Towards micrOMEGAs 5: Freeze-in

Work in progress in collaboration with
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Coming up, soon!

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IRN - Terascale 2017, Montpellier

Outline

· Introduction: from freeze-out to freeze-in

· Types of freeze-in

· Status of micrOMEGAs 5 development and features

· Outlook

Evolution of a particle species

Consider a particle 1 interacting with three other particles through 1+2 ↔ 3+4 in a FLRW Universe. The evolution of its distribution is described by a Boltzmann equation:

$$\frac{d}{dt}\left[\int f_1\left(E,t\right)\frac{g_1 d^3p_1}{\left(2\pi\right)^3}\right] + 3H\int f_1\left(E,t\right)\frac{g_1 d^3p_1}{\left(2\pi\right)^3} =$$

$$-\sum_{\text{spins}}\int\left[f_1 f_2\left(1\pm f_3\right)\left(1\pm f_4\right)\left|\mathcal{M}_{12\to34}\right|^2 - f_3 f_4\left(1\pm f_1\right)\left(1\pm f_2\right)\left|\mathcal{M}_{34\to12}\right|^2\right]$$

$$\times\left(2\pi\right)^2\delta^4\left(p_1+p_2-p_3-p_4\right)\times\prod_{i=1}^{4}\left(\frac{d^3p_i}{\left(2\pi\right)^3 2E_i}\right)$$

Evolution of a particle species

Consider a particle 1 interacting with three other particles through 1+2 ↔ 3+4 in a FLRW Universe. The evolution of its distribution is described by a Boltzmann equation:

Known

Usually the quantity we're most interested in computing, although full distribution carries extremely useful information too.

$$
\frac{d}{dt}\left[\int f_1(E,t)\,\frac{g_1 d^3p_1}{(2\pi)^3}\right] + 3\overbrace{H}^{\text{Hubble parameter}} \overbrace{\int f_1(E,t)\,\frac{g_1 d^3p_1}{(2\pi)^3}}^{\equiv\, n(t)} =
$$

$$
-\sum_{\text{spins}}\int\Big[f_1 f_2 (1\pm f_3)(1\pm f_4)\underbrace{|\mathcal{M}_{12\to34}|^2}_{\text{Matrix elements}} - \underbrace{f_3 f_4 (1\pm f_1)(1\pm f_2)}_{\text{1, 2, 3, 4 distribution functions}}|\mathcal{M}_{34\to12}|^2\Big]
$$

$$
\times (2\pi)^2\,\delta^4(p_1+p_2-p_3-p_4)\times\prod_{i=1}^{4}\left(\frac{d^3p_i}{(2\pi)^3\,2E_i}\right)
$$

Known (model-building part)

Evolution of a particle species

Consider a particle 1 interacting with three other particles through 1+2 ↔ 3+4 in a FLRW Universe. The evolution of its distribution is described by a Boltzmann equation:

Hubble parameter

Known

$\equiv n(t)$

Usually the quantity we're most interested in computing, although full distribution carries extremely useful information too.

$$\frac{d}{dt}\left[\int f_1(E,t)\,\frac{g_1 d^3 p_1}{(2\pi)^3}\right] + 3H\int f_1(E,t)\,\frac{g_1 d^3 p_1}{(2\pi)^3} =$$

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$$\times (2\pi)^2\,\delta^4(p_1+p_2-p_3-p_4)\times\prod_{i=1}^{4}\left(\frac{d^3 p_i}{(2\pi)^3\,2E_i}\right)$$

Matrix elements

Known (model-building part)

1, 2, 3, 4 distribution functions

To compute $f_1(E, t_0)$ in full generality, we also need:

· The distributions of 2, 3, 4 at all temperatures.

· The distribution of 1 at some initial temperature.

(or, eventually, solve a coupled set of 4 such equations knowing $f_{1,2,3,4}$ at some temperature)

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$$\frac{d}{dt}\left[\int f_1(E,t)\,\frac{g_1 d^3p_1}{(2\pi)^3}\right] + 3H\overbrace{\int f_1(E,t)\,\frac{g_1 d^3p_1}{(2\pi)^3}}^{\equiv\, n(t)} =$$

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$$\times (2\pi)^2\,\delta^4(p_1+p_2-p_3-p_4)\times\prod_{i=1}^{4}\left(\frac{d^3p_i}{(2\pi)^3\,2E_i}\right)$$

Usually the quantity we're most interested in computing, although full distribution carries extremely useful information too.

Matrix elements

Known (model-building part)

1, 2, 3, 4 distribution functions

Imagine now actually having a realistic model with lots of particles involved in such reactions. Long story short:

It's a pretty complicated problem to tackle in full generality!

