Recent progress in precision dark matter calculation

Björn Herrmann

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Université Savoie Mont Blanc / CNRS — Annecy, France

GDR/IRN Terascale — 3rd - 5th July 2017 — Montpellier

Dark matter relic abundance — freeze-out picture

Time evolution of number density of the relic particle described by Boltzmann equation
— key ingredient from particle physics: **(co-)annihilation cross-section**

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3Hn - \langle \sigma_{\mathrm{ann}} v \rangle \left(n^2 - n_{\mathrm{eq}}^2 \right)$$

$$\Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_{\mathrm{crit}}} \sim \frac{1}{\langle \sigma_{\mathrm{ann}} v \rangle}$$

(dis)favoured parameter regions…?

$$\Omega_{\mathrm{CDM}} h^2 = 0.1199 \pm 0.0022$$

Relic abundance very precisely known
Planck collaboration 2015

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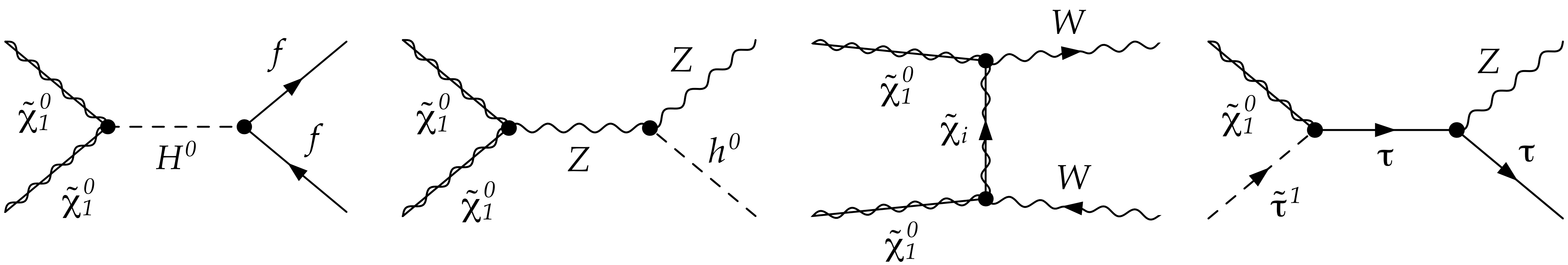
Computational tools allow an efficient calculation of the (neutralino) relic density:
DarkSUSY Bergström, Edsjö, Gondolo *et al.* 2004-2017, **micrOMEGAs** Bélanger, Boudjema, Pukhov *et al.* 2003-2017, **SuperIsoRelic** Arbey, Mahmoudi 2008, …

Motivation for higher order corrections

All processes implemented in public codes — **but only at the (effective) tree-level**



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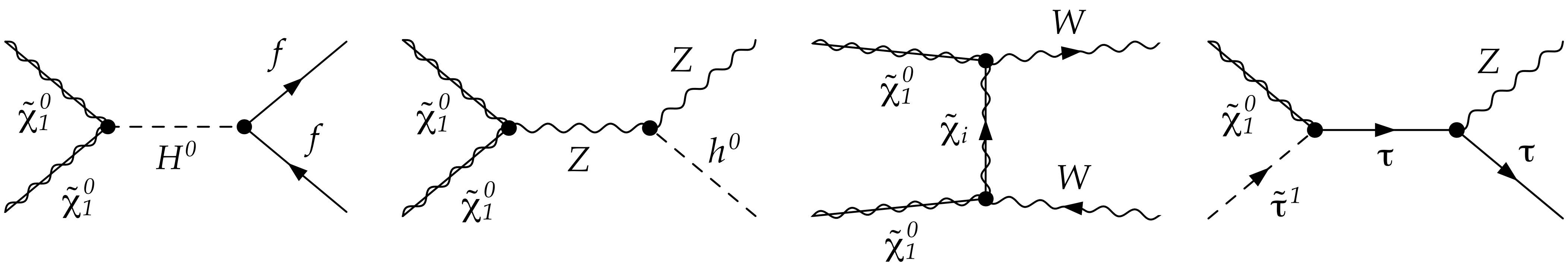
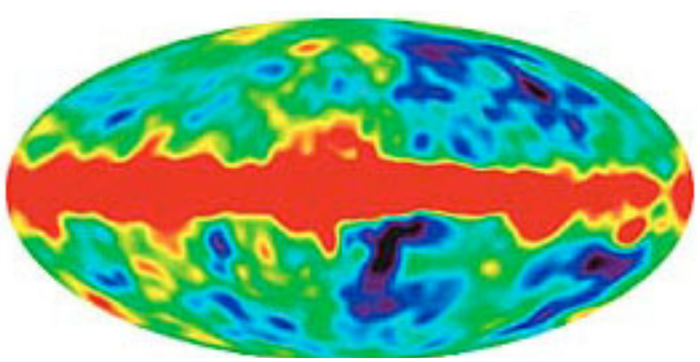


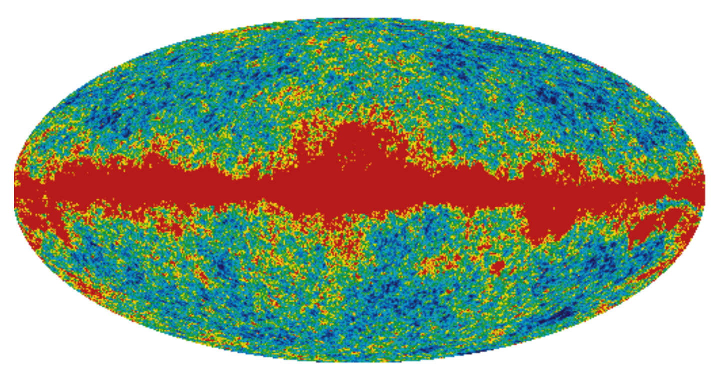
Higher-order loop corrections can give important contributions to cross-sections

In particular, sizeable impact from QCD corrections due to strong coupling constant

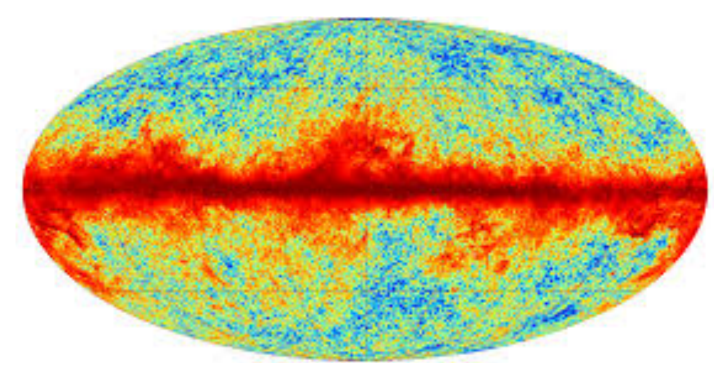
More precise theoretical predictions needed to keep up with experimental improvements



COBE 1989



WMAP 2002



Planck 2013

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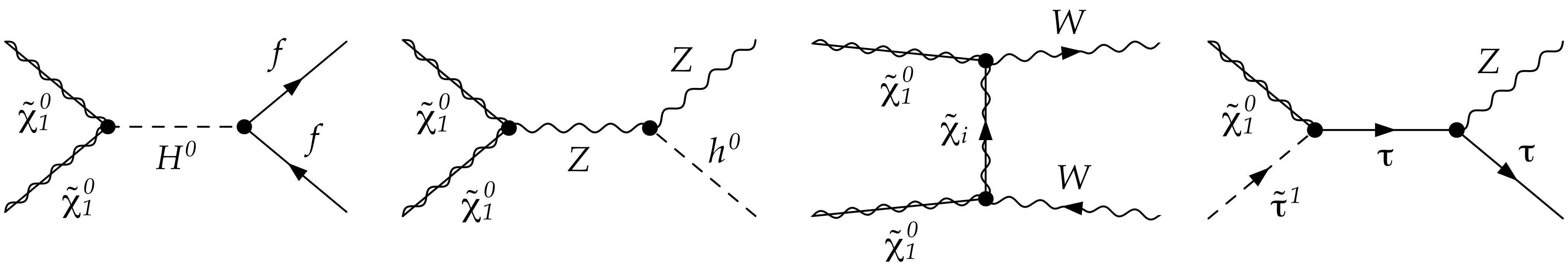

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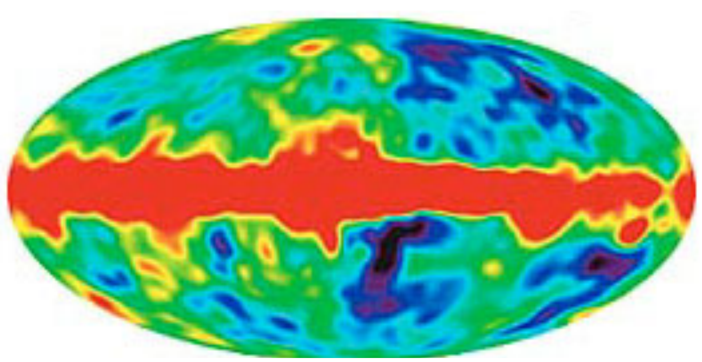

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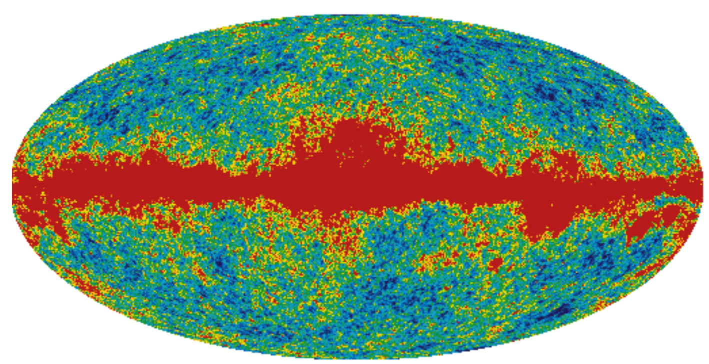

