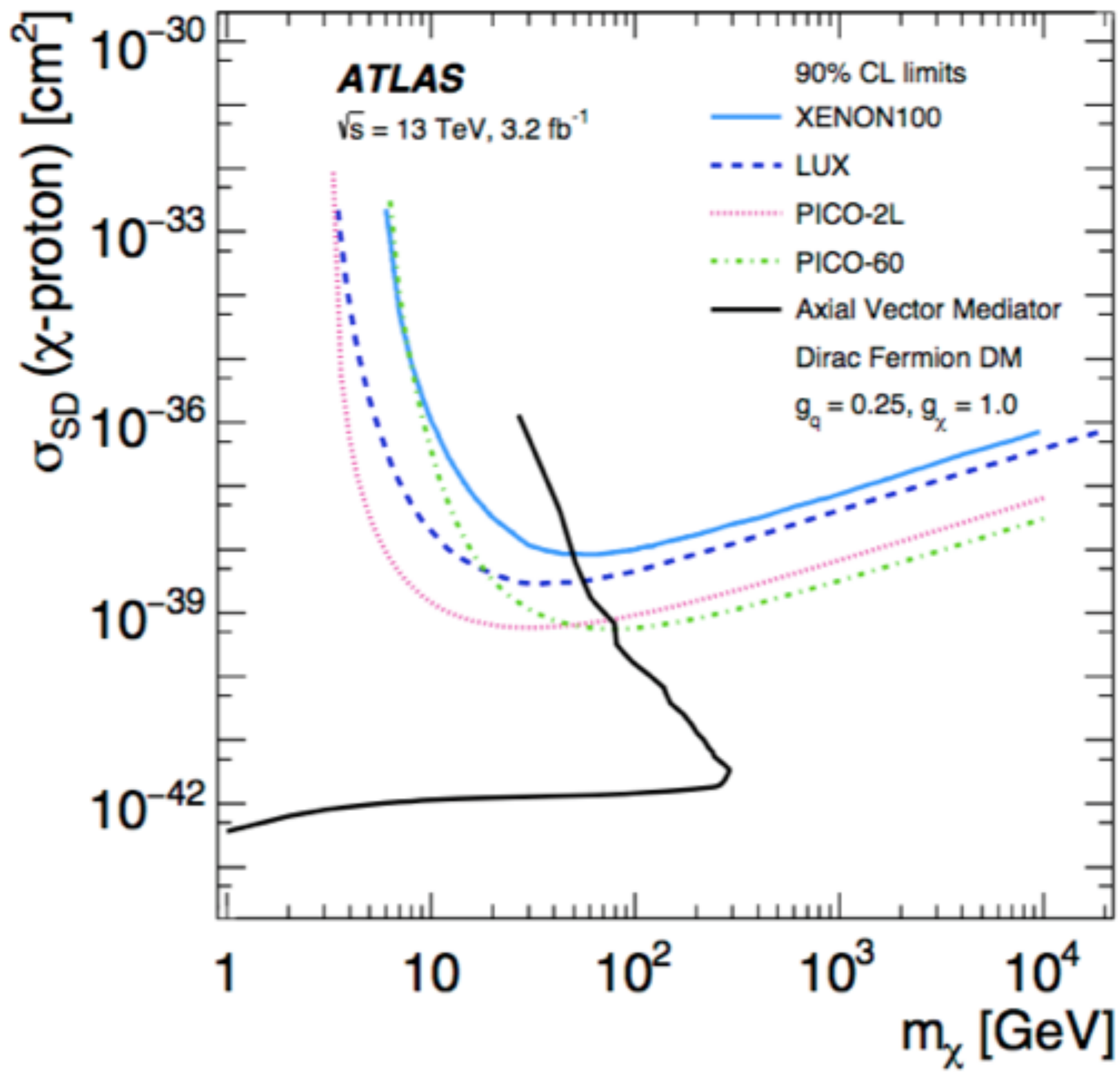
No running - - - - - -

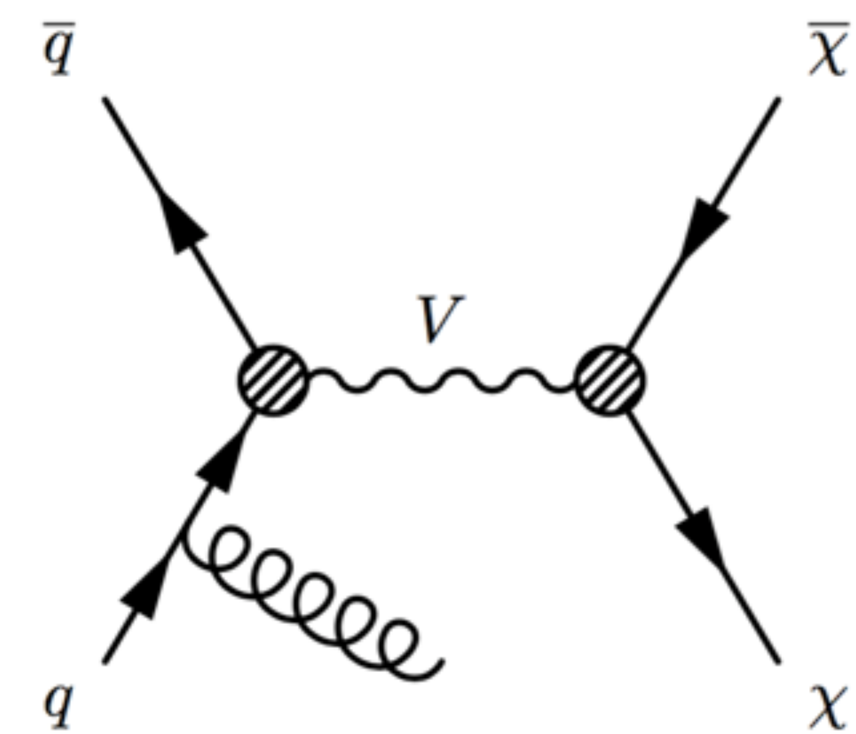
Running ———

Comparing LHC and Direct Detection

LHC mono-X searches

$$\mathcal{L}_{\mathrm{AV}} = g_\chi V_\mu \, \overline{\chi}\gamma^\mu\gamma^5\chi + g_q V_\mu \sum_q \overline{q^i}\gamma^\mu\gamma^5 q^i$$





ATLAS [1604.07773]

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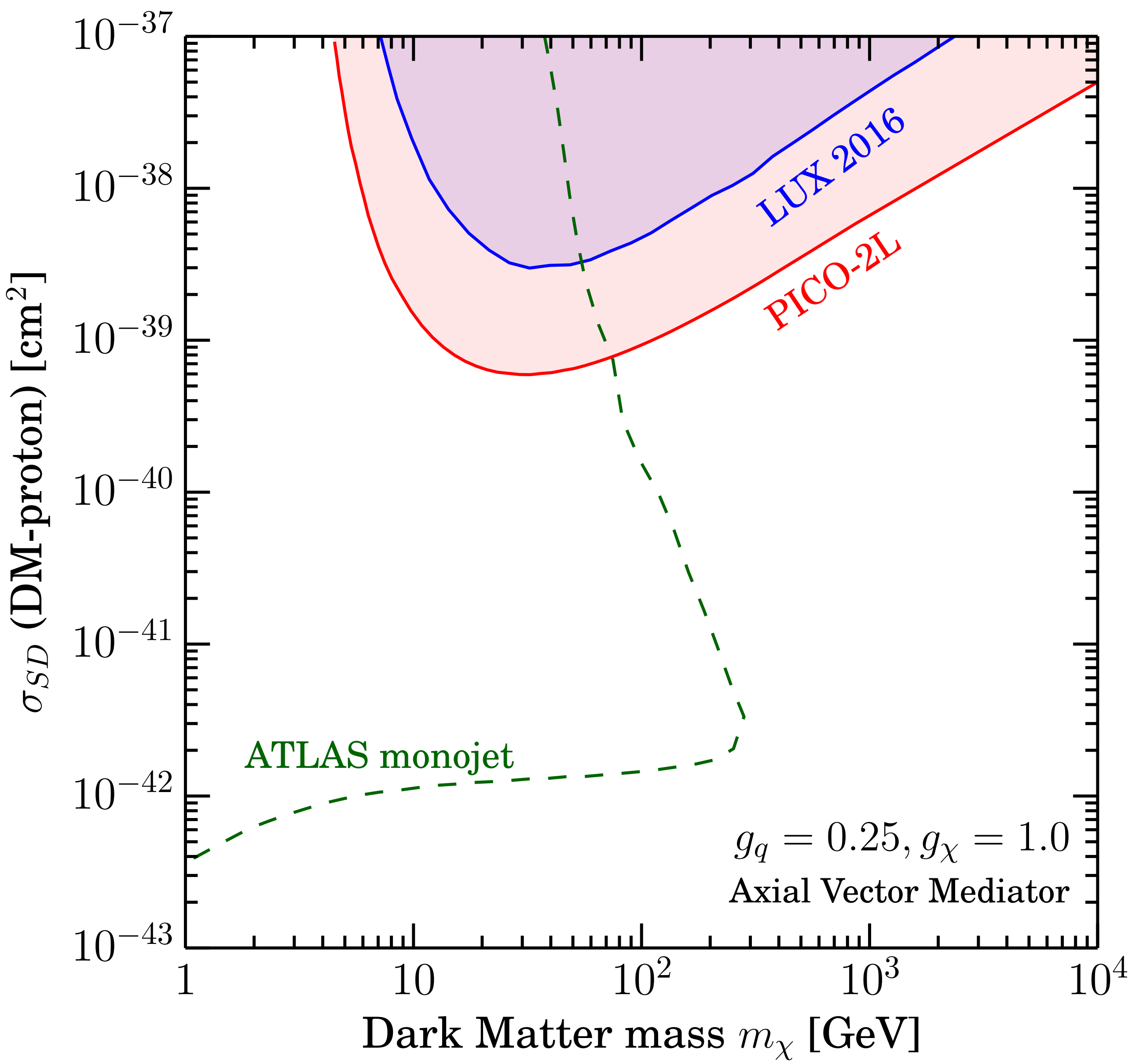