Freeze-out

Given the complexity of the problem, we try to find interesting limits. Assume that particle 1 possesses substantial interactions with 2, 3 and 4. Then:

e.g. Gondolo, Gelmini, Nucl. Phys. B (1991)

$$\frac{d}{dt}\left[\int f_1\left(E,t\right)\frac{g_1 d^3p_1}{\left(2\pi\right)^3}\right] + 3H\overbrace{\int f_1\left(E,t\right)\frac{g_1 d^3p_1}{\left(2\pi\right)^3}}^{\equiv n\left(t\right)} =$$

$$-\sum_{\text{spins}}\int\left[f_1 f_2\left(1\pm f_3\right)\left(1\pm f_4\right)\left|\mathcal{M}_{12\to34}\right|^2 - f_3 f_4\left(1\pm f_1\right)\left(1\pm f_2\right)\left|\mathcal{M}_{34\to12}\right|^2\right]$$

$$\times\left(2\pi\right)^2\delta^4\left(p_1+p_2-p_3-p_4\right)\times\prod_{i=1}^{4}\left(\frac{d^3p_i}{\left(2\pi\right)^3 2E_i}\right)$$

Replace with $\left|\mathcal{M}_{12\to34}\right|^2$

Unitarity

Ignore

Decoupling during radiation domination

$\sim f_3^{eq} \times f_4^{eq} = f_1^{eq} \times f_2^{eq}$

Typically the SM particles, thermalize quickly + Detailed balancing

After quite a bit of algebra, we get the usual expression:

$$\dot{n}_1 + 3Hn_1 = -\left\langle\sigma v\right\rangle\left(n_1 n_2 - n_1^{\text{eq}} n_2^{\text{eq}}\right)$$

where $$\left\langle\sigma v\right\rangle = \frac{\int \sigma v\ dn_1^{\text{eq}}\ dn_2^{\text{eq}}}{\int dn_1^{\text{eq}}\ dn_2^{\text{eq}}}$$

The other side of the spectrum: freeze-in

Assume, now that particle 1 instead possesses feeble (very weak) couplings with 2, 3 and 4.
Reasonable assumption (perhaps): its initial density is negligible.

arXiv:hep-ph/0106249
arXiv:0911.1120 ...and many more

$$\frac{d}{dt}\left[\int f_1(E,t)\frac{g_1 d^3p_1}{(2\pi)^3}\right] + 3H \overbrace{\int f_1(E,t)\frac{g_1 d^3p_1}{(2\pi)^3}}^{\equiv n(t)} =$$

$$-\sum_{\text{spins}}\int\left[\underbrace{f_1 f_2(1\pm f_3)(1\pm f_4)|\mathcal{M}_{12\to34}|^2}_{\text{Ignore}} - \underbrace{f_3 f_4}_{\sim f_3^{eq} \times f_4^{eq}}\underbrace{(1\pm f_1)(1\pm f_2)}_{\text{Ignore}}|\mathcal{M}_{34\to12}|^2\right]$$

$$\times(2\pi)^2\delta^4(p_1+p_2-p_3-p_4)\times\prod_{i=1}^{4}\left(\frac{d^3p_i}{(2\pi)^3\,2E_i}\right)$$

Ignore: Feeble coupling + feeble density

Ignore: Decoupling during radiation domination

$\sim f_3^{eq} \times f_4^{eq}$: Typically the SM particles, thermalize quickly

· No DM annihilation term.

· No equilibrium between 1 and the other particles.

Freeze-out vs freeze-in: some remarks

· Naively, the freeze-in equation is simpler than the freeze-out one: quite similar but only including the DM production term.

· When working in full generality (i.e. for a general particle physics model), though, things are much more involved:

- Equilibrium erases all memory: no dependence on the initial conditions.

Good or bad, depending on perspective!

- In freeze-out, no need to keep track of the decays of heavier particles (unless they happen late, i.e. after freeze-out).

Equilibrium is restored extremely fast

- On the model-building side: more than one particles in the spectrum can be feebly coupled, need to write down Boltzmann equations for them too.

Need to keep track of the evolution of all states and the way they contribute to the DM abundance.

Will clarify this more in a minute

· The most complicated case: the intermediate regime.