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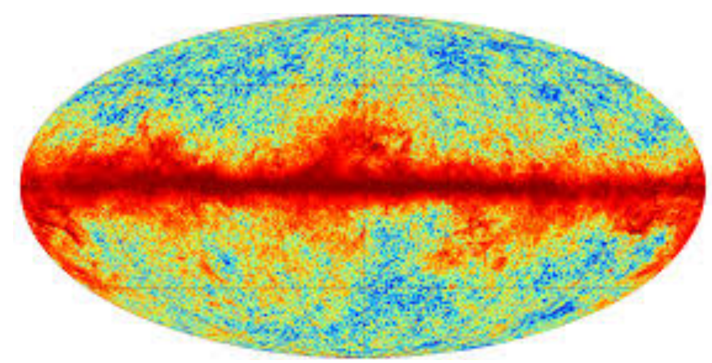
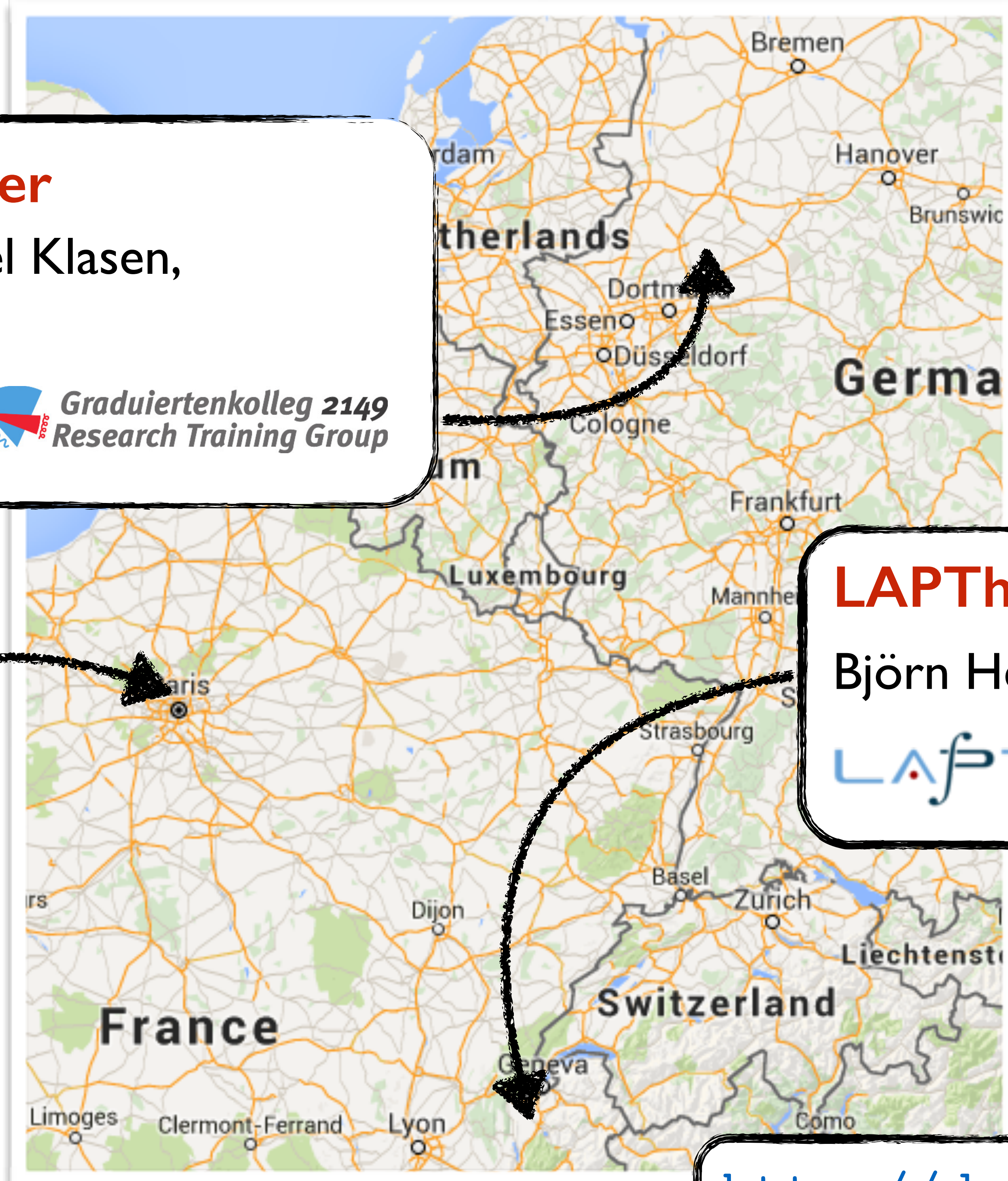


Planck 2013

DM@NLO project — **Provide calculation of σ_{ann} including QCD corrections**

— Extension to public codes (e.g. micrOMEGAs, DarkSUSY)...

DM@NLO — Collaboration



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Research Training Group

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CNRS

LPTHE Paris

Julia Harz

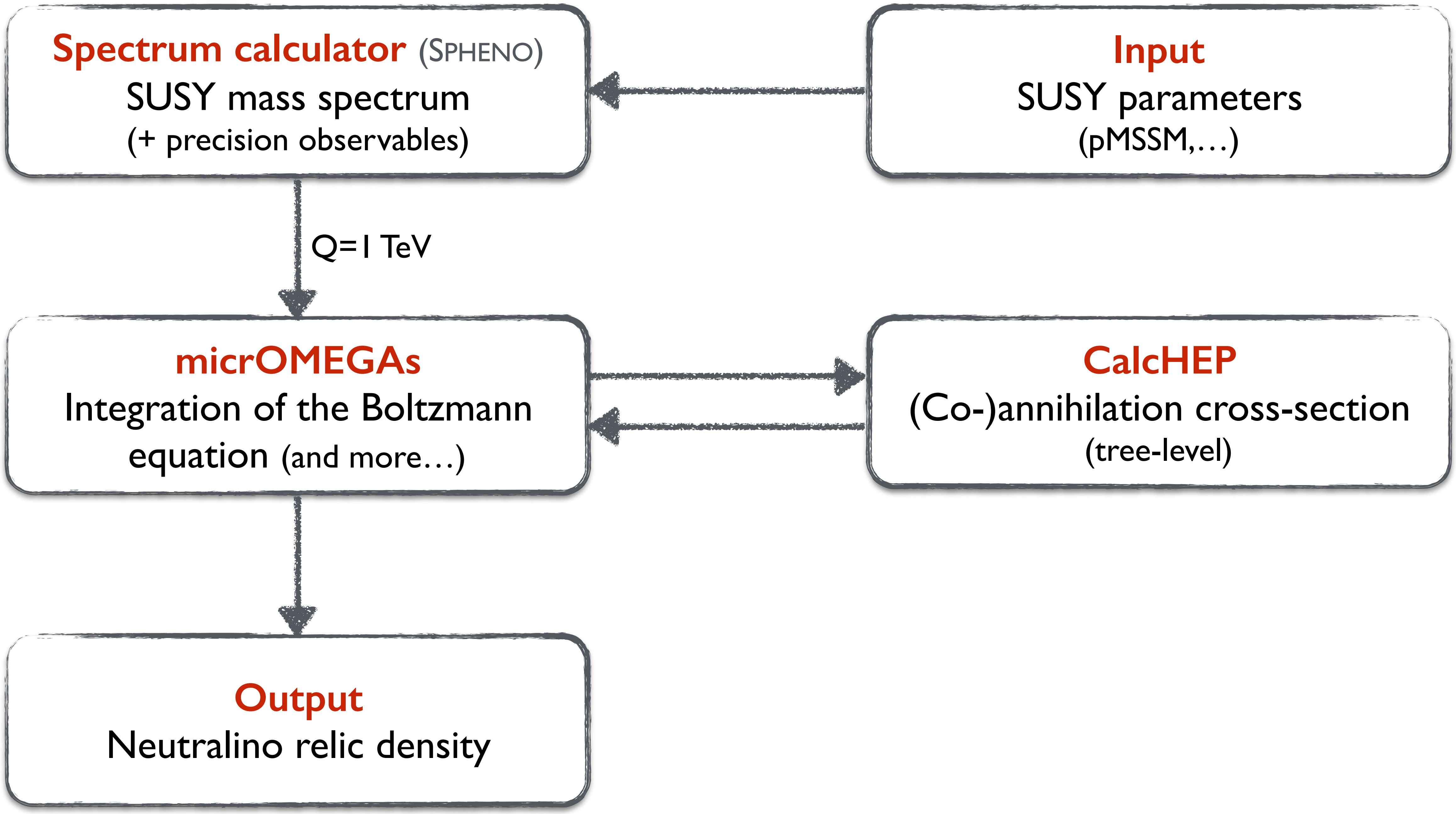
Institut Lagrange de Paris

UPMC Sorbonne Universités

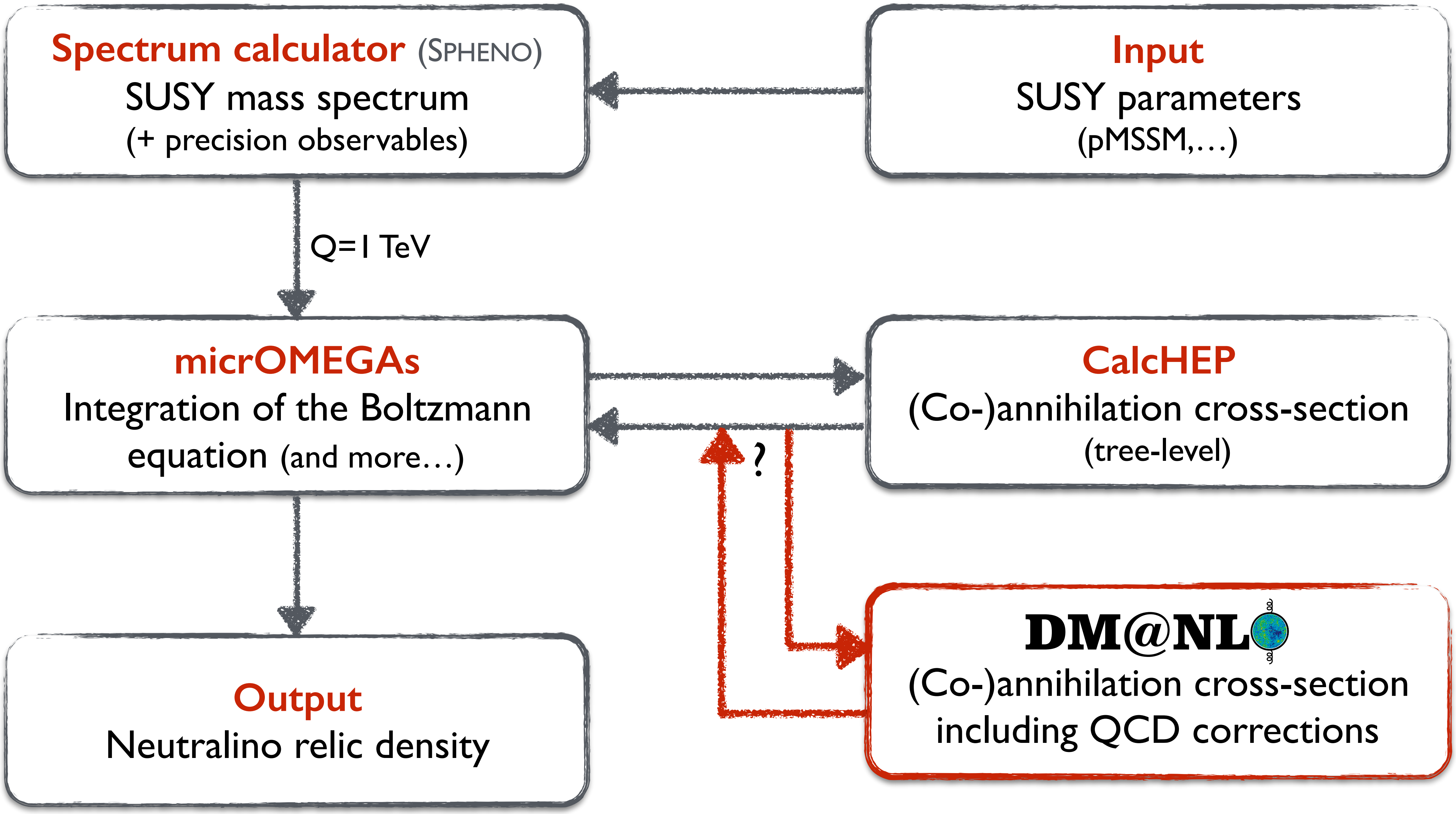
CNRS

http://dmnlo.hepforge.org

DM@NLO — Setup


Spectrum calculator (SPHENO)
SUSY mass spectrum
(+ precision observables)
Input
SUSY parameters
(pMSSM,...)
Q=1 TeV
micrOMEGAs
Integration of the Boltzmann equation (and more...)
CalcHEP
(Co-)annihilation cross-section
(tree-level)
Output
Neutralino relic density