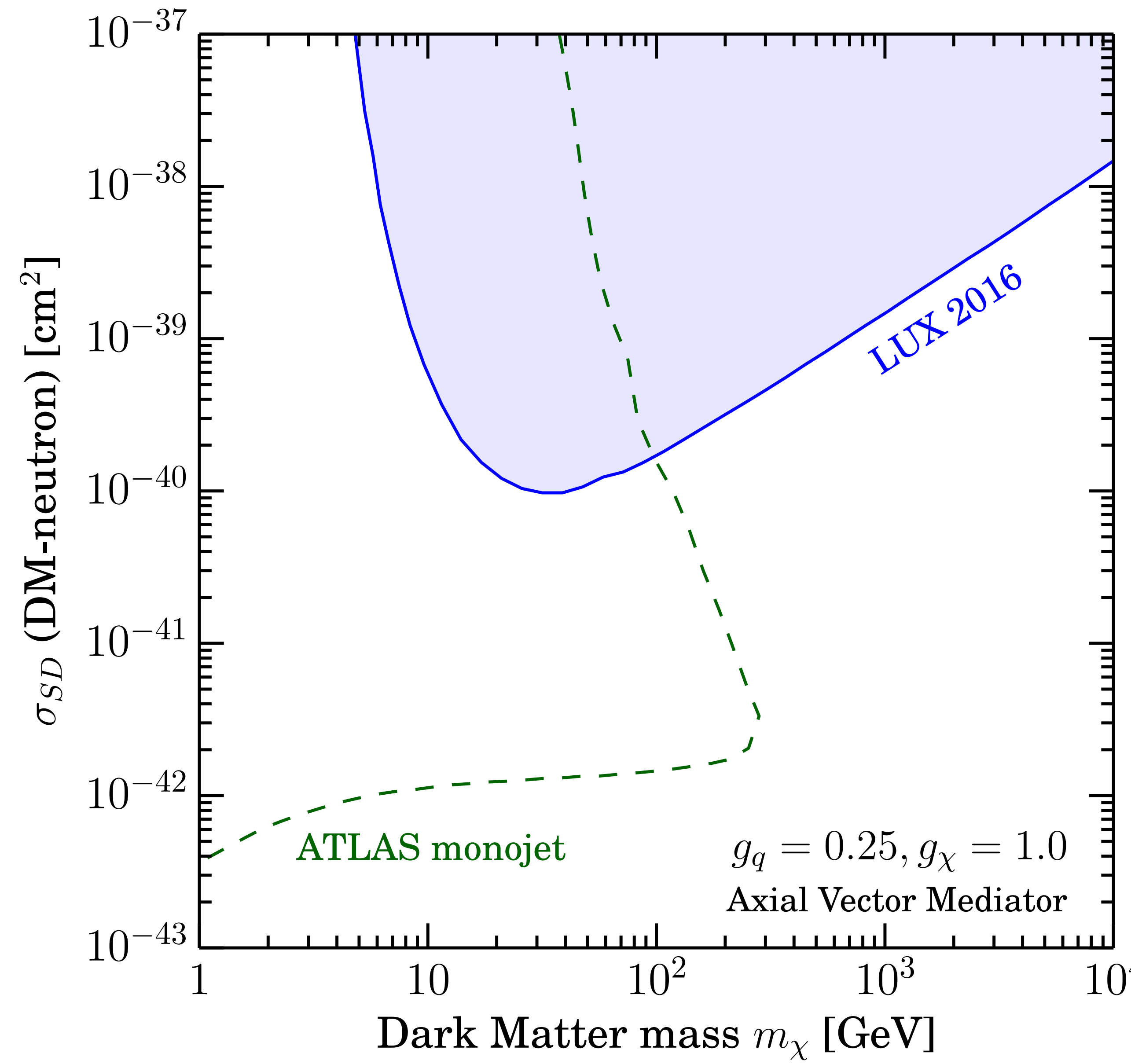

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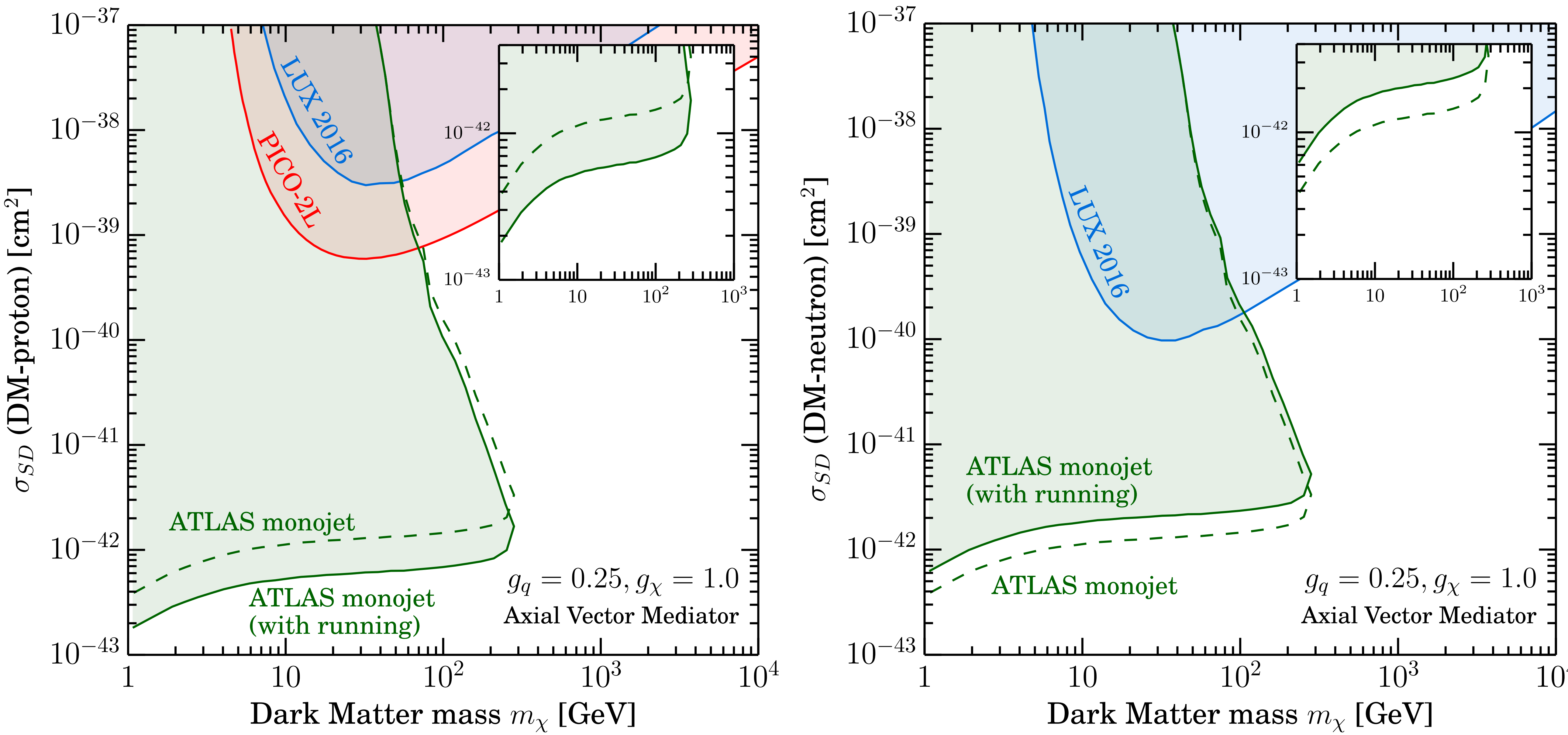
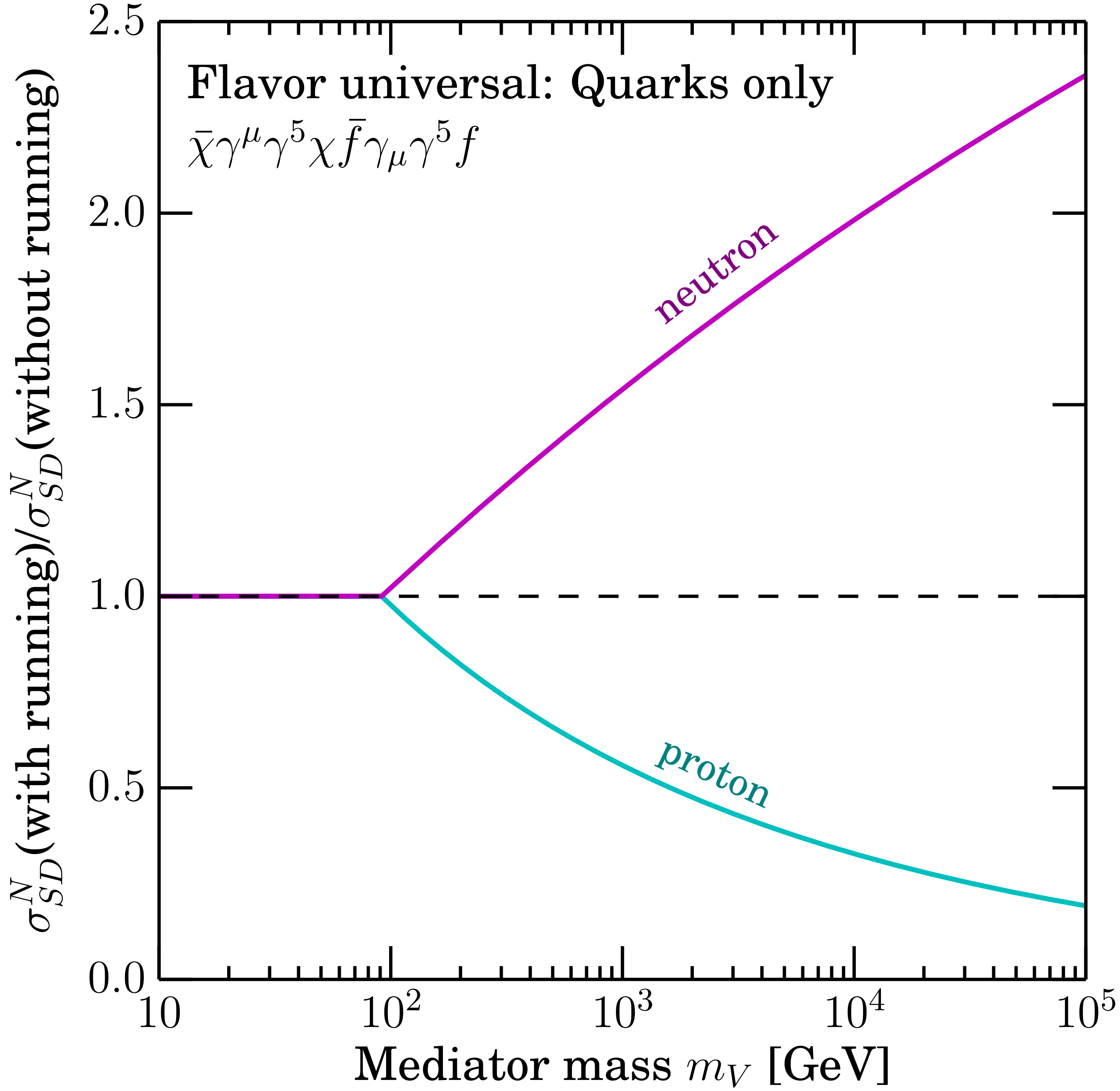