Basic types of freeze-in : scattering

The basic premise of the freeze-in mechanism is that DM interacts extremely weakly with the Standard Model particles. One can then envisage two basic scenarios:

I) Dark matter could be produced directly from 2 → 2 processes, annihilations of Standard Model (or other bath) particles: a + b → χ + χ

$$\dot{n}_\chi + 3Hn_\chi = \int \prod_{i=a,b,\chi,\bar{\chi}} \frac{d^3 |\vec{p}_i|}{(2\pi)^3 2E_i} (2\pi)^4 \delta^4 (p_a + p_b - p_\chi - p_{\bar{\chi}}) |\mathcal{M}|^2 f_a f_b$$

Or, eventually, by defining $\tilde{K}_1 (s, T) \equiv \frac{1}{4p_{a,b}^{CM} T} \int dE_+ dE_- f_a f_b$

Reduces to Bessel function of the 1st kind for Maxwell-Boltzmann statistics.

$$\boxed{\dot{n}_\chi + 3Hn_\chi = \frac{T}{128\pi^6} \int \frac{ds}{\sqrt{s}} p_{\chi\bar{\chi}}^{CM} p_{ab}^{CM} \tilde{K}_1 (s, T) |\mathcal{M}|^2 d\Omega}$$

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· No good argument to assume Maxwell-Boltzmann distribution.

Can induce ~factor 2 difference in the DM abundance computation for B-E statistics, example coming up…

· Not necessarily IR dominated mechanism, details depend on model.

- "Ultraviolet freeze-in": 1410.6157 (NR operators)
- Freeze-in from heavy bath particles (in progress)

Basic types of freeze-in : decays - 1

The basic premise of the freeze-in mechanism is that DM interacts extremely weakly with the Standard Model particles. One can then envisage two basic scenarios:

IIa) Dark matter could be produced from the decay of a heavier particle in equilibrium with the thermal bath: Y → χ + χ (but χ is a FIMP)

$$\dot{n}_\chi + 3Hn_\chi = \int \prod_{i=Y,\chi,\bar{\chi}} \frac{d^3\left|\vec{p}_i\right|}{(2\pi)^3\, 2E_i} (2\pi)^4 \delta^4\left(p_Y - p_\chi - p_{\bar{\chi}}\right) \left|\mathcal{M}\right|^2 f_Y^{\mathrm{eq}}$$

Or, eventually, by defining $\hat{K}_1\left(m_Y, T\right) \equiv \frac{1}{m_Y T}\int dE_Y \sqrt{E_Y^2 - m_Y^2} f_Y^{\mathrm{eq}}$

$$\boxed{\dot{n}_\chi + 3Hn_\chi = \Gamma_{Y\to\chi\bar{\chi}} n_Y^{\mathrm{eq}} \frac{\hat{K}_1\left(m_Y, T\right)}{K_1\left(m_Y, T\right)}}$$

· The heavy particle can be Z2-even or Z2-odd.

"Mediator" - decays into DM pairs.

"NLOP" - decays into single DM particles.

· Since the heavy particles are in equilibrium, no need to write down any dedicated Boltzmann equation.

Unless freeze-in after Y freeze-out, *cf* next slide

Basic types of freeze-in : decays - 2

The basic premise of the freeze-in mechanism is that DM interacts extremely weakly with the Standard Model particles. One can then envisage two basic scenarios:

IIb) Dark matter could be produced from the decay of a heavier particle which is *not* in equilibrium with the thermal bath: Y → χ + χ (both χ and Y are FIMPs)

$$\dot{n}_\chi + 3Hn_\chi = \int \prod_{i=Y,\chi,\bar{\chi}} \frac{d^3 \left|\vec{p}_i\right|}{(2\pi)^3 \, 2E_i} (2\pi)^4 \, \delta^4 \left(p_Y - p_\chi - p_{\bar{\chi}}\right) \left|\mathcal{M}\right|^2 f_Y^{\mathrm{eq}}$$

Slightly trickier case, since one needs to write down a Boltzmann equation for the decaying particle as well