DM@NLO — Setup



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DM@NLO
(Co-)annihilation cross-section including QCD corrections
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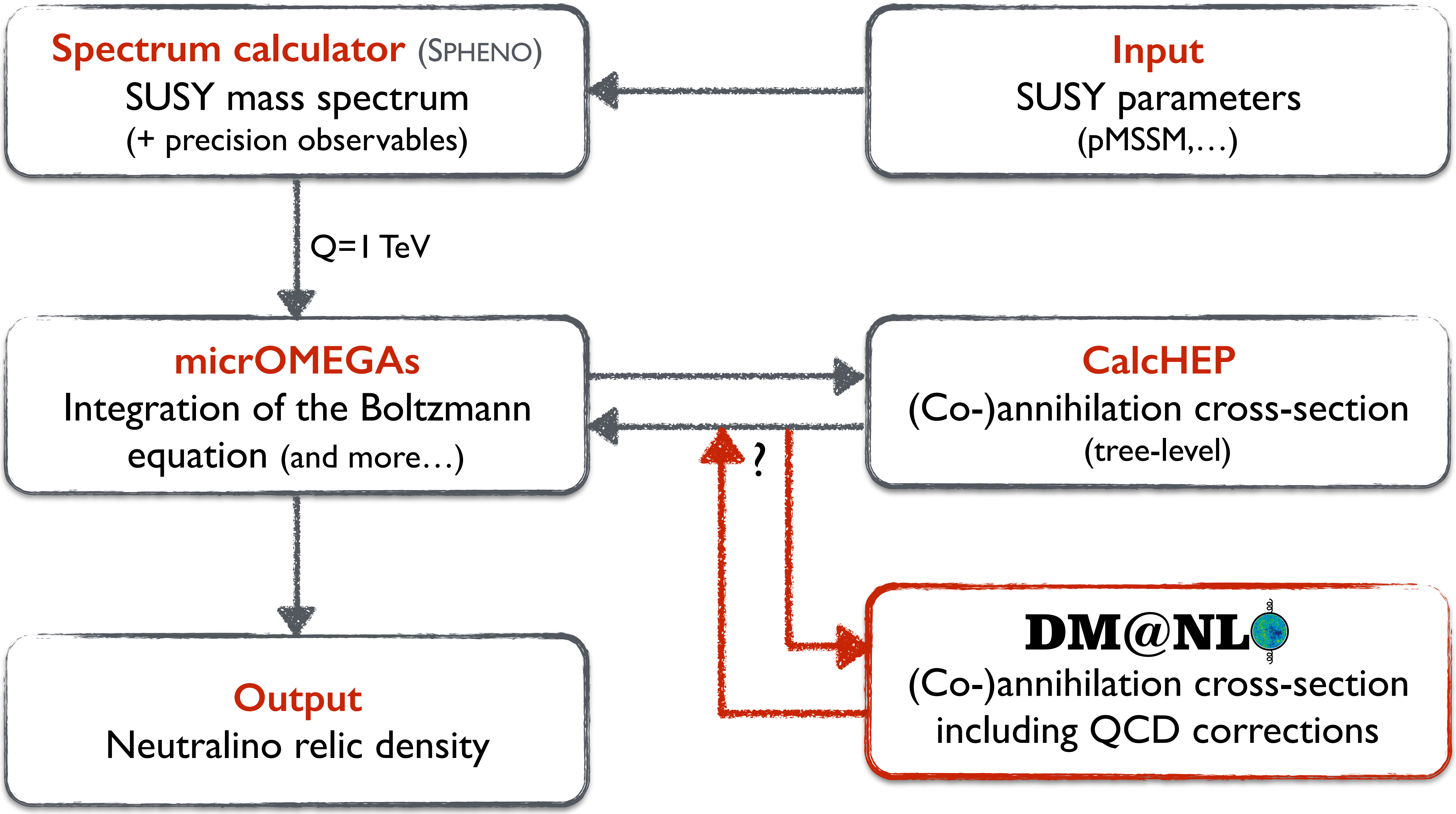
DM@NLO — Setup



Similar setup for use with DarkSUSY in development…

J. Edsjö, J. Harz, B. Herrmann, C. Niblaeus — *in progress…*

DM@NLO — Status

Provide a **next-to-leading order calculation** (in QCD) for the neutralino (co-)annihilation cross section (and thus for the neutralino relic density)

$$\tilde{\chi}\tilde{\chi}' \to q\bar{q}'$$

$$\tilde{\chi}\tilde{q} \to q'H/q'V$$

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→ **numerically implemented, results published**
Herrmann et al. 2009-2016

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→ **work in progress…**
to be published 2017-2018

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Definition and implementation of a dedicated **renormalization scheme**

Infrared treatment — phase space slicing and dipole subtraction à la Catani-Seymour

Resummation of **Coulomb corrections** for stop-stop annihilation

Application of the results to **direct detection**

Interfaces to **micrOMEGAs** (since 2008) and **DarkSUSY** *(work in progress)*

Outline

Motivation

Corrections to the neutralino (co)annihilation cross-section and impact on relic density

Application to direct dark matter detection

Scale dependence and theoretical uncertainty

Conclusion

M. Klasen, K. Kovařík, P. Steppeler — Phys.Rev. D94: 095002 (2016) — arXiv:1607.06396 [hep-ph]

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, P. Steppeler — Phys. Rev. D 93: 114023 (2016) — arXiv:1602.08103 [hep-ph]

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke — Phys. Rev. D 91: 034012 (2015) — arXiv:1410.8063 [hep-ph]

J. Harz, B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 91: 034028 (2015) — arXiv:1409.2898 [hep-ph]

B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke, P. Steppeler — Phys. Rev. D 89: 114012 (2014) — arXiv:1404.2931 [hep-ph]

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, Q. Le Boulc'h — Phys. Rev. D 87: 054031 (2013) — arXiv:1212.5241 [hep-ph]

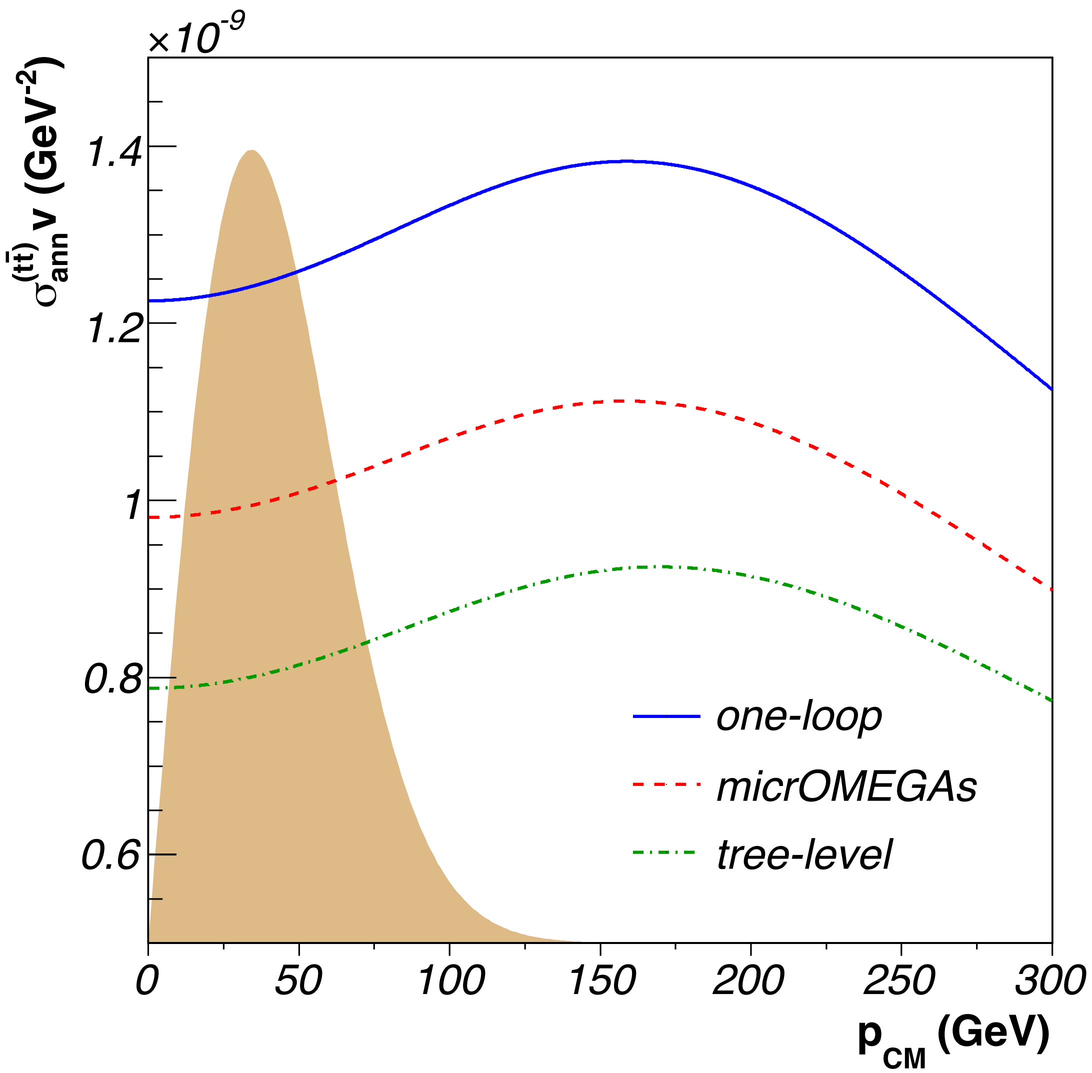
B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 79: 061701 (2009) — arXiv:0901.0481 [hep-ph]

B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 80: 085025 (2009) — arXiv:0907.0030 [hep-ph]