Isospin violation

$$\mathcal{L}_{\mathrm{AV}} = g_\chi V_\mu \,\overline{\chi}\gamma^\mu\gamma^5\chi + g_q V_\mu \sum_q \overline{q^i}\gamma^\mu\gamma^5 q^i$$



$$\mathcal{C}_A^{(N)} = g_q \left[\sum_{q=u,d,s} \Delta_q^{(N)}\right] + \frac{3g_q}{2\pi}\left(\Delta_d^{(N)} + \Delta_s^{(N)} - \Delta_u^{(N)}\right)\left[\alpha_t \ln(m_V/m_Z) - \alpha_b \ln(m_V/\mu_N)\right]$$

Other interactions

Name	Operator	Coefficient
D1	$\bar{\chi}\chi\bar{q}q$	m_q/M_*^3
D2	$\bar{\chi}\gamma^5\chi\bar{q}q$	im_q/M_*^3
D3	$\bar{\chi}\chi\bar{q}\gamma^5 q$	im_q/M_*^3
D4	$\bar{\chi}\gamma^5\chi\bar{q}\gamma^5 q$	m_q/M_*^3
D5	$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu q$	$1/M_*^2$
D6	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	$1/M_*^2$
D7	$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$	$1/M_*^2$
D8	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	$1/M_*^2$

Standard SI

Standard SD

Goodman et al. [1008.1783]

Other interactions

Name	Operator	Coefficient
D1	$\bar{\chi}\chi\bar{q}q$	m_q/M_*^3
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D6	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu q$	$1/M_*^2$
D7	$\bar{\chi}\gamma^\mu\chi\bar{q}\gamma_\mu\gamma^5 q$	$1/M_*^2$
D8	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}\gamma_\mu\gamma^5 q$	$1/M_*^2$

'Velocity suppressed'

Goodman et al. [1008.1783]

Other interactions

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D1	$\bar{\chi}\chi\bar{q}q$	m_q/M_*^3
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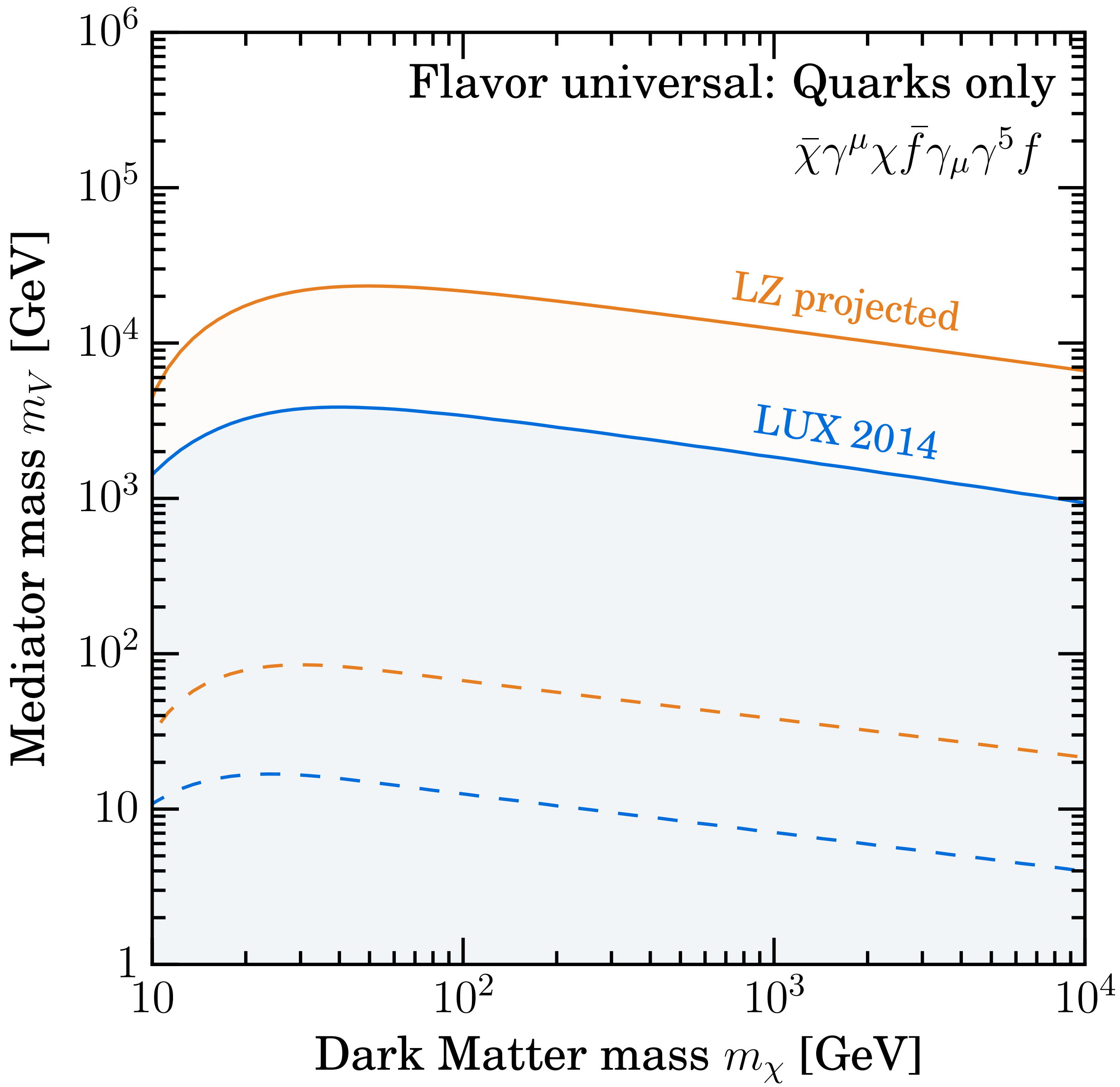
Goodman et al. [1008.1783]



Comparing LHC and direct detection is not always straightforward!

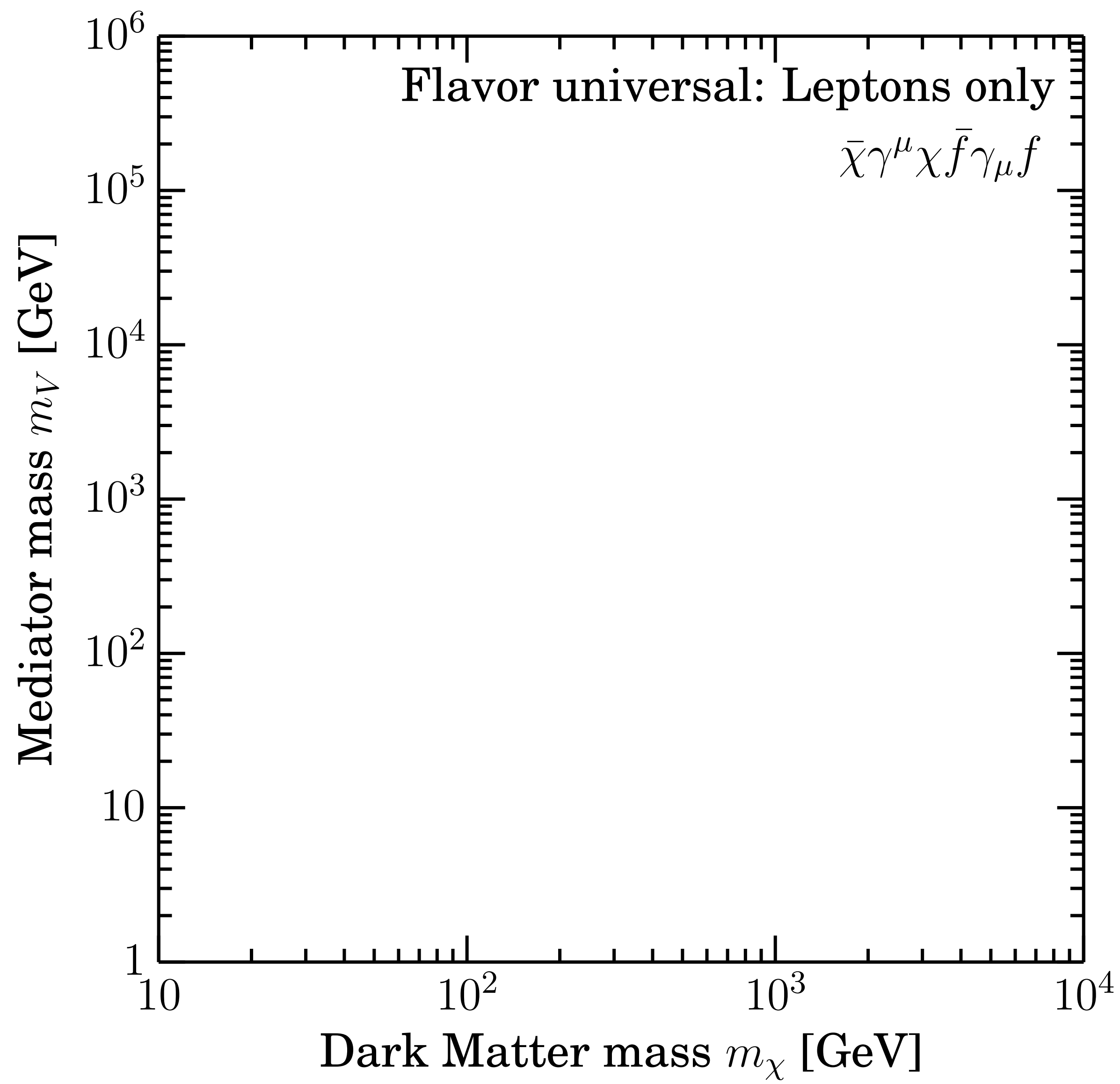
Leptophilic Dark Matter

D'Eramo, BJK, Panci [1702.00016]