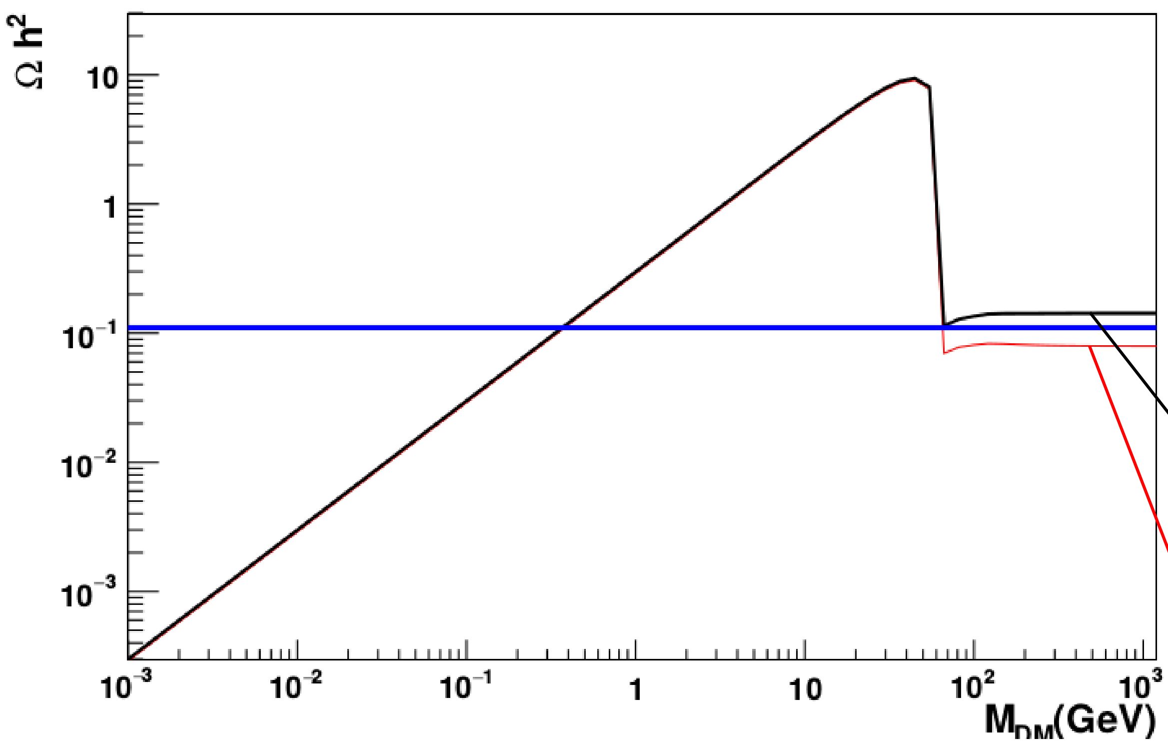
In progress...

A first result

Numerous tests in progress, first result for the singlet scalar DM model

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{\mu_s^2}{2} s^2 + \frac{\lambda_s}{4} s^4 + \lambda_{Hs} H^\dagger H s^2$$

Model first studied in the context of freeze-in by C. Yaguna in arXiv:1105.1654



Example for parameter choices as:

· $\lambda_{Hs} = 10^{-11}$

· m_h = 125 GeV

Assuming Bose-Einstein statistics for the SM particles.

Assuming Maxwell-Boltzmann statistics.

Statistics can be important, taken into account in this version of micrOMEGAs. Price to pay: code relatively slower.

In practice - routines

Using micrOMEGAs 5.0 for freeze-in calculations will require some insight from the user:

Freeze-out and freeze-in modules will be completely separated: when computing the relic abundance, you'll need to *assume* a production mechanism.

Basic freeze-in routines:

· `decayAbundance(TR,M,w,Ndf,eta)`: freeze-in from decays of particle of mass `M`, `Ndf` degrees of freedom and partial width `w` into DM. The particle is assumed in equilibrium with the SM (`TR`: Reheating temperature).

Freeze-in DM abundance from decays of a heavy particle in equilibrium with the SM.

· `SMabundanceFreezeIn(TR,name,&err)`: freeze-in through scattering of SM particles.

Freeze-in through annihilation of SM particles.

· `abundanceFreezeIn(TR,Process,&err)`: freeze-in through scattering for a given `Process`.(for individual SM contributions or for contributions from BSM particles in thermal equilibrium with the SM).

Freeze-in through annihilation of BSM particles.
+ Individual contributions, if needed.

In practice – words of caution

Using micrOMEGAs 5.0 for freeze-in calculations will require some insight from the user:

> Freeze-out and freeze-in modules will be completely separated: when computing the relic abundance, you'll need to *assume* a production mechanism.

i. e. you need to know what you're doing to some extent:

· For the moment, there is no internal check about which particles are in equilibrium with which (although freeze-out would give an enormous relic density at feeble coupling).

· Contributions from scattering/decays have to be computed separately and added up in the end.

· For freeze-in through decays or through annihilation of BSM states, the user must specify by hand all processes contributing to the DM abundance (annihilations of SM automatically taken into account in `SMabundanceFreezeIn` routine).

· BSM particles contributing to freeze-in through scattering are taken to be in equilibrium with the SM.

Outlook

· micrOMEGAs 5.0 will be able to handle feebly coupled dark matter candidates and compute their predicted abundance according to the freeze-in mechanism.

· Most major freeze-in scenarios will be covered, for this first version a bit of intuition from the user will be required.

So consult your local DM theorist!

· Our hope is that this will facilitate phenomenological studies (and model-building endeavours!) and help establish stronger connections between the early Universe phenomenology of FIMPs and their observational signatures.

· Under discussion: compute the full DM distribution function (interest for cosmology). Not sure if will be included in next release.

· To appear within the next couple of months (everyone needs some vacation!).

The ultimate goal (for the next versions): a unified treatment of all cases,with a smooth passages amongst the various regimes.

aka the most general form of the Boltzmann equation