B. Herrmann, M. Klasen — Phys. Rev. D 76: 117704 (2007) — arXiv:0709.0043 [hep-ph]

Corrections to neutralino (co-)annihilation and impact on the relic density

Neutralino pair annihilation into top quarks



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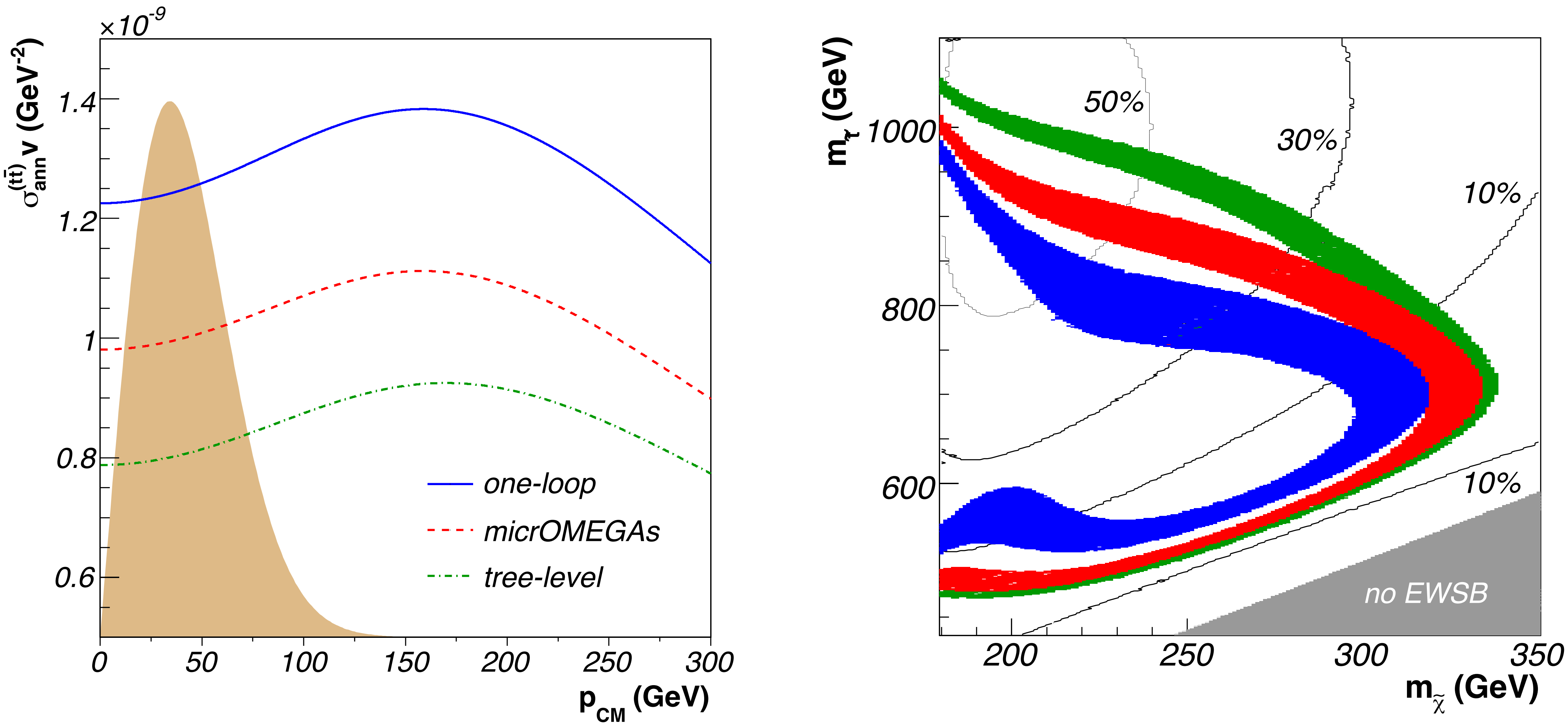

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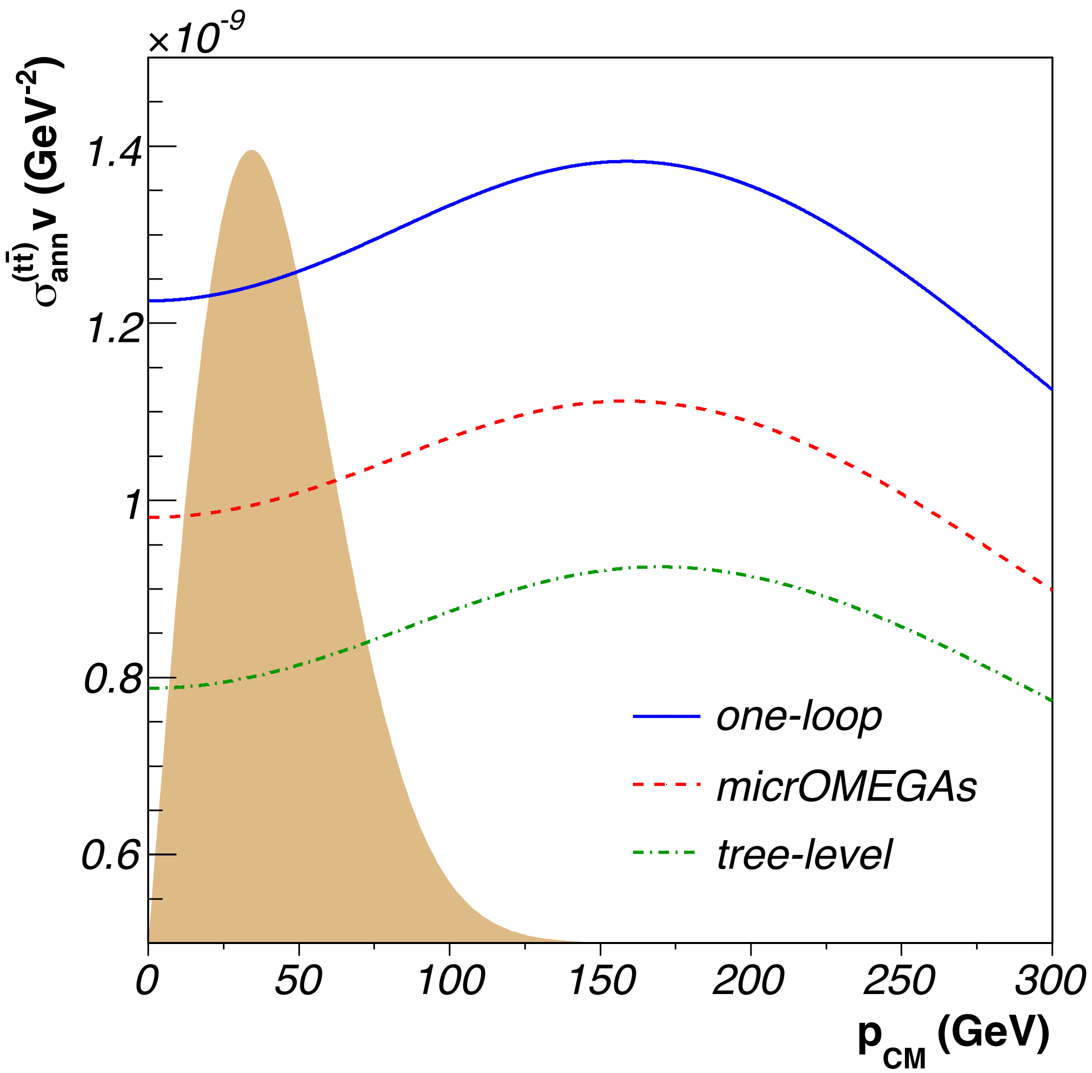
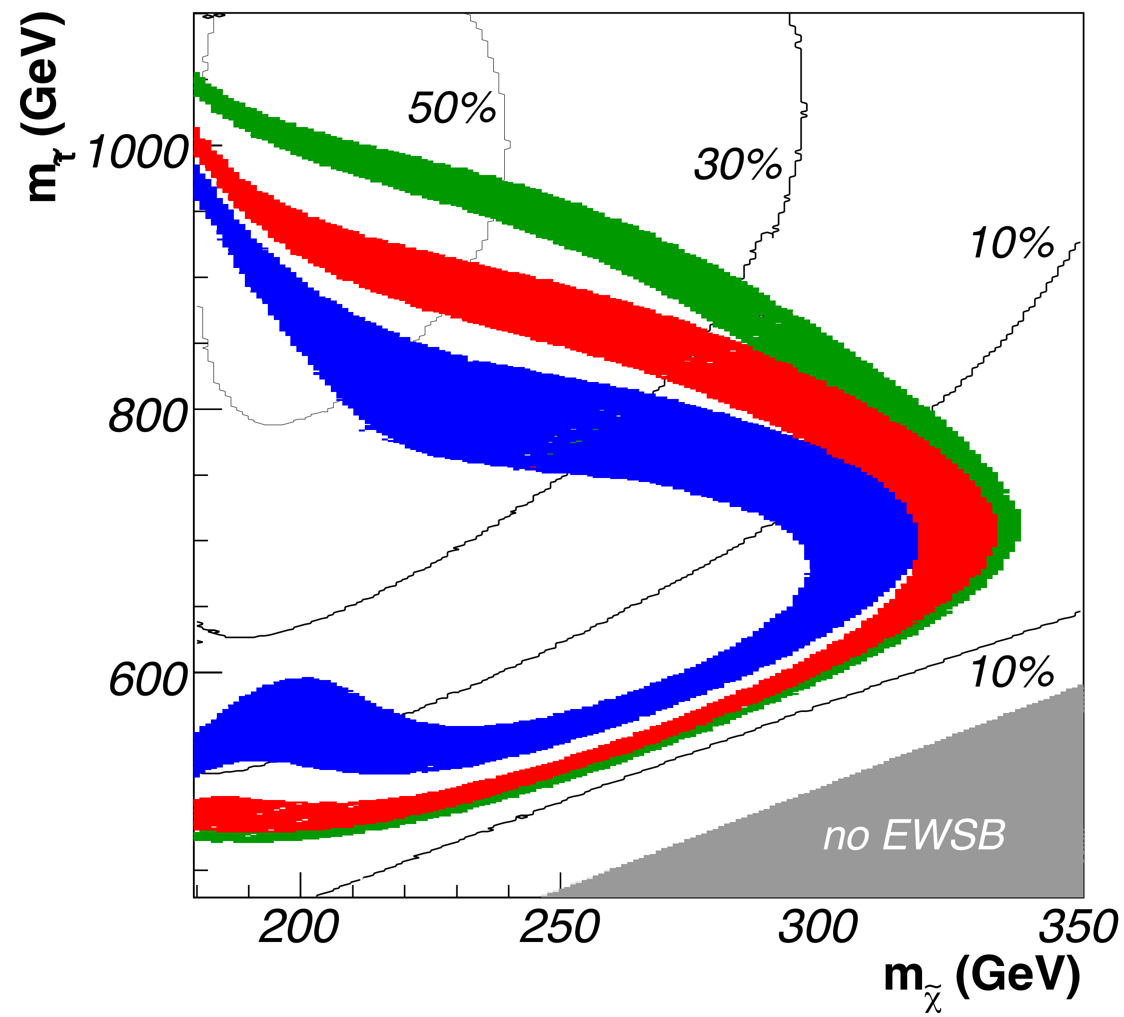