Leptophilic DM

Fox et al. [0811.0399], Kopp et al. [0907.3159], Bell et al. [1407.3001], Freitas & Westhoff [1408.1959], Chen et al. [1501.04486] and many others…

Couple Dark Matter only to SM leptons and evade many standard bounds…



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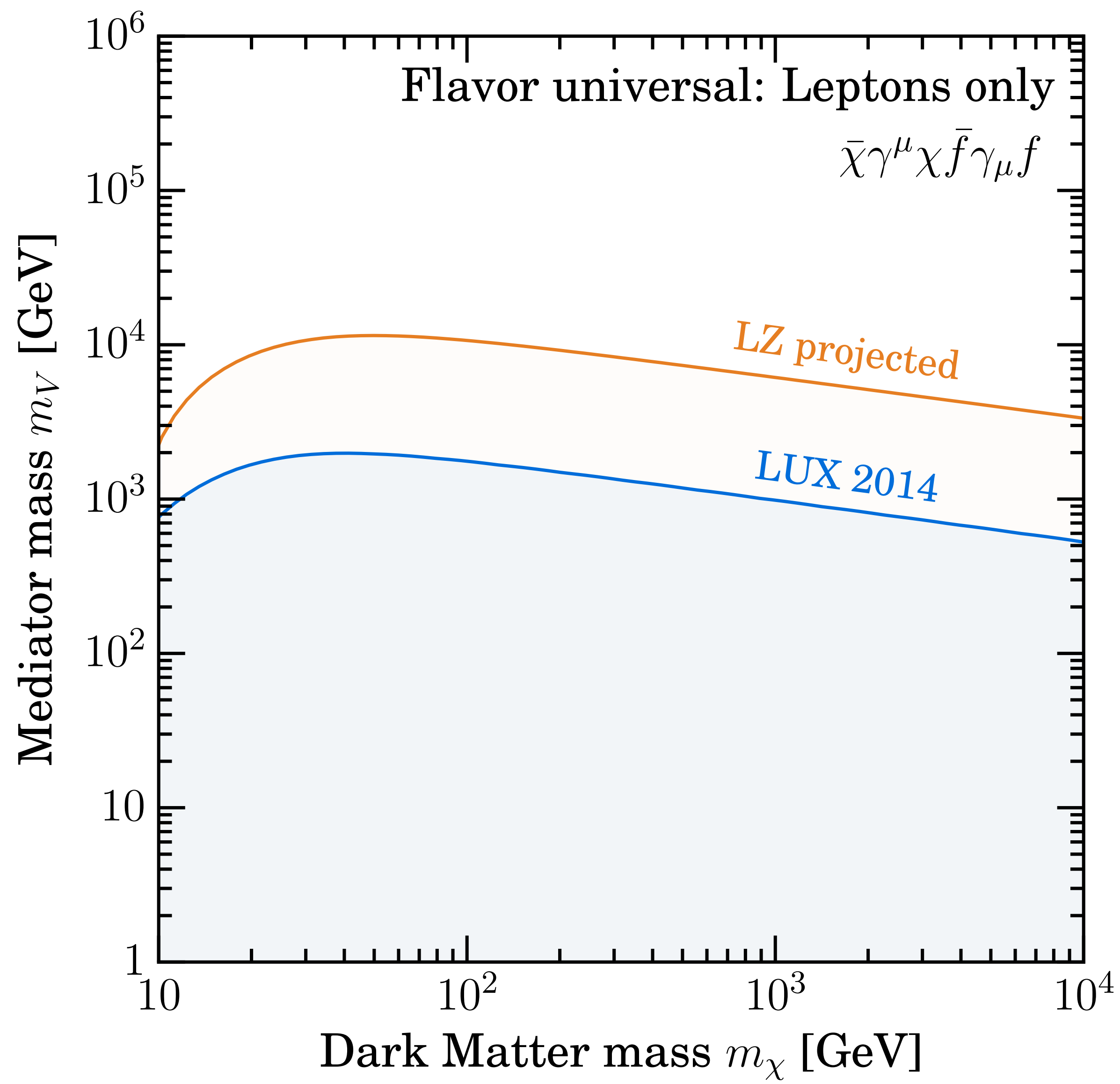
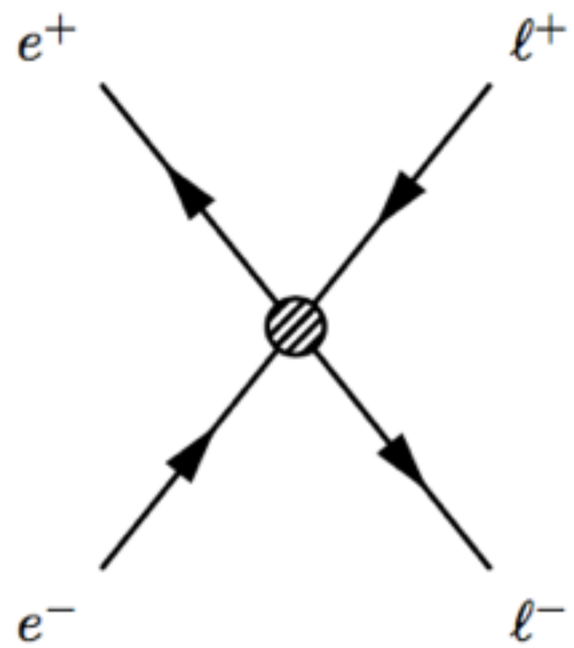
Fox et al. [0811.0399], Kopp et al. [0907.3159], Bell et al. [1407.3001], Freitas & Westhoff [1408.1959], Chen et al. [1501.04486] and many others…

Couple Dark Matter only to SM leptons and evade many standard bounds…unless you account for operator mixing!



LEP-II

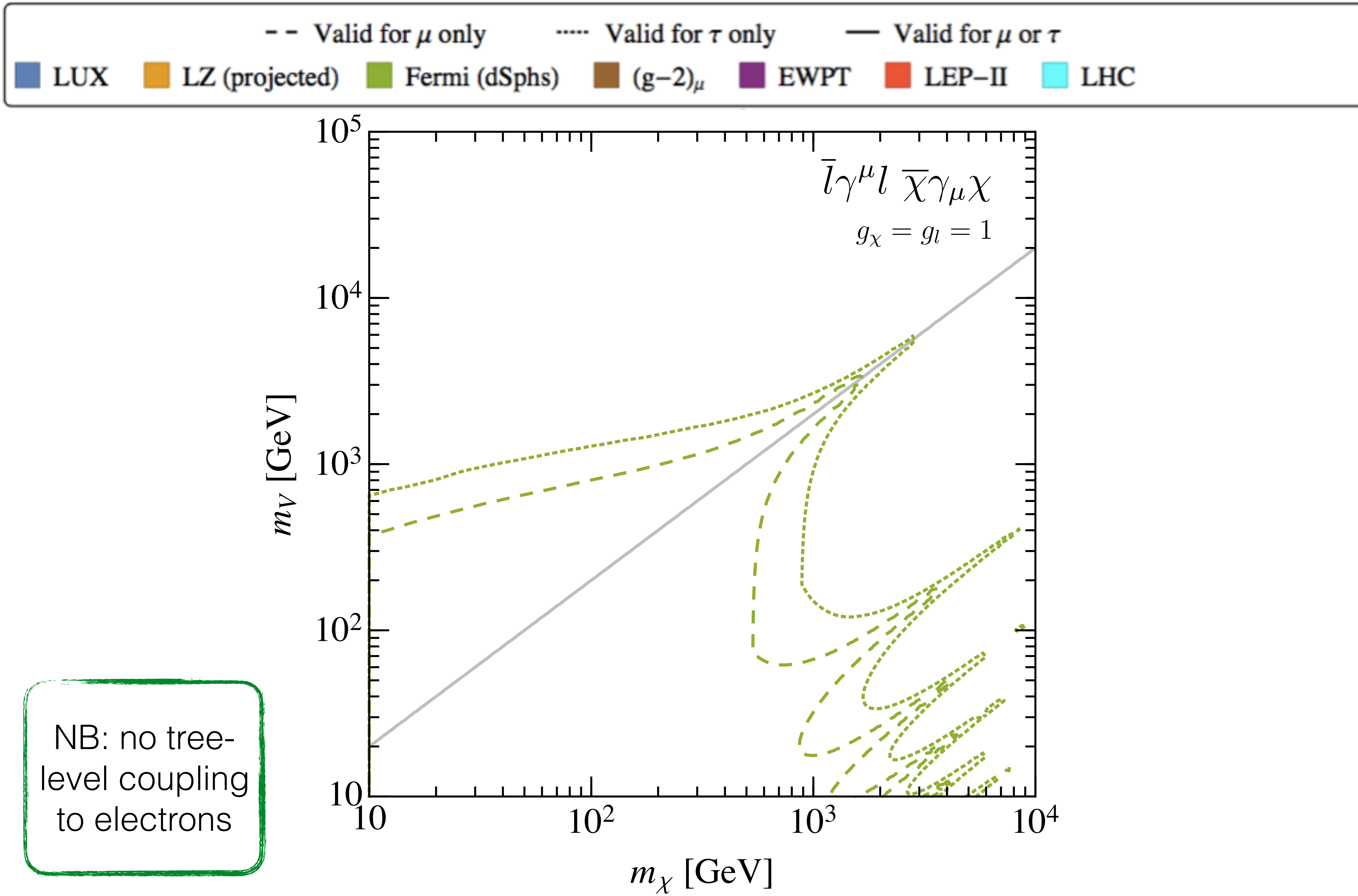
Strong constraints on leptophilic DM come from LEP-II [hep-ex/0312023]