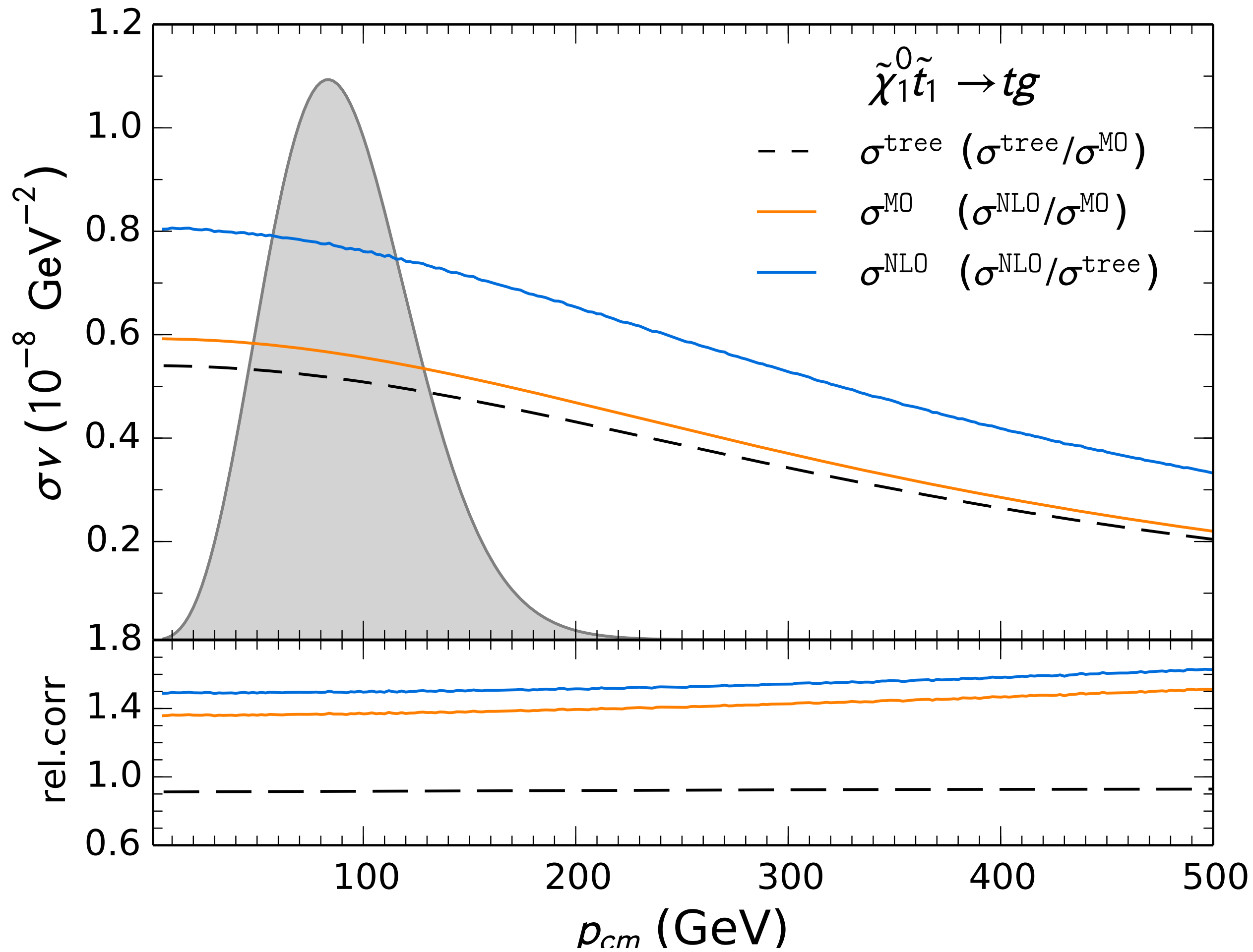


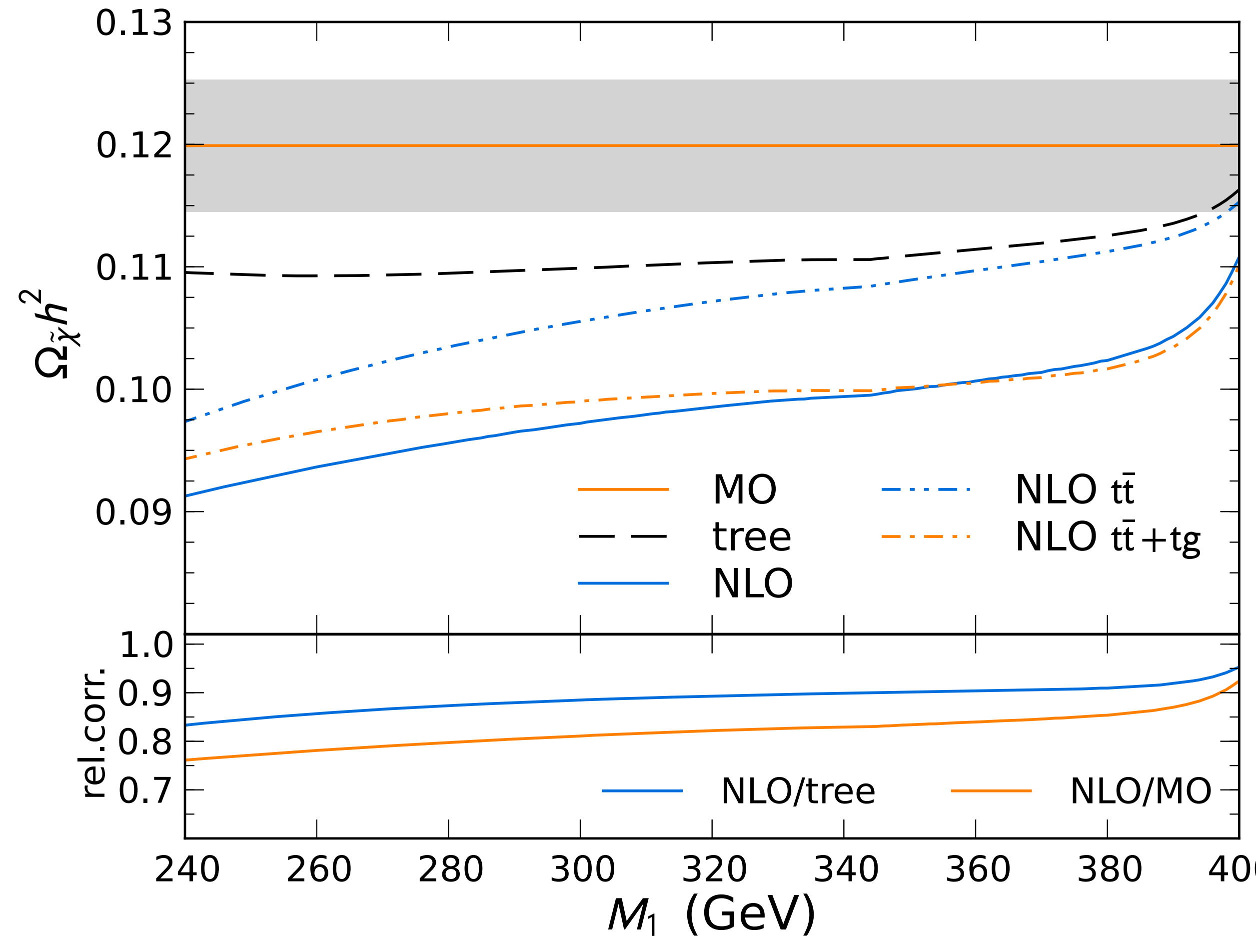
Annihilation cross-section enhanced by up to 50% by radiative corrections
Corrections can lead to **important shifts for preferred regions** (e.g. ~200 GeV for m_{stop})

Effective Yukawa couplings (as e.g. in micrOMEGAs) very good approximation around Higgs-resonances, **but other sub-channels can be dominant** (here: Z^0/squark-exchange)

B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 80: 085025 (2009) — arXiv:0907.0030 [hep-ph]
B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke, P. Steppeler — Phys. Rev. D 89: 114012 (2014) — arXiv:1404.2931 [hep-ph]

Neutralino-stop co-annihilation





Relative corrections of up to 40-50% observed for the co-annihilation cross-section, leading to a **numerically important shift** for the predicted **neutralino relic density** (up to about 25% — more than Planck uncertainty!)

Co-annihilation into **SM-like Higgs** and gluon most important (other final states generally subdominant)

Harz, Herrmann, Klasen, Kovařík, Le Boulc'h — Phys. Rev. D 87: 054031 (2013) — arXiv:1212.5241 [hep-ph]
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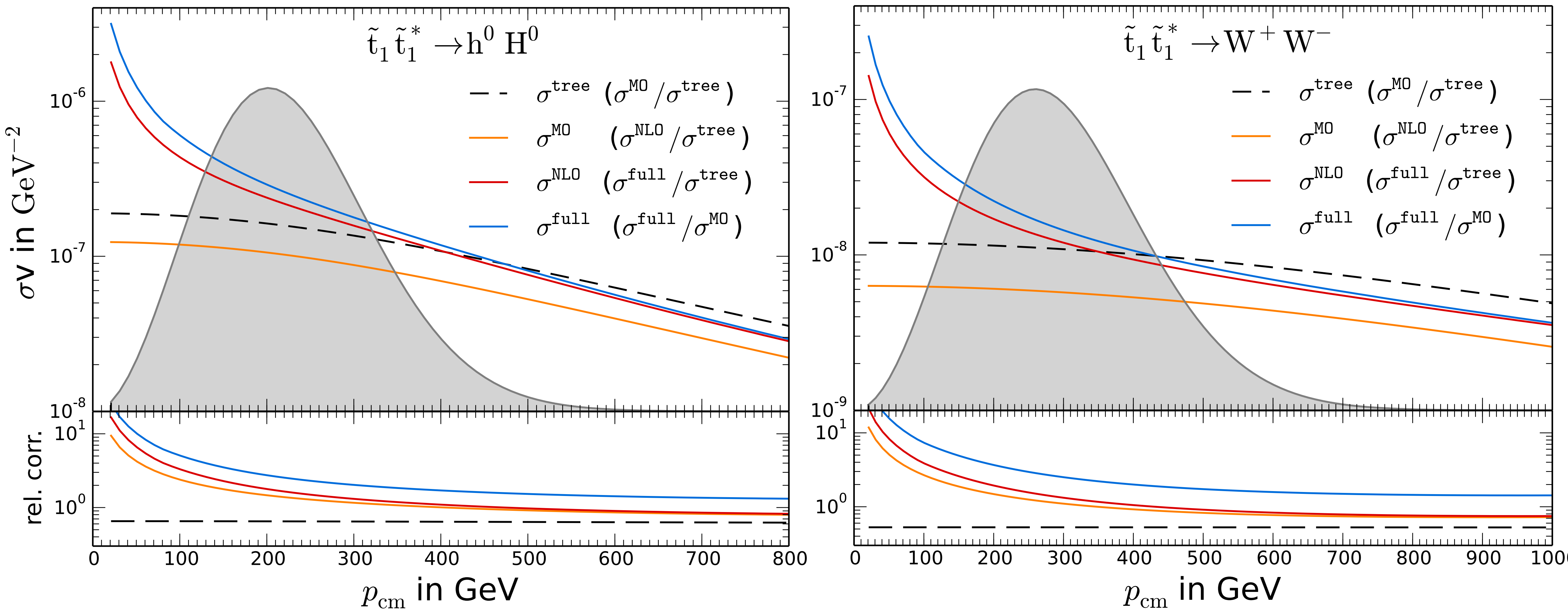
Stop pair annihilation — electroweak final states



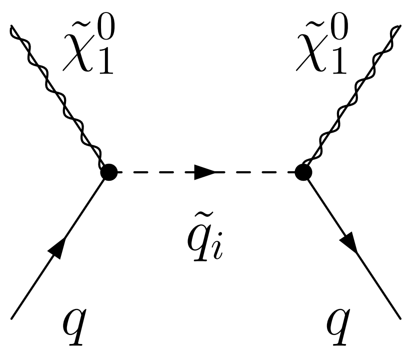
Coulomb corrections **dominant for small values of p_{cm}** (Coulomb singularity), while fixed-order corrections dominant for high-momentum region

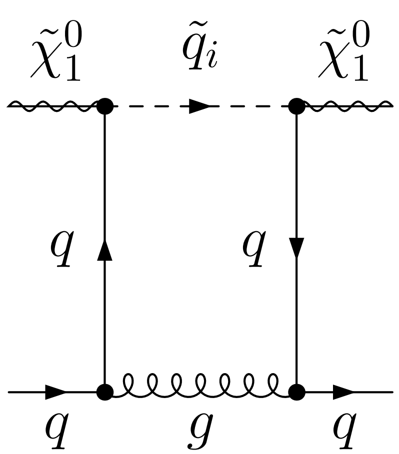
Resulting relic density receives corrections of up to 40% (more important than Planck uncertainty!)

Harz, Herrmann, Klasen, Kovařík, Meinecke — Phys. Rev. D 91: 034012 (2015) — arXiv:1410.8063 [hep-ph]
Harz, Herrmann, Klasen, Kovařík, Schmiemann — work *in progress*…

Application to direct detection

Corrections to direct dark matter detection

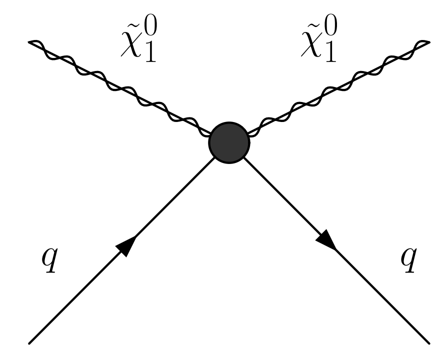


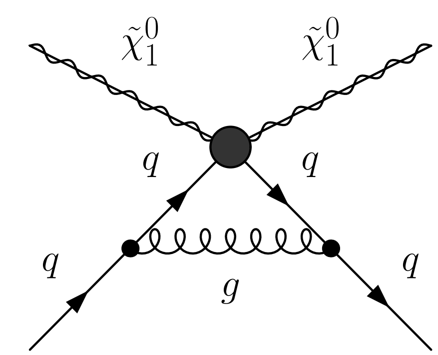


Full theory
(LO + NLO)
valid at Q~1 TeV

Matching
at Q~1 TeV

Effective theory
(LO + NLO)
valid at Q~5 GeV





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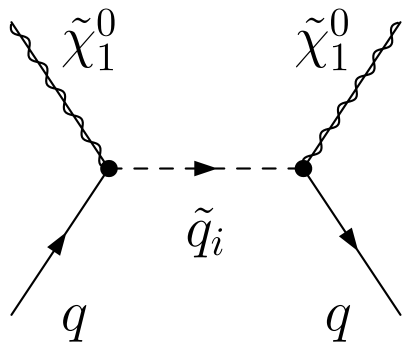

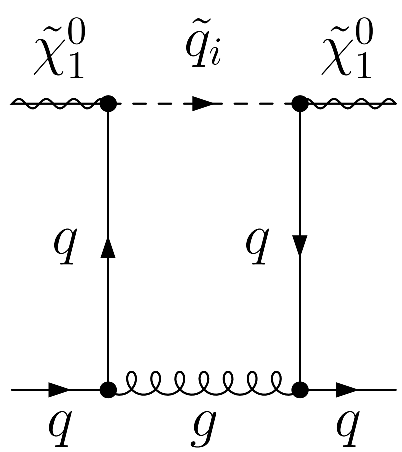

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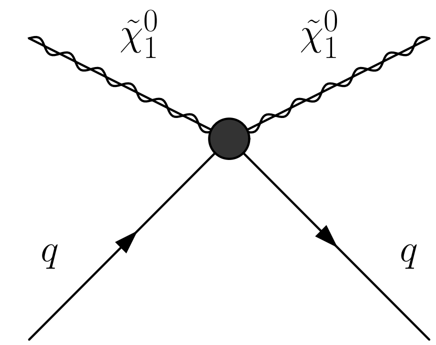
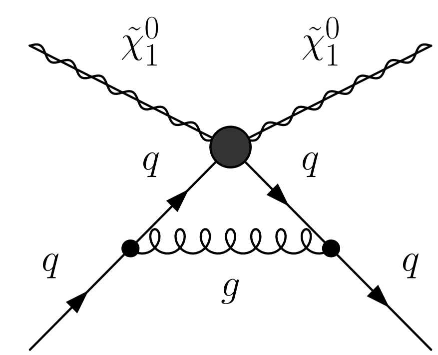




Renormalization (same scheme as before) in order to treat ultraviolet divergencies

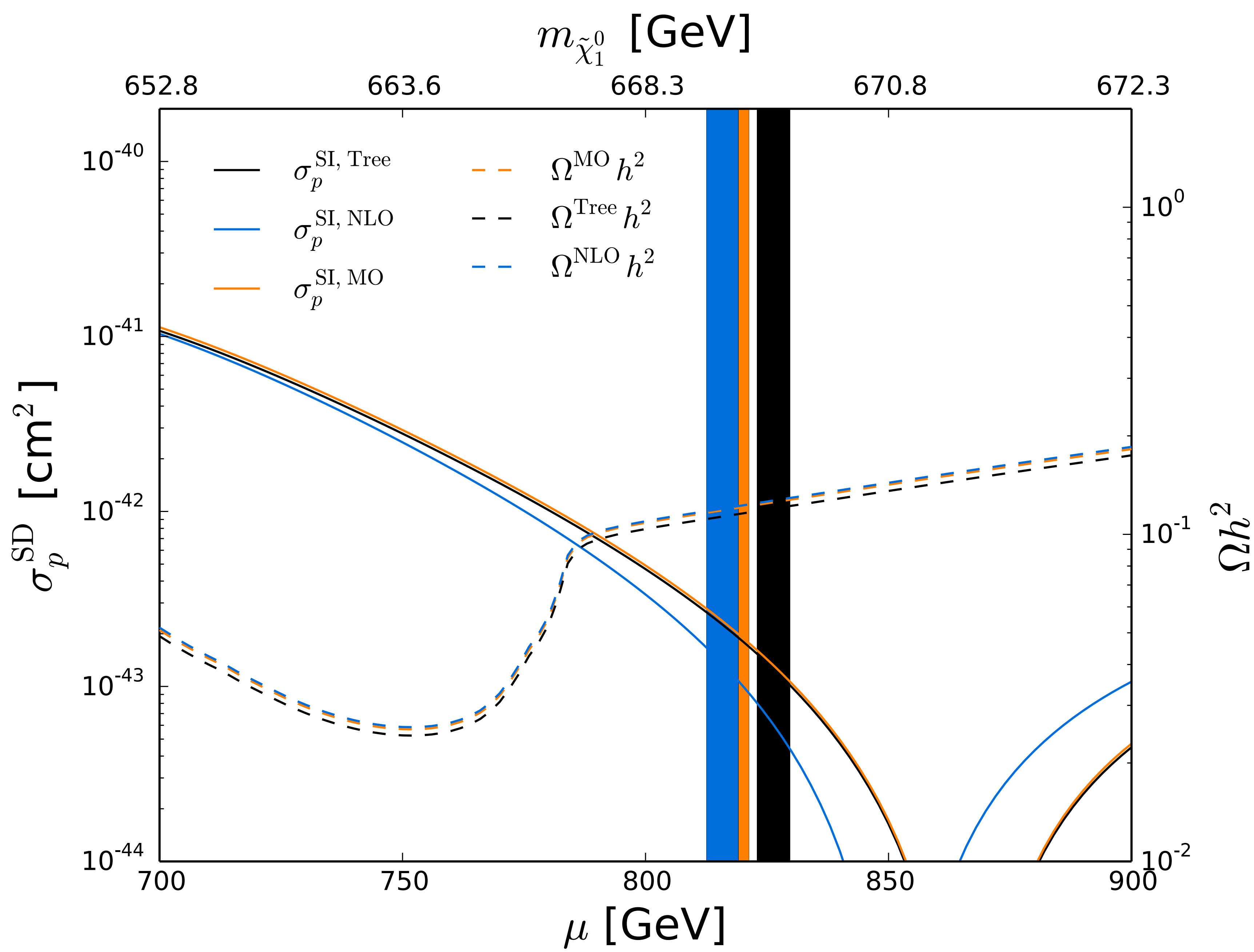
Infrared divergencies cancel between the different contributions

Dedicated **integral reduction procedure applicable to zero-velocity limit**

Renormalization group running of effective theory from Q~1 TeV to Q~5 GeV

Klasen, Kovařík, Steppeler — Phys.Rev. D94: 095002 (2016) — arXiv:1607.06396 [hep-ph]

Corrections to direct dark matter detection



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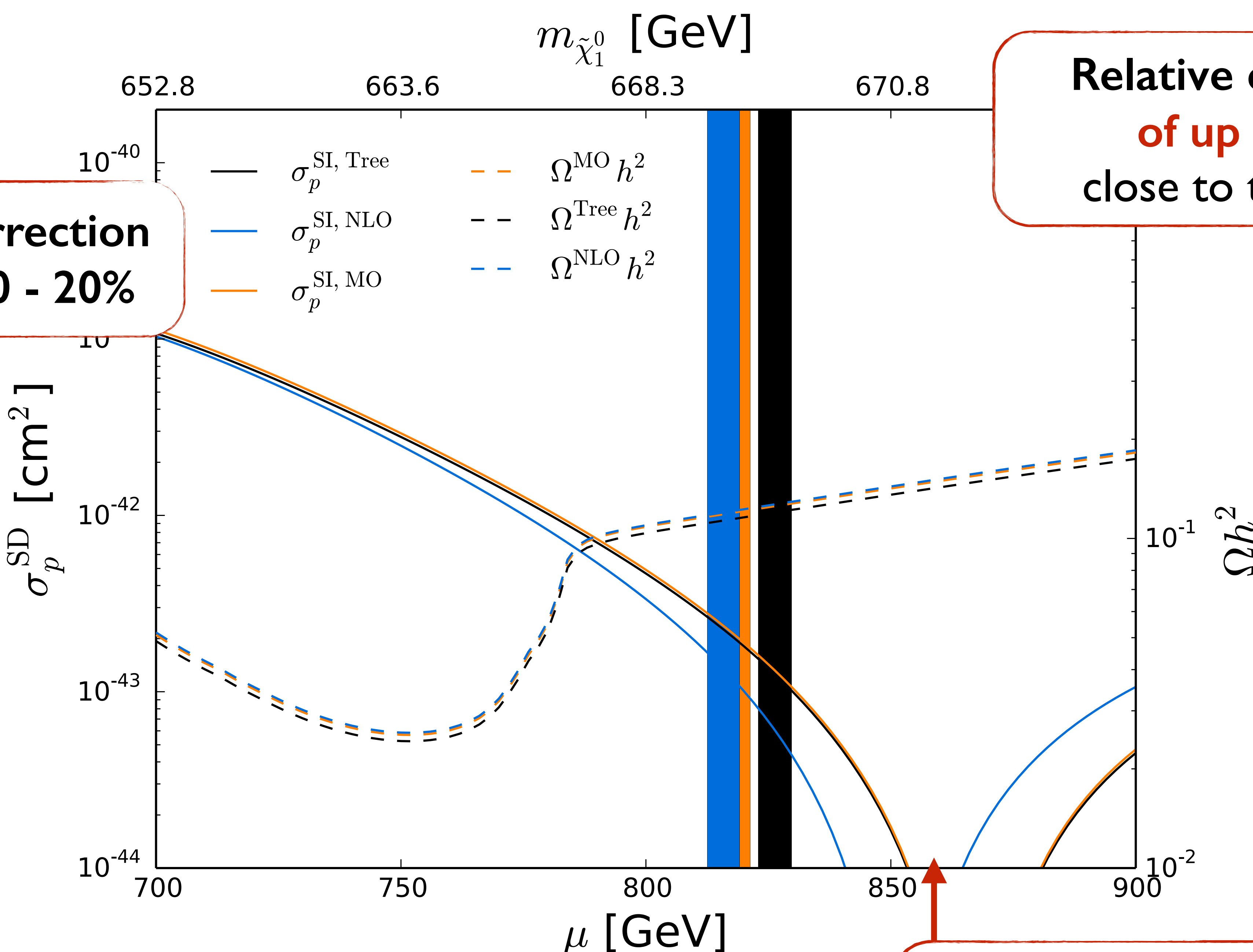

Relative correction of about 10 - 20%

Relative correction of up to 50% close to the "drop"

Destructive interference "higgsino-bino switch"