Constraints on 4-fermion interaction bounds mediator mass to be greater than 3-4 TeV

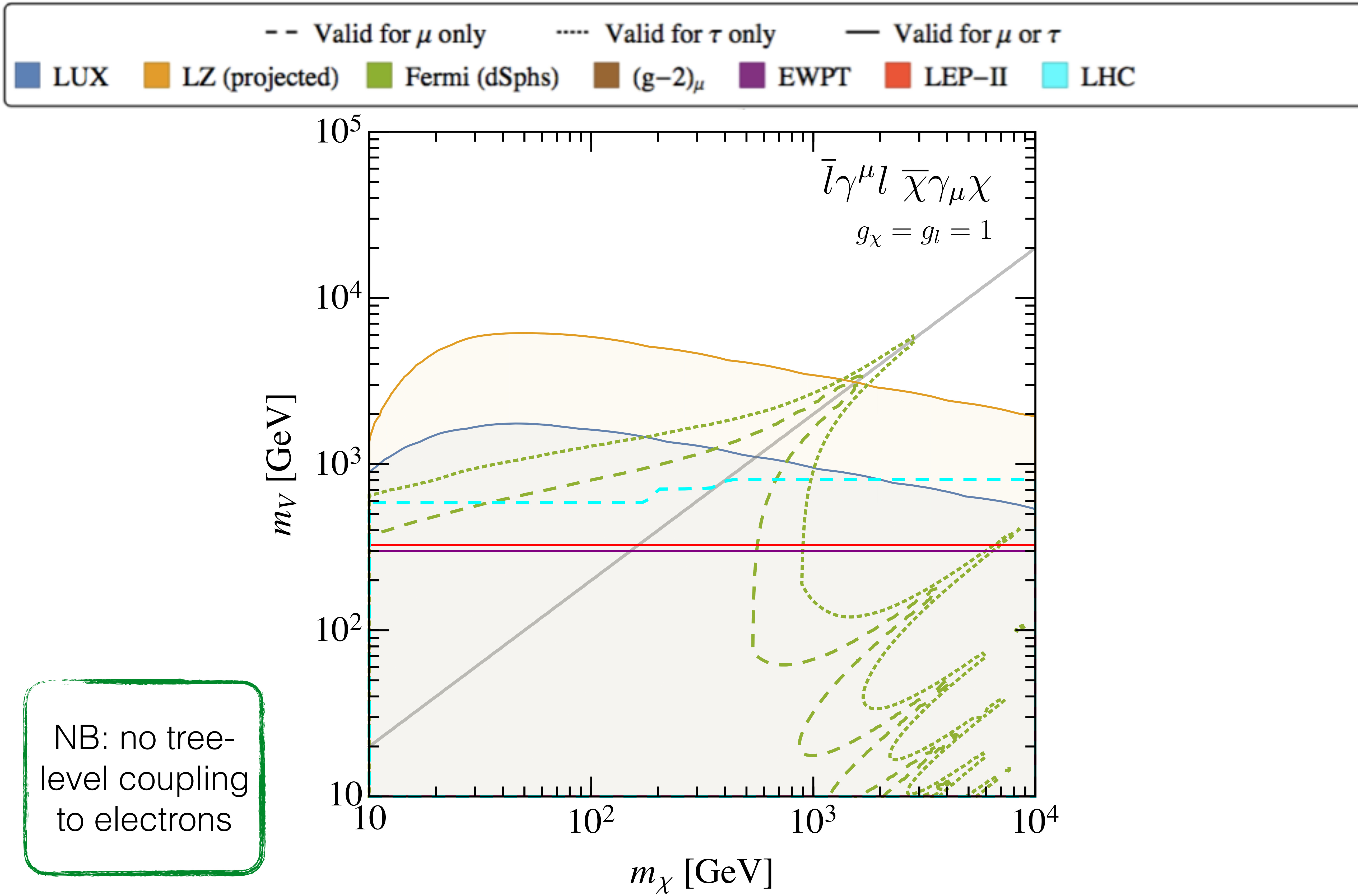
We focus on the case where DM couples only to either the muon or tau, at some high scale $\Lambda_{\rm UV} = 10\ \rm TeV$ …

Leptophilic: vector-vector



NB: no tree-level coupling to electrons

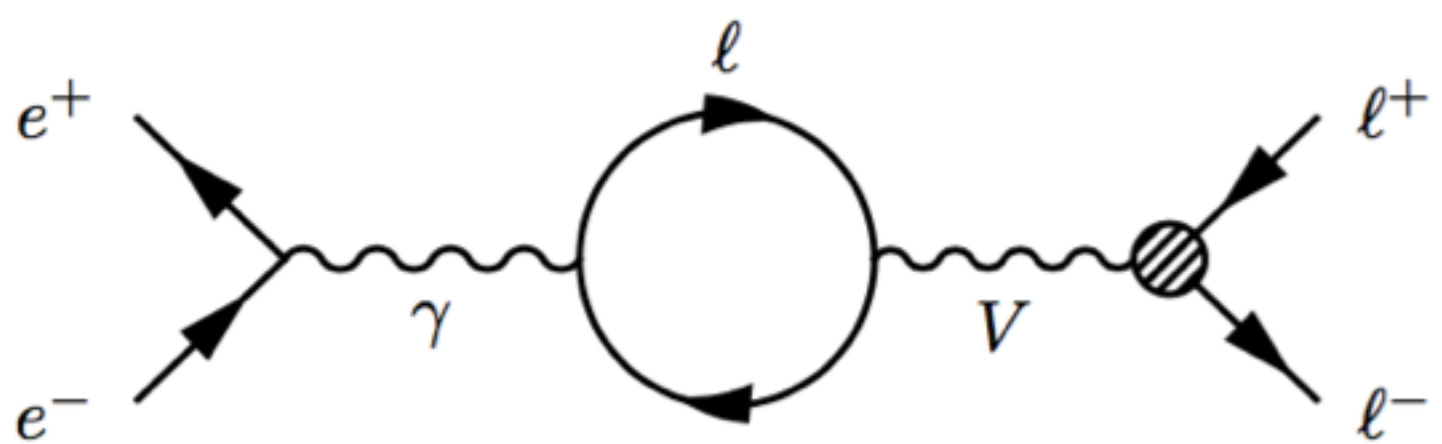
Leptophilic: (vector) - (vector)



NB: no tree-level coupling to electrons

Loop-induced LEP-II constraints

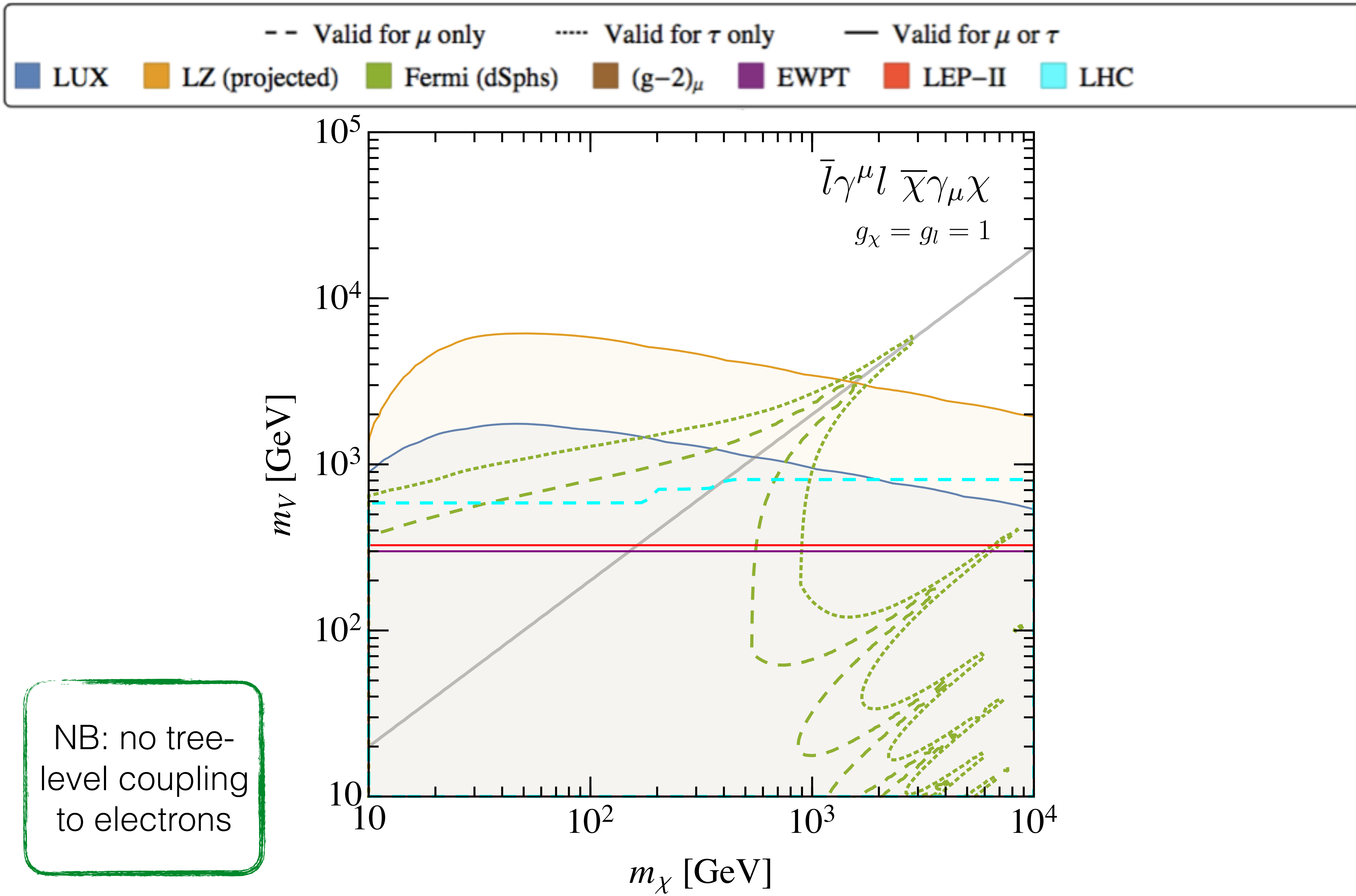
Still get an extra contribution to 4-fermion interactions $\bar{e}e\bar{\ell}\ell$ even if we don't couple to electrons at tree-level