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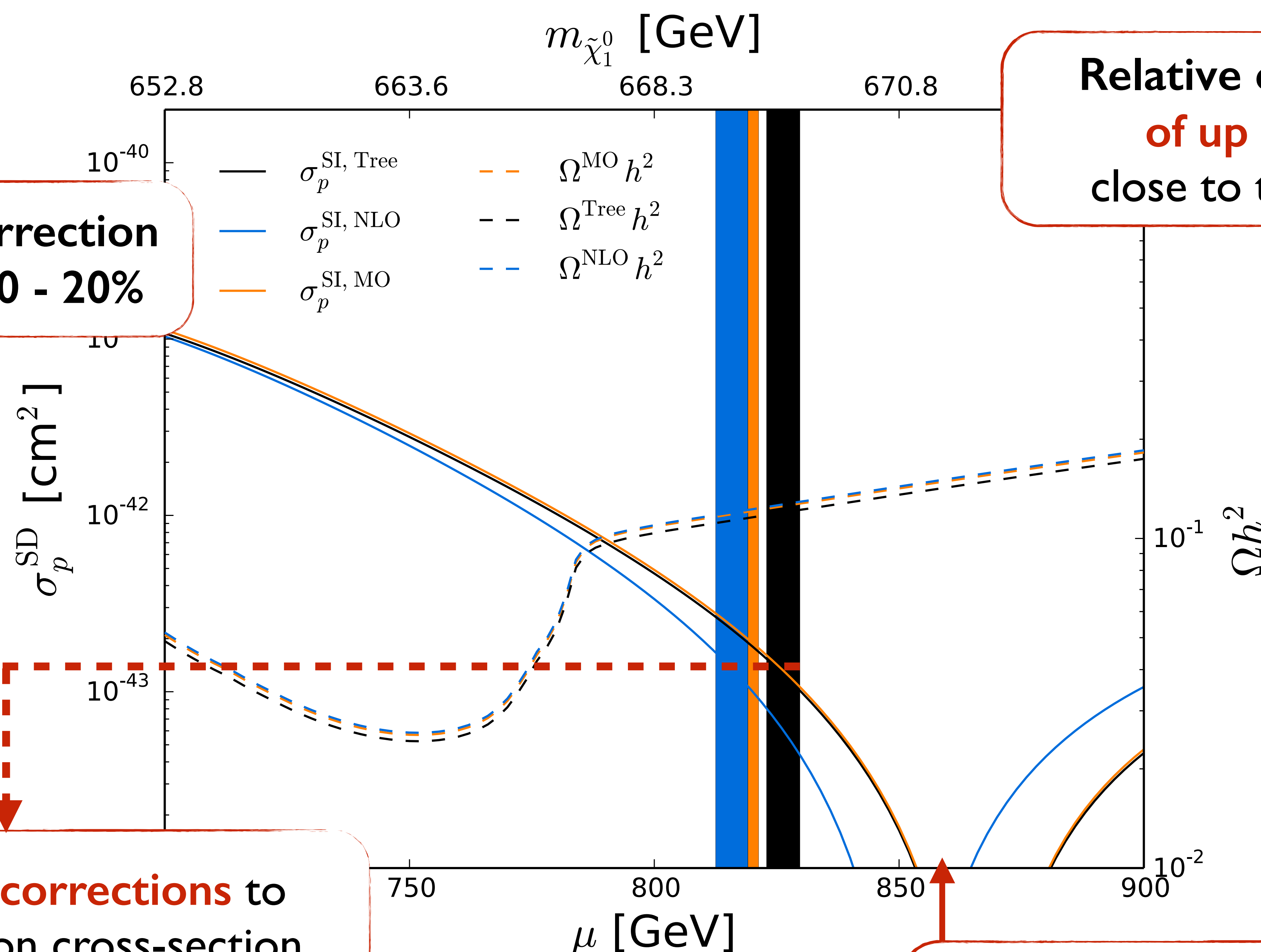

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Effect of corrections to DM-nucleon cross-section and to relic density **are of the same order**

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Klasen, Kovařík, Steppeler — Phys.Rev. D94: 095002 (2016) — arXiv:1607.06396 [hep-ph]

Scale dependence and theoretical uncertainty

Interlude — a few technical details

Loop diagrams include UV-divergent integrals → **Renormalization!**

Hybrid on-shell/$\overline{\mathrm{DR}}$ renormalization scheme for the squark sector (3rd generation), which is applicable to all (co)annihilation processes

$$\boxed{\begin{array}{c} A_t, A_b \\ m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{b}_2} \end{array}} \longrightarrow \boxed{\begin{array}{c} M^2_{\tilde{Q}}, M^2_{\tilde{U}}, M^2_{\tilde{D}} \\ m_{\tilde{t}_2}, \theta_{\tilde{t}}, \theta_{\tilde{b}} \end{array}}$$

Independent "input" parameters

Dependent parameters

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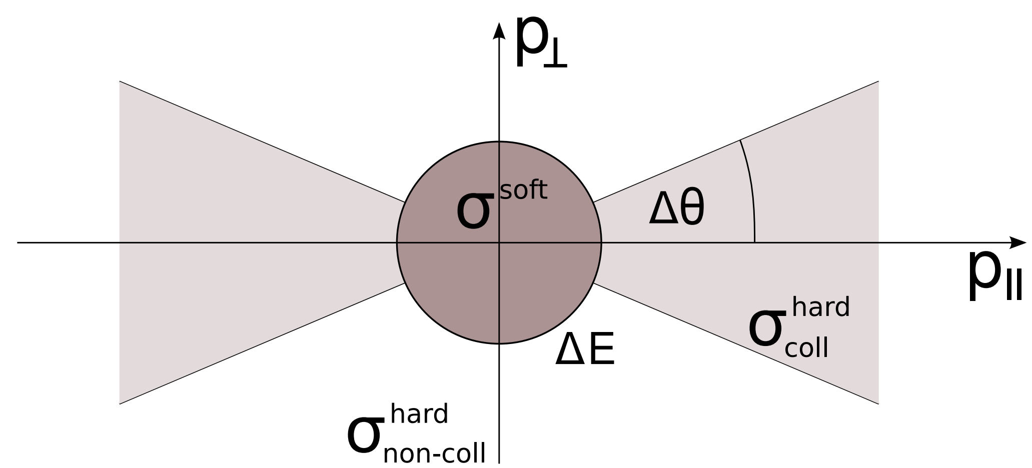
Independent "input" parameters | Dependent parameters

Loop diagrams contain **IR-divergencies** (soft and/or collinear), which vanish when taking into account the real emission of a gluon (2→3 processes)

Dipole Subtraction Method and **Phase Space Slicing**
Catani, Seymour (2001)

$$\boxed{\sigma_{\rm NLO} = \int_3 \left[{\rm d}\sigma^{\rm R}\Big|_{\epsilon=0} - {\rm d}\sigma^{\rm A}\Big|_{\epsilon=0} \right] + \int_2 \left[{\rm d}\sigma^{\rm V} + \int_1 {\rm d}\sigma^{\rm A} \right]_{\epsilon=0}}$$

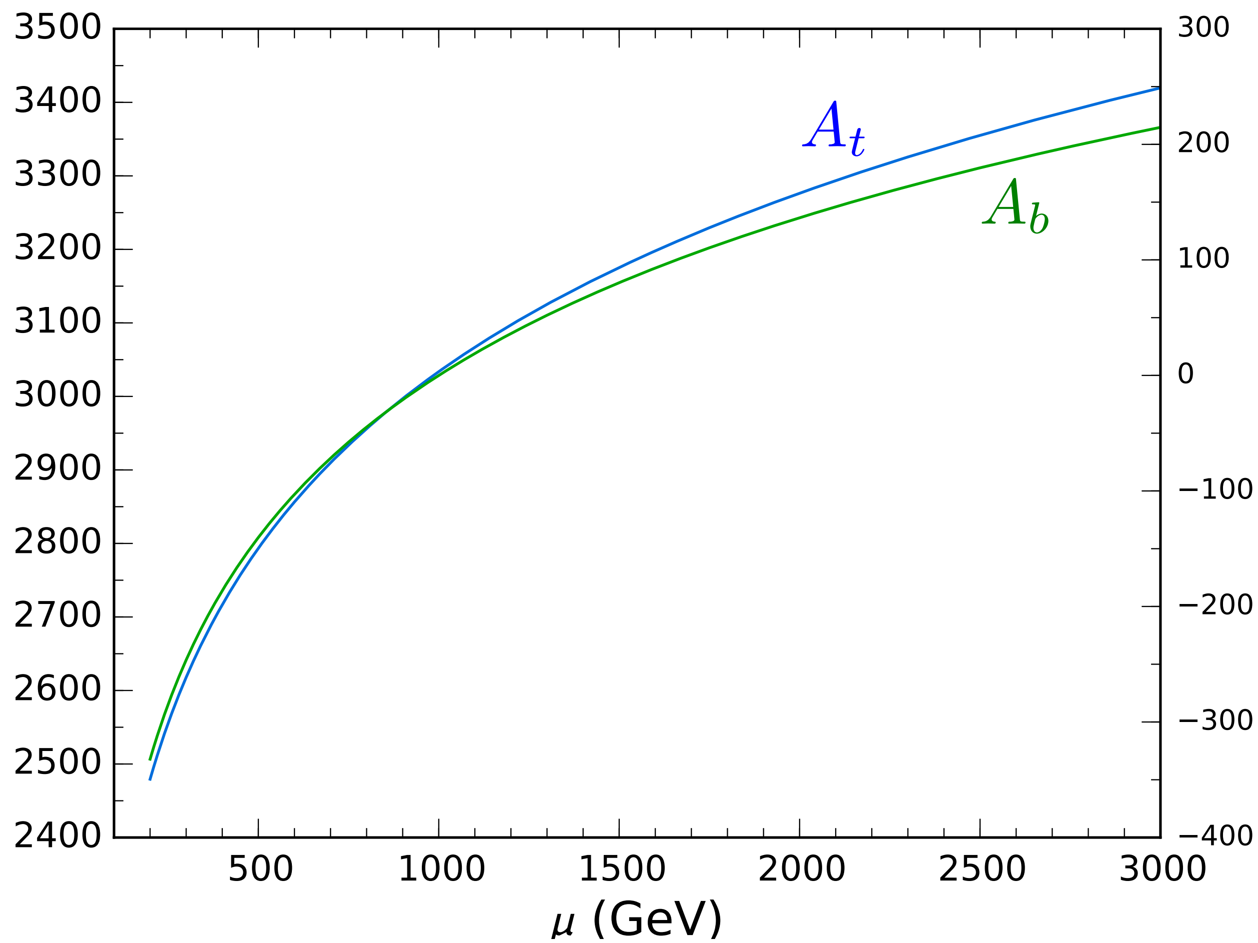


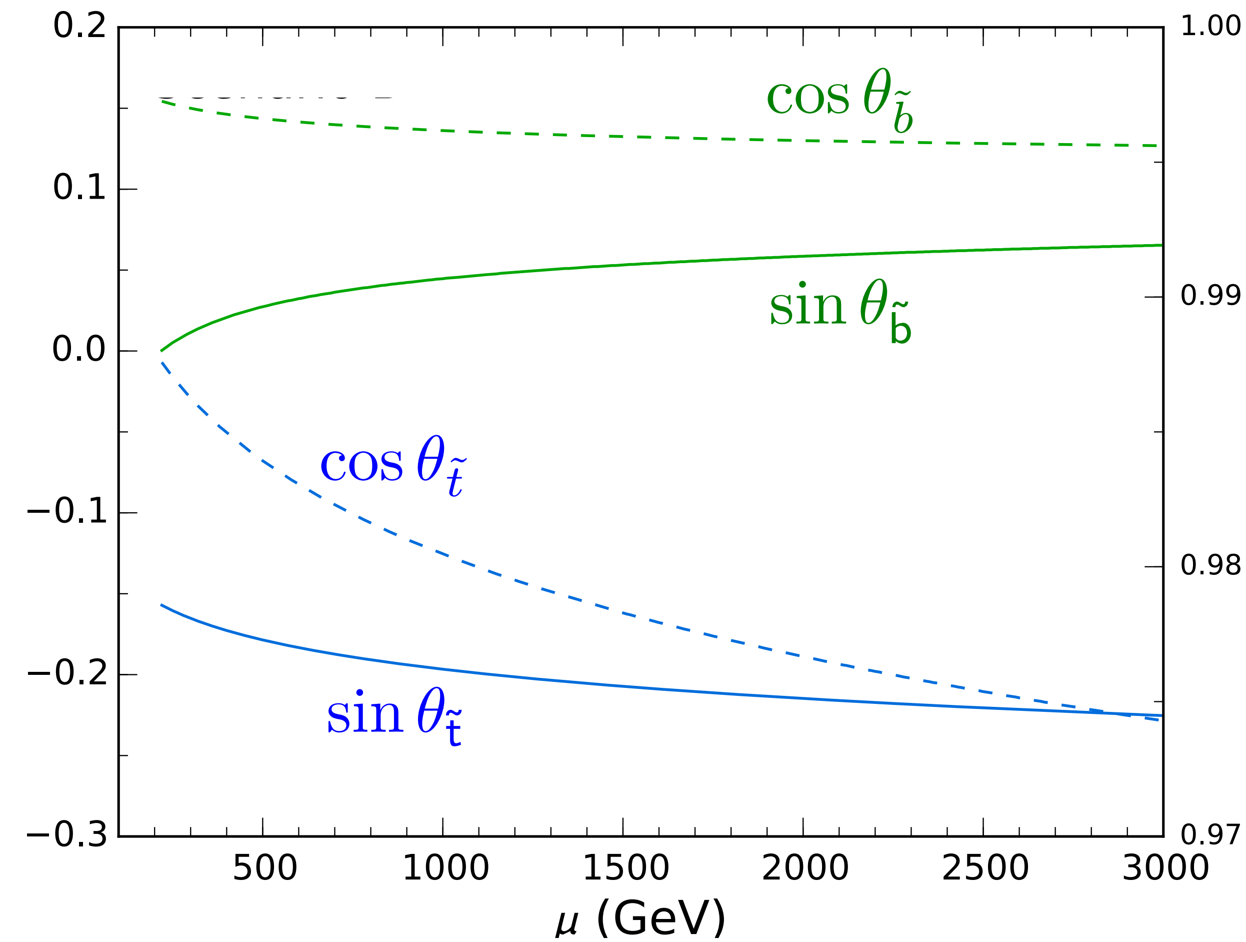
Scale dependence and theoretical uncertainty

Evaluation of theoretical uncertainty by **varying** (unphysical) **renormalization scale**
— hybrid on-shell / DRbar renormalization scheme designed for neutralino (co-)annihilation

$\mu_R = 500 \ldots 2000$ GeV → $A_t, \quad A_b, \quad \theta_{\tilde{t}}, \quad \theta_{\tilde{t}}, \quad \alpha_s, \quad m_b$

scale-dependent parameters





J. Harz, B. Herrmann, M. Klasen, K. Kovařík, P. Steppeler — Phys. Rev. D 93: 114023 (2016) — arXiv:1602.08103 [hep-ph]

Scale dependence and theoretical uncertainty

Evaluation of theoretical uncertainty by **varying** (unphysical) **renormalization scale**
— hybrid on-shell / DRbar renormalization scheme designed for neutralino (co-)annihilation

$\mu_R = 500 \ldots 2000$ GeV $\longrightarrow$ $A_t, \quad A_b, \quad \theta_{\tilde{t}}, \quad \theta_{\tilde{t}}, \quad \alpha_s, \quad m_b$

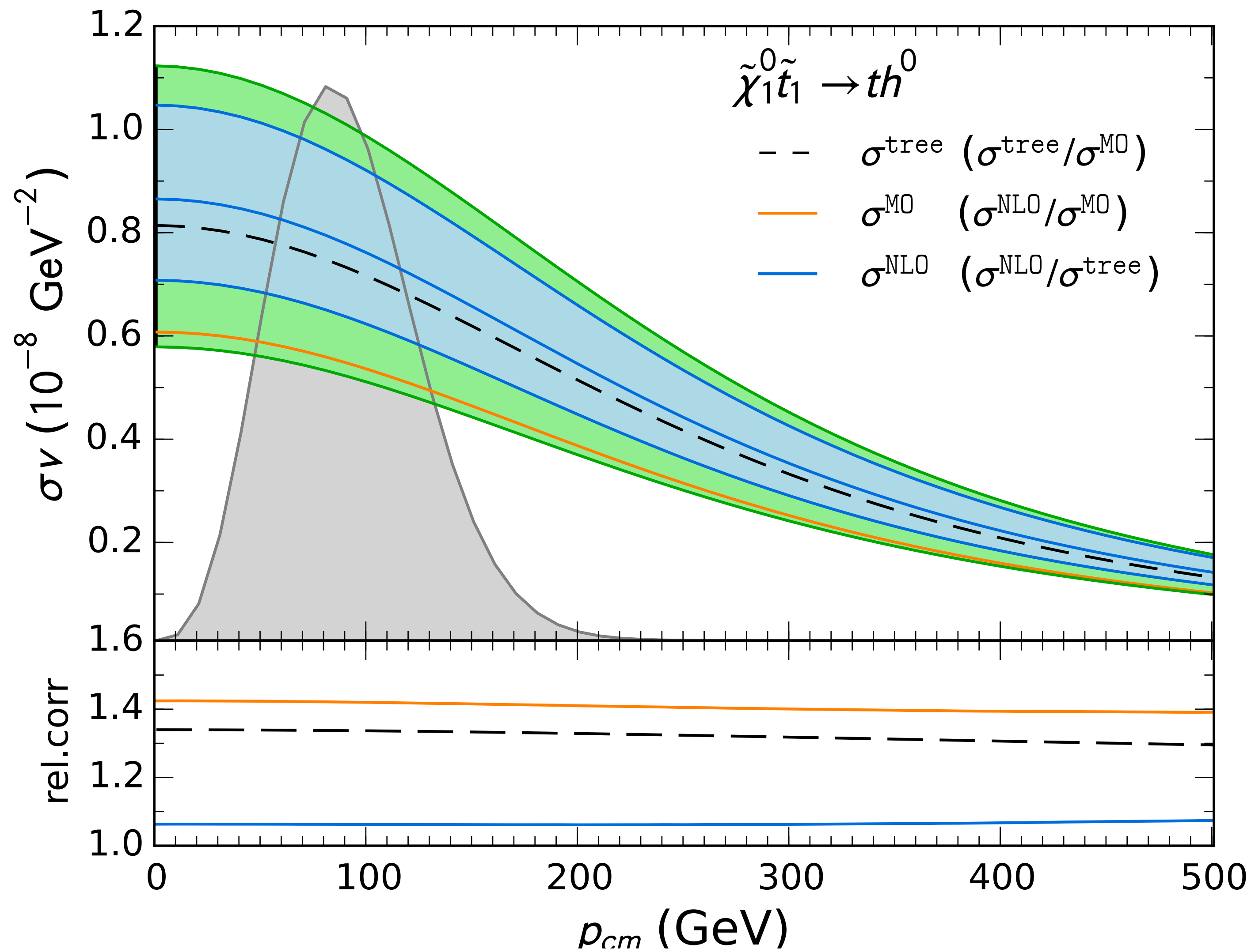
scale-dependent parameters

Within the scale uncertainty, the **tree-level result agrees** with the NLO calculation and the micrOMEGAs value

Scale uncertainty reduced at the one-loop level w.r.t. to tree-level result (as expected)

— main effect from **mixing angle** and **trilinear coupling**

— dependence of α_s subdominant



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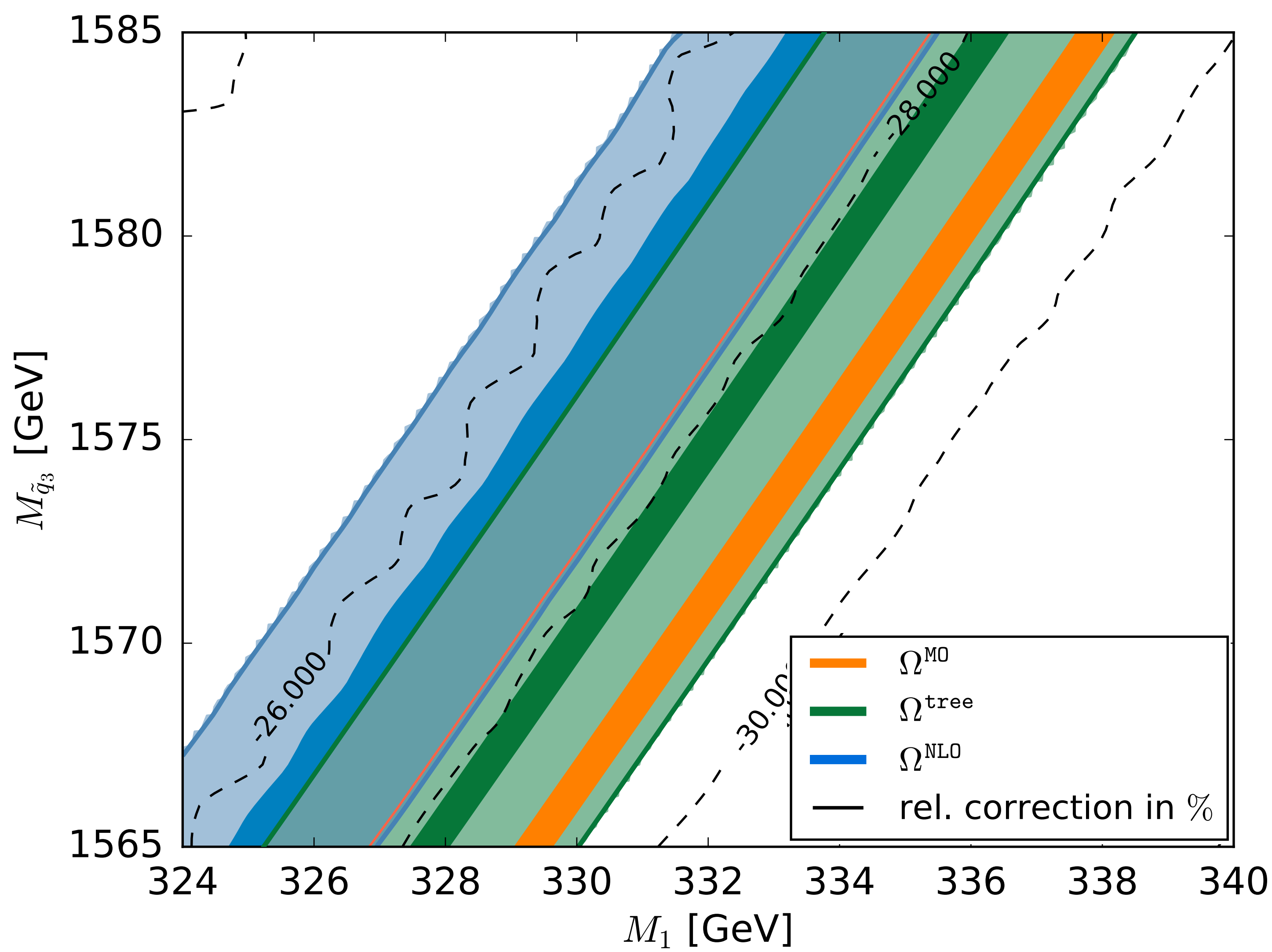

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Impact of corrections on the relic density more important than current exp. uncertainty

— Higher-order corrections important when extracting parameters from cosmological data

Analysis of the theory uncertainty shows that the relic density cannot always be predicted with a precision of 2% similarly to the experimental result

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— include other new physics' models

— implement dipole subtraction scheme for all process classes

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