Calculate loop contribution using RUNDM

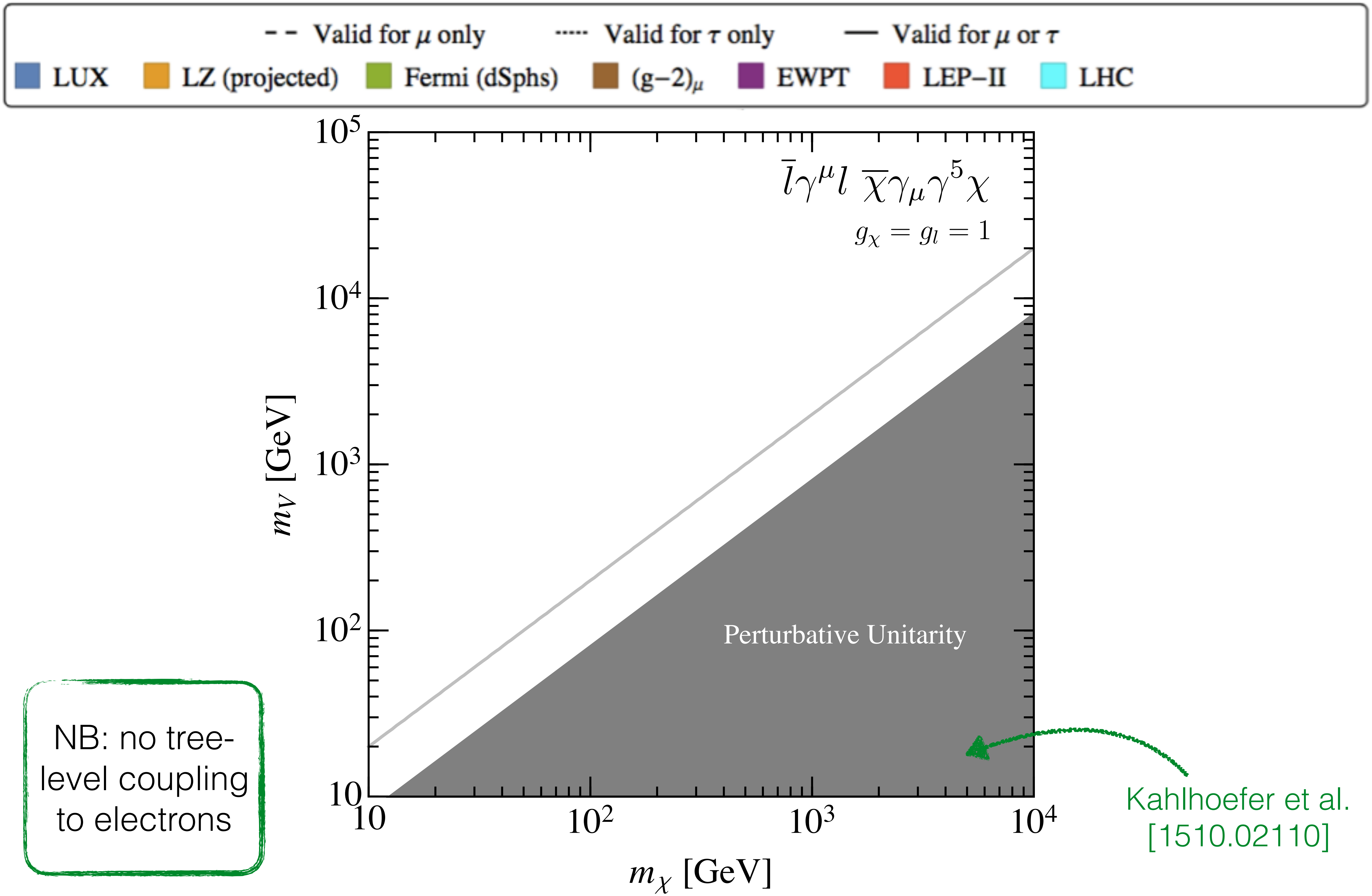
Mediator mass constrained to be larger than ~300 GeV.

Leptophilic: (vector) - (vector)



NB: no tree-level coupling to electrons

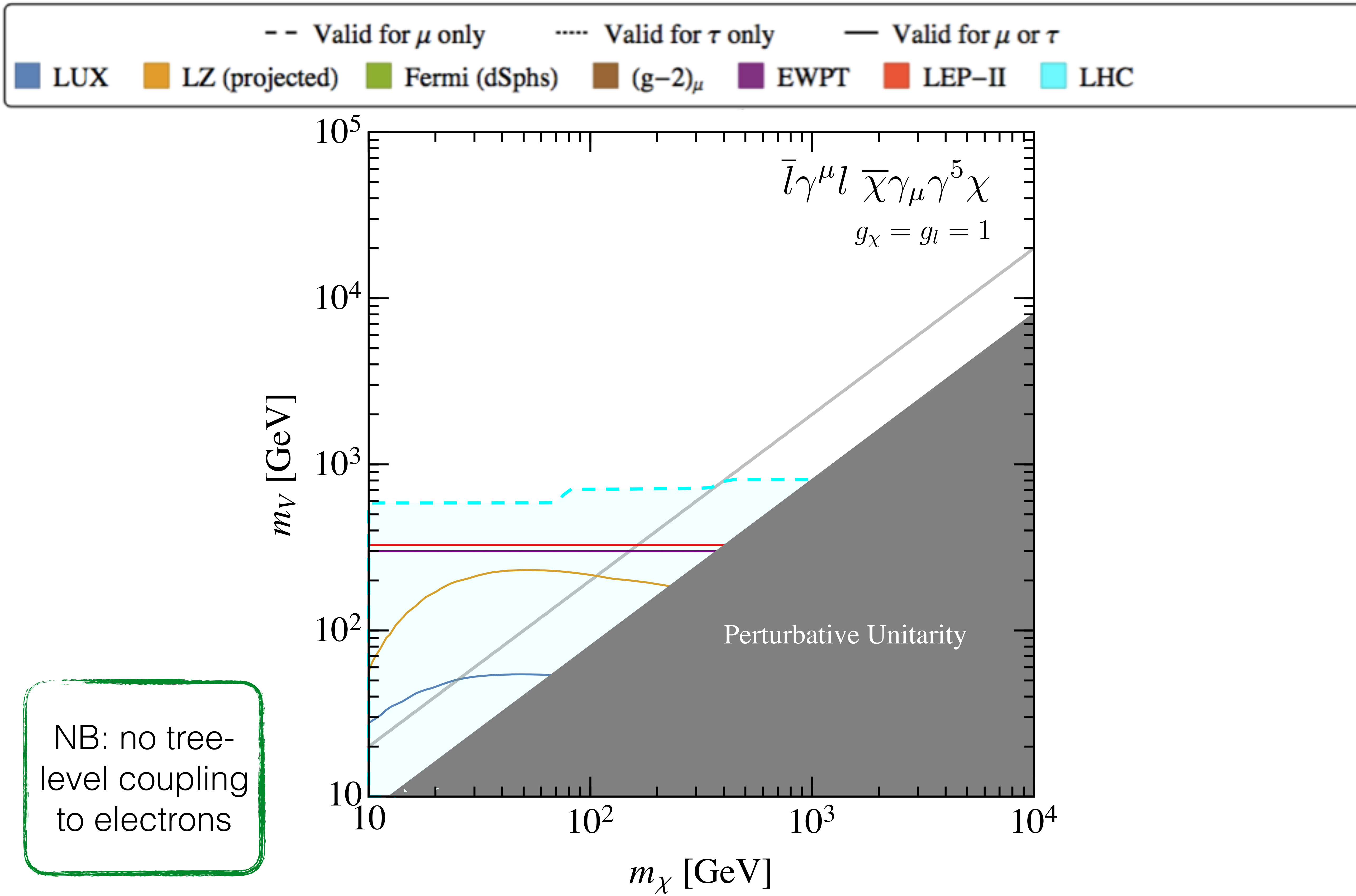
Leptophilic: (vector) - (axial vector)



NB: no tree-level coupling to electrons

Kahlhoefer et al. [1510.02110]

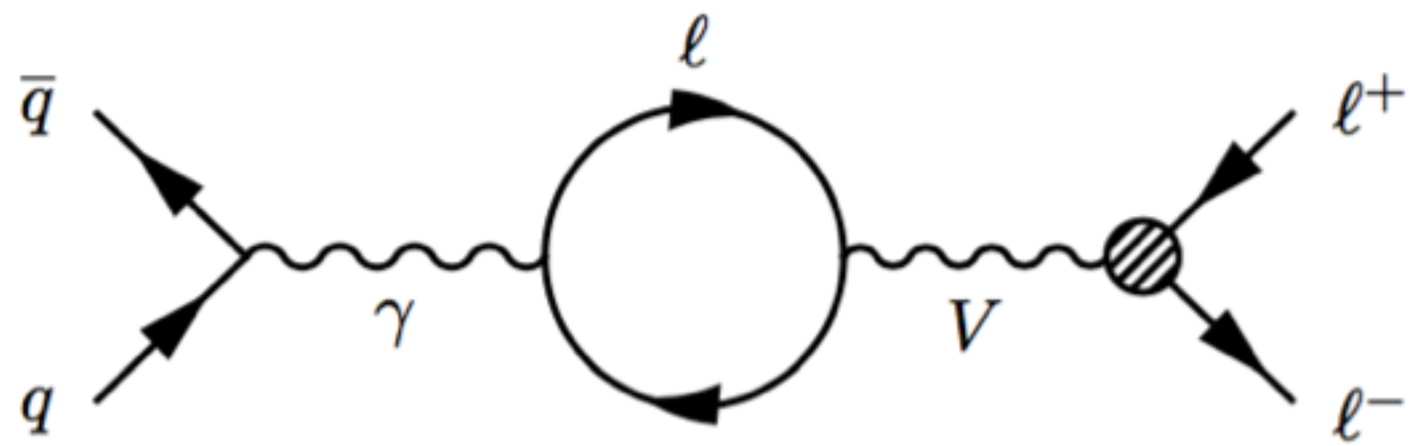
Leptophilic: (vector) - (axial vector)



NB: no tree-level coupling to electrons

Loop-induced LHC constraints

Strongest constraints come from dilepton resonance searches

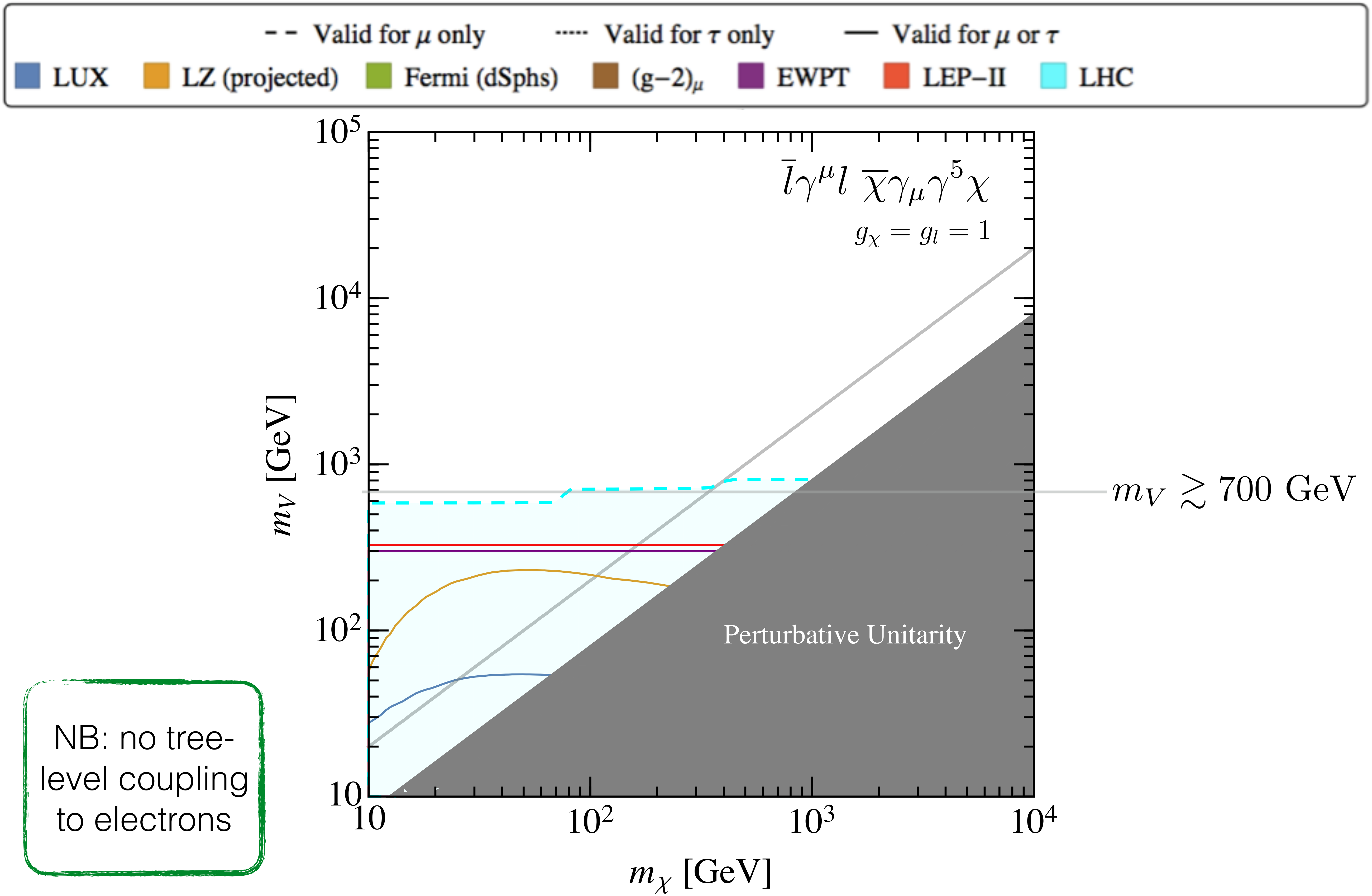


$$\sigma_{pp\to l^+l^-} = \frac{\pi\, BR_{V\to l^+l^-}}{3s}\sum_q C_{q\bar{q}}(m_V^2/s)\ \left(g_{Vq}^2 + g_{Aq}^2\right)$$

$$g_q(\mu) \propto \frac{\alpha_Y}{\pi}\left(\sum_{l=e,\mu,\tau} g_{V\,l}\right)\log\left(\Lambda_{\rm UV}/\mu\right)$$

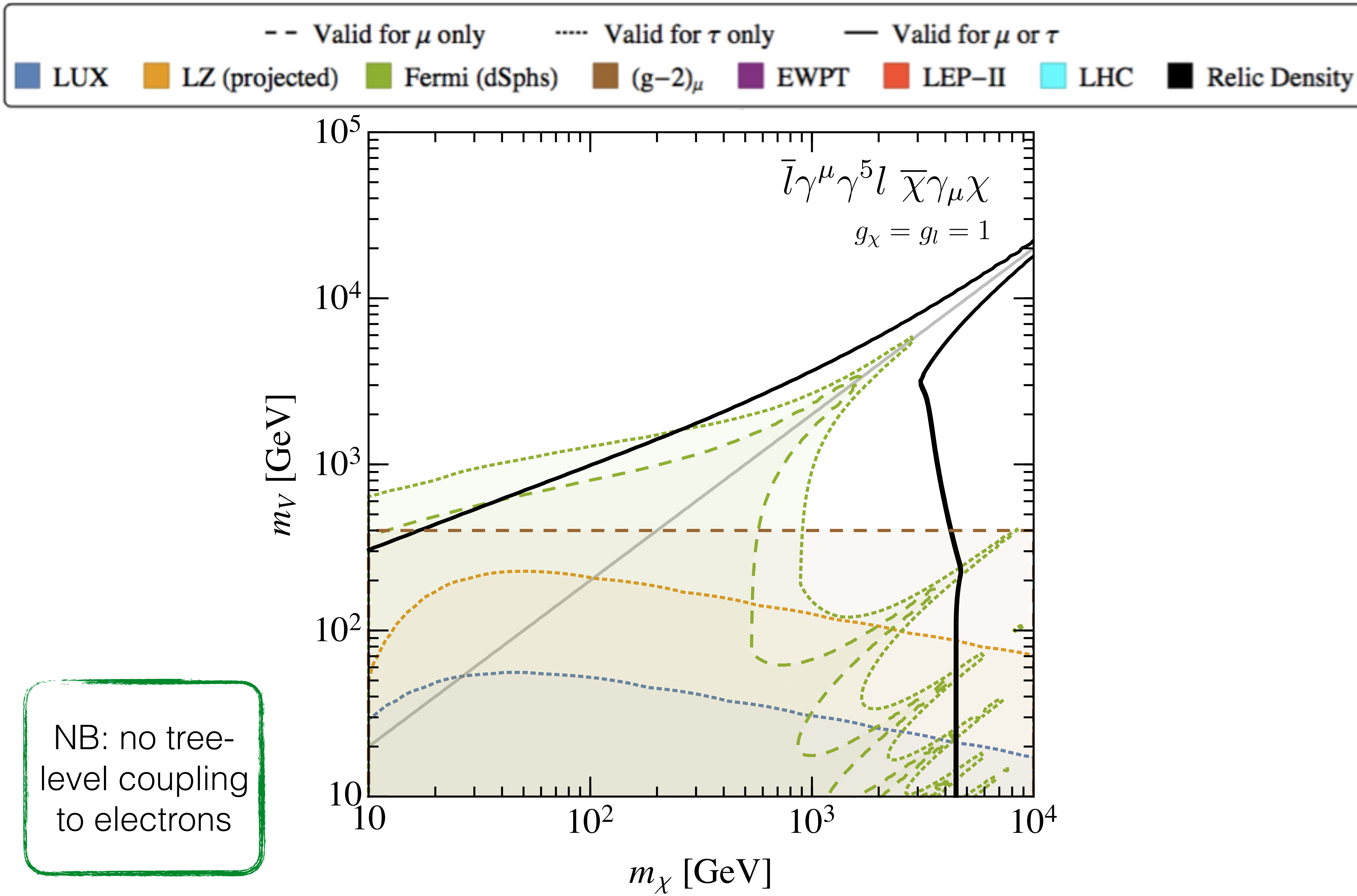
Also get constraints from monophoton [1103.0240] and 4-lepton searches [1403.5657]

Leptophilic: (vector) - (axial vector)



NB: no tree-level coupling to electrons

Leptophilic: (axial vector) - (vector)



NB: no tree-level coupling to electrons

Aside: DM-electron scattering

DM-electron scattering implies electron-electron scattering…

Constraints on 4-fermion interaction from LEP-II requires mediator mass to be greater than ~ 3 TeV for O(1) couplings to electrons

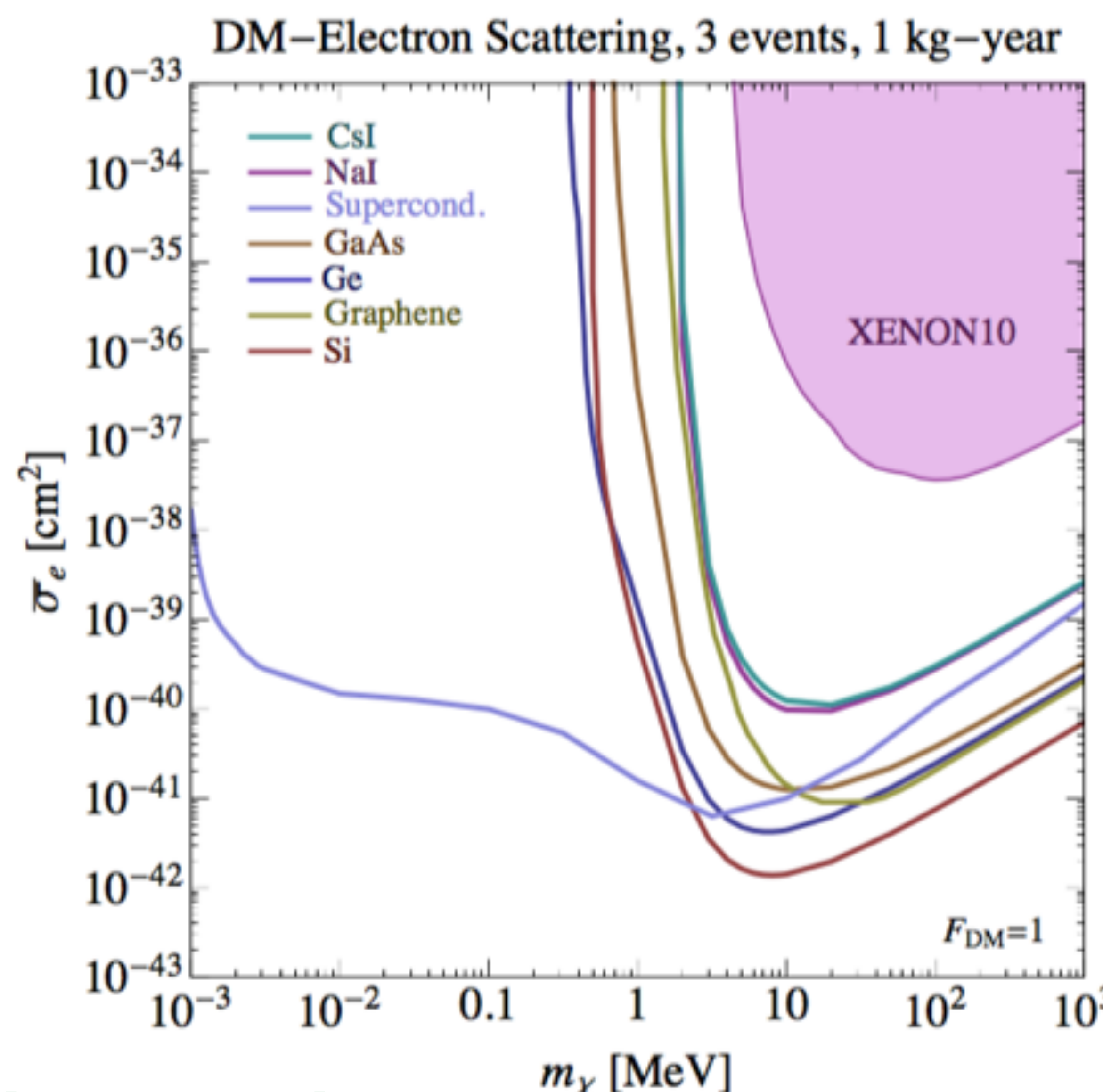
$$\sigma_e \sim \frac{\mu_{\chi e}^2}{\pi}\frac{1}{m_V^4}$$

$$\sigma_e \lesssim 10^{-49}\ \mathrm{cm}^2$$

for tree-level coupling to electrons…

$$\sigma_e \lesssim 10^{-46}\ \mathrm{cm}^2$$

for loop-level coupling to electrons…



[1608.08632]

Summary

- Need to carefully connect Dark Matter searches at different energy scales
- Depends only on SM loops - ***you have to run!***
- Can be done consistently with RUNDM code (tinyurl.com/runDM1)
- Leads to bounds where you weren't expecting them
- Important for understanding complementarity between direct detection and LHC searches (esp. isospin violation)
- In leptophilic models, operator mixing means that hadronic constraints can be the strongest!

Summary

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- In leptophilic models, operator mixing means that hadronic constraints can be the strongest!

Thank you!

Backup slides

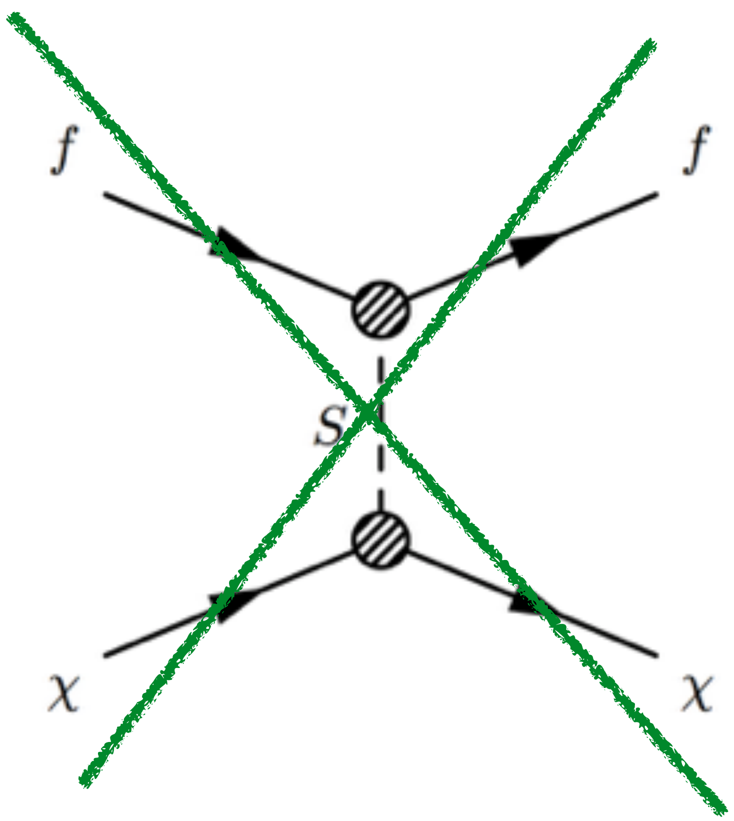
Scalar Mediator

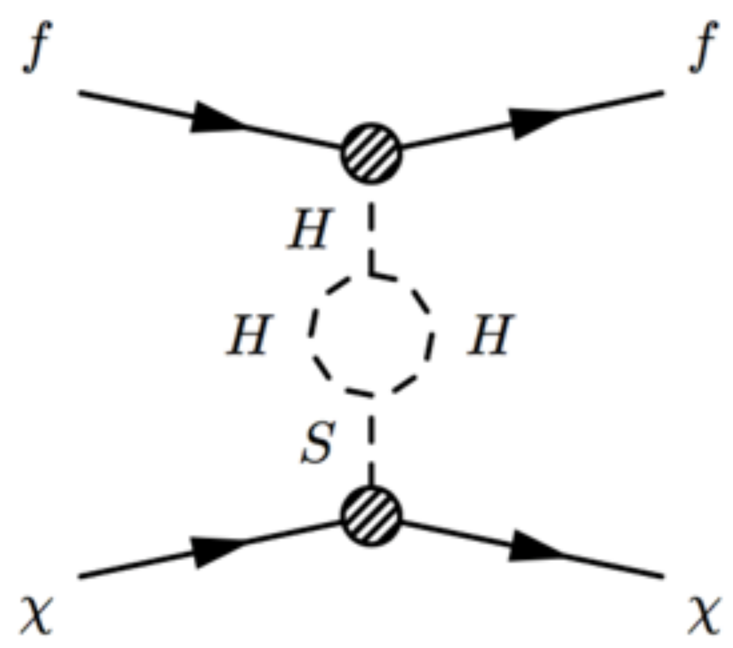
Interactions through a scalar mediator appear at dimension-7, with rates typically suppressed by the quark mass

$$O_{gg}^{S} = \frac{\alpha_s}{\Lambda^3}\,\bar{\chi}\chi\, G_{\mu\nu}G^{\mu\nu}\,, \quad O_{qq}^{SS} = \frac{m_q}{\Lambda^3}\,\bar{\chi}\chi\,\bar{q}q\,,$$

Crivellin, D'Eramo, Procura [1402.1173]

Buckley et al. [1410.6497]





A 750 GeV Scalar Mediator

A 750 GeV Portal:
LHC Phenomenology
and Dark Matter Candidates

Francesco D'Eramo[a,b], Jordy de Vries[c], Paolo Panci[d]

750 GeV Resonance
Rest In Peace
2015-2016

D'Eramo, de Vries, Panci [1601.01571]

$$\mathcal{L}_{\rm EFT}^{m_S<\mu<\Lambda} = \sum_{q=u,d,s,c,b,t} \frac{c_{yq}\, y_q}{\Lambda} S\, (\bar{q}_L H q_R + \text{h.c.}) + \frac{c'_{GG}\, \alpha_s}{\Lambda} S\, G^{A\,\mu\nu} G^A_{\mu\nu} \ ,$$

$$\mathcal{C}_q(\mu_N) \simeq -5.86\, \mathcal{C}_{GG}(m_S) \ ,$$
$$\mathcal{C}_{GG}(\mu_N) \simeq 4.01\, \mathcal{C}_{GG}(m_S) \ .$$

Substantial RG effects!